United States
Department of
Agriculture

Forest Service

**Southern
Research Station**

Research Paper
SRS-19

New Methods, Algorithms, and Software for Rapid Mapping of Tree Positions in Coordinate Forest Plots

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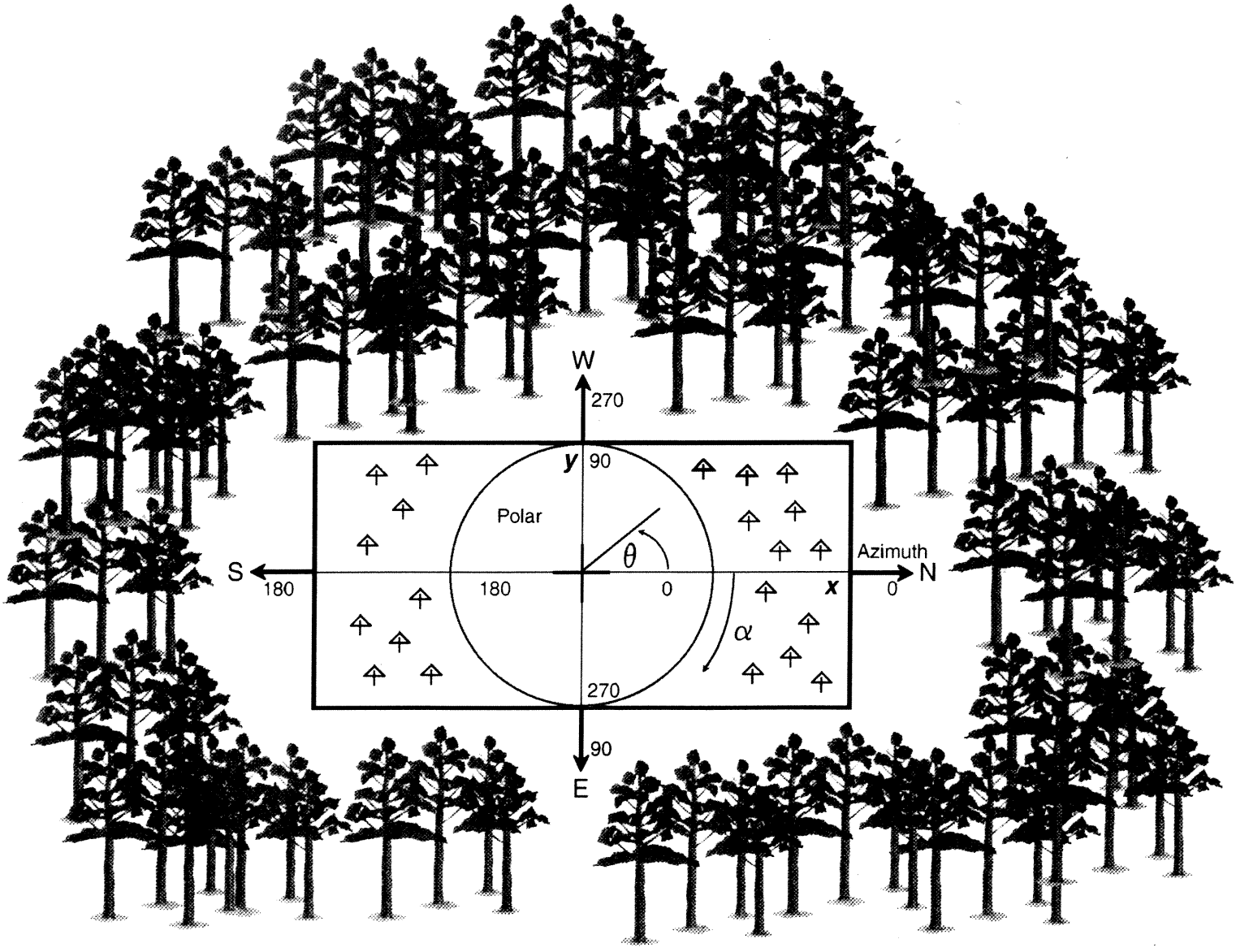

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Abstract

The theories and methodologies for two new tree mapping methods, the Sequential-target method and the Plot-origin radial method, are described. The methods accommodate the use of any conventional distance measuring device and compass to collect horizontal distance and azimuth data between source or reference positions (origins) and target trees. Conversion equations are presented to convert field-derived azimuth coordinates to plot-center coordinates, permitting plots of all tree positions relative to the geometric center of the plot. Plot rotation algorithms for Polar X and Polar Y plotting methods allow the rotation of all plotted positions to any orientation within a rectangular mapping frame and permit corrections for magnetic declination. Additional algorithms are provided to calculate horizontal distance and azimuth between plotted trees and plot-center or between any two tree positions in the plot. All algorithms were incorporated into TreeMapper, a computer program for DOS. The methods and software were tested on a forested plot. Low mean differences between actual and calculated values indicated the accuracy of the methods. The mapping methods and software, originally developed to map trees in research plots for spatial and epidemiological studies of oak wilt disease, have wide applications in forest inventory, management, ecology, spatial-modeling, and other field activities and tasks that require accurate information about tree positions and spatial distribution patterns and ways to keep track of tree attributes and treatments over time. More detailed explanations of the methodologies for downloading data and running TreeMapper will be provided in a second paper.

Keywords: *Ceratocystis fagacearum*, field plotting, oak wilt, spatial structure, stem mapping, tree inventory surveys, tree mapping.

Introduction

The construction of accurate maps of forest stands and woodland plots is required for many aspects of forest management, inventory, and research. Natural resource managers use forest maps for inventory analysis, timber sales, harvest plans, best management practices (BMP) reports, soil and range conservation management, timber management, wildlife management, and many other natural resource applications. Many applied fields of land management, including forestry, use cartographic surveys of objects in the landscape to plan, develop, and manage land-use areas. The construction of accurate maps is a prerequisite for planning and executing most types of activities carried out by USDA Forest Service personnel in forested areas within and outside of the national forests. Timber industry personnel in the State and private sectors also need accurate maps of forest stands to conduct work activities associated with timber management and harvesting on State and private forests.

Research scientists in forestry and related sciences commonly install field plots in order to evaluate forest management practices, test new pest suppression alternatives, investigate ecosystem functions, conduct forest health and inventory analyses, and facilitate studies in many other related disciplines. The task of installing research field plots often requires that the treatments applied or attributes of individual trees are recorded in map form. Consequently, labeling and mapping trees in the research plots are often the first tasks when establishing new field plots. Maps of field plots allow one to keep track of the locations of trees receiving specific treatments so they can be found repeatedly during subsequent data collection visits to the plots.

Many algorithms and variations of protocols for mapping forest plots have been used for numerous forestry applications, ranging from ecological investigations of spatial patterns for development of management-oriented (growth and yield) models to analysis of biotic interactions (competition) and spatial structure (Baskent and Jordan 1995, Glitzenstein and others 1986, Moeur 1993, Platt and Hermann 1986). Methods for mapping field plots have used a variety of approaches to determine the locations of static objects in the landscape, including triangulation (Quigley and Slater 1994), double right-angle prism (Reed and others 1989), distance-bearing (Moeur 1993), and least-squares mapping methods (Hall 1991, Rohlf and Archie 1978). Some of these methods require teams of two or more field workers, several distance measurements, or both to determine tree positions within each plot. Other methods are limited in application to plots of specific shapes. The standard mapping convention, commonly used for many forestry applications, employs a rectangular coordinate system with the origin in the lower left corner of the plot (Warren and Cook 1984). This convention places all plotted trees into Quadrant I of a typical two-dimensional Cartesian plot, making all x- and y-coordinate values positive for each

position. All of these methods vary considerably in accuracy, time consumption, and difficulty of execution.

The Sequential-target mapping method and the Plot-origin radial mapping method were developed to overcome the limitations of current mapping options. These new methods improve the ease and flexibility of mapping methods and equipment needed for tree plot surveys; minimize the time needed by foresters, private land owners, and researchers to produce maps of tree plots to scale; and eliminate the need for expensive and complicated global positioning system (GPS) equipment for mapping small-area plots. They can be applied to two-dimensional (planar) or three-dimensional (topographic) mapping of forest plots that are smaller than most geographic information system (GIS)-scale plots, generally < 1 square mile (mi^2). The methods offer procedures for surveying trees in the two most common field situations: (1) closed or dense stands where substantial physical obstruction occurs among trees to be surveyed and (2) relatively small open stands where all trees of interest are visible from a central reference position (origin). Although these mapping methods originally were developed to facilitate the establishment of field research plots for studies of oak wilt disease, they can be used on any forested plot. A preliminary report has been published (Wilson 1993).

The objectives of this research follow: (1) devise two new methods for rapidly mapping tree positions in two-dimensional and three-dimensional coordinate field plots; (2) develop methods for rotating mapped positions to any desired orientation relative to the plotting axes to accommodate all positions in the mapping field, correct for magnetic declination, or both; (3) derive new algorithms required for mathematically transforming field coordinates in the two mapping methods, for map rotations, and for calculating horizontal distance and azimuth values between any two trees in the plot as well as between plotted trees and plot-center; and (4) develop software to efficiently execute the algorithms described in this paper and provide output in the form of readily useable and printable two-dimensional maps and supporting spatial data on trees in forested plots.

This paper is the first of two describing new tree-mapping methods. It explains how to survey and collect mapping data in the field; gives details on the theory behind the mapping methods, algorithms, and software; demonstrates their use in a small sample; evaluates their accuracy; and explores management implications. The second paper will explain how to download, edit, and input the field data into specially designed mapping software (TreeMapper) that utilizes the algorithms described here to create maps and other desired outputs. Terms used to describe the mechanics of the mapping methods are defined in the appendix.

Materials and Methods

General Methodology

Field data—The sample field data used to demonstrate the mechanics of the tree mapping methods were taken from 20 trees in an oak wilt research plot near Lampasas, TX. The plot was located in a stand of live oaks (*Quercus fusiformis* Small) positioned adjacent to the expanding edge of an oak wilt infection center caused by *Ceratocystis fagacearum*. The plot originally was established to examine the rate disease symptoms spread through the stand relative to the rate toxic fungal metabolites move in advance of the fungus.

Field survey methodology—Most forest-plot mapping methods in forestry-related applications use either Cartesian rectangular coordinates (x, y) or geopositional coordinates (latitude, longitude) collected from field surveys as input data for mapping algorithms. The data generally are collected with compass, theodolite, electronic measuring devices (EMD's), or GPS equipment—which depends on telemetry data from orbiting satellites. The methods described in this paper use vector data acquired between reference and target positions during field surveys. These data are used to calculate spatial information on relative tree positions from plot-center coordinates and to generate a two-dimensional map of the forest plot that may be printed for field use. An initial scout of the forested plot area is recommended before the field survey to determine the general distribution of trees that will be included in the plot. This preview provides an opportunity to determine the most appropriate survey method and the best path for traversing the plot to systematically cover the plot area. All trees to be included in the survey should be marked and numbered before the survey to facilitate the accurate identification of individual trees during and after the survey. Simple conventional metal tags to more elaborate bar code labels with encoded information may be used for this purpose. Site attributes, tree characteristics, and treatments for individual trees should be recorded during the mapping survey.

Azimuth coordinates (horizontal vector), consisting of horizontal distance (HD) and azimuth (AZ), are the only measurements that must be recorded during the field survey for two-dimensional mapping with these methods. Slope angle (SA) also must be recorded for three-dimensional mapping. Azimuth may be taken relative to either magnetic north or true north. Compasses should be set with the

appropriate declination when true north values are desired. Magnetic declination values may be obtained from a United States Geological Survey (USGS) isogonic chart or topographic quadrangle map of the locality containing the survey area of interest. If declination is unknown during the field survey, the final plot map may be rotated later to correct for declination when this information is known. Horizontal vector data from field surveys are converted into rectangular coordinates relative to plot center by using a series of coordinate conversion algorithms to construct the final plot map.

The specific field techniques used to collect azimuth coordinates during field surveys depend on the mapping method chosen for the survey. The criteria used to decide which mapping method is most appropriate and applicable to the survey task will be discussed in more detail under 'Methodology and Theory' in the sections describing each mapping method. Once the survey method has been chosen, field measurements may begin.

The Foresight-inline survey method is used with the Sequential-target mapping procedure. With this survey method, azimuth coordinates are taken sequentially among trees included in the survey. All measurements must be made in precisely the same way to avoid cumulative plotting errors on the final plot map. For example, all azimuth coordinates from reference trees (origins) to target trees must be taken on all target trees at the same height above the ground as the surveyor instrument. Several other techniques may be used to ensure that all measurements are precise and consistent. One technique involves always measuring from the center of the reference tree to the near face of the target tree by resting the surveyor device against the bark of the reference tree with the front of the device in the center of the reference tree and perpendicular to the line of sight in the direction of the target tree. Then, the radius of the target tree is added to the distance. The recorded distance is the HD between the center of the reference tree and the center of the target tree. In a simpler technique, the surveyor device is positioned behind the reference tree; the front of the device is aligned with the near face of the reference tree; and the HD to the face of the target tree is measured. Some error will occur if the trees differ significantly in diameter. To minimize AZ errors with these techniques, the surveyor instrument can be aimed on target trees near the edge of the bole on the same side from which measurements are taken on the reference tree. This approach is useful if either the reference tree or the target tree has a large diameter or if the trees are close together. Alternatively, a measurement can be taken from both sides of the reference tree, aiming at the center of the target tree, then averaging the measurements to obtain the best AZ measurement. Horizontal distance and AZ components of azimuth coordinates also may be taken in separate measurements. Finally, HD could be measured using one of the techniques described above, while the AZ component could be taken at a position directly between the reference and target tree. Regardless of the technique selected, one technique should be used throughout the survey.

Radial surveys are used with the Plot-origin radial mapping method in which all azimuth coordinates are taken from a central fixed location (origin) within the plot. A surveyor device should be mounted onto a sturdy tripod directly over this marked, fixed position. The surveyor device may be positioned over the stake using a surveyor's plumb. The tripod should rotate 360° to allow measurements in all directions.

Equipment requirements—A wide range of surveying equipment from simple compass, steel tape, and clinometers to theodolites and sophisticated EMD's may be used with the tree mapping methods described in this paper. Horizontal distance and AZ measurements may be written down and entered into a spreadsheet or recorded with a data logger and downloaded to a computer. Laser survey instruments were found to be very quick and accurate. Hundreds of trees can be surveyed in just a few hours with these instruments. The Criterion 400 Survey Laser (Laser Technology Inc., Englewood, CO) was used in this research. This infrared survey laser gives distance measurements to 0.1 meters (m) (accuracy ± 9.4 centimeters [cm]) and measures distances from 3.7 to 457 m without a reflective prism (Anonymous 1992). Shorter distances may be measured by taking the difference between HD measurements to the target and source tree from a position behind the source and in line with the target tree or by using the optional engineering diode that measures distances less than 3 m. The instrument records angular measurements (AZ and vertical angle) to 0.1° (6 minutes) with an accuracy of ± 0.2 to 0.3° and a range of 0 ± 60° of vertical angle using a tilt sensor encoder and fluxgate compass. Horizontal distance is calculated automatically in sloping terrain from slope distance and vertical angle, eliminating the need for clinometric measurements.

The specific procedures used to record and store azimuth coordinates on various survey lasers during field surveys may vary considerably. The following sequence of steps used for recording data in this study applies to the Criterion 400 Survey Laser. These steps merely serve as an example of the protocols that may be required for this process. The Criterion 400 Survey Laser provides two separate submenus

under the Survey Menu in its built-in instrument software that permit data acquisition during field surveys (Anonymous 1992). The Basic Measurements submenu provides horizontal vector data (HD and AZ) or azimuth coordinates, on one screen. The user may scroll through subsequent screens that show these values individually along with slope distance, percent slope, and inclination (vertical angle in degrees). The horizontal vector data screen provides all the information needed for two-dimensional planar mapping. Data collected on the Basic Measurements submenu must be downloaded through the battery port into a data logger or field computer immediately after a valid measurement is taken because the programming does not allow direct storage into the memory of the instrument. The data are downloaded using the Enter button on the instrument keypad. The accuracy of the data should be checked before storage. Horizontal vector data also may be gathered by using the Unit Survey submenu. This submenu requires selection of initial setting options, such as survey number, unit (measurement number), point generation (for auto numbering sequence of foresight or backsight surveys), survey reference (point from another survey or coordinate reference), and choice of foresight or user defined survey methods, before horizontal vector data can be acquired. All data collected are stored in the laser surveyor's memory. A foresight-only survey with inline generated values and coordinate type reference must be selected as options for open surveys with the Sequential-target mapping method. The User defined numbering system option should be used with the Plot-origin radial mapping method. The From/To screen must be edited for the proper origin-target numbering sequence before each measurement when using the User defined numbering system, but the Foresight-inline numbering system automatically generates the proper numbering sequence. The horizontal vector screen is reached by scrolling through the x-, y-, and z-coordinate screens or using the Review button. Values may be edited using the Delete or Clear buttons on the keypad before a replacement measurement is taken. The next measurement is taken by editing the From/To screen as needed and using Review to reaccess the horizontal vector screen.

Software requirements—TreeMapper 1.0 for DOS is a newly developed computer program written in Visual Basic that incorporates the algorithms described in this paper to produce two-dimensional maps of forest plots from azimuth coordinates collected during the field survey (Lester and Wilson 1995). The program converts azimuth coordinates to plot-center coordinates and graphically plots tree positions (to scale) on screen. Figure 1 provides a comparison of the overall sequence of coordinate conversions used in the program to calculate plot-center coordinates for the Sequential-target and Plot-origin radial mapping methods. Input data may be entered in the program manually or imported from a data logger or EMD. Downloaded data from the data logger or survey laser must be edited to eliminate all data except origin-reference, HD, and AZ values. Because a single computer screen pixel at 640 × 480 video graphics adapter (VGA) monitor-resolution is used as the mapping unit, the maximum dimensions of the mapping field are limited to plots of 640 × 480 units (feet, meters, chains) in size at 1× scale factor. Higher resolution screens will display the

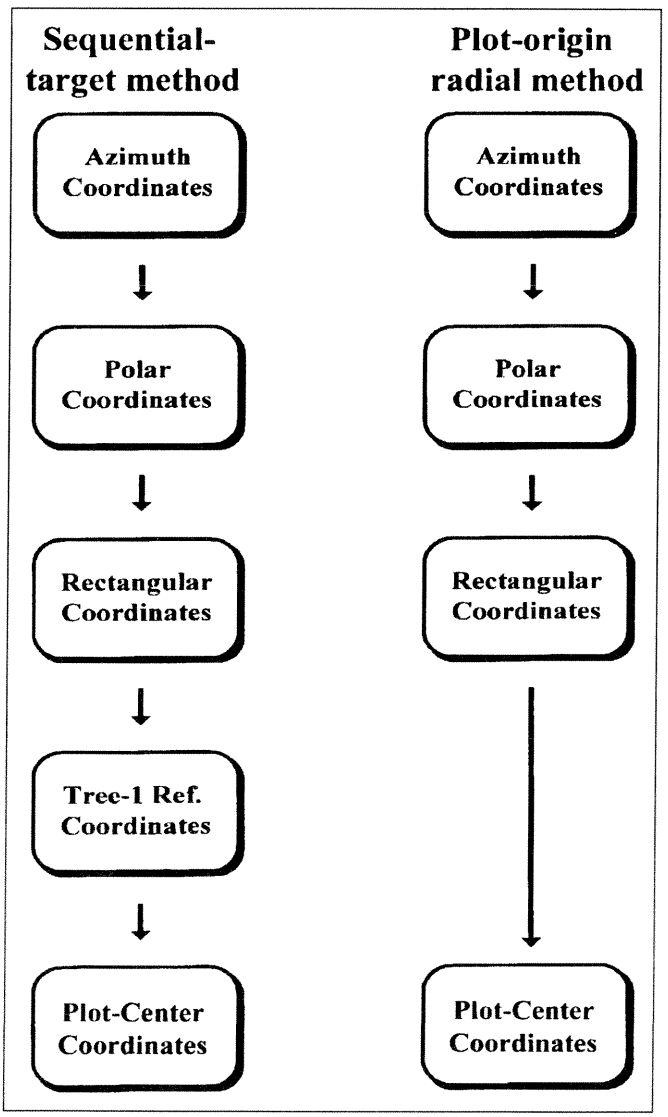

Figure 1—Flowchart illustrating the sequence of coordinate conversions used by TreeMapper to calculate plot-center coordinates for the Sequential-target and Plot-origin radial mapping methods.

plot maps at the same resolution and size and will not fill the entire screen. Large forest plots may be mapped after reduction with small-scale factors. The program allows for reduction and magnification of the plot from 0.1 up to 10×-scale factor. Map sizing allows plot adjustments to fit all plotted points into the map frame and to improve the resolution of plotted tree positions in small plots. Plots of tree positions may be rotated up to 90° in two directions using Polar X or Polar Y plotting methods to improve fit within the map frame. Horizontal distance and AZ between any two positions not directly measured may be calculated. Maps may be printed using high resolution, LaserJet-compatible printers or may be saved to disk in graphics file formats (e.g., PCX files) that are easily imported into other computer graphics software. The program requires a 386 or higher International Business Machines (IBM) compatible computer, VGA monitor, a minimum of 2 megabytes (MB) of random access memory (RAM), and 2 MB of free hard disk space.

Sequential-Target Mapping Method

Methodology and theory—The Sequential-target method is useful for mapping irregularly shaped or elongated forest plots with dense stands or stands containing abundant obstructions between trees. The method may be used in open surveys to map the locations of individual trees (or other static objects) within a plot or in closed surveys to map the boundaries of plots, timber stands, or property lines. Survey data for the Sequential-target method are collected between source and target trees in sequential numerical order as marked during the presurvey tree-labeling process. This survey method, referred to as a Foresight-inline survey, considers the source tree as the origin. The azimuth coordinates refer to the position of the target tree relative to the source tree. The target tree becomes the new origin (source tree) from which azimuth coordinates are taken for each subsequent target tree. Consequently, the sequence of (source-to-target) azimuth coordinate measurements taken using the Foresight-inline survey method correspond to tree combinations [(1-2), (2-3), (3-4)...] and represent relative coordinates. Backsight azimuth coordinates may be taken from target to source trees as needed to check the precision of HD and AZ in foresight data, but backsight data are not used for mapping. Figure 2 shows a comparison of the spatial orientation of the data-acquisition sequence used with the Foresight-inline survey method in Sequential-target mapping to the orientation used with the Radial survey method in Plot-origin radial mapping.

Survey data stored in the memory of an EMD or data logger may be downloaded to a computer for conversion into a format that can be plotted. The Sequential-target mapping method converts field-derived azimuth coordinates into polar, rectangular, tree-1 reference, and finally plot-center coordinates through a series of algorithms for each conversion. Condition formulas define which equations are used based on the input values of coordinates under

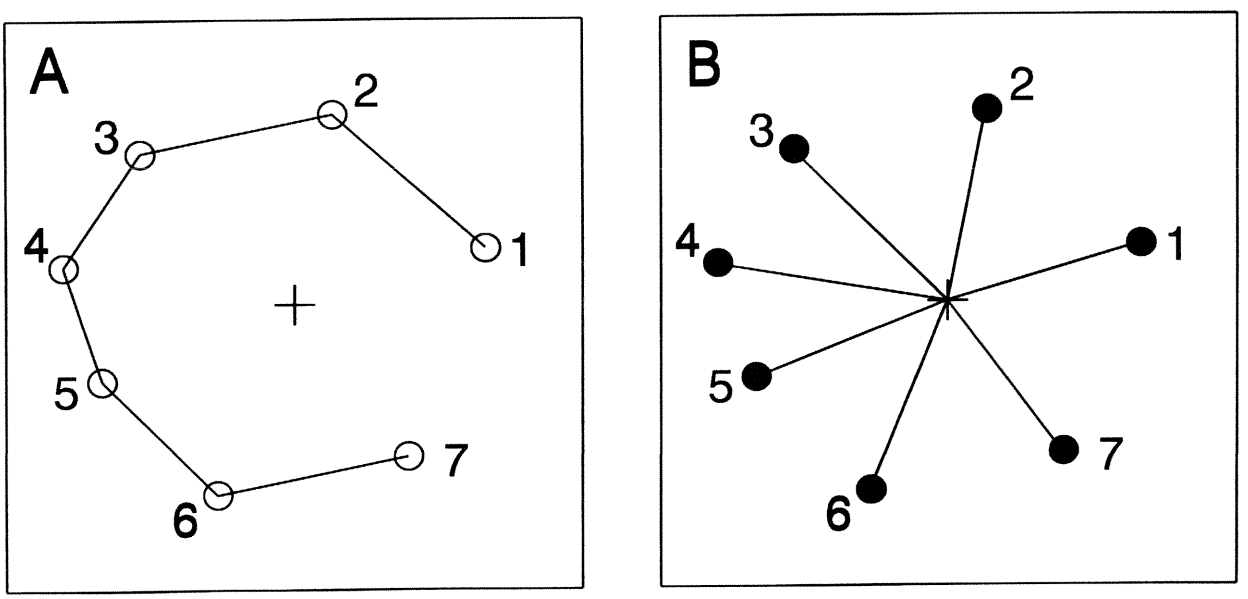

Figure 2—Sequence of measurements for acquisition of azimuth coordinates (HD, AZ) during field surveys using the (a) Foresight-inline survey method and (b) Radial survey method.

Table 1—Incremental data showing the relationship between azimuth bearings and polar bearings using polar X and polar Y tree plotting methods[a]

Azimuth	Polar X	Polar Y	Azimuth	Polar X	Polar Y
0	360	90	190	170	260
10	350	80	200	160	250
20	340	70	210	150	240
30	330	60	220	140	230
40	320	50	230	130	220
50	310	40	240	120	210
60	300	30	250	110	200
70	290	20	260	100	190
80	280	10	270	90	180
90	270	0	280	80	170
100	260	350	290	70	160
110	250	340	300	60	150
120	240	330	310	50	140
130	230	320	320	40	130
140	220	310	330	30	120
150	210	300	340	20	110
160	200	290	350	10	100
170	190	280	360	0	90
180	180	270			

[a] Bearing values (in degrees). Azimuth (compass) bearings (AZ) increase in the clockwise direction from a variable reference position either on the positive x-axis (polar X) or on the positive y-axis (polar Y), while polar bearings (θ) increase in the counterclockwise direction from a fixed reference position on the positive x-axis.

conversion. Polar coordinates (r, θ) and rectangular coordinates (x, y), like azimuth coordinates, are relative coordinates that only indicate the position of target trees relative to the previous source tree. However, tree-1 reference coordinates and plot-center coordinates are absolute coordinates that tie all tree positions together relative to a common reference point. Tree-1 reference coordinates indicate the positions of trees relative to tree-1. Plot-center coordinates indicate the positions of trees relative to the geometric center of the plot. Plot-center coordinates are used to produce the final plot map.

Forest plots are produced from plot-center coordinates using either Polar X or Polar Y plotting methods. Polar X and Polar Y plotting methods produce plots whose positive x-axis or positive y-axis rays point to north when no plot rotation is used. In addition to Polar X and Polar Y plots, plot axis rotations allow all plotted positions to be rotated through any desired angle (α) up to 90° in either a clockwise (eastward) or counterclockwise (westward) direction. The maximum angular rotation is 90° because a 90° westward rotation of a Polar X plot is equivalent to a Polar Y plot with no rotation. Conversely, a Polar Y plot rotated 90° eastward is equivalent to a Polar X plot with no angular rotation, and a 45° eastward rotation of a Polar Y plot is equivalent to a 45° westward rotation of a Polar X plot. When plots are rotated, the direction of north rotates with the points to maintain the proper orientation of points relative to north. Consequently, the rays of the positive x-axis or y-axis no longer point to north. Hence, the strict definition of a Polar X or Y plot no longer exists.

Azimuth to polar coordinate conversions—The initial step in the Sequential-target mapping method employs algorithms that take horizontal vector or azimuth coordinates (HD, AZ) as input data and convert them to polar coordinates (r, θ). The polar coordinate system is based on the vector quantities of radial distance or magnitude (r) and polar angle or direction (θ). It provides a constant or standardized format for expressing coordinates in relation to a fixed orientation relative to the positive x-axis, from which bearings are measured in a counterclockwise direction. Azimuth coordinates, by comparison, are based relative to north on the x-axis, or more commonly y-axis, and have bearings that progress clockwise from the respective reference axis. Horizontal distance and radial distance are equivalent, but bearing values change in this coordinate conversion. Table 1 shows the relationship between AZ and polar bearings for the Polar X and Polar Y plotting methods. Conversion of azimuth coordinates to polar coordinates increases the flexibility of the potential mapping manipulations that are possible by allowing: (1) a choice of

plotting methods to produce different map orientations relative to north on the x- or y-axis, (2) rotations of plots (plotted positions) up to 90° clockwise or counterclockwise for any orientation relative to these axes, and (3) corrections for magnetic declination from true north.

A direct conversion of azimuth coordinates to rectangular coordinates would preclude the ability to easily select the axis designated as north or to rotate plots by requiring that separate case-specific condition formulas be used for calculating the positions of trees for every possible mapping orientation selected relative to the plotting axes. This would complicate the mapping process and create difficult logistic problems in developing computer code in software designed to instruct the production of plot maps by the computer.

Two procedures, the Polar X and Polar Y plotting methods, were developed to increase the versatility of mapping orientations. The method used to convert azimuth coordinates to polar coordinates depends on the orientation chosen for the plot with respect to the axis selected (as magnetic or true north) and whether plot rotation is required. The Polar X method (without plot rotation) considers the positive x-axis ray to point to magnetic north, while the Polar Y method considers the positive y-axis ray to indicate magnetic north. These axes indicate true north when the plots are rotated to correct for declination (fig. 3). Plot rotations through an angle (α) permit map orientations relative to north that are not fixed on the x- or y-axis, which can be demonstrated in a typical clockwise (eastward) polar X plot rotation of 45° (fig. 4). Figure 5 illustrates how rotation of all plotted points in an eastward axes rotation would appear. A plot axes rotation may be necessary if the long-axis of the plot varies significantly from the long axis of the fixed mapping area or if all plotted points will not fit completely within the map frame. A plot also may be rotated to provide a different map orientation relative to the mapping field. A rotation of the plot in an appropriate direction and angle ensures that all of the trees present in the forest plot are represented in the final plot map. In addition, plot rotations may be necessary to correct input data for declination or to reorient the map to designate an axis as true north. The data may be corrected for declination before mapping or the plot may be rotated by the declination amount. In the former case, data converted from magnetic to true north before input into a mapping algorithm would result in a map with N indicating true north. In the latter case, a rotation of the plot through an axis rotation angle (equivalent to the declination) would result in a map in which the positive x- or y-axis ray points to true north, and N rotates with the plotted points and indicates magnetic north. An eastward plot rotation would be required for correction of an east declination of equivalent magnitude and a westward plot rotation would be necessary to correct a west declination.

Table 2 provides equations used to convert azimuth coordinates to polar coordinates, with or without plot rotations, using the Polar X and Polar Y plotting methods. Polar X plots use Equations 1 through 5 and Polar Y plots use Equations 6 through 11. Condition formulas are used to determine which corresponding conversion equation to use to derive polar bearings. Axes rotation angles (α) only apply to conditions and conversion equations when plots are rotated.

Polar to rectangular coordinate conversions—Conversion of polar coordinates (r, θ) to rectangular coordinates (x, y) is required for two-dimensional mapping with the Cartesian coordinate system. Because azimuth coordinates of target tree positions are taken sequentially relative to the last reference point (source tree or position) using the Foresight-inline survey method, all azimuth coordinates as well as calculated polar and rectangular coordinates are relative coordinates. Hence, the origin is redesignated with each new measurement. An initial plot of all relative tree positions must be constructed before the absolute positions of trees relative to a common origin can be calculated. The TreeMapper software plots the positions using a series of offsets that redesignate the new origin each time until all positions are plotted. The accumulated positions of trees then are stored as an array in screen coordinates for subsequent conversions.

The equations used for converting polar coordinates to rectangular coordinates depend on the plotting quadrant within which the target position will be plotted. The plotting quadrant indicates the polarity of the resulting x and y coordinates and is determined by the magnitude of the polar angle bearing as indicated by the condition formulas (table 3). The conversion equations for quadrants II-IV (Equations 14 through 19) are equivalent to Equations 12 and 13 in quadrant I, because the sine and cosine functions take into account quadrant positions for different polar bearing magnitudes. However, the equations appear in this alternative form for future comparison with conversion equations 106 through 121.

Rectangular to tree-1 reference coordinate conversions—The screen coordinates saved in computer memory as an array following polar to rectangular coordinate conversions are essentially tree-1 reference coordinates. Tree-1 reference coordinates are determined by designating tree-1 as the origin with coordinates (0, 0) and adding the relative rectangular coordinates of each successive target tree to the coordinates of the source tree to generate the accumulative

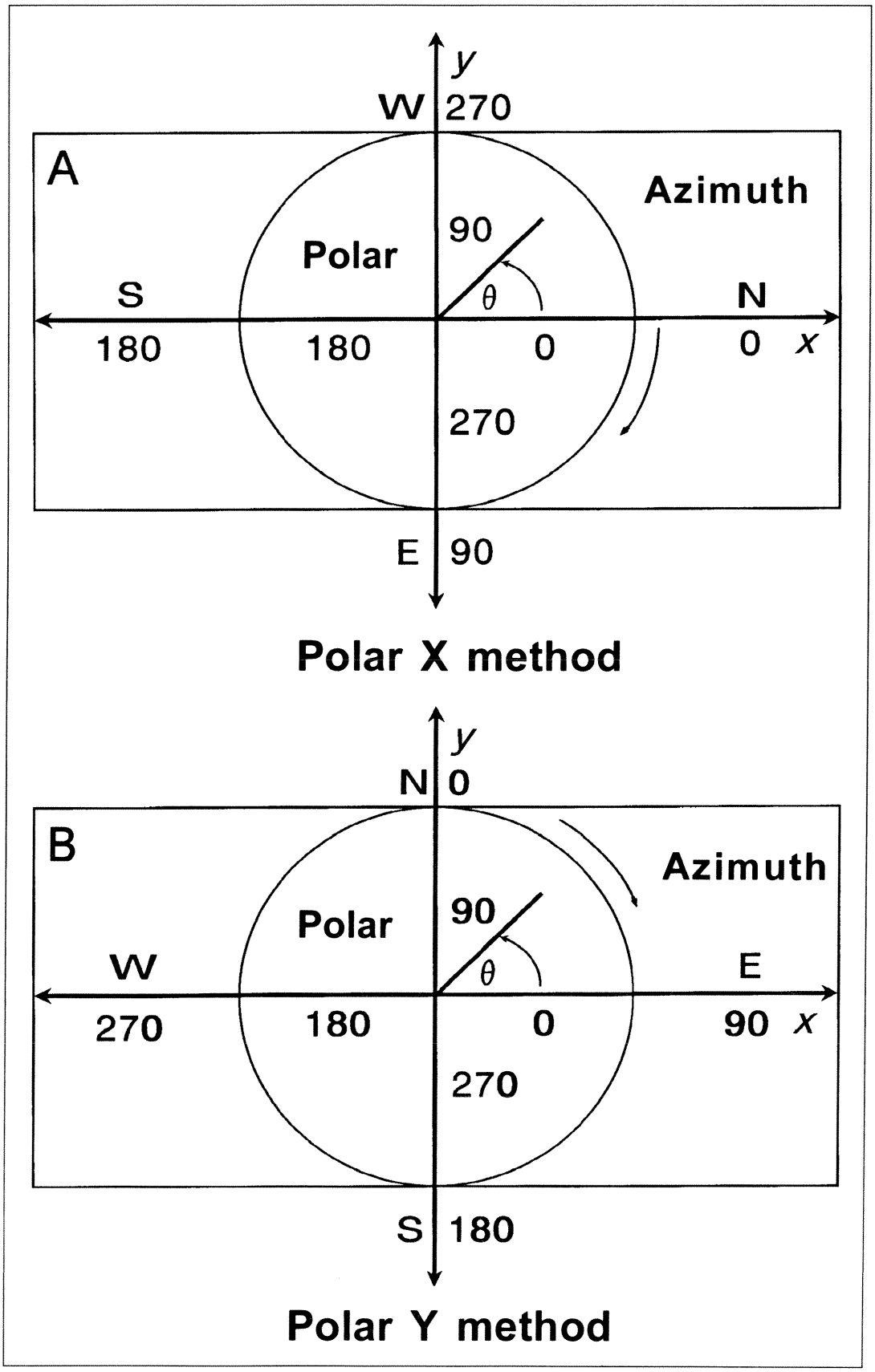

Figure 3—Relationship of azimuth to polar bearings and the orientation of north with respect to plot axes using the (a) Polar X plotting method and (b) Polar Y plotting method.

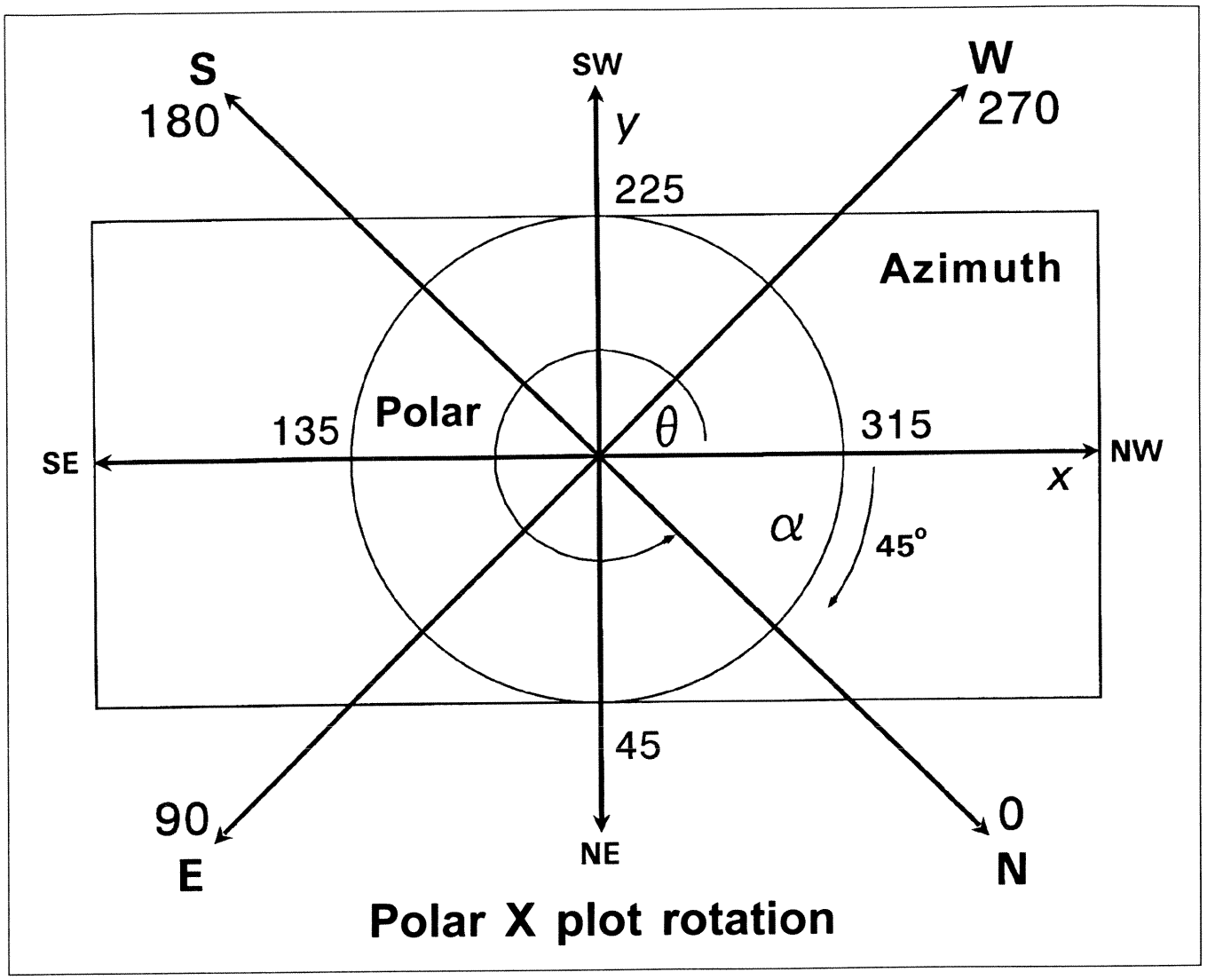

Figure 4—Typical 45° east (clockwise) axes rotation of a Polar X plot with concomitant changes in azimuth and polar bearings.

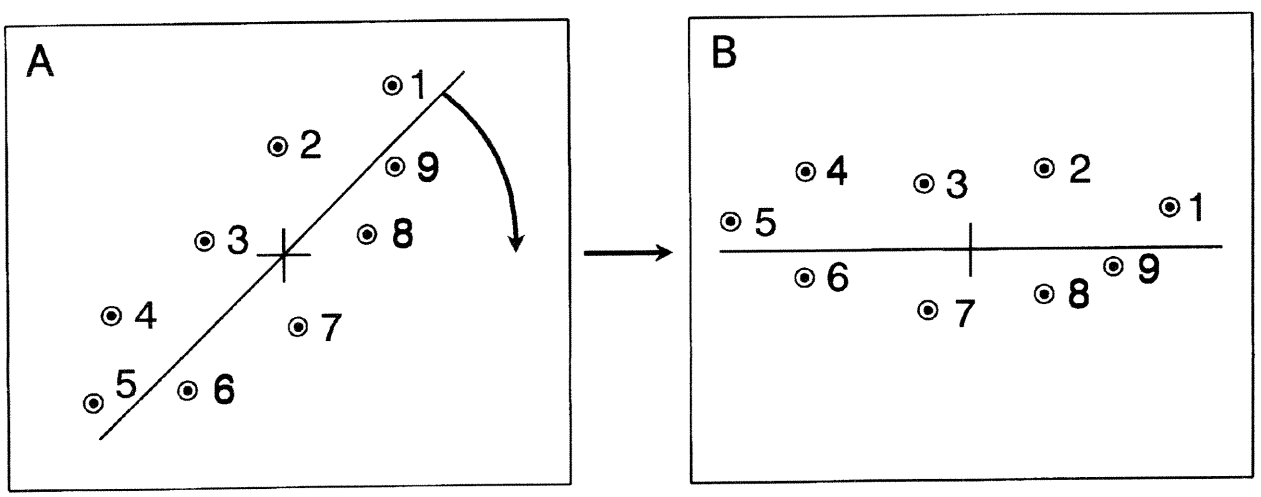

Figure 5—Angular displacement of plotted positions associated with a 45° east (clockwise) rotation of a Polar X plot. Positions are indicated (a) before rotation and (b) after rotation.

Table 2—Equations for converting field-derived azimuth coordinates (HD, AZ) to polar coordinates (r, θ) for Sequential-target mapping of trees using polar X and polar Y plotting methods

Plotting methods	Plot rotation[a]	Condition formulas	Conversion equations[b]	Equation number
Polar X	None	All azimuth values	$\theta = 360 - AZ$	(1)
	West	$AZ < \alpha$	$\theta = \alpha - AZ$	(2)
		$AZ \geq \alpha$	$\theta = (360 - AZ) + \alpha$	(3)
	East	$AZ \leq 360 - \alpha$	$\theta = (360 - AZ) - \alpha$	(4)
		$AZ > 360 - \alpha$	$\theta = (360 - AZ) - \alpha + 360$	(5)
Polar Y	None	$AZ \leq 90$	$\theta = 90 - AZ$	(6)
		$AZ > 90$	$\theta = 360 - (AZ - 90)$	(7)
	West	$AZ \leq 90 + \alpha$	$\theta = (90 - AZ) + \alpha$	(8)
		$AZ > 90 + \alpha$	$\theta = 360 - (AZ - 90) + \alpha$	(9)
	East	$AZ \leq 90 - \alpha$	$\theta = (90 - AZ) - \alpha$	(10)
		$AZ > 90 - \alpha$	$\theta = 360 - (AZ - 90) - \alpha$	(11)

[a] West = counterclockwise rotation of points (positions) within plot (relative to the fixed plot frame); East = clockwise rotation of points within plot.

[b] Symbols: θ = polar coordinate angle (bearing) between the positive x-axis and the ray from the origin (source position) to the target tree position measured in the counterclockwise direction; AZ = azimuth; α = axis rotation angle of plotted points.

Table 3—Equations for converting polar coordinates (r, θ) to rectangular coordinates (x, y) with the Sequential-target mapping method

Plotting quadrant[a]	Condition formulas	Conversion equations[b]	Equation number
I	$0 \leq \theta \leq 90$	$x = r \cos \theta$	(12)
		$y = r \sin \theta$	(13)
II	$90 < \theta \leq 180$	$x = -r \cos (180 - \theta)$	(14)
		$y = r \sin (180 - \theta)$	(15)
III	$180 < \theta \leq 270$	$x = -r \cos (\theta - 180)$	(16)
		$y = -r \sin (\theta - 180)$	(17)
IV	$270 < \theta \leq 360$	$x = r \cos (360 - \theta)$	(18)
		$y = -r \sin (360 - \theta)$	(19)

[a] The polarity of x-coordinate (abscissa) and y-coordinate (ordinate) for points in each quadrant (quarter-plane) follow: Quadrant I: $x > 0$, $y > 0$; Quadrant II: $x < 0$, $y > 0$; Quadrant III: $x < 0$, $y < 0$; Quadrant IV: $x > 0$, $y < 0$.

[b] Conversion of polar coordinates (r, θ) to rectangular coordinates (x, y); r = radial distance or magnitude; θ = polar angle or direction (bearing).

coordinates of each tree in the plot relative to tree-1. The conversion of rectangular coordinates of each sequential tree position in the plot into tree-1 reference coordinates provides absolute coordinates that tie together all tree positions in the plot relative to a common reference position (tree-1).

Tree-1 reference to plot-center coordinate conversions—A plot of tree-1 reference coordinates using coordinate summing results in a tree-1 reference map in which tree-1 is the origin positioned at the center of the mapping field. The distribution of tree positions on a tree-1 reference map is typically skewed to one side of the plotting area (fig. 6A). The skewed distribution may result in the plotting of some points outside the mapping frame. This condition is corrected by converting tree-1 reference coordinates into plot-center coordinates. Plot-center coordinates are tree positions relative to the plot's geometric center (fig. 6B). The conversion involves calculating the geometric center of all tree positions in the tree-1 reference plot followed by repositioning the geometric center along with all plotted points to the origin at the center of the mapping field.

The geometric center of a tree-1 reference plot is found by searching through all of the coordinates in the tree-1 reference plot to find the highest (maximum) and lowest (minimum) x and y values. These extreme values of x and y are usually found in the coordinates of several different trees. TreeMapper instructs the computer to determine these maximum and minimum values using a bubble-sort procedure. The midpoint between the extreme x and extreme y values in the plot data set identifies the coordinates of the geometric center (GC). The coordinates of the GC are calculated by using the following formula:

$$GC_{coord} = \left(\frac{X_{max} + Y_{min}}{2}, \frac{Y_{max} + Y_{min}}{2} \right).$$

The GC coordinates then are used to determine the correctional offset needed to reposition all plotted positions to the center of the mapping field. Correctional offset values, obtained by multiplying the GC coordinates (x and y values) by (-1), are added to each corresponding x- and y-coordinate of the tree-1 reference plot to obtain plot-center coordinates. This equation is applicable only when the plot survey contains more than three trees because calculations with less than four trees may result in coordinates that are not in the center of the plot.

Algorithms for AZ and intertree HD calculations—Forest management and modeling studies investigating correlations and interpretations of spatial patterns, structure, or density of forest stands with respect to environmental conditions, growth and yield, productivity, ecosystem functions, and interaction dynamics often require information on the positions of trees relative to each other and to plot-center. In addition, movement within plots is facilitated by knowledge of the bearing and HD from a reference position near a current position to a target position where subsequent work tasks are planned. The Criterion 400 Survey Laser has a navigation function that will steer the operator to the target with an audible beep or tone when the operator enters the coordinates of the origin and target positions.

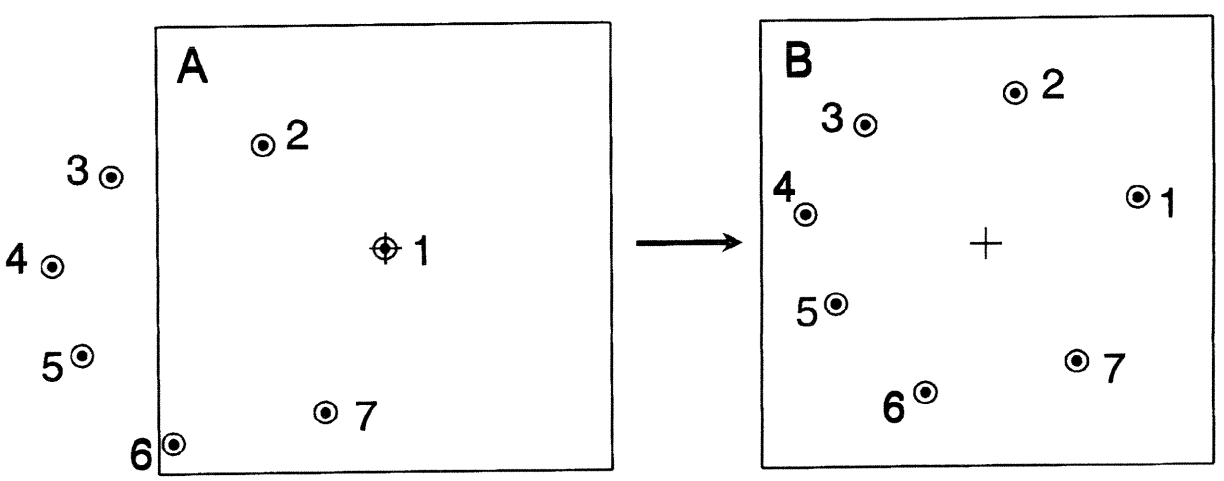

Figure 6—Positional offset of plotted positions during conversion from (A) tree-1 reference coordinates to (B) plot-center coordinates.

Unknown HD's between plotted positions may be determined using the Pythagorean theorem with plot-center absolute coordinates. TreeMapper calculates unknown HD's between any two points of interest in a Distance Calculator submenu. Horizontal distance between trees in the plot also may be calculated directly from polar coordinates using the following equation derived from the law of cosines:

$$d = \sqrt{r_1^2 + r_2^2 - 2\, r_1\, r_2 \cos(\theta_2 - \theta_1)}\ ,$$

where

d = HD between any two plotted tree positions,
r = radial distance or magnitude, and
θ = polar angle or direction (bearing), the vector components of polar coordinates acquired from a common origin.

However, this equation only may be used when all azimuth coordinates (from which all polar coordinates are calculated) are taken from the same reference position (origin) as in the Plot-origin radial mapping method. Consequently, this method for calculating HD between trees does not apply to the Sequential-target mapping method because the reference position is not fixed but moves with each new measurement. This equation is noted as an alternative to show that HD's could be derived by other means. However, this equation is not used here because it would add inefficiency to the software program that calculates both tree positions and HD's between trees from plot-center coordinates in both mapping methods. In addition, tree coordinates derived from sequential mapping using the Foresight-inline survey method cannot be determined directly from the law of cosines.

Azimuth calculations are somewhat more complex because they must take into account plotting methods, plot rotations, and the relative magnitude of x and y plot-center coordinates. Table 4 presents equations for calculating the AZ of target trees from plot-center for Sequential-target mapping with Polar X (Equations 20 through 42) and Polar Y (Equations 43 through 65) plotting methods, with or without plot rotations. Computers calculate trigonometric functions in radians. Consequently, the TreeMapper software must multiply the results of the conversion equations by the constant $180/\pi$ (57.29577) to convert radians to degrees. Table 5 provides analogous equations for calculating the AZ of targets trees from any tree position in the plot using Polar X (Equations 66-85) and Polar Y (Equations 86 through 105) plotting methods. Condition formulas that determine which azimuth equation to use to calculate the AZ between any two tree positions in the plot are selected on the basis of the relative magnitude of x and y coordinates of the source and target tree positions.

Plot-Origin Radial Mapping Method

Methodology and theory—The Plot-origin radial method is useful for mapping relatively small, roughly circular plots, such as variable-radius plots, in generally open stands where all trees of interest are in clear view from a central location. Survey data for this method are collected at a suitable fixed position near the center of the plot using the Radial survey method. This central reference position, referred to as the reference origin or Plot-origin, is the position from which azimuth coordinates of all target tree positions are measured. Horizontal vector data may be taken in either a clockwise or a counterclockwise direction. Azimuth coordinates taken in sequence with the Radial survey method correspond to tree combinations [(0-1), (0-2), (0-3), . . .] and represent absolute coordinates because all positions are determined relative to a fixed reference point (see fig. 2). The fixed reference point is the origin, but not necessarily the geometric center and plot-center.

Azimuth to rectangular coordinate conversions—Coordinate conversions from azimuth coordinates to polar and rectangular coordinates may be executed with the same algorithms used in the Sequential-target method (Equations 1 through 19). These algorithms allow plot rotations of plot-origin radial plots. However, in this method the TreeMapper software plots rectangular coordinates relative to the common reference origin. Hence, all conversion coordinates, including azimuth, polar, and rectangular, are absolute coordinates because the reference origin is a static position from which all field coordinates are recorded. Common-reference plotting in the TreeMapper software is achieved by offsets back to the reference origin after each tree position is plotted.

Alternatively, rectangular coordinates of Polar X and Polar Y plots (by strict definition) without plot rotations may be calculated using the equations presented in table 6. However, the plots may not be rotated. Unlike Equations 14 through 19, Equations 106 through121 cannot be reduced to a pair of equations (for x and y coordinates) applicable to all plotting quadrants, because the condition formulas have changed with respect to the quadrants with which they are applicable. These changes result from a shift in the position of north associated with the use of Polar X and Polar Y plotting methods; consequently, AZ values are shifted accordingly.

Table 4—Equations for calculating azimuth of target trees from plot-center with the Sequential-target mapping method using polar X and polar Y plotting methods with and without plot rotation

Plotting methods	Plot rotation[a]	Condition formulas	Azimuth equations[b]	Equation number
Polar X	None	$x = 0$ and $y = 0$	$AZ = 0$	(20)
		$x = 0$ and $y < 0$	$AZ = 90$	(21)
		$x = 0$ and $y > 0$	$AZ = 270$	(22)
		$x > 0$ and $y \geq 0$	$AZ = 360 - \arctan \lvert y / x \rvert$	(23)
		$x < 0$ and $y \geq 0$	$AZ = 180 + \arctan \lvert y / x \rvert$	(24)
		$x < 0$ and $y < 0$	$AZ = 180 - \arctan \lvert y / x \rvert$	(25)
		$x > 0$ and $y < 0$	$AZ = \arctan \lvert y / x \rvert$	(26)
	West	$x = 0$ and $y = 0$	$AZ = 0$	(27)
		$x = 0$ and $y < 0$	$AZ = 90 + \alpha$	(28)
		$x = 0$ and $y > 0$	$AZ = 270 + \alpha$	(29)
		$x > 0$ and $y \geq$		
		when $\alpha \leq \arctan \lvert y / x \rvert$	$AZ = 360 - \arctan \lvert y / x \rvert + \alpha$	(30)
		when $\alpha > \arctan \lvert y / x \rvert$	$AZ = \alpha - \arctan \lvert y / x \rvert$	(31)
		$x < 0$ and $y \geq 0$	$AZ = 180 + \arctan \lvert y / x \rvert + \alpha$	(32)
		$x < 0$ and $y < 0$	$AZ = 180 - \arctan \lvert y / x \rvert + \alpha$	(33)
		$x > 0$ and $y < 0$	$AZ = \arctan \lvert y / x \rvert + \alpha$	(34)
	East	$x = 0$ and $y = 0$	$AZ = 0$	(35)
		$x = 0$ and $y < 0$	$AZ = 90 - \alpha$	(36)
		$x = 0$ and $y > 0$	$AZ = 270 - \alpha$	(37)
		$x > 0$ and $y \geq 0$	$AZ = 360 - \arctan \lvert y / x \rvert - \alpha$	(38)
		$x < 0$ and $y \geq 0$	$AZ = 180 + \arctan \lvert y / x \rvert - \alpha$	(39)
		$x < 0$ and $y < 0$	$AZ = 180 - \arctan \lvert y / x \rvert - \alpha$	(40)
		$x > 0$ and $y < 0$		
		when $\alpha \leq \arctan \lvert y / x \rvert$	$AZ = \arctan \lvert y / x \rvert$	(41)
		when $\alpha > \arctan \lvert y / x \rvert$	$AZ = 360 - (\alpha - \arctan \lvert y / x \rvert)$	(42)
Polar Y	None	$x = 0$ and $y = 0$	$AZ = 0$	(43)
		$x = 0$ and $y < 0$	$AZ = 180$	(44)
		$x = 0$ and $y > 0$	$AZ = 0$	(45)
		$x > 0$ and $y \geq 0$	$AZ = 90 - \arctan \lvert y / x \rvert$	(46)
		$x < 0$ and $y \geq 0$	$AZ = 270 + \arctan \lvert y / x \rvert$	(47)
		$x < 0$ and $y < 0$	$AZ = 270 - \arctan \lvert y / x \rvert$	(48)
		$x > 0$ and $y < 0$	$AZ = 90 + \arctan \lvert y / x \rvert$	(49)
	West	$x = 0$ and $y = 0$	$AZ = 0$	(50)
		$x = 0$ and $y < 0$	$AZ = 180 + \alpha$	(51)
		$x = 0$ and $y > 0$	$AZ = \alpha$	(52)
		$x > 0$ and $y \geq 0$	$AZ = (90 - \arctan \lvert y / x \rvert) + \alpha$	(53)
		$x < 0$ and $y \geq 0$		
		when $\alpha \leq \arctan \lvert y / x \rvert$	$AZ = 270 + \arctan \lvert y / x \rvert + \alpha$	(54)
		when $\alpha > \arctan \lvert y / x \rvert$	$AZ = \alpha - (90 - \arctan \lvert y / x \rvert)$	(55)
		$x < 0$ and $y < 0$	$AZ = 270 - \arctan \lvert y / x \rvert + \alpha$	(56)
		$x > 0$ and $y < 0$	$AZ = 90 + \arctan \lvert y / x \rvert + \alpha$	(57)
	East	$x = 0$ and $y = 0$	$AZ = 0$	(58)
		$x = 0$ and $y < 0$	$AZ = 180 - \alpha$	(59)
		$x = 0$ and $y > 0$	$AZ = 360 - \alpha$	(60)
		$x > 0$ and $y \geq 0$		
		when $\alpha \leq 90 - \arctan \lvert y / x \rvert$	$AZ = 90 - \arctan \lvert y / x \rvert - \alpha$	(61)
		when $\alpha > 90 - \arctan \lvert y / x \rvert$	$AZ = 360 - [\alpha - (90 - \arctan \lvert y / x \rvert)]$	(62)
		$x < 0$ and $y \geq 0$	$AZ = 270 + \arctan \lvert y / x \rvert - \alpha$	(63)
		$x < 0$ and $y < 0$	$AZ = 270 - \arctan \lvert y / x \rvert - \alpha$	(64)
		$x > 0$ and $y < 0$	$AZ = 90 + \arctan \lvert y / x \rvert - \alpha$	(65)

[a] West = counterclockwise rotation of points within plot (relative to grid frame); East = clockwise rotation of points within plot.

[b] Symbols: AZ = azimuth; α = plot-rotation angle. Computer calculations must include a correction factor consisting of multiplying the values resulting from arctan functions by the constant $180/\pi$ (57.29577) to convert radians to degrees because computers calculate trigonometric functions in radians.

Table 5—Equations for calculating azimuth of target trees from any tree position in the plot with the Sequential-target mapping method using polar X and polar Y plotting methods with and without plot rotation

Plotting methods	Plot rotation[a]	Condition formulas	Azimuth equations[b]	Equation number
Polar X	None	$x_2 = x_1$ and $y_2 < y_1$	$AZ = 90$	(66)
		$x_2 = x_1$ and $y_2 > y_1$	$AZ = 270$	(67)
		$x_2 > x_1$ and $y_2 \geq y_1$	$AZ = 360 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(68)
		$x_2 < x_1$ and $y_2 \geq y_1$	$AZ = 180 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(69)
		$x_2 < x_1$ and $y_2 < y_1$	$AZ = 180 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(70)
		$x_2 > x_1$ and $y_2 < y_1$	$AZ = \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(71)
	West	$x_2 = x_1$ and $y_2 < y_1$	$AZ = 90 + \alpha$	(72)
		$x_2 = x_1$ and $y_2 > y_1$	$AZ = 270 + \alpha$	(73)
		$x_2 > x_1$ and $y_2 \geq y_1$		
		when $\alpha \leq \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = 360 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert + \alpha$	(74)
		when $\alpha > \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = \alpha - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(75)
		$x_2 < x_1$ and $y_2 \geq y_1$	$AZ = 180 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert + \alpha$	(76)
		$x_2 < x_1$ and $y_2 < y_1$	$AZ = 180 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert + \alpha$	(77)
		$x_2 > x_1$ and $y_2 < y_1$	$AZ = \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert + \alpha$	(78)
	East	$x_2 = x_1$ and $y_2 < y_1$	$AZ = 90 - \alpha$	(79)
		$x_2 = x_1$ and $y_2 > y_1$	$AZ = 270 - \alpha$	(80)
		$x_2 > x_1$ and $y_2 \geq y_1$	$AZ = 360 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert - \alpha$	(81)
		$x_2 < x_1$ and $y_2 \geq y_1$	$AZ = 180 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert - \alpha$	(82)
		$x_2 < x_1$ and $y_2 < y_1$	$AZ = 180 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert - \alpha$	(83)
		$x_2 > x_1$ and $y_2 < y_1$		
		when $\alpha \leq \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(84)
		when $\alpha > \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = 360 - (\alpha - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert)$	(85)
Polar Y	None	$x_2 = x_1$ and $y_2 < y_1$	$AZ = 180$	(86)
		$x_2 = x_1$ and $y_2 > y_1$	$AZ = 0$	(87)
		$x_2 > x_1$ and $y_2 \geq y_1$	$AZ = 90 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(88)
		$x_2 < x_1$ and $y_2 \geq y_1$	$AZ = 270 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(89)
		$x_2 < x_1$ and $y_2 < y_1$	$AZ = 270 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(90)
		$x_2 > x_1$ and $y_2 < y_1$	$AZ = 90 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	(91)
	West	$x_2 = x_1$ and $y_2 < y_1$	$AZ = 180 + \alpha$	(92)
		$x_2 = x_1$ and $y_2 > y_1$	$AZ = \alpha$	(93)
		$x_2 > x_1$ and $y_2 \geq y_1$	$AZ = (90 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert) + \alpha$	(94)
		$x_2 < x_1$ and $y_2 \geq y_1$		
		when $\alpha \leq \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = 270 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert + \alpha$	(95)
		when $\alpha > \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = \alpha - (90 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert)$	(96)
		$x_2 < x_1$ and $y_2 < y_1$	$AZ = 270 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert + \alpha$	(97)
		$x_2 > x_1$ and $y_2 < y_1$	$AZ = 90 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert + \alpha$	(98)
	East	$x_2 = x_1$ and $y_2 < y_1$	$AZ = 180 - \alpha$	(99)
		$x_2 = x_1$ and $y_2 > y_1$	$AZ = 360 - \alpha$	(100)
		$x_2 > x_1$ and $y_2 \geq y_1$		
		when $\alpha \leq 90 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = 90 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert - \alpha$	(101)
		when $\alpha > 90 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert$	$AZ = 360 - [\alpha - (90 - \arctan \lvert y_2 - y_1 / y_2 - x_1 \rvert)]$	(102)
		$x_2 < x_1$ and $y_2 \geq y_1$	$AZ = 270 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert - \alpha$	(103)
		$x_2 < x_1$ and $y_2 < y_1$	$AZ = 270 - \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert - \alpha$	(104)
		$x_2 > x_1$ and $y_2 < y_1$	$AZ = 90 + \arctan \lvert y_2 - y_1 / x_2 - x_1 \rvert - \alpha$	(105)

[a] West = counterclockwise rotation of points within plot (relative to grid frame); East = clockwise rotation of points within plot.

[b] Symbols: AZ = azimuth; α = plot-rotation angle. Computer calculations must include a correction factor consisting of multiplying the values resulting from arctan functions by the constant $180/\pi$ (57.29577) to convert radians to degrees because computers calculate trigonometric functions in radians.

Table 6—Equations for conversion of field-derived azimuth coordinates (HD, AZ) to rectangular (plot-center) coordinates (x, y) without plot rotations using Plot-origin radial mapping with polar X and polar Y plotting methods

Plotting quadrant[a]	Plotting method[b]	Condition formulas	Conversion equations[c]	Equation number
I	Polar X	$270 < AZ \leq 360$	$x = HD \cos(360 - AZ)$	(106)
			$y = HD \sin(360 - AZ)$	(107)
II		$180 < AZ \leq 270$	$x = -HD \cos(AZ - 180)$	(108)
			$y = HD \sin(AZ - 180)$	(109)
III		$90 < AZ \leq 180$	$x = -HD \cos(180 - AZ)$	(110)
			$y = -HD \sin(180 - AZ)$	(111)
IV		$0 \leq AZ \leq 90$	$x = HD \cos AZ$	(112)
			$y = -HD \sin AZ$	(113)
I	Polar Y	$0 \leq AZ \leq 90$	$x = HD \cos(90 - AZ)$	(114)
			$y = HD \sin(90 - AZ)$	(115)
II		$270 < AZ \leq 360$	$x = -HD \cos(AZ - 270)$	(116)
			$y = HD \sin(AZ - 270)$	(117)
III		$180 < AZ \leq 270$	$x = -HD \cos(270 - AZ)$	(118)
			$y = -HD \sin(270 - AZ)$	(119)
IV		$90 < AZ \leq 180$	$x = HD \cos(AZ - 90)$	(120)
			$y = -HD \sin(AZ - 90)$	(121)

[a] The polarity of x-coordinate (abscissa) and y-coordinate (ordinate) for points in each quadrant (quarter-plane) follow: Quadrant I: $x > 0$, $y > 0$; Quadrant II: $x < 0$, $y > 0$; Quadrant III: $x < 0$, $y < 0$; Quadrant IV: $x > 0$, $y < 0$.
[b] Polar X plotting designates the positive x-axis ray as magnetic north; Polar Y plotting designates the positive y-axis ray as magnetic north.
[c] Conversion of Azimuth coordinates (HD, AZ) to rectangular (plot-center) coordinates (x, y); HD = horizontal distance between source (origin) or plot-center and target tree; AZ = azimuth (bearing).

Rectangular to plot-center coordinate conversions— Because rectangular coordinates indicate tree positions relative to the reference origin, which is rarely at the geometric center of the plot, rectangular to plot-center conversions are needed to offset the plotted positions from the reference origin to plot-center. These conversions are achieved using the same methods for calculating the geometric center of a Sequential-target plot, with a correctional offset to reposition all plotted positions to the center of the mapping field. Orientation of Plot-origin radial maps in the field, like Sequential-target maps, requires that the geometric center of the plot is located and marked as plot center. Horizontal vector survey data may be input into the TreeMapper software on a notebook computer while in the field to determine plot-center coordinates; this reference position then can be marked before leaving the plot.

The geometric center of the plot can be located easily in the field by using an EMD near the reference origin to take repeated horizontal vector readings to targets of known plot-center coordinates and moving the reference position until it matches these plot-center coordinates. The process is facilitated by first identifying the physical position required to meet the proper AZ bearing to a target with horizontal vector readings taken in a circular path around the target, followed by a straight line correction on that bearing to identify the proper HD. Additional conformational readings to other targets of known plot-center coordinates should confirm the position of plot-center. Alternatively, plots could be set up using rectangular coordinates relative to the reference origin from which azimuth coordinates were taken, but all plotted positions would not necessarily fit within the mapping frame unlike plot-center maps, which position plotted points around the center of the mapping field.

Azimuth and intertree distance calculations—Horizontal distance of plotted points from plot-center and intertree HD's between any two plotted positions in the plot may be calculated from plot-center absolute coordinates using the distance calculator in TreeMapper. Azimuth from plot-center to plotted positions can be determined using equations 20 through 65 and AZ between plotted positions can be calculated using equations 66 through 105, depending on the plotting method and rotation used.

Three-Dimensional Mapping

Topographic maps of forest plots are needed where slope or elevation have significant effects on forest inventory assessments, management and harvesting decisions, construction activities, and research. The Sequential-target and Plot-origin radial mapping methods for two-dimensional plotting may be extended to three-dimensional plotting to create topographic maps. The theory and methodology for applying these mapping methods to topographic mapping are explained here to show that this is not a limitation of the methods. However, TreeMapper for the disk operating system (DOS) does not have topographic mapping capability due to the memory limitations of the DOS environment. Application of this capability will require development of comparable software in the memory-rich Windows environment.

The addition of a Z-coordinate to indicate an elevation change (ΔZ) between the source position and the target position is required to generate (x, y, z) rectangular coordinates necessary for three-dimensional mapping. Measuring SA, HD, and AZ with the EMD during field surveys provides three-dimensional azimuth coordinates (HD, AZ, SA) as input data for mapping. The change in elevation (ΔZ) then may be calculated from the SA using the following equation:

$$\Delta Z = HD \tan SA$$

where

SA = slope angle equivalent to the arctan of percent slope (expressed as a decimal fraction).

The z-coordinate then is determined by adding the ΔZ to the z-coordinate of the source (reference) position from which the measurements were taken. If the source position is the reference origin, as in the Plot-origin radial method, the z-coordinate will be the same as the ΔZ because the reference origin is the base elevation from which all plotted positions are referenced. Although there is an offset of x- and y-coordinates to plot center, the z-coordinate does not change. When the Sequential-target method is used, the ΔZ must be added to the z-value of each successive reference position to determine the cumulative z-coordinate of each successive target position relative to tree-1 (the relative base elevation). An absolute plot-center z-coordinate for each position then may be calculated by determining the midpoint in the two extreme (highest and lowest) tree-1 reference z-values and adding a correctional offset to each tree-1 z-coordinate to arrive at plot-center z-coordinates for each plotted position.

The coordinates of plot center with the z-coordinate added become (0, 0, 0) for three-dimensional mapping, in which the elevation geometric center, located at plot-center, is defined as the base elevation where Z = 0.

Slope distance (SD) is useful and necessary information for certain field mensuration tasks requiring actual distance measurements between plotted points on a slope. Slope distance, the actual distance between two plotted positions with an elevation change, may be calculated with the following equation:

$$SD = HD \sec SA .$$

Results demonstrating three-dimensional mapping algorithms are not included because the sample data set was not obtained from a location that had significant relief.

Results

Test Plot Orientation

Field sample data taken from the small, relatively open oak wilt research plot near Lampasas, TX were used to illustrate data inputs, conversions, and outputs from algorithms used to produce maps with the Sequential-target and Plot-origin radial mapping methods. Most of the research trees in this plot were oriented along a north-south axis. Thus, the Polar X plotting method without axes rotation was selected for both mapping methods because the x-axis is the long axis of the mapping field.

Sequential-Target Mapping Method

Table 7 presents the results of data conversions for 20 trees in this plot. Relative azimuth, polar, and rectangular coordinates between origin and target trees are listed in the same sequential order as that of the horizontal vector data recorded in the field. The HD component in azimuth coordinates and the r component in polar coordinates were equivalent, but AZ to θ polar bearing conversions changed.

Azimuth to rectangular coordinate conversions—Azimuth coordinates were converted into a standardized polar coordinate format to permit plot rotations and then Cartesian rectangular coordinates to permit plotting. Comparisons of azimuth and polar coordinate data demonstrate that only the angular bearing component of the azimuth coordinate changes with this conversion, while HD to target is unaffected. No other changes occur in the conversion because the polar coordinate system has a fixed reference

Table 7—Conversion of azimuth coordinates to polar and rectangular coordinates using Sequential-target mapping with the Polar X plotting method without plot rotation

		Relative coordinates[d]		
Tree origin-target[a]	Equations used[b]	Azimuth[c] (HD, AZ)	Polar (r, θ)	Rectangular (x, y)
1–2	1, 16, 17	(8.6, 152.1)	(8.6, 207.9)	(-7.6, -4.0)
2–3	1, 14, 15	(8.5, 213.3)	(8.5, 146.7)	(-7.1, 4.7)
3–4	1, 14, 15	(2.6, 210.8)	(2.6, 149.2)	(-2.2, 1.3)
4–5	1, 16, 17	(11.9, 110.5)	(11.9, 249.5)	(-4.2, -11.1)
5–6	1, 14, 15	(13.8, 251.1)	(13.8, 108.9)	(-4.5, 13.1)
6–7	1, 14, 15	(6.9, 217.4)	(6.9, 142.6)	(-5.5, 4.2)
7–8	1, 14, 15	(0.4, 228.1)	(0.4, 131.9)	(-0.3, 0.3)
8–9	1, 12, 13	(13.4, 318.2)	(13.4, 41.8)	(10.0, 8.9)
9–10	1, 14, 15	(22.8, 208.7)	(22.8, 151.3)	(-20.0, 10.9)
10–11	1, 14, 15	(3.9, 261.8)	(3.9, 98.2)	(-0.6, 3.9)
11–12	1, 16, 17	(11.1, 97.8)	(11.1, 262.2)	(-1.5, -11.0)
12–13	1, 18, 19	(1.4, 22.7)	(1.4, 337.3)	(1.3, -0.5)
13–14	1, 18, 19	(11.1, 77.5)	(11.1, 282.5)	(2.4, -10.8)
14–15	1, 18, 19	(1.8, 49.3)	(1.8, 310.7)	(1.2, -1.4)
15–16	1, 14, 15	(51.6, 209.0)	(51.6, 151.0)	(-45.1, 25.0)
16–17	1, 16, 17	(4.8, 167.9)	(4.8, 192.1)	(-4.7, -1.0)
17–18	1, 18, 19	(24.0, 50.1)	(24.0, 309.9)	(15.4, -18.4)
18–19	1, 18, 19	(10.7, 55.6)	(10.7, 304.4)	(6.0, -8.8)
19–20	1, 16, 17	(0.7, 100.3)	(0.7, 259.7)	(-0.1, -0.7)

[a] Survey data for 20 trees were taken sequentially between source and target trees with each target tree becoming the source tree for each successive azimuth coordinate measurement.
[b] Equation number (in order) for conversions of: 1) azimuth to polar coordinates and 2) polar to rectangular coordinates (x and y values).
[c] Azimuth coordinates were taken in the field between source trees (origin) and target trees using the Sequential-target survey method.
[d] Symbols: HD = horizontal distance; AZ = azimuth; r = radial distance (magnitude) equivalent to horizontal distance; θ = polar bearing (direction); x, y = Cartesian coordinates.

starting point (the positive x-axis) from which all bearings were measured. The fixed reference point allows one to define a polar coordinate equivalent for any AZ, regardless of the reference point chosen for the azimuth coordinate. The use of polar coordinates allows the standardization of azimuth coordinates regardless of plotting orientation. The polar-to-rectangular conversion transforms polar vector data into the Cartesian coordinate format. This rectangular coordinate format greatly facilitates computer plotting and HD calculations by allowing the use of trigonometric functions, grid plotting, and the Pythagorean theorem.

Rectangular to plot-center coordinate conversions—All relative tree positions in the plot must be tied together by a common reference position to obtain absolute coordinates by which any two positions can be directly compared. The conversion to absolute coordinates by assigning tree-1 as the reference origin (0, 0) and adding the relative rectangular coordinates of each successive target coordinate resulted in a sum positional change relative to tree-1. Tree-1 coordinates are absolute because all points are based on a single reference position (tree-1). The midpoint between the extremes of the x- and y-coordinates among all coordinates in the tree-1 reference data set determine the geometric center. In the sample data set, the geometric center of the tree-1 reference plot was determined to be (-44.2, 12.2). Hence, the correctional offset values (44.2, -12.2) were added to each corresponding x- and y-coordinate in the data set to arrive at plot-center coordinates (table 8). Absolute plot-center coordinates allowed calculations of HD and AZ of each tree from plot-center.

Table 8—Conversion of rectangular coordinates to tree-1 reference and plot-center coordinates using Sequential-target mapping with polar X plotting, no plot rotation, and with associated horizontal distance and azimuth of target trees from plot center

Tree no.	Tree-1 reference coordinates[a]	Plot-center coordinates[b]	Equations used[c]	Distance from plot-center[d]	Azimuth from plot-center
1	(0.0, 0.0)	(44.2, -12.2)	26	45.9	15.4
2	(-7.6, -4.0)	(36.6, -16.2)	26	40.0	23.9
3	(-14.7, 0.7)	(29.5, -11.5)	26	31.7	21.3
4	(-16.9, 2.0)	(27.3, -10.2)	26	29.1	20.5
5	(-21.1, -9.1)	(23.1, -21.3)	26	31.4	42.7
6	(-25.6, 4.0)	(18.6, - 8.2)	26	20.3	23.8
7	(-31.1, 8.2)	(13.1, - 4.0)	26	13.7	17.0
8	(-31.4, 8.5)	(12.8, - 3.7)	26	13.3	16.1
9	(-21.4, 17.4)	(22.8, 5.2)	23	23.4	347.2
10	(-41.4, 28.3)	(2.8, 16.1)	23	16.3	279.9
11	(-42.0, 32.2)	(2.2, 20.0)	23	20.1	276.3
12	(-43.5, 21.2)	(0.7, 9.0)	23	9.0	274.4
13	(-42.2, 20.7)	(2.0, 8.5)	23	8.7	283.2
14	(-39.8, 9.9)	(4.4, - 2.3)	26	5.0	27.6
15	(-38.6, 8.5)	(5.6, - 3.7)	26	6.7	33.5
16	(-83.7, 33.5)	(-39.5, 21.3)	24	44.9	208.3
17	(-88.4, 32.5)	(-44.2, 20.3)	24	48.6	204.7
18	(-73.0, 14.1)	(-28.8, 1.9)	24	28.9	183.8
19	(-67.0, 5.3)	(-22.8, - 6.9)	25	23.8	163.2
20	(-67.1, 4.6)	(-22.9, - 7.6)	25	24.1	161.6

[a] Tree-1 reference coordinates were determined by accumulative summing of relative rectangular coordinates starting with tree-2 (tree-1 becomes the new origin).

[b] Plot-center coordinates were obtained by adding the correctional offset (44.2, -12.2) to the coordinates of each tree in the tree-1 reference plot. The coordinates of the geometric center of the tree-1 reference plot was (-44.2, 12.2).

[c] Equation number for calculation of azimuth to target tree from plot center.

[d] Target distance was calculated using the Pythagorean theorem: $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. The numbers indicate the reference equations used to determine the tree azimuth from plot-center.

Relative and absolute coordinates with plot-rotation—Use of polar coordinates provides the flexibility of rotating the axes of a plot to any desired orientation. However, the process for conversion of relative azimuth coordinates to rectangular coordinates changes slightly when plot-rotations are employed. A correctional adjustment in the polar bearing (directional) component (θ) of the polar coordinate accommodates the necessary change to allow determination of the new rectangular coordinates of each point after rotation. The data set was divided into two parts (table 9): (1) trees 1 through 10 were rotated 45° westward (counterclockwise), and (2) trees 11 through 20 were rotated 45° eastward (clockwise). This division was done to show the effects of plot rotations in opposite directions on conversion coordinates. The rectangular coordinates reflect the change in the polar bearing as the points were rotated in an arc of magnitude and direction equivalent to the angular rotation indicated by the plot rotation.

The conversion process from rectangular to tree-1 reference and plot-center coordinates with rotated plots was identical to the process used when no plot-rotation was applied. The only exception was that tree-10 (not tree-1) was used as the reference origin (0, 0) for trees 11-20 from which tree-1 reference coordinates were determined. There were two correctional offsets, one for each part of the divided data set, because each part was essentially treated as a separate plot. Table 10 shows the position of each tree relative to the geometric center for each of two subplots.

Table 9—Coordinate-plot conversion data generated by polar X plotting method using Sequential-target mapping with axes rotation of plot

Tree origin-target	Plot rotation Degrees	Plot rotation Direction	Equations used[a]	Relative coordinates[b] Azimuth (HD, AZ)	Relative coordinates[b] Polar (r, θ)	Relative coordinates[b] Rectangular (x, y)
1–2	45	West	3, 16, 17	(8.6, 152.1)	(8.6, 252.9)	(-2.5, -8.2)
2–3			3, 16, 17	(8.5, 213.3)	(8.5, 191.7)	(-8.3, -1.7)
3–4			3, 16, 17	(2.6, 210.8)	(2.6, 194.2)	(-2.5, -0.6)
4–5			3, 18, 19	(11.9, 110.5)	(11.9, 294.5)	(4.9, -10.8)
5–6			3, 14, 15	(13.8, 251.1)	(13.8, 153.9)	(-12.4, 6.1)
6–7			3, 16, 17	(6.9, 217.4)	(6.9, 187.6)	(-6.8, -0.9)
7–8			3, 14, 15	(0.4, 228.1)	(0.4, 176.9)	(-0.4, 0.0)
8–9			3, 12, 13	(13.4, 318.2)	(13.4, 86.8)	(0.8, 13.4)
9–10			3, 16, 17	(22.8, 208.7)	(22.8, 196.3)	(-21.9, -6.4)
10–11	45	East	4, 12, 13	(3.9, 261.8)	(3.9, 53.2)	(2.3, 3.1)
11–12			4, 16, 17	(11.1, 97.8)	(11.1, 217.2)	(-8.8, -6.7)
12–13			4, 18, 19	(1.4, 22.7)	(1.4, 292.3)	(0.5, -1.3)
13–14			4, 16, 17	(11.1, 77.5)	(11.1, 237.5)	(-6.0, -9.4)
14–15			4, 16, 17	(1.8, 49.3)	(1.8, 265.7)	(-0.1, -1.8)
15–16			4, 14, 15	(51.6, 209.0)	(51.6, 106.0)	(-14.2, 49.6)
16–17			4, 14, 15	(4.8, 167.9)	(4.8, 147.1)	(-4.0, 2.6)
17–18			4, 16, 17	(24.0, 50.1)	(24.0, 264.9)	(-2.1, -23.9)
18–19			4, 16, 17	(10.7, 55.6)	(10.7, 259.4)	(-2.0, -10.5)
19–20			4, 16, 17	(0.7, 100.3)	(0.7, 214.7)	(-0.6, -0.4)

[a] Numbers indicate (in order) the equation number used for (1) azimuth to polar coordinate conversions and (2) polar to rectangular coordinate conversions.
[b] Target coordinates relative to origin; Azimuth (HD, AZ), Polar (r, θ), Rectangular and plot-center (x, y).

Intertree distance and azimuth calculations—The HD and AZ from one tree to another can be calculated from plot-center coordinates of the source (reference position) and the target position. The distance-azimuth calculator utility in the TreeMapper software serves this function. Table 11 lists this spatial information between different combinations of source and target positions. Once the HD and AZ are known, an EMD can be used to locate the desired position.

Plot-Origin Radial Mapping Method

Azimuth to rectangular coordinate conversions—The results of converting field-derived azimuth coordinates to plot-center coordinates using the Plot-origin radial mapping method were very similar to results with the Sequential-target method, but with a few noted exceptions. Unlike the Sequential-target mapping method, all coordinates in the Plot-origin radial conversion process are absolute coordinates from the beginning because all azimuth coordinates were recorded relative to a common, static reference origin during the Radial field survey. The use of a common reference origin (position 0) from which all azimuth coordinates were recorded is reflected in the origin-target column of table 12. Because all conversion coordinates are plotted with respect to the reference origin, there is no need for tree-1 reference coordinates. The conversion to polar coordinates permits plot rotations with this mapping method; however, examples of plot rotation and calculations of intertree HD and AZ were not included here.

Rectangular to plot-center coordinate conversions—Plot-center coordinates derived from the Plot-origin radial mapping method were identical to those calculated using the Sequential-target mapping method (see table 8). The HD and AZ values from plot-center for each plotted position also were in agreement between the two methods because these calculations were based on identical plot-center coordinates.

Table 10—Conversion of rectangular coordinates to plot-center coordinates using the polar X plotting method and Sequential-target mapping with plot rotation

Tree no.	Plot rotation		Tree-1 reference coordinates[a]	Plot-center coordinates[b]
	Degrees	Direction		
1	45	West	(0.0, 0.0)	(19.2, 10.7)
2			(-2.5, -8.2)	(16.7, 2.5)
3			(-10.8, -9.9)	(8.4, 0.8)
4			(-13.3, -10.5)	(5.9, 0.2)
5			(-8.4, -21.3)	(10.8, -10.6)
6			(-10.0, -20.5)	(9.2, -9.8)
7			(-16.8, -21.4)	(2.4, -10.7)
8			(-17.2, -21.4)	(2.0, -10.7)
9			(-16.4, -8.0)	(2.8, 2.7)
10			(-38.3, -14.4)	(-19.1, -3.7)
11	45	East	(2.3, 3.1)	(18.7, -6.9)
12			(-6.5, -3.6)	(9.9, -13.6)
13			(-6.0, -4.9)	(10.4, -14.9)
14			(-12.0, -14.3)	(4.4, -24.3)
15			(-12.1, -16.1)	(4.3, -26.1)
16			(-26.3, 33.5)	(-9.9, 23.5)
17			(-30.3, 36.1)	(-13.9, 26.1)
18			(-32.4, 12.2)	(-16.0, 2.2)
19			(-34.4, 1.7)	(-18.0, -8.3)
20			(-35.0, 1.3)	(-18.6, -8.7)

[a] Tree-1 reference coordinates were determined by accumulative summing of relative rectangular coordinates starting with tree-2 (for trees 1–10) and tree 11 (for trees 11–20) with trees 1 and 10, respectively, as the new origins in this split data set.

[b] Plot-center coordinates were obtained by adding the correctional offsets: (19.2, 10.7) for trees 1–10 and (16.4, -10.0) for trees 11–20 to the coordinates of each tree in the tree-1 reference plot. The coordinates of the geometric centers of each tree-1 reference split-plot data set are (-19.2, -10.7) and (-16.4, 10.0), respectively.

Table 11—Determinations of intertree distance and azimuth between tree positions using plot-center coordinates for a plot created with polar X plotting and Sequential-target mapping with no plot rotation

Tree source-target	Plot-center coordinates		Equations used[a]	Target distance from source tree	Target azimuth from source tree
	Source (x_1, y_1)	Target (x_2, y_2)			
1–4	(44.2, -12.2)	(27.3, -10.2)	69	17.0	186.7
2–8	(36.6, -16.2)	(12.8, -3.7)	69	26.9	207.7
3–7	(29.5, -11.5)	(13.1, -4.0)	69	18.0	204.6
5–14	(23.1, -21.3)	(4.4, -2.3)	69	26.7	225.5
7–10	(13.1, -4.0)	(2.8, 16.1)	69	22.6	242.9
9–15	(22.8, 5.2)	(5.6, -3.7)	70	19.4	152.6
11–19	(2.2, 20.0)	(-22.8, -6.9)	70	36.7	132.9
12–17	(0.7, 9.0)	(-44.2, 20.3)	69	46.3	194.1
13–18	(2.0, 8.5)	(-28.8, 1.9)	70	31.5	167.9
14–20	(4.4, -2.3)	(-22.9, -7.6)	70	27.8	169.0
15–12	(5.6, -3.7)	(0.7, 9.0)	69	13.6	246.5
17–13	(-44.2, 20.3)	(2.0, 8.5)	71	47.7	14.3
18–4	(-28.8, 1.9)	(27.3, -10.2)	71	57.4	12.2
19–16	(-22.8, -6.9)	(-39.5, 21.3)	69	32.8	239.4
20–6	(-22.9, -7.6)	(18.6, -8.2)	71	41.5	0.8
16–8	(-39.5, 21.3)	(12.8, -3.7)	71	58.0	25.5
14–9	(4.4, -2.3)	(22.8, 5.2)	68	19.9	337.8
17–7	(-44.2, 20.3)	(13.1, -4.0)	71	62.2	23.0
10–3	(2.8, 16.1)	(29.5, -11.5)	71	38.4	45.9

[a] Target distance was calculated using the Pythagorean theorem: $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. The numbers indicate the equations used to determine the target azimuth from the source tree.

Table 12—Conversion of azimuth coordinates to polar and rectangular coordinates using the Plot-origin radial mapping method with polar X plotting and no plot rotation

		Absolute coordinates[c]		
Tree origin-target	Equations used[a]	Azimuth[b] (HD, AZ)	Polar (r, θ)	Rectangular[d] (x, y)
0–1	1, 12, 13	(70.1, 349.7)	(70.1, 10.3)	(69.0, 12.5)
0–2	1, 12, 13	(61.9, 352.1)	(61.9, 7.9)	(61.3, 8.5)
0–3	1, 12, 13	(55.9, 346.3)	(55.9, 13.7)	(54.3, 13.2)
0–4	1, 12, 13	(54.0, 344.4)	(54.0, 15.6)	(52.0, 14.5)
0–5	1, 12, 13	(48.0, 355.9)	(48.0, 4.1)	(47.9, 3.4)
0–6	1, 12, 13	(46.4, 339.1)	(46.4, 20.9)	(43.3, 16.5)
0–7	1, 12, 13	(43.2, 331.3)	(43.2, 28.7)	(37.9, 20.7)
0–8	1, 12, 13	(43.0, 330.7)	(43.0, 29.3)	(37.5, 21.0)
0–9	1, 12, 13	(56.2, 327.8)	(56.2, 32.2)	(47.6, 29.9)
0–10	1, 12, 13	(49.2, 304.0)	(49.2, 56.0)	(27.5, 40.8)
0–11	1, 12, 13	(52.2, 301.1)	(52.2, 58.9)	(27.0, 44.7)
0–12	1, 12, 13	(42.2, 307.0)	(42.2, 53.0)	(25.4, 33.7)
0–13	1, 12, 13	(42.6, 308.8)	(42.6, 51.2)	(26.7, 33.2)
0–14	1, 12, 13	(36.8, 322.5)	(36.8, 37.5)	(29.2, 22.4)
0–15	1, 12, 13	(36.9, 325.2)	(36.9, 34.8)	(30.3, 21.1)
0–16	1, 14, 15	(48.3, 252.2)	(48.3, 107.8)	(-14.8, 46.0)
0–17	1, 14, 15	(49.1, 246.7)	(49.1, 113.3)	(-19.4, 45.1)
0–18	1, 14, 15	(27.0, 261.3)	(27.0, 98.7)	(-4.1, 26.7)
0–19	1, 12, 13	(18.0, 276.3)	(18.0, 83.7)	(-2.0, -17.9)
0–20	1, 12, 13	(17.2, 276.3)	(17.2, 83.7)	(-1.9, -17.1)

[a] Numbers indicate (in order) the equation number for conversions of (1) azimuth to polar coordinate conversions, and (2) polar to rectangular coordinate conversions (x and y values).
[b] Azimuth coordinates were taken in the field at plot-center (origin) using the Plot-center radial survey method.
[c] Symbols: HD = horizontal distance; AZ = azimuth; r = radial distance (magnitude) equivalent to horizontal distance; θ = polar bearing (direction); x, y = x-coordinate and y-coordinate, respectively, in the Cartesian coordinate system.
[d] Rectangular coordinates are equivalent to plot-center coordinates (although not necessarily the geometric center) because all survey azimuth coordinates were taken from a single central reference position in the plot.

Mapping Plot-Center Coordinates

Absolute plot-center coordinates were used to construct the final plot maps for both Sequential-target and Plot-origin radial mapping methods. Figure 7 presents the Polar X plot map for the 20 sample trees. The map displays a plot of live oak trees that was located in an open stand on the expanding edge of an oak wilt infection center. Asymptomatic healthy trees were threatened by imminent infection through root grafts and common root systems with symptomatic diseased trees. The positions of the 20 trees in the sample data set were numbered in the order surveyed. The map was produced at a scale factor of 1.6× to improve resolution between trees, yet maintain all plotted positions in the mapping frame. A scale-bar was provided to indicate a measure of HD. Plot-center is indicated by the cross hairs in the center of the mapping field. Magnetic north was placed on the positive x-axis because the plot used the Polar X plotting method with no plot rotation. The orientation could have been corrected for true north using a plot rotation of approximately 10° east (the declination for Lampasas, TX). The arrow to north would still indicate magnetic north, but it would be rotated 10° eastward (clockwise) away from the positive x-axis, and the positive x-axis would then point to and indicate true north. All plotted positions would be rotated clockwise with the plotting axes and magnetic north bearing to maintain the proper orientation.

Methods Accuracy and Performance

The accuracy and performance of the two mapping methods were tested by directly measuring azimuth coordinates between different combinations of tree positions not measured during the initial field survey and comparing these

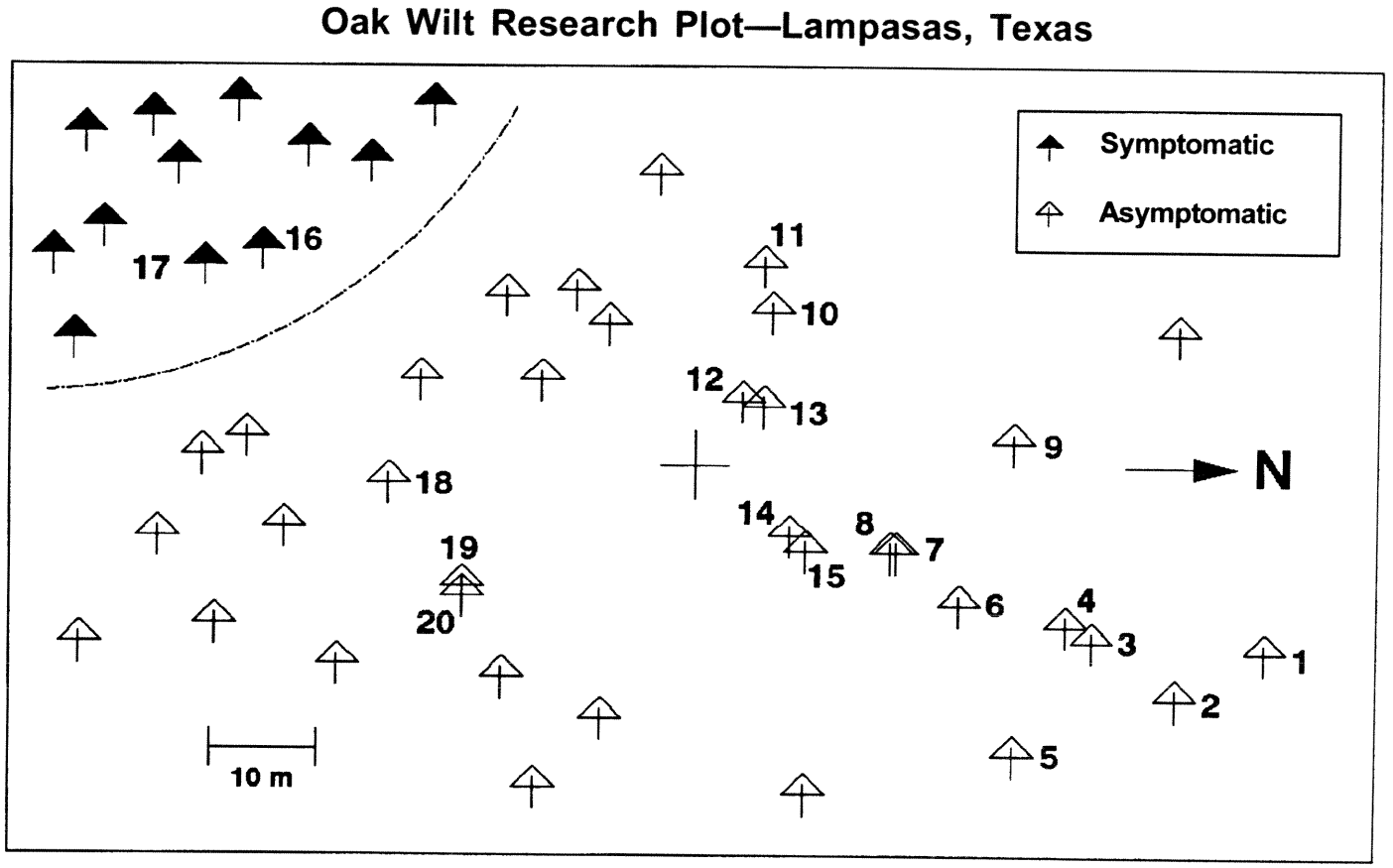

Figure 7—Map of oak wilt research plot (Lampasas, TX) constructed from identical plot-center coordinates derived from the Sequential-target and Plot-origin radial mapping methods. The first 20 trees included in the sample data set were numbered. Scale factor = 1.6×, Scale bar = 10 meters.

values to calculated values derived from TreeMapper using the original survey data. The Sequential-target and Plot-origin radial mapping methods essentially produced identical plot-center coordinates and thus had comparable performance. Mean differences between actual and calculated values were 1.25 ± 1.63 m (±1SEM) for HD measurements and 1.76 ± 2.5° (±1SEM) for AZ measurements, indicating the overall accuracy of the mapping methods.

The major sources of error in the methods occurred during field measurements. The magnitude of error depended on the precision of the data acquisition methods, the diligence in using proper and consistent technique, and the recognition of obviously bad field readings due to obstacles interfering with the transmission path of the EMD. The variability in tree diameters introduced error in both HD and AZ. Measurements taken on the side of the tree at a tangential position perpendicular to the target caused an error in AZ of ±1 to 2° depending on tree diameter and target distance. This error was corrected in this study by taking AZ readings in the direct line between each source and target tree. Target distance from the reference position did not significantly affect AZ readings from the EMD because laser beam divergence on this instrument is very small; as low as 3 milliradians (mrad) or 0.1719°, which represents an error of only 1.37 m over a maximum range shot of 457 m without a reflective prism. Azimuth readings to the center of the bole on the target tree were preferred because target width can vary considerably depending on tree diameter and HD from the source tree.

Tree diameters also affect HD measurements depending on where the EMD is positioned relative to the center of the source tree. The ideal measurement reflects the HD from the center of the source tree to the center of the target tree. This measurement was achieved in this study by taking HD readings from the near face of the reference tree to the near face of the target tree. Horizontal distance measurements also were affected significantly when the target tree leaned heavily, a common occurrence with live oaks, preventing an accurate measure to the position where the tree emerges

from the ground. The EMD was aimed low at the base of the target tree when taking HD readings to reflect the ground position at the center of the trunk. When the source and target tree are spaced by a distance shorter than the range of operation of the EMD, the HD can be determined by taking two distance measurements. The operator takes a position behind the source tree directly opposite the target tree within the operating range of the instrument and then takes a HD measurement to the source tree. A slight offset to one side allows a shot to the target tree. The HD then can be determined by subtraction. A similar method without the offset can be used to acquire AZ readings between source and target trees that are too close together for operation of the EMD. The accuracy of HD and AZ values often can be improved when these measurements are acquired from two separate laser shots, sometimes at different locations on the reference tree depending on its growth characteristics.

Errors from field measurements were theoretically cumulative with the Sequential-target method because azimuth coordinates were taken from tree to tree with a moving source or origin for each measurement. By contrast, errors were not cumulative with the Plot-origin radial method because the reference origin was a fixed position from which all azimuth coordinates in the survey were recorded. Nevertheless, no significant difference was found in the accuracy of the two methods ($P < 0.001$). The errors generated with the Sequential-target method using the Foresight-inline survey method were generally small and tended to be insignificant due to the precision and accuracy provided by the survey laser and to the use of consistently applied surveying methods.

The final ground resolution of tree locations relative to each other, as reflected by the plot map and supporting coordinates, primarily depends on the word-length of the computer as determined by the programmer's capability for extending precision, which is limited by the computer language used. The magnitude of ground resolution also is affected by the units of measure. Large units of measure would result in the same proportional ground resolution as a smaller unit of measure, but the absolute resolution in terms of physical distance would be smaller resulting in greater absolute resolution for smaller units of measure. However, larger units of measure often are required for very large plot areas to ensure all plotted positions fit within the mapping frame.

Discussion

The Sequential-target and Plot-origin radial mapping methods provide a means for rapidly producing accurate maps of forest plots to scale and for quantifying the positions and spatial distributions of trees in those plots. These methods do not require as many direct field measurements as other mapping methods and thus allow more trees to be mapped faster with fewer personnel. They only require azimuth coordinates from each surveyed tree using appropriate survey equipment. This horizontal vector data, recorded either from a static reference origin or between individual plot positions, then can be converted into plot-center coordinates to map the plot in the center of the mapping field. Queries about the spatial relationships between individual tree positions, plot center, or both then are possible using the vector calculator utility in the TreeMapper software.

These mapping methods are versatile for several reasons: (1) the variety of instruments that may be used to acquire the input data; (2) the use of surveying methods for data acquisition that do not require expensive and complicated GPS equipment; (3) the rapidity of map construction and coordinate output; (4) the polar coordinate conversion step with two plotting methods (Polar X and Y); (5) the capability of interpositional HD and AZ calculations; (6) the accommodation of two forest plot survey methods, the Foresight-inline and Radial survey methods, to cover the two most common field survey situations encountered during field surveys; and (7) plot rotations that allow one to rotate the plot in any direction to customize the orientation with respect to the mapping field. Plot rotations are useful in making sure all plotted positions are included in the fixed-area rectangular mapping frame, especially for irregularly shaped or elongated plots. They allow plotting corrections for declination and provide the ability to select unique map orientations to achieve perspectives relative to specific features that will be emphasized on the map. They also provide the ability to calculate interpositional azimuth coordinates between any two positions in the plot and are useful for supplying the information needed to guide movements between positions during routine work tasks in the field.

The TreeMapper software for DOS, developed to rapidly execute the algorithms of each mapping method, enables the rapid acquisition of accurate spatial data about individual tree positions in a plot and provides this information with the plot map output. A number of features have been included in the software to facilitate the mapping process. Mapping parameter options include sequential or radial mapping, polar X or polar Y plotting, axes rotations up to 90° in two directions, correction for declination, map scale factor adjustment for enlargements or reductions, and English, chain, or metric scale units. The map label options are

position numbering, symbol type, and title or headings. Printing options include support for 185 printer drivers, landscape or portrait printing orientation, and normal or reverse color. The vector calculator utility permits calculations of HD and AZ between any two positions, plot center, or both. TreeMapper, in addition to mapping capabilities, provides optional printouts of the output data of all coordinate conversions along with the map. The map and supporting coordinate data may be used in the field to locate the geometric plot-center and individual trees in the plot and to keep track of treatments, attributes, and responses of individual trees for research and management applications. The software also can be loaded onto a laptop computer to permit acquisition of this information when the field plot is originally constructed.

The use of vector data from EMD's in these mapping methods provides several advantages over the use of telemetry data from satellites via GPS instruments. Although GPS instruments may be used to determine the latitude-longitude (lat-lon) coordinates of any position in a forest plot, geopositional coordinates generally do not provide sufficient resolution to separate the positions of individual trees within small-area plots, especially when the trees are spaced close together. Lat-lon values to 0.01 seconds are needed to achieve 1-m accuracy. However, most GPS equipment does not measure positions with that level of accuracy. By comparison, EMD's with submeter accuracy easily allow unequivocal resolution of individual trees. The difficulty is further compounded by the common inability to obtain geopositional readings under a dense canopy. When GIS maps are required (e.g., for very large plots) coordinates are usually obtained by using an EMD and converting these coordinates to geopositional coordinates using GIS software to solve the accuracy problem. The output from vector data usually will provide higher resolution than telemetry data, but a printed map of stands with high tree density often will require enlargements by increasing the scale factor to provide adequate resolution to distinguish closely spaced trees.

Forest Management Implications

The mapping methods described have been useful in numerous oak wilt studies including work on early diagnosis, epidemiology, disease suppression, and others outlined for future research (see Wilson 1995). They have been used to set up research plots, keep track of treatments for data collection over time, and to measure the rate of movement of fungal pathogens through forest stands. These mapping methods are applicable to other work tasks that require accurate information about tree positions, intertree HD's, and spatial patterns and distributions and the ability to keep track of tree attributes over time. These tasks include forest inventory surveys, management operations, research studies of spatial models and forest dynamics, road building, timber harvesting, wildlife management, and recreation planning. Some specific applications of the types of spatial information provided by the mapping methods may be found in growth and physiological modeling studies investigating the effects of intertree shading on stand dynamics (see Kuuluvainen and Pukkala 1989, Urban and others 1991, Wang and Jarvis 1990), predicting individual tree characteristics from intertree interactions (see Reed and Burkhart 1985, Samra and others 1989), and estimating the influences of site characteristics, tree distribution, and canopy structure on physiological processes to simulate tree growth and primary production (see Leemans and Prentice 1987, Mohren and others 1984, Sievänen and others 1988). The methods also could be particularly useful in repeated horizontal-point and plot survey sampling for estimating stand components of growth and developing volume growth estimation models (see Eriksson 1995, Gregoire 1993, May 1988, Roesch and others 1989, Schreuder and others 1993, Van Deusen and others 1986). Even wider applications of these methods extend to the mapping of positions for any stationary objects in the landscape, including buildings, towers, landmarks, signs, road intersections, and any other objects with a definable position.

The USDA Forest Service is now beginning to keep track of individual trees in forest inventory surveys in the Western States to prevent the theft, harvest, and illegal sale of high-value trees. This trend will continue as increasing demand for forest products greatly augments the value of standing timber. The increasing need to produce maps of forest stands and keep track of individual trees demonstrates the need for these new, easy-to-use mapping algorithms and methods with versatile software that use data directly from an EMD or other surveyor device to rapidly produce maps and spatial output data on trees and other objects in the forested landscape.

Acknowledgment

The author gratefully acknowledges Drs. Ted D. Leininger and Nathan M. Schiff for reviewing drafts of the manuscript and Don G. Lester for writing the computer code for the TreeMapper 1.0 software for DOS and double checking the values presented.

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Appendix

Definitions of Terms

Angle of rotation. The angle (α) through which a plot is rotated in a clockwise (eastward) or counterclockwise (westward) direction.

Azimuth (AZ). The magnetic compass direction from a reference position to a target position.

Azimuth coordinates. Field-derived horizontal vector measurements of an attribute (target tree) position relative to a reference point (source), often the origin, given as horizontal distance and azimuth (HD, AZ).

Elevation distance (ED). The vertical rise or fall in elevation between a reference and target position indicated by changes in the z-coordinate in three-dimensional mapping; equivalent to SD sin SA.

Foresight-inline survey. A sequential field survey method that utilizes each consecutive target position as the reference origin from which the coordinate position of a subsequent target is recorded.

Geometric center. The position within a tree-1 reference plot that represents the midpoint between the extreme x and y (maxima and minima) values among all tree-1 reference coordinates for positions included in the plot.

Horizontal distance (HD). The level distance, equivalent to SD cos SA, between two geographic positions plotted on a horizontal plane passing between the two points at a constant elevation; most apparent from an aerial viewpoint above both positions.

Horizontal vector. Field data equivalent to azimuth coordinates.

Inclination. Slope angle or angle of slope.

Plot center. Plot geometric center and origin, or position from which plot-center coordinates of all plot positions are referenced, to be distinguished from plot reference origin.

Plot-center coordinates. Coordinates of a plotted position relative to the plot geometric center, which is located at the plot origin in the center of the mapping field.

Plot-origin radial mapping. Plot mapping method that utilizes Radial survey data from a common reference origin to calculate and map tree positions.

Plot-reference origin. Reference position from which all field coordinate data is acquired using the Radial survey method, to be distinguished from the plot-center position.

Plot rotation. The rotation of all plotted positions on the plot map through an angle (α) for the purpose of adjusting or customizing the orientation of the plot relative to the mapping frame.

Polar coordinates (r, θ). Vector-type coordinates of a tree or object position indicated in terms of a radial or horizontal distance (magnitude) and counterclockwise angular displacement (direction) relative to the positive x-axis.

Polar X plotting. Method for plotting tree or object positions by designating the positive x-axis ray to indicate the direction of magnetic or true north as a base plot-orientation, allowing plot rotations relative to this map orientation.

Polar Y plotting. Method for plotting tree or object positions by designating the positive y-axis ray to indicate the direction of magnetic or true north as a base plot-orientation, allowing plot rotations relative to this map orientation.

Radial distance (r). Magnitude (distance) component of polar coordinates.

Radial survey. Field survey method employed in the Plot-origin radial mapping procedure that takes all tree or object coordinates relative to a common position (the plot reference origin).

Rectangular coordinates. Cartesian coordinates that define an object position relative to a reference origin at the intersection of two or three mutually perpendicular plotting axes (x, y, and/or z) with positive or negative values to indicate opposing directions on each axis.

Sequential-target mapping. Plot mapping method that utilizes Foresight-inline survey data from sequential reference origins to calculate and map tree positions.

Slope angle (SA). Angle of slope between reference position and target position, also called vertical angle or inclination.

Slope distance (SD). The direct distance between two object positions at different elevations measured on an inclined (sloping) line through the two positions.

Theta (θ). Directional component of polar coordinates indicating counterclockwise angular displacement relative to the positive x-axis.

Tree-1 reference coordinates. The relative coordinates of a target position relative to a common reference position (tree-1) determined by coordinate summing of rectangular coordinates using Sequential-target mapping methods.

Wilson, A.D. 2000. New methods, algorithms, and software for rapid mapping of tree positions in coordinate forest plots. Res. Pap. SRS-19. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 27 p.

The theories and methodologies for two new tree mapping methods, the Sequential-target method and the Plot-origin radial method, are described. The methods accommodate the use of any conventional distance measuring device and compass to collect horizontal distance and azimuth data between source or reference positions (origins) and target trees. Conversion equations are presented to convert field-derived azimuth coordinates to plot-center coordinates, permitting plots of all tree positions relative to the geometric center of the plot. Plot rotation algorithms for Polar X and Polar Y plotting methods allow the rotation of all plotted positions to any orientation within a rectangular mapping frame and permit corrections for magnetic declination. Additional algorithms are provided to calculate horizontal distance and azimuth between plotted trees and plot-center or between any two tree positions in the plot. All algorithms were incorporated into TreeMapper, a computer program for DOS. The methods and software were tested on a forested plot. Low mean differences between actual and calculated values indicated the accuracy of the methods. The mapping methods and software, originally developed to map trees in research plots for spatial and epidemiological studies of oak wilt disease, have wide applications in forest inventory, management, ecology, spatial-modeling, and other field activities and tasks that require accurate information about tree positions and spatial distribution patterns and ways to keep track of tree attributes and treatments over time. More detailed explanations of the methodologies for downloading data and running TreeMapper will be provided in a second paper.

Keywords: *Ceratocystis fagacearum*, field plotting, oak wilt, spatial structure, stem mapping, tree inventory surveys, tree mapping.