United States Department of Agriculture

Forest Service

Southern Forest Experiment-Station

New Orleans, Louisiana

Research Paper SO-264

Growth and Yield Predictions for Thinned and Unthinned Slash Pine Plantations on Cutover Sites in the West Gulf Region

Stanley J. Zarnoch, Donald P. Feduccia, Virgil C. Baldwin, Jr. and Tommy R. Dell

SUMMARY

A growth and yield model has been developed for thinned and unthinned slash pine (*Pinus elliottii* Engelm. var. *elliottii.*) plantations on problem-free cutover sites in the west gulf region. These sites were originally virgin stands where clearcutting was followed by grazing and repeated burning that controlled the woody vegetation, thus allowing plantation establishment without site preparation. The model was based on the moment-percentile method using the Weibull distribution for tree diameters. This technique was applied to unthinned and thinned stand projections and, subsequently, to the prediction of residual stands immediately after thinning. Generally, initial thinnings were from below but at later ages a good distribution of tree diameters across the plot was also a criterion. The data base upon which the model's parameters were estimated was obtained from several studies throughout the region and contained stands well into the 40-year age class. The growth and yield prediction system was tested against the data used in its development and, generally, the predicted stand- and yield-table variables averaged within 5 percent of the observed values. In addition, predicted trends for unthinned and thinned plantations and some comparisons of results of these management alternatives are given under specific site and stand conditions.

June 1991

Growth and Yield Predictions for Thinned and Unthinned Slash Pine Plantations on Cutover Sites in the West Gulf Region

Stanley J. Zarnoch, Donald P. Feduccia, Virgil C. Baldwin, Jr. and Tommy R. Dell

INTRODUCTION

Slash pine (*Pinus elliottii* Englem. var. *elliottii.*) has been extensively planted in the west gulf region on problem-free cutover forest sites, which were virgin stands where clearcutting followed by grazing with repeated burning controlled the woody vegetation. This procedure had permitted plantation establishment without site preparation. The stand projection system called USLYCOWG (Dell and others 1979) was developed to estimate the yield of unthinned plantations of this type by predicting the parameters of a Weibull diameter distribution based model. However, no provision was made for estimating thinned-stand yields. USLYCOWG has been used throughout the west gulf region to predict unthinned-stand yields, but most of the data used to develop the model was obtained in stands less than 25 years old. Estimates of yields beyond this age, which are needed for sawtimber management, are questionable. The objective of the research reported here was to develop a stand projection system for estimating yields of thinned and unthinned slash pine stands in the west gulf region. An expanded data pool contained information on stands well into the 40-year age class. In addition, an approach, called parameter recovery (Matney and Sullivan 1982, Hyink and Moser 1983), was used in modeling the diameter distributions.

PLANTATION DATA

The study's data pool consisted of 507 unthinned-stand yield observations and 543 thinned-stand growth-period observations from 0.10- to 0.25acre plots in slash pine plantations established on problem-free cutover forest land. The length of the growth period was generally 5 years but ranged from 3 to 11 years. The plots were not located in areas where survival was poor or where heavy insect, disease, or other damage was present. For the most part thinning was from below, but at later ages a good distribution of tree diameters across the plot was also a criterion. In addition, there were 530 residual-stand observations of post thinning conditions.

Diameter at breast height (d.b.h.) to the nearest 0.1 inch was taken for all trees at each observation on a plot. The site index equation of Zarnoch and Feduccia (1984) (fig. 1) was used to calculate site index for each plot from a sample of dominant and codominant trees. Upper-stem height and outside-bark diameter measurements were made on sample trees, and volume per acre was determined by the height accumulation method (Lohrey 1967[1], Lohrey and Dell 1969). This process utilized height measurements from breast height to treetop along the bole at diameters that were multiples of 2 inches (i.e., 2 inches, 4 inches, 6 inches). The distributions of observations by various age, site, and density combinations are summarized for unthinned-stand (tables 1 through 3), thinned-stand (tables 4 through 9), and residual-stand (tables 10 through 13) observations.

MODEL DEVELOPMENT

The moment-percentile method was used to estimate the parameters of the Weibull distribution. This technique was then applied to unthinned- and thinned-stand projection situations and, subsequently, to the

[1] Lohrey, Richard E. 1967. Unpublished special report, "Description and use of two new computer programs for summarizing sample plot data. " On tile with: USDA Forest Service, Southern Forest Experiment Station, Pineville, Louisiana.

Stanley J. Zamoch is mathematical statistician, Southeastern Forest Experiment Station, Athens, GA' 30602; Donald P. Feduccia is chief of forest management, Office of Forestry, Baton Rouge, LA 70821; Virgil C. Baldwin, Jr. is research forester, Alexandria Forestry Center, Pineville, LA 71360; and Tommy R. Dell is project leader, Institute for Quantitative Studies, Southern Forest Experiment Station, New Orleans, LA 70113.

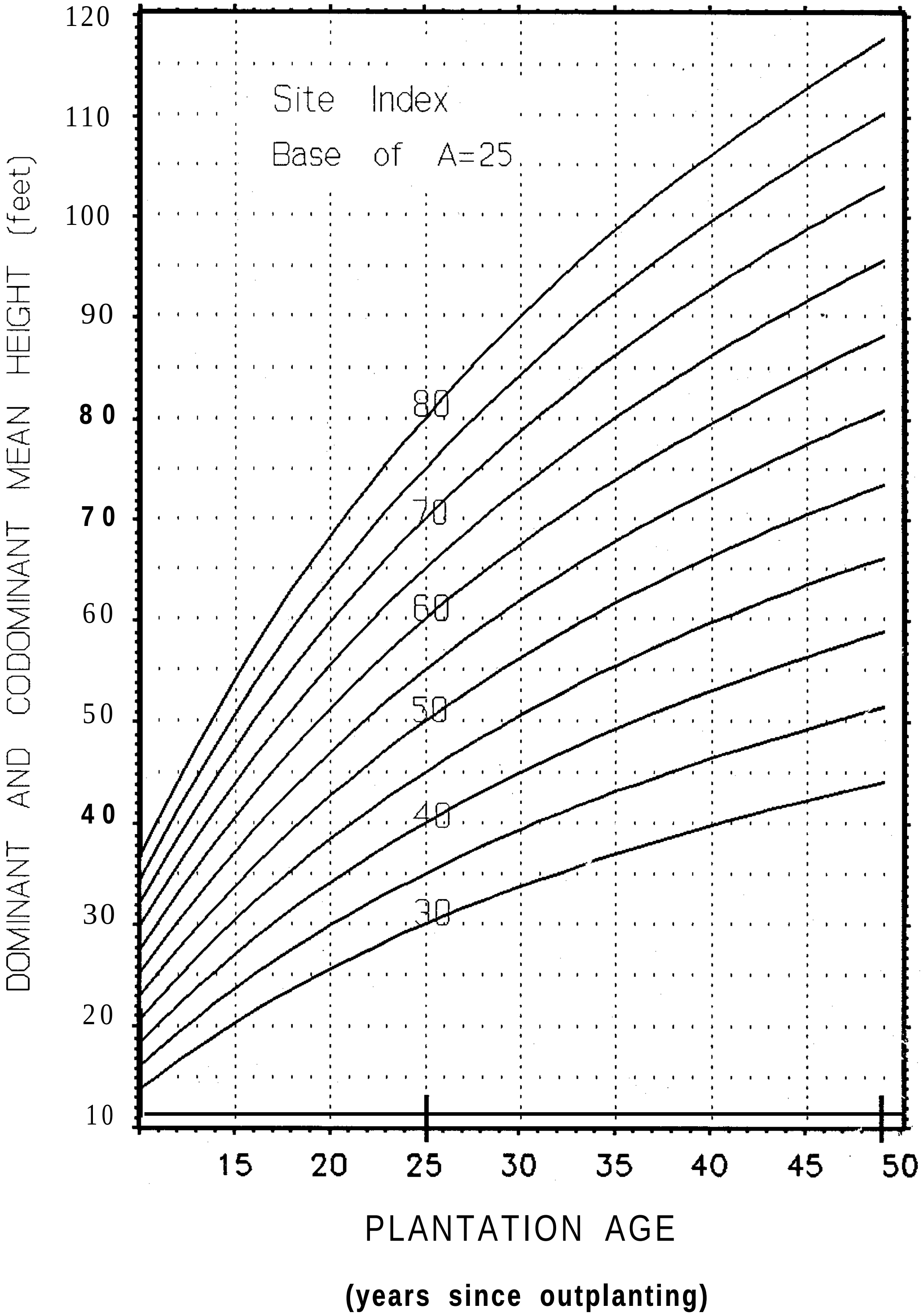

Figure 1 .--*Height of dominant and codominant trees by plantation age for site indices of 30 through 80 based on the equation* $\log (H_{DC}) = \log (S) + 2.922293\{[1/SQRT(25)]-[1/SQRT(A)]\}$ *(Zarnoch and Feduccia 1984). Note that log implies logatithm to the base 10. Also observe that 49 years since outplanting corresponds to SO years since seed.*

Table 1.--*Distribution of unthinned-stand yield observations by plantation age and site index*

Age class	Site index class (feet)												
	28-32	33-37	38-42	43-47	48-52	53-57	58-62	63-67	68-72	73-77	78-82	83-87	Total
Years	Number												
8-12	1	2	3		1	2	4	15	37	16	4		a5
13-17	2	3		10	11	22	12	51	61	34	5	2	213
18-22				5	6	16	la	15	14	9	3		86
23-27				1	10	12	11	7	12	3			56
28-32					4	a	5	4	12				33
33-37				1	1	5	1	1	15				24
38-42							1	1	6	1			9
43-47							1						1
Total	3	5	3	17	33	65	53	94	157	63	12	2	507

Table 2. --*Distribution of unthinned-stand yield observations by plantation 'age and planting density*

Age class	Planting density class (trees per acre)						
	251-500	501-750	751-1000	1001-1250	1251-1500	1501-1750	Total
Years	Number						
a-12	6	6	40	33			a5
13-17	14	13	144	42			213
18-22	11	11	40	16	3	5	86
23-27	a	a	24	9	3	4	56
28-32		5	la	9	1		33
33-37		2	14	a			24
38-42			7	2			9
43-47			1				1
Total	39	45	288	119	7	9	507

Table 3.--*Distribution of unthinned-stand yield observations by planting density and site index*

Planting density class	Site index class (feet) 28-32	33-37	38-42	43-47	48-52	53-57	58-62	63-67	68-72	73-77	78-82	83-87	Total
Trees per acre	Number												
251-500				5	4	10	3	2	4	6	4	1	39
501-750					3	18	8	2	5	5	3	1	45
751-1000	3	5	3	12	23	33	30	64	85	26	4		288
1001-1250					3	4	9	21	55	26	1		119
1251-1500							1	2	4				7
1251-1750							2	3	4				9
Total	3	5	3	17	33	65	53	94	157	63	12	2	507

Table 4.--*Distribution of thinned-stand observations by plantation age and site index*

Age class	Site index class (feet) 43-47	48-52	53-57	58-62	63-67	68-72	73-77	78-82	Total
Years	Number								
8-12						13	3	2	18
13-17	1	7	7	8	30	55	40	2	150
18-22	2	4	10	17	26	48	7		114
23-27	3	4	14	14	21	39	15	1	111
28-32	2	9	16	11	6	43	24	2	113
33-37					1	27	3		31
38-42					3	3			6
Total	a	24	47	50	87	228	92	7	543

Table 5.--*Distribution of thinned-stand observations by plantation age and residual basal area immediately after thinning*

Age class	Residual basal area (ft^2/acre)						
	21-40	41-60	61-80	81-100	101-120	121-140	Total
Years	Number						
8-12			8	38	2		18
13-17	1	12	49	65	21	2	150
18-22	2	10	32	41	22	7	114
23-27	2	12	29	50	16	2	111
28-32	2	13	27	47	19	5	113
33-37		4	5	12	10		31
38-42					2	4	6
Total	7	51	150	223	92	20	543

Table 6.--*Distribution of thinned-stand observations by plantation* age *and residual trees immediately after thinning*

Age class	Residual trees per acre										
	1-100	101-200	201-300	301-400	401-500	501-600	601-700	701-800	801-900	901-1000	Total.
Years	Number										
8-12					1	10	4	1	1	1	18
13-17			20	55	46	26	3				150
18-22		12	46	30	14	6	6				114
23-27	5	60	32	11	2	1					111
28-32	31	62	12	5	3						113
33-37	11	20									31
38-42		4	2								6
Total	47	158	112	101	66	43	13	1	i	1	543

Table 7.--*Distribution of thinned-stand observations by residual basal* area *immediately after thinning and site index*

Residual basal area	Site index class (feet)								
	43-47	48-52	53-57	58-62	63-67	68-72	73-77	78-82	Total
Ft^2/acre	-----	-----	-----	-----	Number	-----	-----	-----	-----
21-40	2	2	2	1					7
41-60		7	18	11	5	10			51
61-80	3	5	7	11	19	77	26	2	150
81-100	3	3	10	17	42	93	53	2	223
101-120		4'	6	7	18	41	13	3	92
121-140		3	4	3	3	7			20
Total	8	24	47	50	87	228	92	7	543

Table 8.--*Distribution of thinned-stand observations by residual basal* area *and residual trees immediately after thinning*

Residual basal area	Residual trees per acre										
	1-100	101-200	201-300	301-400	401-500	501-600	601-700	701-800	801-900	901-1000	Total
Ft^2/acre	-----	-----	-----	-----	-----	Number	-----	-----	-----	-----	-----
21-40	3	2	1		1						7
41-60	19	15	8	6	2	1					51
61-80	22	41	38	29	8	11	1				150
81-100	3	73	49	42	32	18	3	1	1	1	223
101-120		25	14	22	15	9	7				92
121-140		2	2	2	8	4	2				20
Total	47	158	112	101	66	43	13	1	1	1	543

Table 9.--*Distribution of thinned-stand observations by residual trees immediately after thinning and site index*

Residual trees	Site index class (feet) 43-47	48-52	53-57	58-62	63-67	68-72	73-77	78-82	Total
Trees/acre					Number				
l-100		2	9	2	1	23	8	2	47
101-200	1	3	8	14	21	79	31	1	158
201-300	3	2	14	8	20	50	15		112
301-400	1	7	6	13	16	38	20		101
401-500	1	3	6	6	17	19	12	2	66
501-600	2	4	1	5	12	14	5		43
601-700		3	3	2		2	1	2	13
701-800						1			1
801-900						1			1
901-1000						1			1
Total	8	24	47	50	87	228	92	7	543

Table 10.--*Distribution of residual stand observations by basal area before thinning and basal area after thinning; the first figure in each entry is for the first thinning and the second is for the second through fifth thinnings*

Basal area before thinning	Basal area after thinning (ft^2/acre) 21-40	41-60	61-80	81-100	101-120	121-140	Total
Ft^2/acre				Number			
41-60	0/5	0/4					0/9
61-80	1/1	1/24	1/2				3/27
81-100		lb/9	8/77	5/9			23/95
101-120		3/0	12/26	16/104	3/14		34/144
121-140		2/2	8/3	27/45	14/36	1/0	52/86
141-160			7/0	10/3	6/7	6/2	29/12
161-180			4/0	4/0	2/0	3/0	13/0
181-200				2/0		1/0	3/0
Total	1/6	16/39	40/108	64/161	25/57	11/2	157/373

Table 11.--*Distribution of residual stand observations by basal area before thinning and trees per acre before thinning; the first figure in each entry is for the first thinning and the second is for the second through fifth thinnings*

Basal area before thinning	Trees per acre before thinning					
	1-200	201-400	401-600	601-800	801-1000	Total
Ft^2/acre	Number					
41-60	0/7	0/2				0/9
61-80	0/18	0/9	1/0	2/0		3/27
81-100	0/38	0/47	8/9	12/1	3/0	23/95
101-120	0/38	0/70	9/36	20/0	5/0	34/144
121-140	0/7	0/45	8/26	41/7	3/1	52/86
141-160		0/6	4/4	18/2	7/0	29/12
161-180			4/0	4/0	5/0	13/0
181-200					3/0	3/0
Total	0/108	0/179	34/75	97/10	26/1	157/373

Table 12.--*Distribution of residual stand observations by basal area before thinning and plantation age; the first figure in each entry is for the first thinning and the second is for the second through fifth thinnings*

Basal area before thinning	Age class (years)						
	8-12	13-17	18-22	23-27	28-32	33-37	Total
Ft^2/acre	Number						
41-60			0/1	0/3	0/5		0/9
61-80		3/0	0/5	0/7	0/11	0/4	3/27
81-100	6/0	16/16	1/26	0/19	0/29	0/5	23/95
101-120	9/0	21/20	4/33	0/40	0/35	0/16	34/144
121-140	3/0	42/10	4/22	3/27	0/22	0/5	52/86
141-160		23/0	2/0	1/11	3/1		29/12
161-180		10/0	3/0				13/0
181-200		l/0	2/0				3/0
Total	18/0	116/46	16/87	4/107	3/103	0/30	157/373

Table 13.--Distribution *of* residual stand observations by *trees per acre before thinning and plantation age; the first figure in each entry is for the first thinning and the second is for the second through the fifth thinnings*

Trees per acre before thinning	Acre class (years)						
	8-12	13-17	18-22	23-27	28-32	32-37	Total
Trees/acre	-----Number-----						
1-200				0/17	0/61	0/30	0/108
201-400		0/13	0/52	0/74	0/40		0/179
401-600		28/26	3/34	1/13	2/2		34/75
601-800	8/0	76/6	9/1	3/3	1/0		97/10
801-1000	10/0	12/1	4/0				26/1
Total	18/0	116/46	16/87	4/107	3/103	0/30	157/373

prediction of residual stands immediately after thinning. Total tree height and volume equations required for stand summaries are discussed.

Moment-Percentile Estimators

The three-parameter Weibull function (Bailey and Dell 1973) was selected as a model for the distribution of diameters. The Weibull probability density function is defined as:

$$f(x) = \begin{cases} \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} e^{-\left(\frac{x-a}{b}\right)^{c}} & \text{for } a \geq 0,\ b > 0,\ c > 0 \text{ and } x \geq a \\ 0 & \text{elsewhere} \end{cases} \qquad (1)$$

where

a = location parameter,
b = scale parameter,
c = shape parameter, and
x = diameter outside bark (inches) at a height of 4.5 feet (d.b.h.), with the cumulative distribution function:

$$F(x) = 1 - e^{-\left(\frac{x-a}{b}\right)^{c}} \qquad (2)$$

It can be shown that the second moment about the origin is defined as:

$$E(X^2) = \int_a^\infty x^2 f(x)\, dx = a^2 + 2ab\,\Gamma(1+\tfrac{1}{c}) + b^2\,\Gamma(1+\tfrac{2}{c}) \qquad (3)$$

where

$\Gamma(z)$ = the gamma function evaluated at z (Dell and others 1984).

The expectation in equation 3 defines the average squared diameter measured in square inches, which when multiplied by the constant

$$\pi \left[\frac{1}{2}\right]^2 \left[\frac{1}{12}\right]^2 = 0.005454,$$

and the number of trees per acre, gives the basal area per acre (@*/acre). If the sample estimate of the second moment is defined as:

$$E[\hat{X^2}] = \frac{1}{n}\sum_{i=1}^{n} x_i^2,$$

then taking the square root gives the quadratic mean stand diameter:

$$\left[E(\hat{X^2})\right]^{0.5} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}.$$

Percentile definitions involve letting x_p equal the diameter that is the 100, percentile of the Weibull distribution where $0 \leq p \leq 1$. Equation 2 can then be solved for the scale parameter *b*:

$$b = \frac{x_p - a}{[-\ln(1-p)]^{\frac{1}{c}}} \tag{4}$$

Combining equations 3 and 4 and rearranging gives:

$$a^2 + 2a(x_p - a)\frac{\Gamma(1+\frac{1}{c})}{[-\ln(1-p)]^{\frac{1}{c}}} + (x_p - a)^2 \frac{\Gamma(1+\frac{2}{c})}{[-\ln(1-p)]^{\frac{2}{c}}} \cdot E(X^2) = 0, \tag{5}$$

which is solved by numerical techniques for the shape parameter c, given a specified p and estimated a, $E(X^2)$, and x_p. In the study described here, the location parameter was estimated by a simple projection function, yielding a two-parameter Weibull distribution, with a being fixed. The value of p was set at 0.93 because (1) the estimators of a and $E(X^2)$ tie down the left tail and center of the distribution, respectively, (2) the upper right tail is where most of the value is for distributions of tree diameters, and (3) the 93rd percentile has been found useful in estimating Weibull parameters in other situations (Zanakis 1979). Hence, equation 5 becomes:

$$a^2 + 2a(x_{.93} - a)\frac{\Gamma(1+\frac{1}{c})}{(2.65926)^{\frac{1}{c}}} + (x_{.93} - a)^2 \frac{\Gamma(1+\frac{2}{c})}{(2.65926);} \cdot E(X^2) = 0, \tag{6}$$

and equation 4 becomes:

$$b = \frac{X_{.93} - a}{(2.65926)^{\frac{1}{c}}} \qquad (7)$$

Solving for c in equation 6 and substituting its value into equation 7 gives b, which then completely defines the Weibull distribution.

Unthinned Stands

In order to obtain the Weibull diameter distribution parameters for a given stand by the moment-percentile method of parameter recovery, three stand-level attributes must be predicted and these estimates used to solve for a, b, and c as described previously. After considerable modeling effort, the stand attributes X_{min}, B, and $X_{.93}$ were selected where:

X_{min} = d.b.h. (inches) of the smallest diameter tree on a study plot,

B = basal area (ft^2/acre), and

$X_{.93}$ = d.b.h. (inches) that is the observed 93rd percentile in the diameter distribution of a study plot.

The projection equations used to estimate these quantities over time form a system of nonlinear functions of age, site, and number of trees surviving. The stand attribute X_{min} is inherently weak in that it is an order statistic and is a decreasing function of sample size. Thus, in forestry applications, X_{min} will decrease when plot size increases, and when plots of unequal size are used, as in the present case, the definition of X_{min} becomes questionable unless it is modeled as a function of plot size or more directly by the number of trees in the sample. However, this problem has generally been dismissed in previous growth and yield research because only a crude estimate of X_{min} is needed; therefore, this issue was not pursued further.

The basal area yield equation was modeled as:

$$B = \alpha_1 H_{DC}^{\alpha_2} T_s^{\alpha_3} e^{\alpha_4 A^{-1}} \qquad (8)$$

where

H_{DC} = average height (ft) of dominant and codominant trees,

T_s = number of trees surviving per acre,

A = age of plantation (years); i.e., number of growing seasons since field planting, and

α_i = parameters to be estimated (i = 1,2,3,4).

The quadratic mean diameter $\overline{X}_q$, defined as the d.b.h. of the tree of average basal area, can be derived by algebraic manipulation from equation 8, yielding:

$$\overline{X}_q = \delta_1 H_{DC}^{\delta_2} T_s^{\delta_3} e^{\delta_4 A^{-1}} \tag{9}$$

where

$$\delta_1 = \left(\frac{\alpha_1}{0.005454}\right)^{\frac{1}{2}},$$

$$\delta_2 = \frac{\alpha_2}{2},$$

$$\delta_3 = \frac{(\alpha_3 - 1)}{2}, \quad \text{and}$$

$$\delta_4 = \frac{\alpha_4}{2}.$$

Given the definition of $\overline{X}_q$ it is reasonable to require that models of X_{min} and $X_{.93}$ have the same functional form as the model of $\overline{X}_q$. Thus we have the nonlinear models:

$$X_{min} = \beta_1 H_{DC}^{\beta_2} T_s^{\beta_3} e^{\beta_4 A^{-1}} \tag{10}$$

and:

$$X_{.93} = \gamma_1 H_{DC}^{\gamma_2} T_s^{\gamma_3} e^{\gamma_4 A^{-1}} \tag{11}$$

With these three stand-level attributes projected for a specific stand, the location parameter is fixed as:

$$a = 0.5 X_{min} \tag{12}$$

and the second moment is estimated by:

$$E(X^2) = \frac{B}{0.005454 T_s}. \tag{13}$$

Experience has indicated that the value of a affects the distribution little because the other parameters adjust themselves accordingly (Zarnoch and Dell 1985). Moreover, because *a* is between zero and X_{min}, it is natural to fix this parameter at the midpoint of this interval. Hence, the shape parameter c is estimated by solving equation 6 after substitution from equations 11, 12, and 13. The scale parameter *b* is found by substituting the value of c and values from equations 11 and 12 into 7.

Equations 8, 10, and 11 were fitted to the plot data. Their estimated coefficients and fit statistics are shown in table 14. An example of the behavior and relationship of these stand attributes is shown in figure 2, where the minimum diameter (X_{min}), quadratic mean diameter (function of B), and the 93rd percentile ($X_{.93}$) are plotted over plantation age for three levels of trees surviving at age 10 where the site index is 60.

The predictor variables consist of age, height of dominant and codominant trees, and number of trees surviving. If values of the latter two stand measurements at a given age are unknown, they must be predicted. Mean height of the dominant and codominant trees at any age can be obtained from a back-solution of the site index equation (Zarnoch and Feduccia 1984). Two forms of this equation are given in table 15. There are three ways to predict survival in unthinned stands. If the number of trees surviving at the starting age, T_1, is unknown, but basal area per acre and site index or mean height of the dominant and codominant trees is known, then the fourth equation in table 14 is applicable. If only the number of trees planted, T_p, is known, then the fifth equation in table 14 can be used to predict T_s. Finally, when projecting changes in number of trees surviving from one age to another, an equation is given in table 14 based on the model:

$$T_2 = T_1 e^{\beta_1 (A_2 - A_1)} \tag{14}$$

where

T_2 = number of trees alive at the projection age,

T_1 = number of trees alive at the initial age,

A_2 = projection age, and

A_1 = initial age.

Figure 3 illustrates survival patterns, over time, based on the fitted model (equation 14) for four stand densities, each beginning at a stand age of 10 years.

Thinned Stands

The thinned-stand model is also based on the moment-percentile parameter recovery approach, estimating the values of stand-level attributes B_2, $X_{min,2}$, $X_{.93,2}$, and T_2 at projection age A, from initial **age A_1** where:

B_2 = basal area (ft^2/acre) at age A_2,
$X_{min,2}$ = d.b.h. (inches) of the smallest diameter tree on a study plot at A_2,
$X_{.93,2}$ = d.b.h. (inches) that is the 93rd percentile in the diameter distribution at A_2 on a study plot, and
T_2 = number of surviving trees at A_2.

The basal-area projection equation originally used by Clutter (1963) and Sullivan and Clutter (1972) was used and is defined as:

$$B_2 = B_1^{\frac{A_1}{A_2}} e^{\left(1-\frac{A_1}{A_2}\right)(\alpha_1 + \alpha_2 S)} \tag{15}$$

where
S = site index at base age 25 and
α_i = parameters to be estimated (i = 1,2).

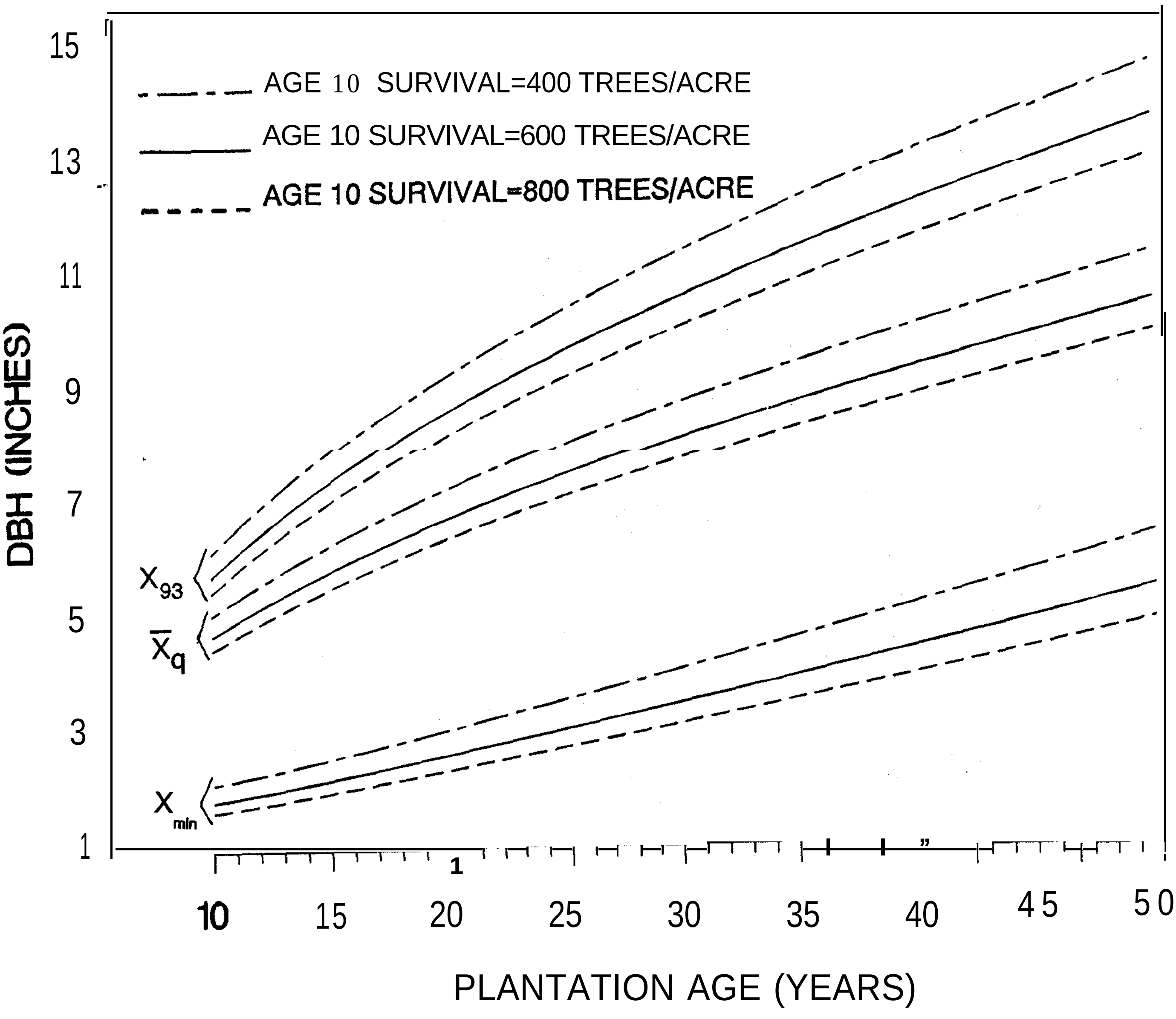

Figure *2.--Predicted diameter-growth relationships by plantation age for unthinned slash pine (site index 60)* with *survival at* age 10 **of 400, 600, and** *800 trees per acre where* X_{min} *is d. b. h. (inches) of the smallest diameter tree on a plot,* $\overline{X}_q$ *is the quadratic mean diameter (inches), and* $X_{.93}$ *is the d.b.h. (inches) that is the observed* 93^{rd} *percentile in the diameter distribution of a plot.*

Table 14.--Equations used for the unthinned-stand yield model components

Equation	Fit statistic"	S.E.
$B = 0.004715\ H_{DC}^{1.506277}\ T_s^{0.623533}\ e^{5.819339A^{-1}}$	0.93	10.40
$X_{min} = 0.011293\ H_{DC}^{1.788576}\ T_s^{-0.401856}\ e^{16.415341A^{-1}}$	0.60	0.86
$X_{.93} = 1.810092\ H_{DC}^{0.659044}\ T_s^{-0.175583}\ e^{0.596322A^{-1}}$	0.95	0.44
$T_s = 5424.369016\ H_{DC}^{-1.924749}\ B^{1.180924}\ e^{-7.092173A^{-1}}$	0.82	72.30
$T_s = T_p^{1.0-0.0034168}$	0.56	116.90
$T_2 = T_1\ e^{-0.021863(A_2-A_1)}$	0.93	45.70

*The fit statistic is the square of the correlation coefficient between the predicted and observed variables.

Table 15. Miscellaneous equations needed for the growth and yield model

Equation
$S = H_{DC}\ 10^{-0.584459+2.922293\ A^{-0.5}}$, base age of 25 years.
$H_{DC} = S\ 10^{0.584459-2.922293\ A^{-0.5}}$
$V = 0.12905 + 0.0028271\ D^2H \cdot 0.10102(10^{-7})(D^2H)^2$
$H = 1.9838\ H_{DC}^{0.913811}\ B^{-0.117585}\ T_s^{0.0754701} e^{0.589077\ A^{-1}\ -2.16139D^{-1}}$

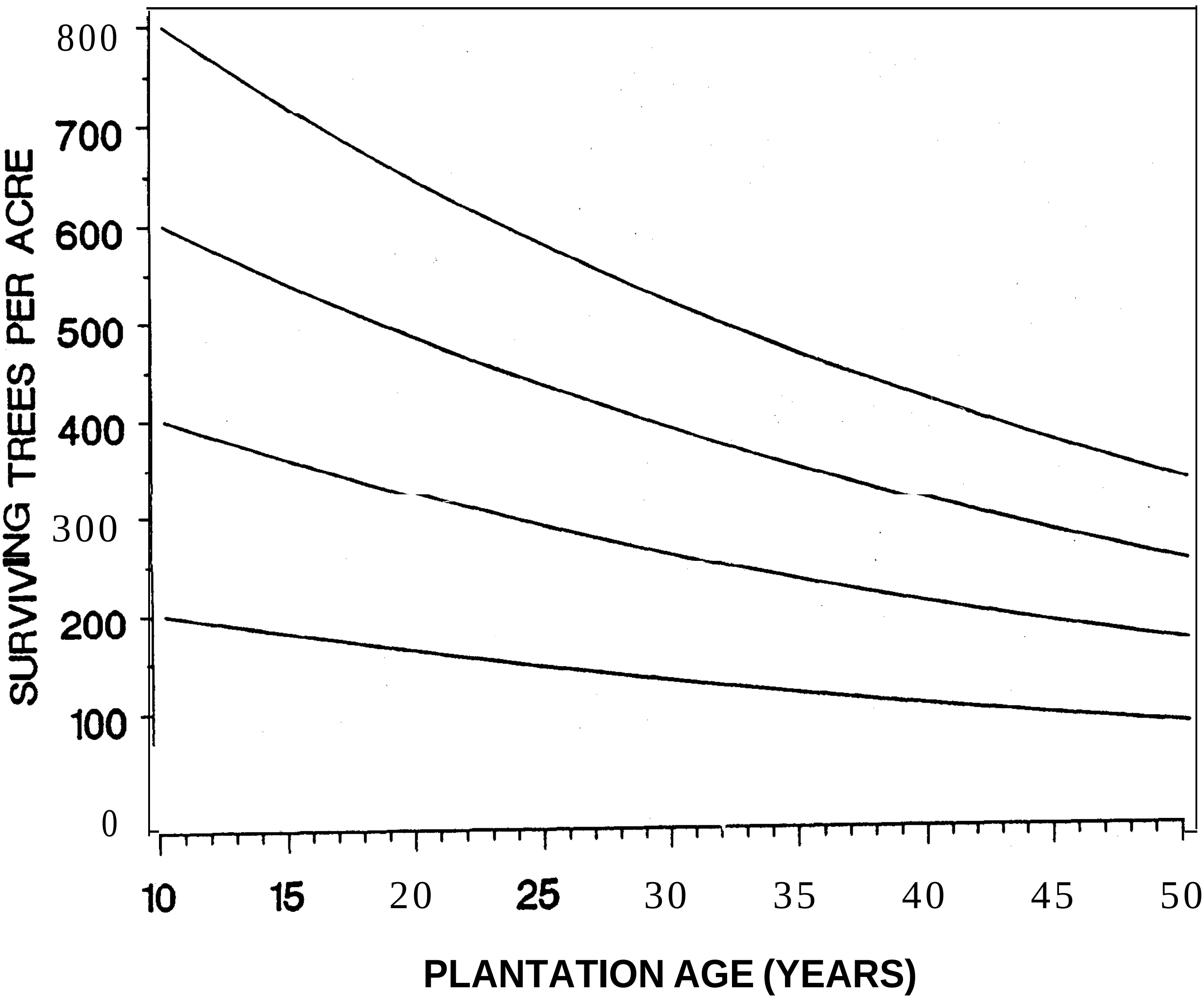

Figure 3 .--*Surviving trees per acre by plantation age for unthinned slash pine for four different starting densities.*

Equation 14 was used to model survival. An estimate of the quadratic mean diameter at projection age $\bar{X}_{q,2}$ was derived by combining equations 14 and 15:

$$\bar{X}_{q,2} = \delta_1 \left(\frac{B_1^{\frac{A_1}{A_2}}}{T_1} \right)^{0.5} e^{(1-\frac{A_1}{A_2})(\delta_2+\delta_3 S) + \delta_4 (A_2-A_1)} \qquad (16)$$

where

$$\delta_1 = 13.540741,$$

$$\delta_2 = \frac{\alpha_1}{2},$$

$$\delta_3 = \frac{\alpha_2}{2}, \text{ and}$$

$$\delta_4 = \frac{-\beta_1}{2}.$$

Again it will be assumed that $X_{min,2}$ and $X_{.93,2}$ have the same functional form as $\bar{X}_{q,2}$; thus, we have:

$$X_{min,2} = \beta_1 \left(\frac{B_1^{\frac{A_1}{A_2}}}{T_1} \right)^{0.5} e^{(1-\frac{A_1}{A_2})(\beta_2+\beta_3 S)+\beta_4 (A_2-A_1)} \qquad (17)$$

and:

$$X_{.93,2} = \gamma_1 \left(\frac{B_1^{\frac{A_1}{A_2}}}{T_1} \right)^{0.5} e^{(1-\frac{A_1}{A_2})(\gamma_2+\gamma_3 S)+\gamma_4 (A_2-A_1)} \qquad (18)$$

Given these 'stand-level attributes projected for a specific stand at age A,, the location parameter is fixed at:

$$a = 0.9\ X_{min,2}, \qquad (19)$$

and the second moment estimated by:

$$E(X^2) = \frac{B_2}{0.005454\ T_2}. \qquad (20)$$

Notice that a is now fixed very near the minimum diameter, while in the unthinned-stand model a was fixed at half the minimum diameter. The justification for this difference is that thinned stands contain fewer trees and often exhibit a smoother distribution of diameters as a result of previous thinnings. It was judged that for thinned stands a was better estimated by the minimum diameter, and the value of a was set closer to this number. The shape parameter c is estimated by solving equation 6 after substitution of values from equations 18, 19, and 20. The scale parameter b is found by substituting values from equations 18 and 19 and the value of c into equation 7. The stand-level attribute model equations 14, 1.5, 17, and 18 were fitted to the plot data;

their estimated coeffkients and fit statistics are shown in table 16. Because thinning affects stand growth, thinned-stand growth models are more complex than models describing the growth of unthinned stands. The necessity of employing basal area (B,) along with age (A, and A_2), site index, and trees surviving (T_1) as input variables illustrates this point. Obviously, residual B_1 and T, define $\bar{X}_{q,1}$, and therefore the quadratic mean diameter can be substituted for one of the other two. More important, though, the fact that B_1 and T_1 define $\bar{X}_{q,1}$ indicates the important role that average residual-tree diameter has in thinned-stand growth models, even though it may not explicitly enter into any of the projection equations.

Residual Stands

The model formulated to predict the diameter distributions of residual stands immediately after thinning from below is also based on the moment-percentile method of parameter recovery. The model used to predict the number of residual trees after thinning (Matney and Sullivan 1982) was:

$$T_r = T_b \left[1.0 - \left(1.0 - \frac{B_r}{B_b} \right)^{\alpha_1} \right]^{\alpha_2} \qquad (21)$$

where

T_b = number of trees per acre before thinning,
B_b = basal area (ft^2/acre) before thinning, and
B_r = a specified residual basal area (ft^2/acre) that will remain after thinning.

It should be noted, that in a previously unthinned stand, T_b would be identical to the T_s input required to make predictions of diameter distribution parameters at that age and that B_b is simply the estimated basal area from equation 8. Conversely, in a previously thinned stand, T_b is identical to T_2 in equation 14, and B_b is estimated using equation 15.

The other two stand attributes were modeled as:

$$X_{min,r} = \beta_1 + \beta_2 A + \beta_3 X_{min,b} + \beta_4 \frac{B_b - B_r}{B_b} \qquad (22)$$

and:

$$X_{.93,r} = \gamma_1 + \gamma_2 X_{.93,b} + \gamma_3 \bar{X}_{q,b} + \gamma_4 \frac{B_b - B_r}{B_b} \qquad (23)$$

where

A = plantation age (years) at time of thinning,
$X_{min,b}$ = d.b.h. (inches) of the smallest diameter tree on a study plot before thinning,
$X_{min,r}$ = d.b.h. (inches) of the smallest diameter residual tree on a study plot after thinning,
B_b = basal area (ft^2/acre) before thinning,
B_r = residual basal area (ft^2/acre) after thinning,
$X_{.93,b}$ = the diameter (inches) that is the 93rd percentile in the diameter distribution before thinning,
$X_{.93,r}$ = the diameter (inches) that is the 93rd percentile in the residual diameter distribution after thinning, and
$\bar{X}_{q,b}$ = the quadratic mean diameter (inches) before thinning.

Given these stand attributes, the location parameter is fixed at:

$$a = 0.9\, X_{min,r}, \qquad (24)$$

Table 16. -- Equations used for the thinned-stand growth projections

Equation	Fit statistic"	S.E.
$B_2 = B_1^{\frac{A_1}{A_2}} e^{(1-\frac{A_1}{A_2})(5.190356+0.006501\ S)}$	0.88	7.30
$X_{min,2} = 11.075838\left(\frac{B_1^{\frac{A_1}{A_2}}}{T_1}\right)^{0.5} e^{(1-\frac{A_1}{A_2})(-4.238939+0.048459\ S)+0.076748(A_2-A_1)}$	0.78	1.35
$X_{.93,2} = 15.132775\left(\frac{B_1^{\frac{A_1}{A_2}}}{T_1}\right)^{0.5} e^{(1-\frac{A_1}{A_2})(4.455365-0.012739\ S)-0.010953(A_2-A_1)}$	0.92	0.72
$T_2 = T_1\ e^{-0.015386(A_2-A_1)}$	0.97	23.70

*The fit statistic is the square of the correlation coefficient between the predicted and observed variables.

because the residual stand contains fewer trees and has a smoother diameter distribution than the unthinned stand. The second moment is estimated by:

$$E(X^2) = \frac{B_r}{0.005454\ T_r}, \qquad (25)$$

and the *b* and c parameters are found after appropriate substitutions into equations 6 and 7.

Models 21, 22, and 23 were fitted to before- and after-thinning data, and the estimated coeffkients and fit statistics are shown in table 17. Originally, these were fitted separately to data from first, second, third, fourth, and fifth thinnings. However, after scrutiny of the coefficients, models for two to five thinnings were considered essentially the same, and so the data was pooled and coefficients refitted. The formation of one set of equations for the first thinning and another for subsequent thinnings is also based on the judgment that the first thinning differs from the rest when thinning is from below: the first thinning gets the stand in shape for repeated subsequent thinnings.

Height-Diameter Equation

The total height of a given tree was modeled as a function of tree diameter, site index, and stand conditions. There were 17,606 height observations from the thinned and unthinned stands that were used together, giving:

$$\ln(H) = 0.684994 + 0.589077\ A^{-1} + 0.913811\ \ln(H_{DC}) - 0.117585\ \ln(B) + 0.0754701\ \ln(T_S) - 2.16139\ D^{-1} \qquad (26)$$

Table 17.-- Equations used to predict the residual stand following thinning

Equation*	R^2 or fit statistic+	S.E.
First thinning		
$X_{min,r} = -1.0388 + 0.053716\ A + 1.171432\ X_{min,b} + 1.322203\ P$	0.61	0.96
$X_{.93,r} = 0.293766 + 0.811760\ X_{.93\ b} + 0.301176\ \bar{X}_{q\ b} + 0.205805\ P$	0.93	0.26
$T_r = T_b \left[1.0 - \left(1.0 - \frac{B_r}{B_b} \right)^{0.889554} \right]^{0.994982}$	0.95	30.60
Second and future thinnings		
$X_{min,r} = -1.28337 + 0.107893\ A + 0.862925\ X_{min,b} + 1.114448\ P$	0.83	1.14
$X_{.93,r} = 0.119026 + 1.058200\ X_{.93\ b} - 0.051883\ \bar{X}_{q,b} + 0.316591\ P$	0.99	0.20
$T_b \left[1.0 - \left(1.0 - \frac{B_r}{B_b} \right)^{0.706519} \right]^{0.742329}$	0.97	17.60

*$P = \frac{(B_b - B_r)}{B_b}$, B_b = basal area (ft^2/acre) before cut, and B_r = residual basal area (ft^2/acre) after cut.

+The fit statistic is the square of the correlation coefficient between the predicted and observed variables.

where
H = estimated total height (ft) of a given tree,
D = d.b.h. (inches) of a given tree, and

the other variables are as previously defined, with $R^2 = 0.94$ and S.E. = 0.0911, both in the logarithmic scale for the dependent variable. (See table 15 for an alternate form). The equation generates estimates of tree heights that are used in the volume-defining function to determine cubic-foot volume per tree, which can be accumulated on a stand basis for estimates of volume per acre.

Volume-Defining Equation

Tree volumes were determined by the height accumulation method. Stump height was set at 0.5 ft. Several volume equations were fitted. These included the typical $V = b_0 + b_1 D^2H$ model and variations with no intercept, with a weighted term, with estimated exponents in a multiplicative model, and with the square of D^2H included as another term. Comparison of these models resulted in selection of the best model:

$$V = 0.12905 + 0.0028271\ D^2H - 0.10102\ (10^{-7})\ (D^2H)^2 \quad (27)$$

where

V = total volume (ft^3) outside bark above a 0.5-ft stump,
D = d.b.h.(inches), and
H = total tree height (ft),

with $R^2 = 0.9865$ and S.E. = 1.1916. (See table 15). This model was used for computing individual-tree total outside-bark volumes and subsequently volume per acre for thinned and unthinned stands.

MODEL TESTING

The growth and yield prediction system was tested against the data used in its development. The tests verified that predicted values were close to those observed.

The prediction phases tested were: yield prediction in an unthinned stand, growth prediction in an untbinned stand, characterization of a residual stand after thinning, and growth prediction in a thinned stand. In each case selected, predicted values of stand and yield table variables were compared with their respective observed values. The same volume defining function was used in each case. Mean predicted, mean observed, correlation coefficient, mean difference (predicted minus observed), and mean percentage difference statistics were calculated. The percentage differences are defined as:

$$100 \left[\frac{\text{PREDICTED} - \text{OBSERVED}}{\text{OBSERVED}} \right]$$

Results of these tests are found in tables 18 through 22. With the general exception of X_{min}, which is highly variable, the stand- and yield-table variables averaged within ± 5 percent of the observed values. This indicates that the system of equations accurately predicts growth and yield in the stands from which it was developed and should provide good results when used to make predictions in similar slash pine plantations.

DISCUSSION

Trends

Prediction trends for unthinned and thinned plantations and some comparisons of results of these management alternatives are given in figures 4 through 11. Generally in these figures, the extremely wide range of site indexes from 40 to 80 are presented. However, in this discussion, we have focused on the more realistic site indexes of 50 and 70. In most cases it was assumed that 700 trees were planted per acre (about an 8- by 8-ft spacing) on lands with site indices (base age 25) of 50 and 70. After prediction of stand conditions at age 10,

Table 18.--Observed vs. predicted values of stand- and yield-table variables for unthinned slash pine plantations (n=507)

Parameter	Mean Predicted	Mean Observed	Correlation coefficient	Mean Difference	Mean Percent difference
X_{min}	2.58	2.57	0.77	0.01	17.60
$X_{.24}$	5.27	5.14	0.94	0.13	3.69
$X_{.63}$	6.66	6.65	0.97	0.01	0.57
$X_{.93}$	8.08	8.08	0.97	0.00	0.37
$\bar{X}_q$	6.3[illegible]	6.32	0.98	0.01	0.63
B	124.84	124.63	0.97	0.21	1.51
VOL	3300.30	3369.71	0.98	-69.42	-0.38

Table 19.--Observed vs. predicted values of stand- and yield-table variables for unthinned slash pine plantations after one growth period (n=255)

Parameter	Mean Predicted	Mean Observed	Correlation coefficient	Mean Difference	Mean Percent difference
X_{min}	2.87	3.10	0.77	-0.22	2.25
$X_{.24}$	5.70	5.66	0.94	0.04	1.56
$X_{.63}$	7.29	7.40	0.96	-0.10	-1.07
$X_{.93}$	8.93	9.05	0.96	-0.13	-0.98
X_q	6.93	7.03	0.97	-0.11	-1.16
B	136.48	137.48	0.93	-1.00	-0.57
T_S	537.28	537.25	0.97	0.03	1.64
H_{DC}	55.25	57.95	0.83	-2.69	-2.23
VOL	3828.00	4172.65	0.96	-344.62	-6.11

Table 20.--Observed vs. predicted values of stand- and yield-table variables immediately after the first thinning (n=157)

Parameter	Mean Predicted	Mean Observed	Correlation coefficient	Mean Difference	Mean Percent difference
X_{min}	2.59	2.59	0.78	0.00	13.58
$X_{.24}$	5.10	5.05	0.86	0.05	2.35
$X_{.63}$	6.30	6.38	0.94	-0.08	-0.91
$X_{.93}$	7.51	7.51	0.96	0.00	0.11
$\bar{X}_q$	6.00	6.05	0.96	-0.05	-0.46
T_S	450.99	449.60	0.97	1.40	1.33
H_{DC}	45.92	46.00	0.99	-0.08	-0.05
VOL	2053.10	2133.68	0.99	-80.63	-2.58

Table 21.--Observed vs. predicted values of stand- and yield-table variables immediately following all thinnings except the first (n=370)

Parameter	Mean Predicted	Mean Observed	Correlation coefficient	Mean Difference	Mean Percent difference
X_{min}	5.68	5.69	0.91	-0.00	10.99
$X_{.24}$	7.94	7.84	0.96	0.10	2.19
$X_{.63}$	9.25	9.29	0.99	-0.04	-0.41
$X_{.93}$	10.54	10.54	1.00	0.00	0.06
T_S	8.89	a.94	0.99	-0.05	-0.43
H_D	219.21 65.84	218.55 66.09	0.99 1.00	-0.26 0.65	-0.37 1.15
DC VOL	2722.50	2809.05	0.99	-86.57	-2.80

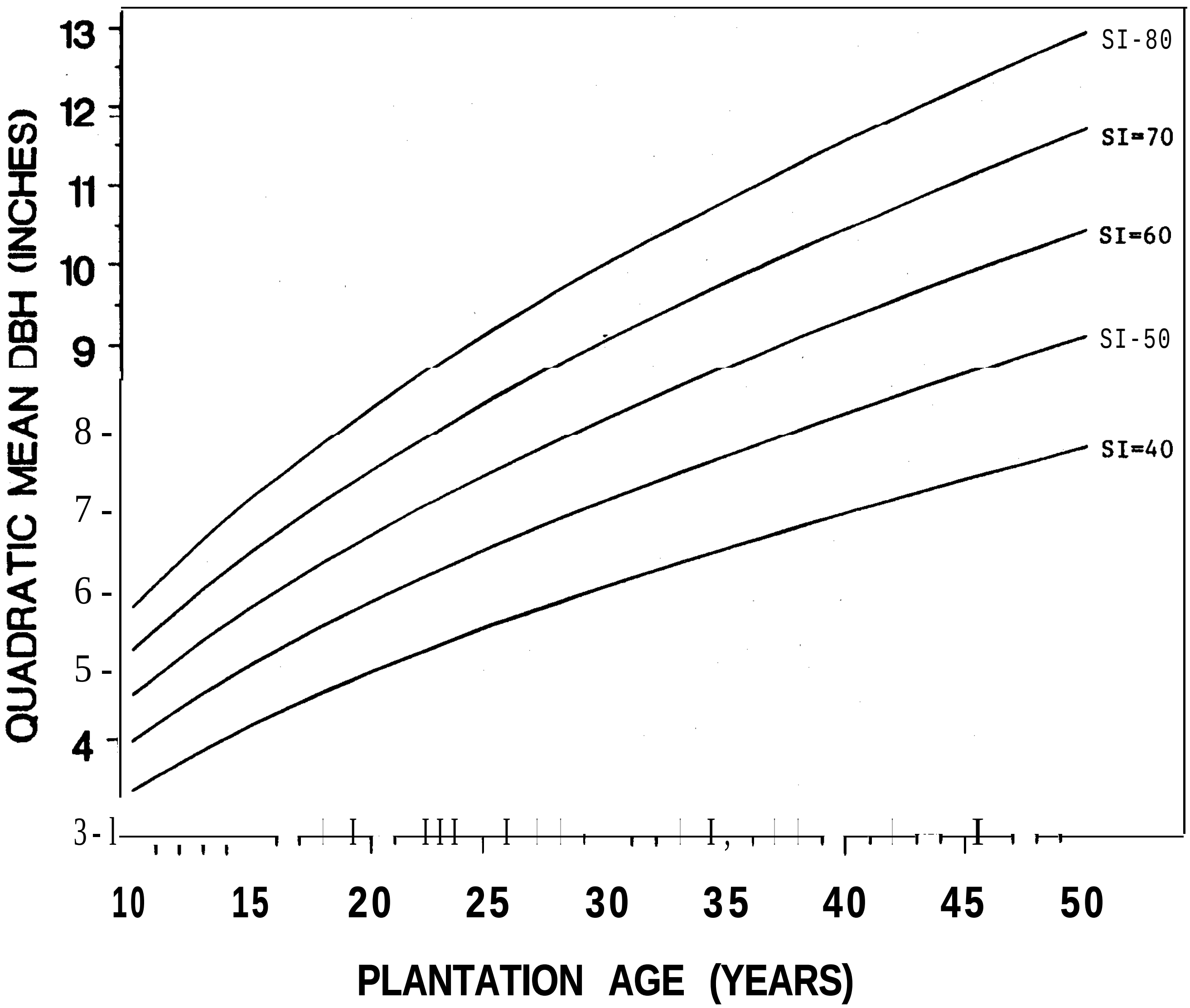

Figure 4.--*Predicted quadratic mean diameter trends for site indices of 40 through 80, each planted with 700 trees per acre.*

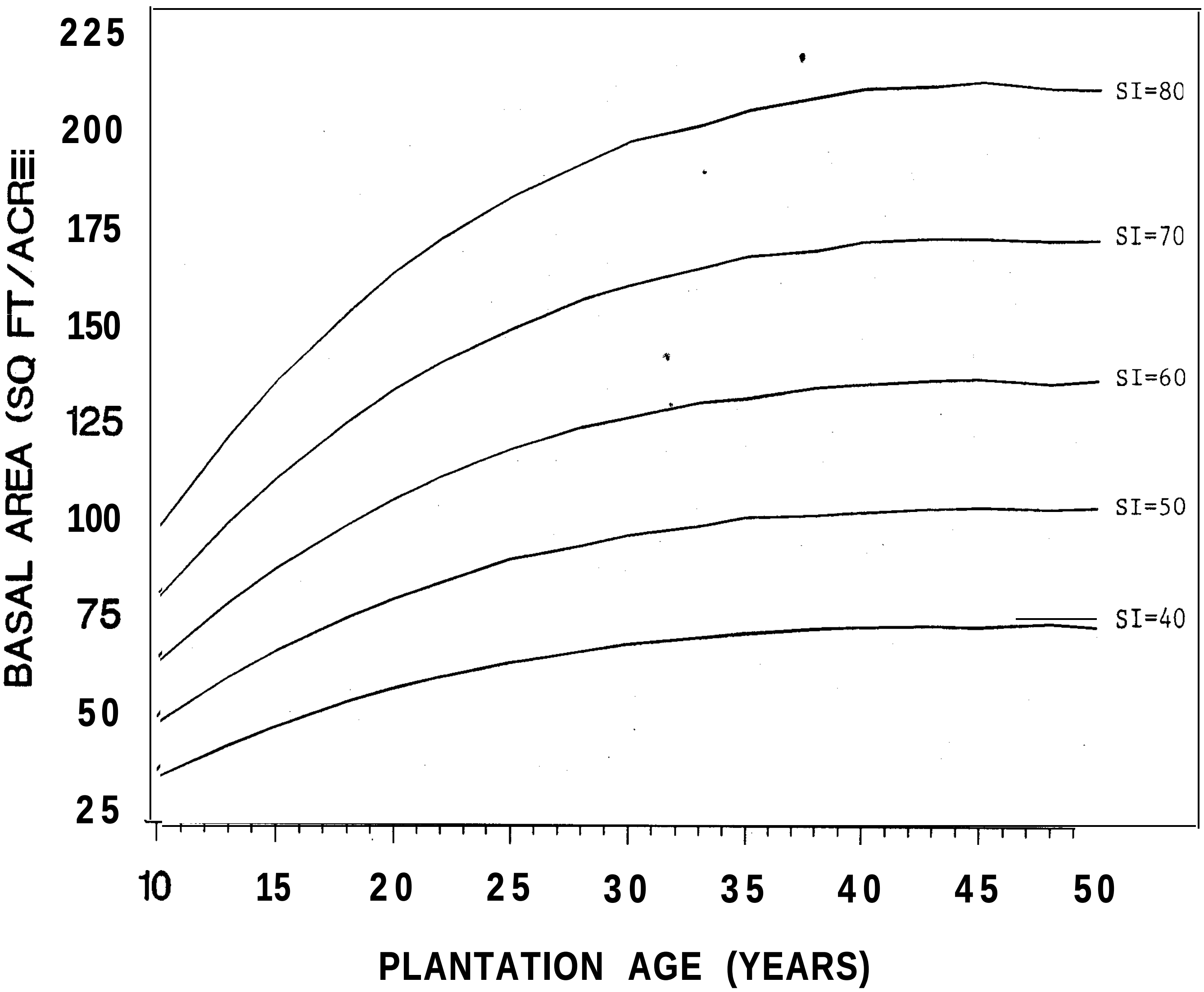

Figure 5 .--Predicted basal area per arce trends for site indices of 40 through 80, each planted with 700 trees per acre.

Table 22.--Observed vs. predicted values of stand- and yield-table variables for slash pine plantations after a 5-year growth period following thinning (n=543)

Parameter	Mean Predicted	Mean Observed	Correlation coefficient	Mean Difference	Mean Percent difference
X_{min}	5.64	5.54	0.88	**0.10**	18.84
$X_{.24}$	8.50	8.03	0.98	0.46	5.67
$X_{.63}$	9.80	9.64	0.98	0.16	1.27
$X_{.93}$	11.02	11.07	0.96	-0.05	-0.48
B_1	9.41	9.26	0.99	0.15	1.32
T_S	267.15 105.07	105.12	0.94	-0.05	0.47
		270.41	0.99	-3.25	-1.97
H_{DC}	69.52	69.35	0.98	0.17	0.31
VOL	3550.90	3693.60	0.94	-142.70	-3.05

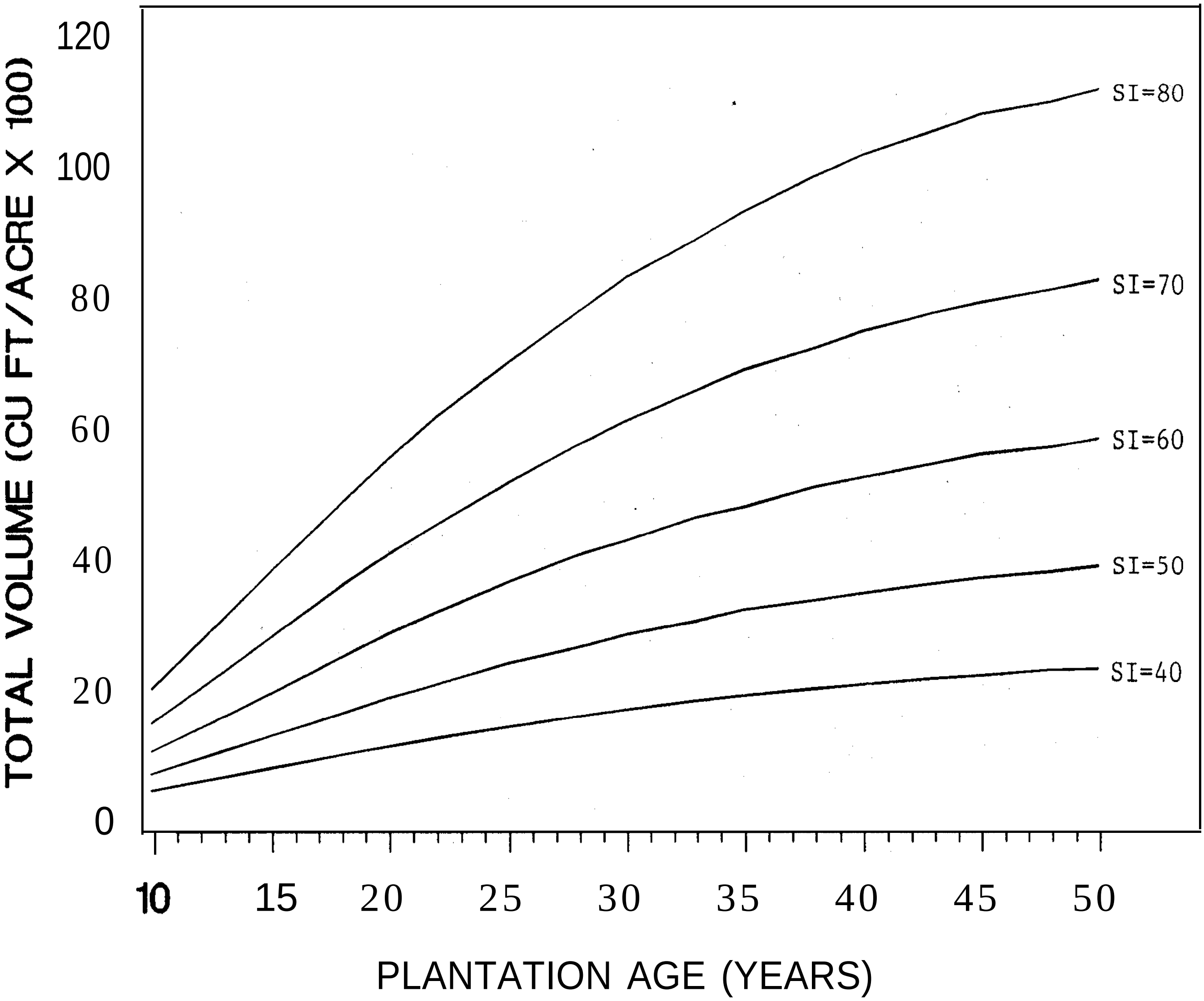

Figure 6.--*Predicted total cubic-foot volume per acre trends for site indices of 40 through 80, each planted with 700 trees per acre.*

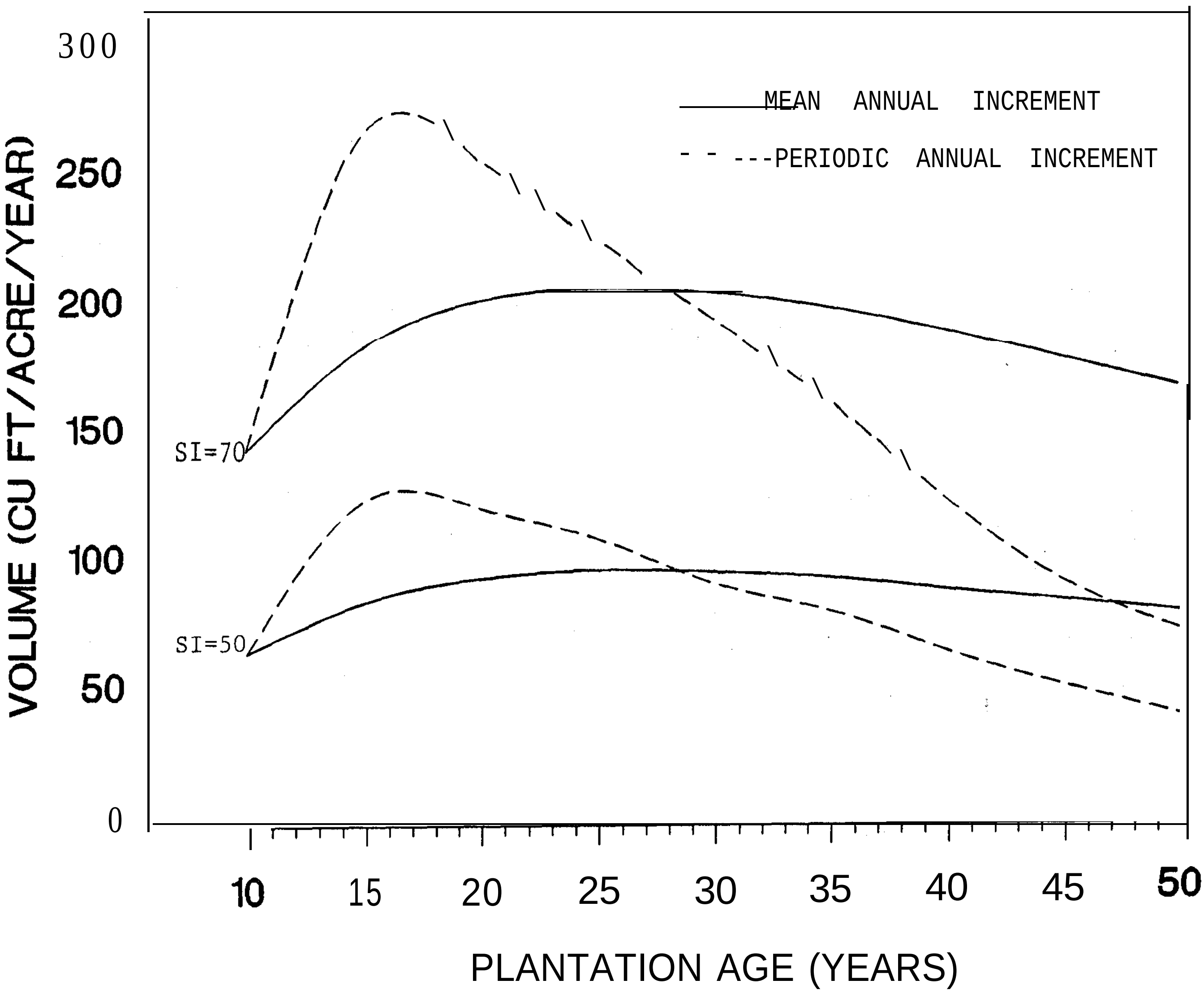

Figure *1.--Predicted mean and periodic annual cubic-foot volume increments for site indices of 50 and 70, each planted with 700 trees per acre.*

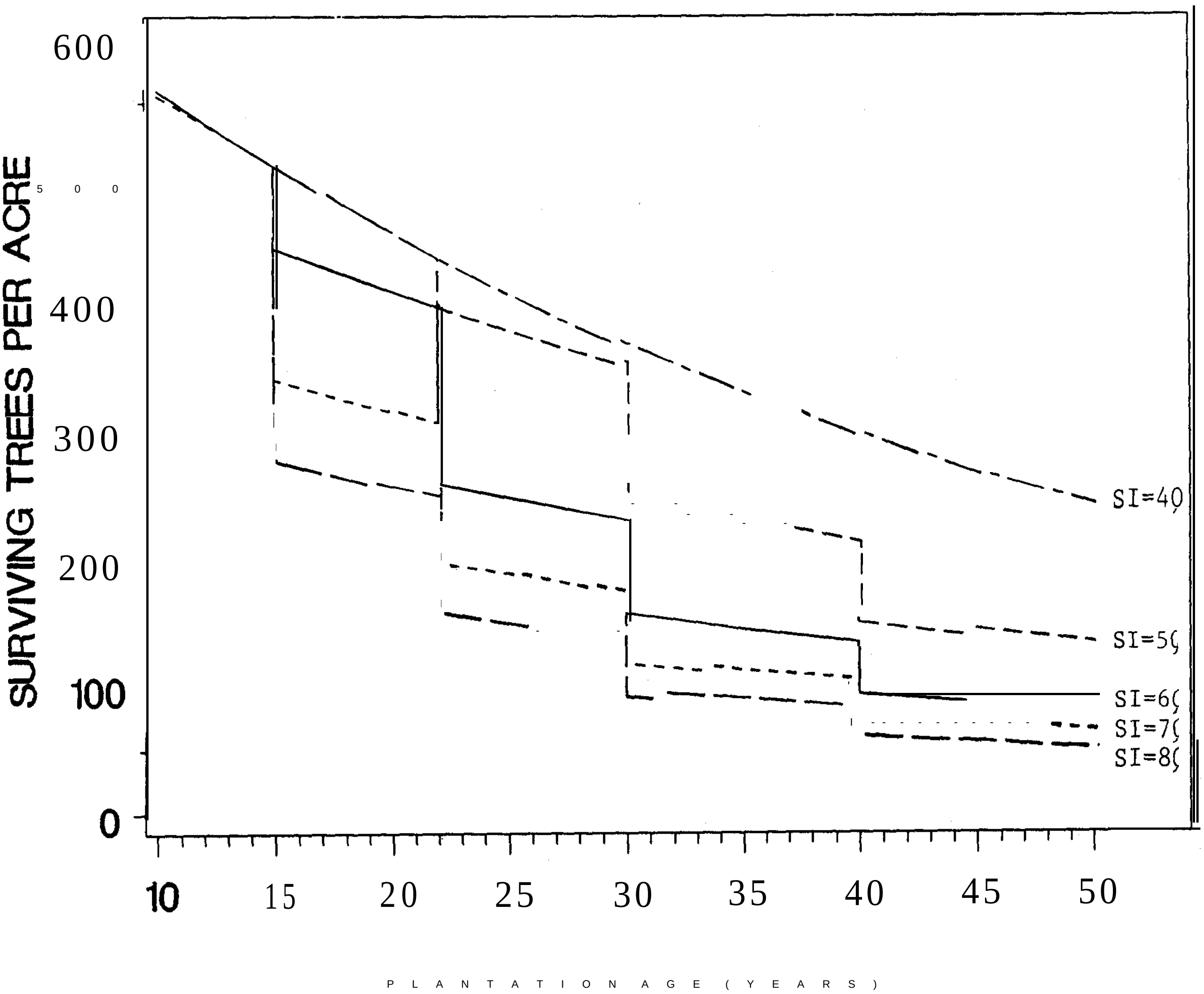

Figure *1.--Predicted survival trends for site indices of 40 through 80, each planted with 700 trees per acre and thinned to 80 ft²/acre of basal area at ages 15, 22, 30, and 40 where possible.*

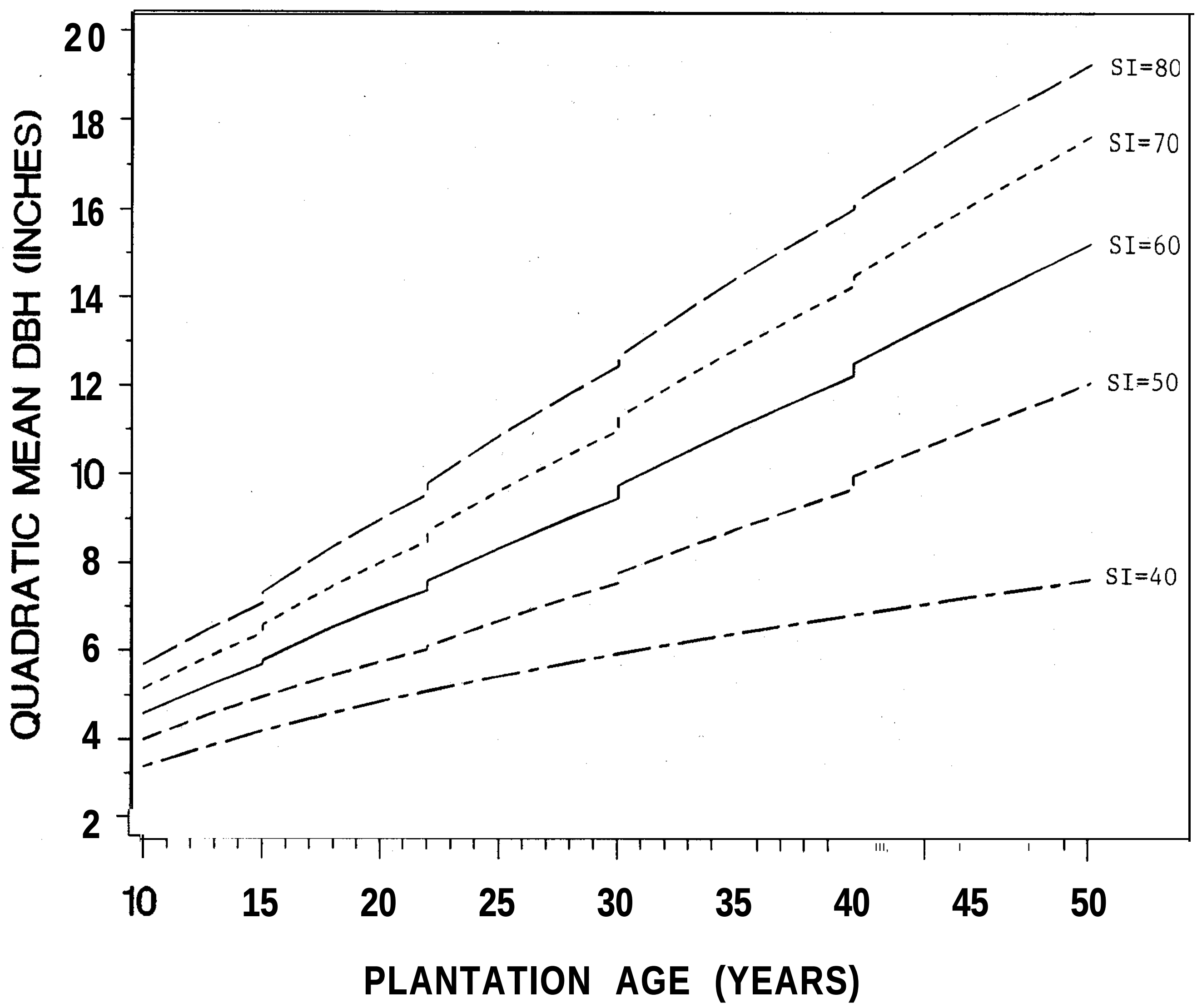

Figure *9.--Predicted quadratic mean diameter trends for site indices of 40 through 80, each planted with 700 trees per acre and thinned to 80 ft^2/acre of basal area at ages 15, 22, 30, and 40 where possible.*

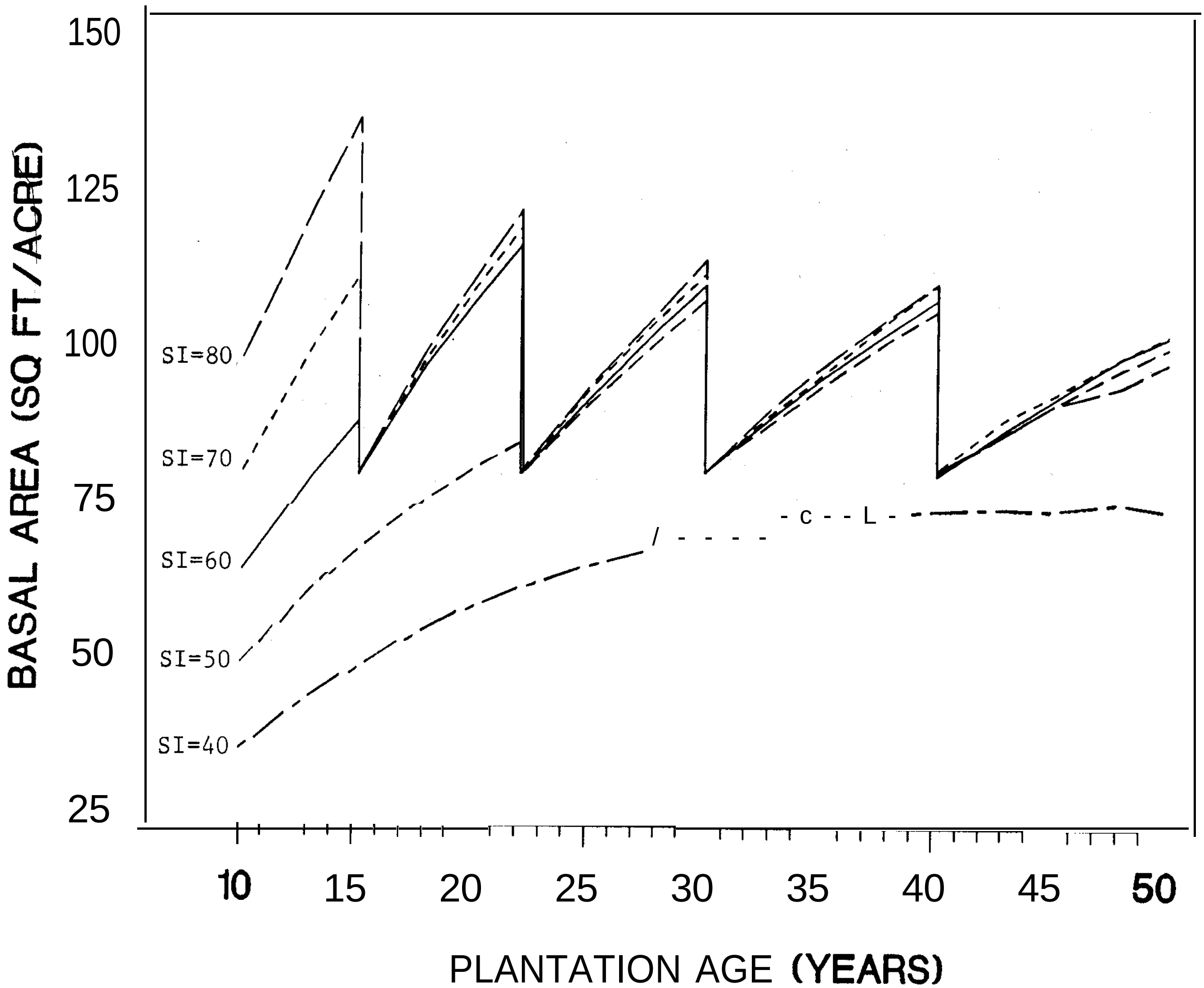

Figure 10 .--*Predicted basal area per acre trends for site indices of 40 through 80, each planted with 700 trees per acre and thinned to 80 ft^2/acre of basal area at ages 15, 22, 30, and 40 where possible.*

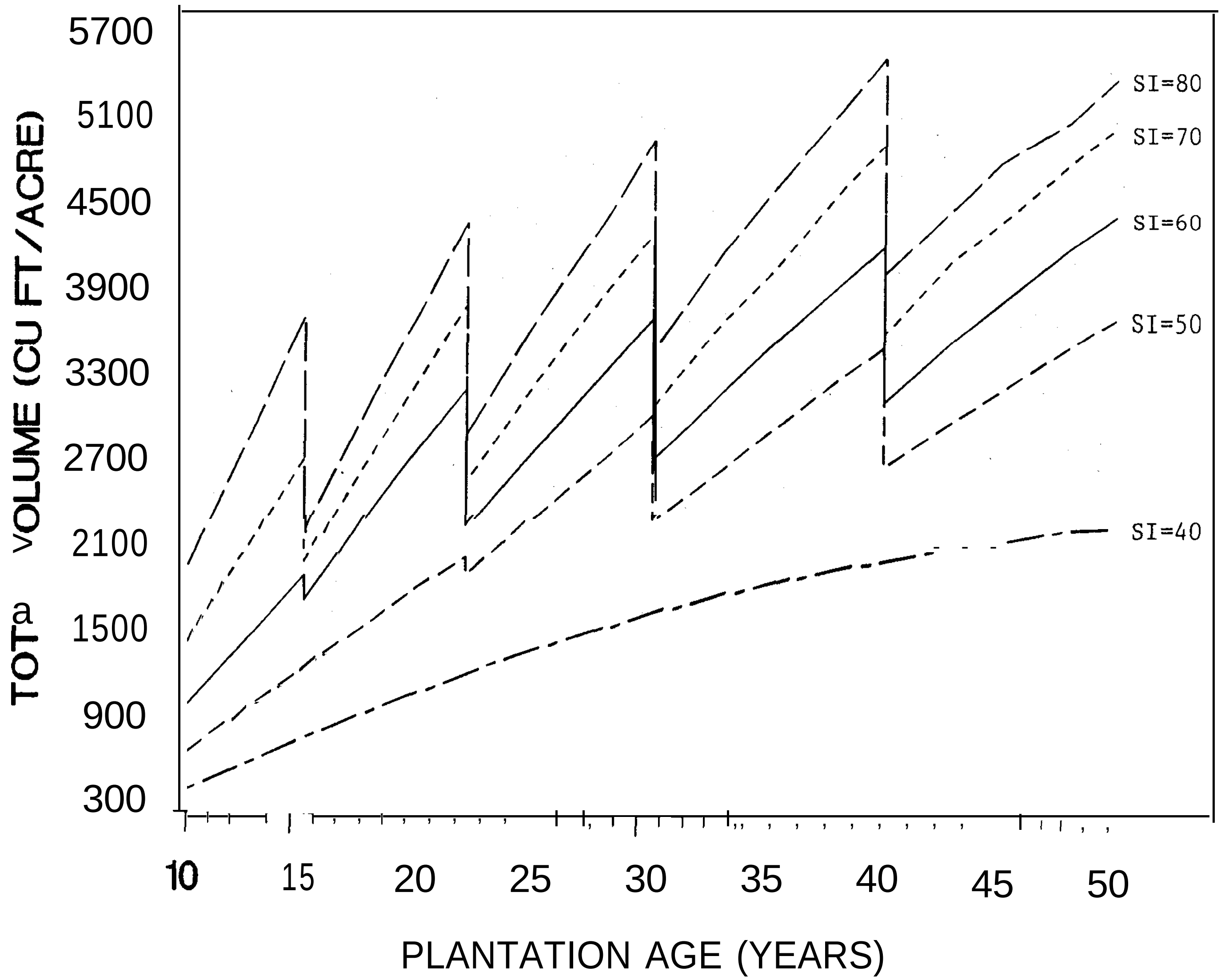

Figure 11.--*Predicted total cubic-foot volume per acre trends for site indices of 40 through 80, each planted with 700 trees per acre and thinned to 80 ft^2/acre of basal area at ages 15, 22, 30, and 40 where possible.*

projections were made to age 40 for unthinned-stand examples or to age 50 for thinned-stand examples. In the thinned-stand examples, the plantation with a site index of 70 was thinned to 80 ft^2 of basal area at 15, 22, 30, and 40 years. The plantation with a site index of 50 was thinned to 80 ft^2 of basal area at 22, 30, and 40 years.

Unthinned Plantation

Mean Diameter--The average gain in mean diameter for site 70 over the site 50 plantation was 2.34 inches by stand age 40 (fig. 4).

Basal Area--Basal area increased with increasing site index (fig. 5) but increased more rapidly on site 70. Basal area was near culmination on both sites by stand age 40.

Total-Stem Volume Yield--Total-stem yields (cubic feet, outside bark) did not culminate before age 40 on either site, but the volume on site 70 at age 40 was 4,024 ft^3/acre greater than the corresponding value for site 50 (fig. 6).

Mean and Periodic Annual Increment--Total-stem volume (cubic-feet outside bark) mean annual increment (MAI) culminated at about stand age 25 on both sites and tapered off more rapidly on site 70. However, at age 25 MA1 was 110.3 ft^3/acre per year greater on site 70 than on the site 50 (fig. 7).

Thinned Plantation

Surviving Trees--At age 22, when the site 70 stand was thinned for a second time, the site 50 stand got its first thinning to the target basal area of 80 ft^2/acre (fig. 8). Obviously, very few trees were cut from the site 50 stand. The site 70 stand had about 200 fewer trees per acre after the thinning at age 22 than did the site 50 stand. From this time on, though, the cuts on site 50 removed more trees than did those on site 70, so the gap closed to about 60 trees per acre. Mortality was not a very important factor in either stand after the thinning regime was initiated.

Mean Stand Diameter--Quadratic mean d.b.h. averaged 3.4 inches higher on site 70 than on site 50. The difference was smaller in the early years but consistently increased over time (fig. 9). Average diameter increased with each thinning because the thinning technique used in the study plantations was modified low thinning.

Basal Area--After each stand had been thinned at least once, basal area yield was about 4 ft^2/acre higher in the site 70 stand than in the site 50 stand just prior to the last two thinnings, and this relationship persisted until almost age 50 (fig. 10).

Total-Stem Volume Yield--Total cubic-foot (outside bark) standing volume was always greater in the site 70 plantation than in the site 50 plantation (fig. 11), even after all thinnings. Total volume at age 50 was 1,338 ft^3/acre greater for site 70 than for site 50, and total volume removed in thinnings was 2,812 ft^3/acre greater for site 70 than for site 50.

Unthinned- and Thinned-Stand Comparison

The unthinned site 70 plantation contained 290 trees per acre at age 40, whereas the corresponding thinned plantation contained 99 trees per acre at that age (before the final thinning). The unthinned plantation contained SO percent more total volume than the thinned plantation-- 7,371 ft^3/acre compared to 4,907 ft^3/acre. However, the average diameter of the trees in the thinned plantation (at age 40) was about 4 inches greater than those in the unthinned stand. For the thinned site 70 stand, the total of the volume harvested in the first three thinnings and the volume available for harvesting just before the age-40 thinning was 3,204 + 4,907 = 8,111 ft^3/acre. This was about 740 ft^3/acre more volume than what was available in the unthinned site 70 stand at age 40.

Computer Program

COMPUTE_P-SLASH, a program that performs calculations and generates yield tables for thinned and unthinned slash pine plantations on cutover sites in the west gulf region, will soon be available. It is written in FORTRAN 77 and will run on most computers. A user's guide, which will accompany the software and explain the program's features, is also being prepared.

LITERATURE CITED

Bailey, R.L.; Dell, T.R. 1973. Quantifying diameter distributions with the Weibull function. Forest Science. 19(2): 97-104.

Clutter, J.L. 1963. Compatible growth and yield models for loblolly pine. Forest Science. 9(3): 354-371.

Dell, T.R.; Feduccia, D.P.; Campbell, T.E. [and others]. 1979. Yields of unthinned slash pine plantations on cutover sites in the west gulf region. Res. Pap. SO-147. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 84 p.

Dell, T.R.; Matney, T.G.; Zarnoch, S.J. 1984. Mathematics and statistics. In: Wenger, K.F., ed. Forestry handbook. New York: John Wiley & Sons: 1253-1318.

Hyink, D.M.; Moser, J.W., Jr. 1983. A generalized framework for projecting forest yield and stand structure using diameter distributions. Forest Science. 29(1): 85-95.

Lohrey, R.E.; Dell, T.R. 1969. Computer programs using height accumulation for tree volumes and plot summaries. Journal of Forestry. 67(8): 554-555.

Matney, T.G.; Sullivan, A.D. 1982. Compatible stand and stock tables for thinned and unthinned loblolly pine stands. Forest Science. 28(1): 161-171.

Sullivan, A.D.; Clutter, J.L. 1972. A simultaneous growth and yield model for loblolly pine. Forest Science. 18(1): 76-86.

Zanakis, S.H. 1979. A simulation study of some simple estimators for the three-parameter Weibull distribution. Journal of Statistical Computation and Simulation. 9(2): 101-l 16.

Zarnoch, S.J.; Feduccia, D.P. 1984. Slash pine plantation site index curves for the west gulf. Southern Journal of Applied Forestry. 8(4): 223-225.

Zarnoch, S.J.; Dell, T.R. 1985. An evaluation of percentile and maximum likelihood estimators of Weibull parameters. Forest Science. 3 l(1): 260-268.

*U.S. GOVERNMENT PRINTING OFFICE: 1992-566-016/40021

Zarnoch, S.J.; Feduccia, D.P.; Baldwin, Jr., V.C.; Dell, T.R. 1991. Growth and yield model predictions for thinned and unthinned slash pine plantations on cutover sites in the west gulf region. Res. Pap. SO-264. New Orleans, LA; U.S. Department of Agriculture, Southern Forest Experiment Station. 32 p.

A growth and yield model has been developed for slash pine plantations on problem-free cutover sites in the west gulf region. The model was based on the moment-percentile method using the Weibull distribution for tree diameters. This technique was applied to untbinned and thinned stand projections and, subsequently, to the prediction of residual stands immediately after thinning.

Keywords: Parameter recovery, *Pinus elliottii* Engelm. *var. elliottii*, size distribution, survival prediction, volume prediction, Weibull distribution.