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Taper Functions for Predicting Product Volumes in Natural Shortleaf Pines

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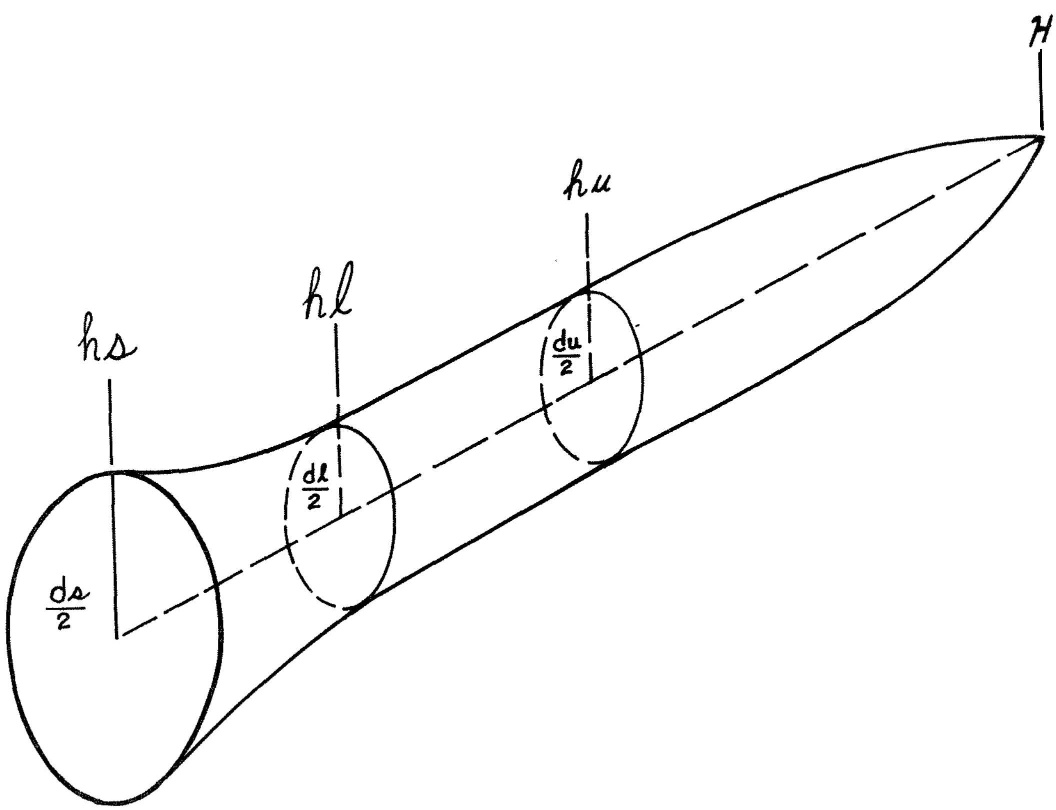

SUMMARY

Taper (stem-profile) functions are presented for natural shortleaf pine (*Pinus echinata* Mill.) trees growing in the West Gulf area, but they should also have applications elsewhere. These functions, when integrated, permit the prediction of volume between any two heights on a stem and, conversely by iteration, the volume between any two diameters on a stem. The integrated equations generally predict cubic-foot volumes that are within ± 1 cubic foot of observed volume and account for at least 97 percent of the variation in volume. Examples are given of use of the functions, and a computer program is available to evaluate the functions for cubic-foot volume and compare them with other volume equations.

ACKNOWLEDGMENTS

The authors are indebted to Mr. Daniel A. Yaussy, currently located at the Northeastern Forest Experiment Station, Columbus, OH, for his conscientious assistance in gathering the field data in southern Arkansas and northern Louisiana. We are further indebted to Dr. Edwin R. Lawson of the Southern Forest Experiment Station, Oxford, MS, for supplying the field data from western Arkansas.

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Table 4 in R. P. SO-234 contains 3 erroneous values. The location of these errors is denoted by the 3 underscored data values in the reproduction of Table 4 below. These errors have been corrected and the correct values appear in the reproduction of Table 4 below.

Table 4. Coefficients for lower and upper stem taper functions by o.b., i.b., and 3 CR classes, natural shortleaf pines, 342 trees.

Equation	n	g_1	g_2	b_1	b_2	b_3	b_4
CR1, o.b.	-.13253541			25.385423	2.279039	-.044477	-23.637118
CR2, o.b.	-.11988464			19.513315	1.772916	-.026344	-18.120387
CR3, o.b.	-.11799179			4.995668	2.091531	-.027642	-10.484750
CR1, i.b.	-.12195134	-.372776	.936758	19.473495	2.066904	-.[illegible]	-17.738120
CR2, i.b.	-.10905991	-.406013	.930204	13.933809	1.593209	-.020028	-12.575889
CR3, i.b.	-.11164159	-.534799	.935277	-1.878825	1.954822	-.023757	-3.780569

The correct coefficients were included in the BASIC program available to evaluate the taper functions. Therefore, Tables 5, 6, and 7 of R. P. SO-234 are correct.

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INTRODUCTION

How would you like to have a prediction system that estimates the tree volumes for almost any assortment of products imaginable? Foresters have long pursued the notion of devising a system of tree stem-volume predictions that are both accurate and universal regarding merchantability specifications and units of measure. Through the use of integrated stem taper functions (also called stem-profile functions), which assume that the tree stem is a solid of revolution, we are coming closer to this goal. Similar to D^2H (d.b.h.^{2} x total or merchantable height) volume functions, such taper functions could be slightly biased if they are developed from one population of trees and used to obtain estimates, perhaps, for a different population. However, any such resulting bias is usually minor and often offset by the utility of these functions in mensurational applications.

Taper functions can be regarded as infinitely variable D^2H volume functions. Essentially, the diameters at any two heights or the heights to any two diameters on the stem can be predicted, which in turn can be converted into various units of volume or surface area. If properly constructed, they resolve the intersection problems that can occur in the construction of a family of D^2H volume functions. However, taper functions are difficult to evaluate without considerable computing hardware, but their saving grace is the variety of output they afford.

This paper reports on a taper function model fitted with data for naturally regenerated shortleaf pine (*Pinus echinata* Mill.) trees in the West Gulf area. Since shortleaf pine has the widest range of the four major southern pines, this information should be of interest in the Central States and the Northeast and also in the East Gulf area and the Southeast. About 96 percent of the shortleaf cubic-foot volume probably occurs in the area served by the Southern and Southeastern Forest Experiment Stations (Sternitzke and Nelson 19701, but outside this area the species can be locally quite important. Kentucky and Missouri together probably contain over 75 percent of the northern shortleaf volume and West Virginia contains about 10 percent.

METHODS

A total of 342 felled shortleaf trees were measured in a rectangular distribution of d.b.h., total height, and crown-ratio classes in north Louisiana and south and west Arkansas. These trees varied in d.b.h. from 1 to 26 inches, in total height from 8 to 107 feet, and in crown ratio from 16 to 86 percent (tables l-3).

The selected trees were single-stemmed, reasonably free of crook or sweep, and visibly undamaged. Samples were drawn from two areas: (1) south Arkansas and north Louisiana (215 trees) and (2) west Arkansas (127 trees). In the first sample, stems were measured for d.o.b., bark thickness, and height at each l-inch taper step from the stump to the zero-d.o.b. tip. In the second sample, which had already been collected, stems were measured for d.o.b., bark thickness, and height to variable taper steps from the stump to the zero-d.o.b. top. Also, in both samples, d.o.b., bark thickness, and height were measured at the stump, breast height, and the live-crown base on each tree.

We used multiple regression (SAS 1979) to fit these d.o.b., d.i.b., and height data to the stem-profile model of Bennett and others (1978) to obtain predicted values of stem diameter given height of the diameter and the tree d.b.h., total height, and crown-ratio class.

The model is:

$$d = \begin{cases} D(h/4.5)^n & \text{if } h_s \le h \le 4.5, \quad \text{(la)} \\ \text{or} & \\ D(H-h)/(H-4.5) + b_1(H-h)(h-4.5)/H^2 + b_2D(H-h)(h-4.5)/H^2 + b_3D^2(H-h)(h-4.5)/H^2 + b_4(H-h)(h-4.5)(2H-h-4.5)/H^3 & \text{if } 4.5 \le h \le H, \quad \text{(lb)} \end{cases}$$

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Table 1.--Distribution of felled natural shortleaf sterns, CR < 36%

D.b.h. class (in)	Total height class (ft)											
	10	20	30	40	50	60	70	80	90	100	110	120
1	3	4										
2		4	2	1								
3		4	7	4	2							
4			2	2	2							
5				1	2							
6				2	2		1					
7						1		1				
8					2							
9						2	1	1				
10					1		1		1			
11						1		1	2			
12								2				
13						1		1	2			
14									1	2		
15									1	1		
16									1			
17										2		
18										1		
19									1			
20												
21										1		
22												
23												
24												
25												
26												
27												
28												
29												
30												
31												
32												
33												
34											Total	= 77

where n and b_i are parameters to be estimated and the other definitions pertaining to this model are:

- d = predicted stem diameter (in), either o.b. or i.b., at height (h),
- h = height above groundline (ft),
- D = diameter at breast height (in),
  - = $D_{o.b.}$ if d is diameter o.b., or
  - = $D_{i.b.}$ if d is diameter i.b., where

$$D_{i.b.} = g_1 + g_2 D_{o.b.}, \text{ and} \tag{2}$$

- H = total tree height, groundline to tip of bud (ft), and
- h_s = stump height above groundline (ft).

Separate equations for d are used for the base of the stem ($h \leq 4.5$ feet) and for the upper stem ($h \geq 4.5$ feet), but the overall model is constrained such that the two stem sections are joined at breast height. These expressions were fitted with both outside- and inside-bark data and for each of three crown-ratio (CR) classes, which were

- CR1: CR < 36 percent,
- CR2: CR = 36 to 50 percent, and
- CR3: CR > 50 percent.

Cubic-foot volumes from the integrated taper function (solid of revolution) are obtained as:

$$V = K_o \int_{h_l}^{h_u} d^2 dh, \tag{3}$$

where $K_o = \pi/576$ and

- d = predicted diameter at height h from any of the four taper sections implied by equations Pa and lb, for a selected CR class,
- h_u = an upper height, and
- h_l = a lower height.

The solutions of equation (3) are not presented herein because of their length and complexity but are available in general form in the appendix of the paper

Table %-Distribution of *felled natural shortleaf stems, CR = 36% to 50%*

D.b.h. class (in)	Total height class (ft)											
	10	20	30	40	50	60	70	80	90	100	110	120
1	5	1										
2	2	3	2									
3		3	2	3								
4		2	4	3	2							
5			1	1	1							
6				3	5	1						
7				3	6	1						
8				2	2	2	1					
9					5							
10					5	5	2					
11					1	5	2	2				
12					4	3	1	1	1			
13						1	1	3				
14						4	1		2			
15						3	2		11			
16						1	1	1	1			
17						1		1	2			
1.8						1		1	2	1		
19							1		1			
20									1			
21									1			
22									2	2		
23												
24									1		1	
25									1			
26												
27												
28												
29												
30												
31												
32												
33												
34											Total = 141	

by Bennett and others (1978). They are specifically available from the authors in a BASIC program to evaluate the integrated shortleaf pine taper functions for cubic-foot volumes (see Appendix).

We also tried the model of Matney and Sullivan (1980), but the model of Bennett and others (1978) performed as well and was fitted more efficiently.

To determine "goodness of fit ,"observed cubic-foot volume values were calculated per tree for the 342 trees by assuming conic sections between consecutive specified taper steps and a cone for the tip and then compared with corresponding predicted values. A program was written in BASIC for an IBM-PC XT microcomputer[1] to produce the predicted values (see Software Announcement box in the Appendix). This program also allows one to calculate "observed" volumes via other available cubic-foot equations for comparison purposes. The "observed" equations involve d.b.h. and total height as independent variables.

[1]Mention of commercial product names does not imply endorsement by the USDA Forest Service nor exclude *other similar products* that may serve as well.

RESULTS

Coefficients for the lower and the upper stem func-Lions by o.b., i.b., and three crown-ratio classes are given in table 4. Note the consistent trends in the coefficients with crown-ratio class. This implies that CR could be incorporated into the functions as a continuous variable. No conceptual problems prevent this (Farrar 1985), and Valenti and Cao (1986) present a technique for incorporating crown ratio into a taper function model. However, addition of crown ratio and possible interactions would greatly increase the complexity of our model and the difficulty in obtaining logical fits. For this reason and to maintain comparability with other species' functions employing this model, crown ratio was left as a discrete variable.

Plottings of predicted stem profiles (d.o.b. by height) for three example trees are given in figure 1. A profile for each of the three CR classes is shown for each of the three example trees starting at a 0.5-foot stump. The characteristics of the example trees are:

(1) D.b.h. = 12 inches, total height = 70 feet, CR = <36 percent, 36 to 50 percent, >50 percent,

Table 3.—*Distribution of felled natural shortleaf stems, CR > 50%*

D.b.h. class (in)	Total height class (ft)											
	10	20	30	40	50	60	70	80	90	100	110	120
1	7	2										
2	3	4										
3		3	3	2								
4		2	3	1	1							
5				1								
6		2	1	1								
7			2		3							
8			2	2	4							
9			1		3	1						
10			1	1	2	1						
11				1	2	2						
12			1	1	1	2	1					
13					1	2	2	1				
14					1	4	1	1				
15						3	3					
16				1	2	1	1	2	1			
17					1	3	3		1			
18								5				
19						1	2					
20							1		1			
21						1			2			
22												
23							1		1			
24												
25								1		2		
26									1			
27												
28												
29												
30												
31												
32												
33												
34											Total = 124	

Table 4.—*Coefficients for lower and upper stem taper functions by o.b., i.b., and 3 CR classes, natural shortleaf pines, 342 trees*

Equation	n	g_1	g_2	b_1	b_2	b_3	b_4
CR1, o.b.	−.13253541			25.385423	2.279039	−.044477	-23.637118
CR2, o.b.	−.11988464			19.513315	1.772916	−.026344	-18.120387
CR3, o.b.	−.11799179			4.995668	2.0915314	−.027642	-10.484750
CR1, i.b.	−.12195134	−.372776	.936758	19.473495	2.066904	−.038933	-17.738120
CR2, i.b.	−.10905991	−.406013	.930204	13.933809	1.593209	−.020028	-11.923614
CR3, i.b.	−.11164159	-534799	.935277	-1.878825	1.9548223	−.023757	-3.780569

(2) D.b.h. = 8 inches, total height = 50 feet, CR = same classes as in (1), and

(3) D.b.h. = 4 inches, total height = 30 feet, CR = same classes as in (1).

Note in figure 1 that a higher CR class (percent) generally suggests lower stem volume due to greater taper in the long crown section, and this is generally borne out in evaluations of the integrated function for volumes (see APPLICATIONS section). However, figure 1 also shows that even though a high CR implies low volume, in certain lower stem sections of a high CR tree the predicted taper is less than that of a similar low CR tree and, hence, a greater volume is suggested for these sections. These relationships were also noted by Dell (1979), Farrar (1987), and Farrar and Murphy (1987).

The "goodness of fit" statistics in tables 5 and 6 show how well the taper functions reproduced the observed o.b. and i.b. cubic-foot stem volumes. Definitions pertaining to these tables are:

n = number of observations,

$\bar{o}$ = mean observed (o) value,

$\bar{d}$ = mean difference between predicted (p) and observed
  = $\Sigma(p - o)/n$,

$\overline{\%d}$ = percent mean difference
  = $[\Sigma((p - o)/o)/n](100)$,

RMSd = root mean squared difference
  = $\sqrt{\Sigma(p - o)^2/n}$,

RMS%d = root mean squared percent difference
  = $\sqrt{\Sigma((p - o)/o)^2/n}\,(100)$,

FI = fit index
  = $1 - [\Sigma(p - o)^2/\Sigma(o \quad \bar{o})^2]$,

Total stem cubic feet = volume of a stem from a 0.1-foot stump to the bud tip for trees with d.b.h. >0 inch,

Merchantable stem cubic feet = volume of a stem above a 0.5-foot stump to a 3-inch d.o.b. top for trees with d.b.h. >3.5 inches, and

Sawtimber stem cubic feet = volume of a stem above a 0.5foot stump to an 8-inch top d.o.b. for trees with d.b.h. >9.5 inches.

DISCUSSION

The fit statistics indicate that the taper functions adequately represent the stem volumes for shortleaf pine. Mean differences ($\bar{d}$) are generally less than 1 cubic foot, indicating a relatively low positive bias. Root mean squared differences (RMSd) are generally less than ±6 cubic feet, indicating good precision. The fit indices (FI), which represent the amount of volume variation accounted for by the taper functions, are all 97 percent or better, also indicating a good fit.

In general, the shortleaf pine taper functions remap the observed volumes slightly better than recently developed functions for natural loblolly pines (Farrar and Murphy 1937) by all criteria except fit index (FI), which is consistently just slightly better for loblolly. The stem taper in shortleaf, particularly in the crown, appears more regular and smooth than that of loblolly and may account for the shortleaf advantage. The better fit indices for loblolly are probably due to the fact that the loblolly trees were larger, and similar deviations produced a better fit index for larger volumes because the volume variances were also larger.

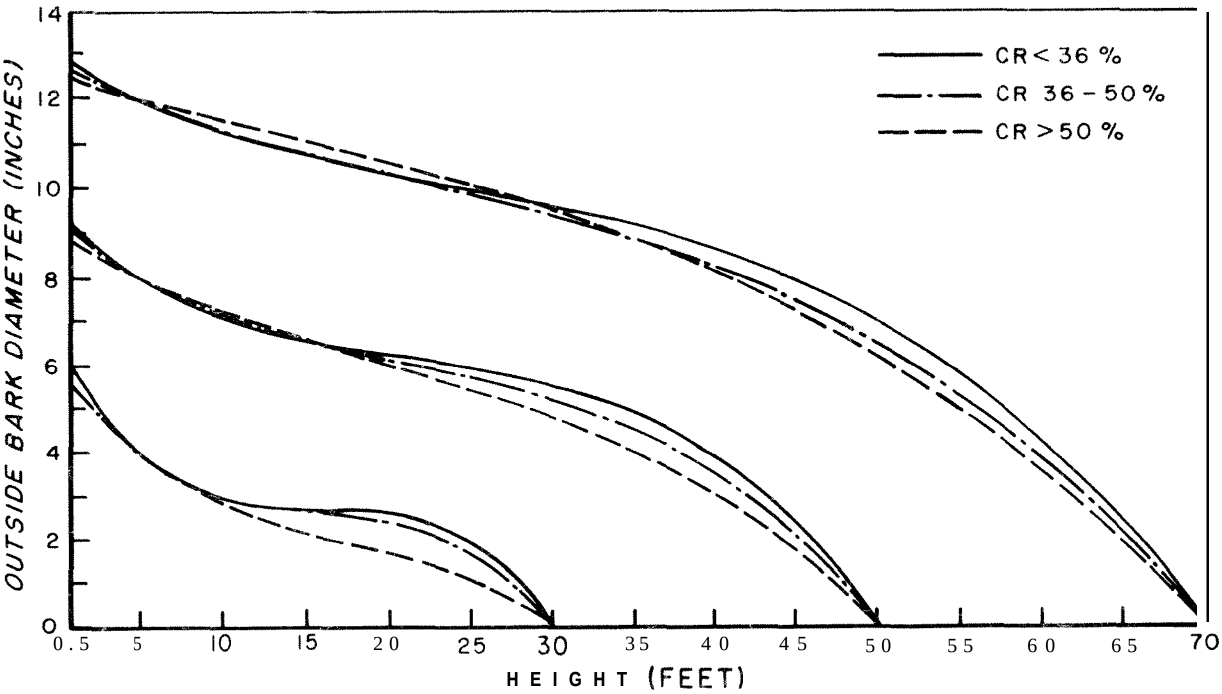

Figure 1.—*Predicted d.o.b. by height and CR for three example shortleaf pine stems.*

Table 5.—*Fit statistics (outside bark) for natural shortleaf pine taper equations, 342 trees measured at 1-inch and varying taper steps*

Equation	n	$\bar{o}$	$\bar{d}$	$\overline{\%d}$	RMSd	RMS%d	FI
		ft^3	ft^3	%	ft^3	%	
Total stem cubic feet, o.b.							
CR1, o.b.	77	18.366	.311	-3.550	2.565	14.685	.9912
CR2, o.b.	141	28.413	.698	-.153	3.978	10.947	.9870
CR3, o.b.	124	33.270	.591	-6.104	4.089	17.973	.9892
Overall	342	27.912	.572	-3.075	3.751	14.638	.9888
Merchantable stem cubic feet, o.b.							
CR1, o.b.	46	29.409	.390	-5.886	3.272	15.090	.9876
CR2, o.b.	120	32.257	.701	-1.930	4.287	12.399	.9849
CR3, o.b.	100	39.979	.553	-1.127	4.521	11.911	.9867
Overall	266	34.668	.591	-2.312	4.222	12.731	.9862
Sawtimber stem cubic feet, o.b.							
CR1, o.b.	24	45.865	1.019	1.523	4.673	11.716	.9731
CR2, o.b.	75	45.174	.710	-.634	5.572	12.667	.9763
CR3, o.b.	70	52.485	.311	.658	5.591	11.167	.9790
Overall	169	48.300	.589	.207	5.462	11.931	.9775

Table 6.-*Fit statistics (inside bark) for natural shortleaf pine taper equations, 342 trees measured at 1-inch and varying taper steps*

Equation	n	$\bar{o}$	$\bar{d}$	$\overline{\%d}$	RMSd	RMS%d	FI
		ft^3	ft^3	%	ft^3	%	
Total stem cubic feet, i.b.							
CR1, i.b.	77	15.510	.231	-4.590	2.470	17.574	.9892
CR2, i.b.	141	23.890	.523	-.994	3.659	17.312	.9854
CR3, i.b.	124	28.031	.437	-7.523	4.102	29.490	.9857
Overall	342	23.504	.426	-4.171	3.605	22.548	.9864
Merchantable stem cubic feet, i.b.							
CR1, i.b.	46	24.988	.280	-4.215	3.175	13.932	.9848
CR2, i.b.	120	27.216	.539	.145	3.949	14.608	.9832
CR3, i.b.	100	33.777	.423	2.543	4.545	16.001	.9826
Overall	266	29.297	.450	.293	4.067	15.036	.9833
Sawtimber stem cubic feet, i.b.							
CR1, i.b.	24	39.501	.585	1.539	4.517	12.653	.9679
CR2, i.b.	75	38.433	.456	.177	5.095	15.033	.9745
CR3, i.b.	70	44.710	.121	2.317	5.573	15.401	.9734
Overall	169	41.185	.335	1.257	5.223	14.875	.9736

Both the loblolly and shortleaf fit statistics are slightly poorer than those reported by Farrar (1987) for natural longleaf stems, where the mean difference was generally less than ±0.1 cubic foot, the RMSd less than ±3 cubic feet, and the fit index greater than 98 percent. This may be due to longleaf stems being more regularly formed with smoother taper and easily defined live crown bases. The stem taper in loblolly crown sections is often quite irregular and approaches a step-function in shape. Shortleaf stem taper appears reasonably regular and smooth, but the base of the live crown is often difficult to locate precisely because live limb size and vigor often decrease gradually down the stem, with no sharp demarcation for a crown base.

APPLICATIONS

Predicted cubic-foot volumes for the example trees in figure 1 are given in table 7. The volume definitions are the same as those in tables 5 and 6 except that, for convenience, a 0.5-foot stump is assumed throughout, as in figure 1. Note that, as suggested, a high CR generally implies a relatively low stem volume. These predicted volumes are output from evaluation programs written in BASIC for an IBM-PC XT microcomputer. See the associated Software Announcement box for reference and source of this program.

One thing is noteworthy in table 7. For the 12-inch 70-foot tree, shortleaf volumes for CR3 (high crown ratio) are slightly greater than those for CR2 (medium crown ratio). This bears out the earlier statement that a larger predicted volume is possible in the lower stems of high crown-ratio trees that have generally poorer upper stem form.

Comparisons of the predicted values in table 7 with similar predictions for loblolly trees (Farrar and Murphy 1987) and longleaf trees (Farrar 1987) indicate that for the same d.b.h., total height, and CR class the shortleaf values are generally somewhat larger than those for loblolly and often somewhat larger than those for longleaf. For this study at least, this would imply that shortleaf generally has better form than loblolly and sometimes has better implied form per CR class than longleaf.

Board-foot volume predictions are also possible by writing a program to utilize equations (1a) and (1b) and a formula log rule such as Doyle or International 1/4-inch. A diagram log rule can also be used by storing its array of values by scaling diameter and log lengths or, better, by using a functional fit of the diagram rule.

To obtain predicted board-foot volumes, equations (1a) and (1b) would first be evaluated to obtain scaling diameters at the top of each simulated log (or fractional log). Then the scaling diameter and log lengths could be passed to the log rule function to obtain predicted board feet per log. Finally, the board-foot volumes per log could be summed to obtain predicted stem volume. Note that equations (1a) and (1b) are evaluated using the cumulative height to the top of each log (or fraction), including stump height and trim allowances. For example, the height of the first log might be 17.3 feet, which would include a 1-foot stump, a 16-foot log, and a 0.3-foot trim allowance. The height to the second log might be 33.6 feet (17.3 + 16.3). An actual evaluation program such as this is beyond the scope of this paper.

If cubic-foot volume equations in terms of d.b.h. and total height are available for a locale, the adequacy of the taper functions for that locale can be easily checked via the available BASIC evaluation program. Either inside- or outside-bark equation values for total volume or for volume to specified top diameters can be compared with those predicted by the taper functions for the d.b.h.-height combinations involved in the volume equations. The program could also be modified to predict volumes for merchantable heights

Table I.—*Predicted cubic-foot volumes for the three example shortleaf stems in figure 1*

D.b.h.	Total height	CR class	Total volume o.b.	Total volume i.b.	Merchantable volume o.b.	Merchantable volume i.b.	Sawtimber volume o.b.	Sawtimber volume i.b.
in	*ft*		ft^3					
12	70	1	29.584	24.784	29.469	24.696	25.585	21.546
		2	28.408	23.495	28.210	23.389	24.127	20.036
		3	28.644	23.596	28.498	23.489	24.378	20.231
8	50	1	9.252	7.504	9.122	7.407	n.a.	n.a.
		2	8.869	7.081	8.721	6.972	n.a.	n.a.
		3	8.452	6.610	8.261	6.484	n.a.	n.a.
4	30	1	1.347	.981	.730	.523	n.a.	n.a.
		2	1.308	.929	.774	.539	n.a.	n.a.
		3	1.077	.691	.684	.444	n.a.	n.a.

n.a. = not applicable.

and thereby aceomodate comparisons with user-supplied equations that are in terms of d.b.h. and merchantable height.

CONCLUSION

In the Midsouth, we now have six integrated taper (stem profile) functions for southern pines that employ the same basic model (Bennett and others 1978) and use the same crown ratio classes as a surrogate for stem form. These functions are:

Planted slash pine-Dell and others (19'79)
Planted loblolly pine—Feduccia and others (1979)
Planted longleaf pine-Baldwin and Polmer (1981)
Natural longleaf pine-Farrar (1987)
Natural shortleaf pine-reported herein
Natural loblolly pine-Farrar and Murphy (1987)

This is not the only available model and others may serve as well, but this model is reasonable in form, easy to fit, and highly serviceable, as affirmed by the above use. The commonality of functional form allows, within reason, direct comparisons between species' profile functions. The most obviously needed improvement would include crown ratio as a continuous, rather than a discrete, variable and thereby improve the utility. However, the present three CR classes seem to be adequate as discriminatory devices for many purposes. They are not unique but were simply the most logical groupings found in the planted slash pine data base (Dell and others 1979) and have been used since in the other species' functions to maintain consistency.

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Appendix

SOFTWARE ANNOUNCEMENT

Upon request, a printout of a BASIC program to evaluate the natural shortleaf pine stem-profile function discussed in the accompanying article is available from the authors. Inputs are tree d.b.h., total height, crown-ratio class, stump height, and top diameter. Outputs are predicted cubic-foot volumes, i.b. and o.b., from the profile function; "observed" volumes from user-supplied volume functions; and basic "goodness of fit" values generated from a comparison of predicted and "observed" values. The example trees from the article are evaluated as examples and test problems. The program was written for the IBM-PC XT microcomputer but should be easily adapted to other BASIC dialects for use with other machines.

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Farrar, Robert M., Jr.; Murphy, Paul A. Taper functions for predicting product volumes in natural shortleaf pines. Res. Pap. SO-234. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station; 1987. 9 p.

Taper (stem-profile) functions are presented for natural shortleaf pine (*Pinus echinata* Mill.) trees growing in the West Gulf area. These functions, when integrated, permit the prediction of volume between any two heights on a stem and, conversely by iteration, the volume between any two diameters on a stem. Examples are given of use of the functions, and a computer program is available to evaluate the functions for cubic-foot volume equations.

Additional keywords: Naturally regenerated shortleaf pine, predicting product volumes, taper function model.