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SUMMARY

Equations are given to estimate current and projected volumes and projected basal areas of uneven-aged shortleaf pine (*Pinus echinata* Mill.) stands managed under the selection system. The independent variables are initial merchantable basal area, initial sawtimber basal area, site index for shortleaf pine (base age 50), and elasped time. The results provide guidelines for basal area, cubic-foot, and board-foot volume production of uneven-aged shortleaf pine stands on different sites in the Interior Highlands of the West Gulf.

INTRODUCTION

Shortleaf pine (*Pinus echinata* Mill.) has the widest range of the southern pines. It is found in 22 states in an area greater than 440,000 square miles (Lawson and Kitchens 1983). It amounts to one-quarter of the southern pine volume, is exceeded only by loblolly pine (*P. taeda* L.), and outranks the combined volumes of slash (*P. elliottii* Engelm.) and longleaf (*P. palustris* Mill.) pines. The greatest concentration of shortleaf pine is found in the Interior Highlands of Arkansas and east Oklahoma (Sternitzke and Nelson 1970). It is the only naturally occurring pine in the Interior Highlands, and it is there that shortleaf management is concentrated.

Despite its importance as a resource, shortleaf has been the most neglected of the major southern pines in terms of research information. This is particularly true for growth and yield information. There is some growth and yield data available for natural even-aged stands (U.S. Department of Agriculture, Forest Service 1929, Brinkman et al. 1965; Schumacher and Coile 1960; Sander and Rogers 1979; Murphy and Beltz 1981; Murphy 1982) and plantations (Smalley and Bailey 1974). Though Murphy and Farrar (1982, 1983) and Farrar et al. (1984) developed information for uneven-aged loblolly-shortleaf pine, their data carne from the West Gulf Coastal Plain where loblolly is predominant and usually preferred over shortleaf. Hence, shortleaf is usually a minor component in these stands. No models exist for uneven-aged stands where shortleaf is the predominant species, as in the Interior Highlands.

This paper presents a system of equations for predicting projected basal areas and current and projected volumes for selection-managed stands of shortleaf pine.

PROCEDURE

Data

The data come from permanent inventor-y plots located in Conway, Garland, Perry, Pulaski, Saline, and Ye11 counties in Arkansas.[1] Over 400 0.2-acre plots have been measured four times, in 1966, 1972, 1978, and 1982. Individual tree records were maintained for al1 trees 5.6 inches d.b.h. and larger. The pertinent information collected on shortleaf pine was the following: (1) d.b.h. to the nearest 0.1 inch; (2) merchantable length on pulpwood trees (5.6 to 8.5 inches d.b.h.), the length from a 0.5-foot stump to a 4-inch top, outside bark, to the nearest 5 feet with a five foot minimum; (3) sawlog length for sawtimber trees (8.6 inches d.b.h. and larger), the length from a 1-foot stump to a 6-inch top, outside bark, to the nearest even foot with a minimum length of 10 feet; (4) and merchantable length for sawtimber trees, the length from a 1-foot stump to a 4-inch top, outside bark, to the nearest 5 feet; (5) tree product class; and (6) tree history. Site index (base age 50) for shortleaf pine was also determined for each plot using Misc. Publ. 50 (U.S.D.A., Forest Service 1929).

[1]We thank Deltic Farm and Timber Co., Inc., of El Dorado, Arkansas, for kindly making these data available.

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Individual tree volumes were calculated in the following manner: Merchantable cubic-foot volumes, inside bark, for shortleaf pines 5.6 inches d.b.h. and larger from a 0.5foot stump to a 4-inch-top, outside bark, were calculated using the equation

$$C_m = 0.00274D^2H_m + 0.2997$$

where

C_m = merchantable cubic-foot volume, i.b.,
D = diameter breast height, and
H_m = merchantable length to the nearest 5 feet from a 0.5-foot stump to a 4-inch top, outside bark.

Sawlog cubic-foot volumes, inside bark, for shortleaf pine sawtimber (8.6 inches d.b.h. and larger) from a 1-foot stump to a 6-inch top, outside bark, were determined by equation

$$C_s = 0.002774D^2H_s + 1.415 ,$$

where

C_s = sawtimber cubic-foot volume,
D = diameter breast height, and
H_s = sawlog length to the nearest even foot from a 1-foot stump to a 6-inch top, outside bark.

Board-foot volumes for the Doyle, Scribner, and International 1/4-inch rules were calculated using the following taper assumptions. For the first 16-foot log, a Girard form class of 82 was used; for the second log, a taper of 2.1 inches was used; and for the third and subsequent logs, a taper of 3.2 inches was used. A 0.3-foot trim allowance was used for each 16-foot log and the top fractional log.

The following plot variables were calculated on a per-acre basis for shortleaf pine:

B_{mi} = the merchantable basal area in trees, 5.6 inches d.b.h. and larger, present at time i,

B_{si} = the sawtimber basal area in trees, 8.6 inches d.b.h. and larger, present at time i,

V_{mi} = merchantable cubic-foot volume, inside bark, in trees, 5.6 inches d.b.h. and larger, from a 0.5-foot stump to a 4-inch top, outside bark, to the nearest 5 feet at time i,

V_{si} = sawtimber cubic-foot volume, inside bark, in trees, 8.6 inches d.b.h. and larger, from a 1-foot stump to a 6-inch top, outside bark, to the nearest even foot at time i,

D_i = board-foot volume, Doyle rule, in trees, 8.6 inches d.b.h. and larger, from a 1-foot stump to a 6-inch top, outside bark, to the nearest even foot at time i,

S_i = board-foot volume, Scribner rule, in trees, 8.6 inches d.b.h. and larger, from a 1-foot stump to a 6-inch top, outside bark, to the nearest even foot at time i,

I_i = board-foot volume, International 1/4-inch rule, in trees, 8.6 inches d.b.h. and larger, from 1-foot stump to a 6-inch top, outside bark, to the nearest even foot at time i.

Also,

Q = site index (shortleaf pine, base age 50) of plot in feet.

Summarizing the data resulted in 1,338 growth observations. To assure that the plots were predominantly shortleaf and uneven-aged in character, only those that were classified as being a pine type and having a pine understory, a pine overstory, and pine reproduction were used. Even with these restrictions, most plots had a minor hardwood component. Observations of plots that were cut during a growth period or had an excess of 10 percent mortality in merchantable basal area were deleted. Because the data were concentrated in a few site index and basal area classes, a restricted set of observations were randomly chosen to obtain a more uniform sample across the range of the data. This selection reduced the total observations used to 149. Table 1 shows the distribution of the plots.

Analysis

A stand leve1 growth and yield prediction system consists of stand volume equations plus one or more basal area projection equations. Directly determined variables, such as present basal area, are used to estimate volumes of current stands. For future volumes, however, some variables—such as future basal area—must themselves be predicted before future volumes can be estimated.

The equation for projected merchantable basal area has been previously used by Moser and Hall (1969) and Murphy and Farrar (1982) for other uneven-aged conditions. It is

$$B_{m2} = [k_1 - \{k_1 - B_{m1}^{k_3}\}e^{k_2k_3t}]^{1/k_3} ,$$

where

B_{m1} = merchantable basal area per acre at time 1,
B_{m2} = merchantable basal area per acre at time 2,
t = elapsed time in years between times one and two, and
k_i = coefficients to be estimated.

Since projected sawtimber volumes are of as much interest as merchantable volume, a sawtimber basal area projection equation was also incorporated into the equation system. It is an uneven-aged adaptation of an equation originally formulated for even-aged conditions (Murphy 1983) and is:

$$B_{s2} = B_{m2}[1 - \{1 - (B_{s1}/B_{m1})^{n_2}\}e^{n_1 n_2 t}]^{1/n_2} ,$$

where

B_{s1} = sawtimber basal area per acre at time 1,
B_{s2} = sawtimber basal area per acre at time 2,
n_i = coeffkients to be estimated,

and the other variables are as previously defined. The equation for merchantable volume is

$$V_{mi} = b_0 B_{mi}^{b_1} e^{b_2 Q} ,$$

where

V_{mi} = merchantable cubic volume per acre at time 1,
B_{mi} = merchantable basal area per acre at time i,
Q = site index (shortleaf pine, base age 50), and
b_i = coefficients to be estimated.

The generalized equation for sawtimber volumes is

$$Z_{si} = c_0 B_{si}^{c_1} e^{c_2 Q} ,$$

Table 1.—*Plot distribution by shortleaf merchantable basal area, sawtimber basal area, and site index (base age 50) set at time one*

Site index	Merchantable basal area	Sawtimber basal area (ft²)					
		<11	11-29	3049	50-69	70-89	Total
ft	*ft²*	*number of plots*					
<45	<11	3					3
	11-29	3	4				7
	3049	...	4	2			6
	50-69	...	.....	2	.....		2
	70-89	1	.....	.....	.....	.....	1
	>89	...	.....	.....	.....	.....	0
	Total	7	8	4	0	0	19
46-55	<11	7					7
	11-29	7	7				14
	3049	2	7	6			15
	50-69	...	1	7	2		10
	70-89	...	.....	1	3	1	5
	> 89	...	.....	.....	.....	.....	0
	Total	16	15	14	5	1	51
56-65	<11	7					7
	11-29	7	7				14
	3049	2	7	7			16
	50-69	...	3	7	2		12
	70-89	...	.....	1	3	.....	4
	>89	...	.....	.....	1	.....	1
	Total	16	17	15	6	0	54
>65	<11	3					3
	11-29	2	5				7
	30-49	1	6	4			11
	50-69	...	1	2	1		4
	70-89	...	.....	.....	.....	.....	0
	>89	...	.....	.....	.....	.....	0
	Total	6	12	6	1	0	25
All sites	<11	20					20
	11-29	19	23				42
	3049	5	24	19			48
	50-69	...	5	18	5		28
	70-89	1	.....	2	6	1	10
	>89	...	.....	.....	1	.....	1
	Total	45	52	39	12	1	149

where

Z_{si} = sawtimber volume per acre of interest at time i

c_i = coefficients to be estimated,

B_{si} = sawtimber basal area at time i,

and the other variables are as previously defined. The volumes of interest are sawtimber cubic-foot volumes per acre and board-foot volumes per acre for the Doyle, Scribner, and International 1/4-inch log rules.

The coefficients were estimated by nonlinear least squares (SAS Institute 1982). The resulting equations are:

$$(1)\ B_{m2} = [1.7819 - (1.7819 - B_{m1}^{0.11699})e^{-0.36313(0.11699)t}]^{(1/0.11699)}$$

$$(2)\ B_{s2} = B_{m2}[1 - \{1 - (B_{s1}/B_{m1})^{1.5036}\}e^{-0.017286(1.5036)t}]^{(1/1.5036)}$$

$$(3)\ V_{mi} = 9.6209B_{mi}^{1.1200}e^{0.0031623Q},$$

$$(4)\ V_{si} = 10.030B_{si}^{1.1554}e^{0.0017908Q},$$

$$(5)\ D_i = 16.327B_{si}^{1.2944}e^{0.0070266Q},$$

$$(6)\ S_i = 37.227B_{si}^{1.2333}e^{0.0046645Q},$$

$$(7)\ I_i = 43.258B_{si}^{1.2363}e^{0.0041047Q},$$

To evaluate the model, the predicted values were compared with the corresponding observed values. The results are in table 2. The basal area projection equations have little bias. Estimates of current merchantable cubic-foot volumes overpredict somewhat, and projected merchantable volumes are negatively biased. The equations for all other volume estimates overpredict slightly for current volumes, and the overpredictions are less pronounced for projected volumes. All the equations were deemed to predict with adeguate precision.

APPLICATION

Use of Tables

To illustrate the use of the accompanying tables, suppose it is decided to manage an uneven-aged shortleaf pine stand on land with a site index of 60 feet on a 7-year cutting cycle. The densities at the start of the cutting cycle are 60 and 45 square feet for merchantable and sawtimber basal areas, respectively. What will be the cyclic harvest under this management regime?

The first step is to obtain estimates of current volume. Looking at table 3, we find that the mer-

Table 2.-Evaluation of prediction equations

Variable	Fit index[1]	Observed mean value	Bias[2]	Absolute deviations: Mean	Absolute deviations: Standard deviation
		---- ft^2 ----			
B_{m2}	0.92	47.2	0.2	5.2	4.0
B_{s2}	0.92	31.2	-1.7	4.4	2.8
		---- ft^3, i.b. ----			
V_{m1}	0.90	--------	-20	97	89
V_{m2}	0.93	[illegible]	22	97	97
V_{s1}	0.93	422	-16	58	62
V_{s2}	0.91	629	-14	93	78
		---- fbm ----			
D_1	0.84	1,482	-58	337	351
D_2	0.85	2,325	-8	431	554
S_1	0.89	2,403	-89	417	453
S_2	0.88	3,676	-49	605	692
I_1	0.89	2,728	-110	487	519
I_2	0.88	4,191	-53	689	776

[1] $I^2 = 1 - \Sigma(y_i - \hat{y}_i)^2/\Sigma(y_i - \bar{y})^2$

[2] $\text{Bias} = \Sigma(y_i - \hat{y}_i)/n$

[3] Absolute deviation $= |y_i - \hat{y}_i| = d_i$

Mean $= \Sigma d_i/n$

Standard deviation $= \sqrt{[\Sigma d_i^2 - (\Sigma d_i)^2/n]/(n - 1)}$

where

y_i = observed value,

$\hat{y}_i$ = predicted value, and

n = 149, the number of observations.

chantable volume per acre is 1,141 cubic feet. Table 4 contains sawtimber volumes, and the estimates are found by interpolating between 40 and 60 square feet of sawtimber basal area. Current volumes are 909 cubic feet; 3,443 board feet (Doyle rule); 5,396 board feet (Scribner rule); and 6,133 board feet (International 1/4-inch rule).

Projected volumes and basal areas are to be determined next, enabling us to then determine projected volumes. Projected basal areas are determined first. An initial merchantable basal area of 60 square feet and a projection period of 7 years yields a projected basal area of 75 square feet (table 5). These initial merchantable and sawtimber basal areas are needed to derive projected sawtimber basal area in table 6.

Table 3.—*Merchantable cubic-foot volume per acre for uneven-aged stands of shortleafpine for different merchantable basal areas and site indexes*

Merchantable basal area	Site index (ft)			
	50	60	70	80
ft^2	ft^3, i.b.			
30	508	525	542	559
40	702	724	748	772
50	901	930	960	991
60	1,105	1,141	1,177	1,215
70	1,313	1,356	1,399	1,444
80	1,525	1,574	1,625	1,677
90	1,740	1,796	1,854	1,914

Table 4.-*Sawtimber volumes per .cre for uneven-aged stands of shortleafpine for different sawtimber basal areas and site indexes*

Sawtimber basal area	Sawlog volume	Board-foot volume		
		Doyle	Scribner	International 1/4-inch
ft^2	ft^3, i.b.	fbm		
Site index 50				
20	349	1,121	1,891	2,156
30	558	1,894	3,118	3,559
40	778	2,749	4,446	5,080
50	1,007	3,670	5,854	6,693
60	1,244	4,647	7,331	8,385
70	1,486	5,673	8,866	10,146
80	1,734	6,743	10,453	11,967
Site index 60				
20	356	1,202	1,981	2,246
30	568	2,032	3,267	3,708
40	792	2,949	4,658	5,292
50	1,026	3,937	6,134	6,974
60	1,266	4,985	7,681	8,737
70	1,513	6,086	9,289	10,571
80	1,765	7,234	10,952	12,469
Site index 70				
20	362	1,290	2,076	2,341
30	579	2,180	3,423	3,864
40	807	3,164	4,881	5,514
50	1,044	4,223	6,427	7,266
60	1,289	5,348	8,047	9,103
70	1,540	6,529	9,732	11,014
80	1,797	7,760	11,475	12,991
Site index 80				
20	369	1,384	2,175	2,439
30	589	2,339	3,586	4,026
40	821	3,394	5,114	5,745
50	1,063	4,531	6,734	7,570
60	1,312	5,737	8,432	9,484
70	1,568	7,004	10,197	11,476
80	1,830	8,325	12,023	13,535

Projected sawtimber basal area is found by interpolating between the projected values for initial sawtimber basal areas of 40 and 50 feet-which are 55 and 65 square feet, respectively. Projected sawtimber basal area is thus 60 square feet. Merchantable cubic-foot volume in 7 years is found by using the projected merchantable basal area (from table 5) and linear interpolation in table 3; it is 1,465 cubic feet. Sawtimber volumes are similarly found by consulting table 4 where sawtimber basal area is 60 square feet. They are 1,266 cubic feet; 4,985 board feet (Doyle rule); 7,681 board feet (Scribner rule); and 8,737 board feet (International 1/4-inch rule).

Basal area growth for the period is 2 square feet per acre per year for both sawtimber and merchantable basal areas. Annual per-acre volume growth is 46 cubic feet for merchantable volume; 51 cubic feet for sawtimber; and 220, 326, and 372 board feet for the Doyle, Scribner, and International 1/4-inch rules, respectively.

Table 5.—*Projected merchantable basal area per acre for uneven-aged shortleaf pine stands for different elapsed times and initial merchantable basal areas*

Elapsed time	Initial merchantable basal area (ft^2)						
	30	40	50	60	70	80	90
years				*ft²*			
1	32	42	52	62	72	82	92
2	34	45	55	64	74	84	93
3	37	47	57	67	76	86	95
4	39	49	59	69	78	87	97
5	41	52	61	71	80	89	98
6	43	54	64	73	82	91	100
7	46	56	66	75	84	93	101
8	48	58	68	77	86	94	102
9	50	61	70	79	88	96	104
10	53	63	72	81	89	97	105

Use of Equations

These growth and yield models can also be used for other purposes. Many private nonindustrial timberlands brought under management are understocked. The problem is to increase stocking while simultaneously providing the landowner a periodic income under a variety of constraints. One common harvesting constraint is that there must be an operable cut of, say, at least a thousand board feet per acre (Doyle rule). The models can be used to derive a management strategy with these objectives and constraints.

For example, suppose a tract of uneven-aged shortleaf pine is to be brought under management. The site index for shortleaf is 70 feet (base age 50) on the property, and the current stand has 45 square feet in merchantable basal area and 25 square feet in sawtimber basal area. The desired management regime is a 7-year cutting cycle and residual densities of 60 and 45 square feet for merchantable and sawtimber basal areas, respectively. How might this property be managed to bring the stand up to these stocking goals while providing a periodic cut that is at least 1,000 board feet (Doyle rule)?

A proposed strategy is to maintain a 7-year cutting cycle and to cut 75 percent of growth and see if the harvesting constraint is followed. In this illustration, equations will be used instead of tables.

The present stand will be allowed to grow 7 years and then be harvested. To determine merchantable basal area in 7 years, equation (1) is used,

$$B_{m2} = [1.7819 - (1.7819 - 45^{0.11699})e^{-0.36313(0.11699)7}]^{(1/0.11699)} ,$$
$$= 61.1 \text{ square feet} .$$

The periodic growth is 61.1 − 45.0 or 16.1 square feet. If 75 percent of periodic growth is to be harvested, then 12.1 square feet of merchantable basal area will be cut, and the residual density is 49.0 square feet. Equation (1) is used again to project basal area for 7 years, and the whole process is repeated until the stocking goal is reached. The following table summarizes the cyclic harvests and residual densities for merchantable basal area:

Merchantable Basal Area

Time	Before cut	Cut	After cut
years		ft²	
0	45.0	...	4s.u
7	61.1	12.1	49.0
14	64.9	11.9	53.0
21	68.7	11.8	56.9
28	72.3	12.3	60.0
35	75.1	15.1	60.0

The residual stocking goal for merchantable basal area is reached in 28 years, and regular cyclic cuts for merchantable basal area and volume occur after that.

The cutting schedule for sawtimber basal area is computed next. The projected sawtimber basal area in 7 years, given initial basal areas of 45 square feet for mechantable trees and 25 square feet for sawtimber trees, is determined by equation (2),

$$B_{s2} = 61.1[1 - \{1 - (25/45)^{1.5036}\}e^{-0.017286(1.5036)7}]^{(1/1.5036)} ,$$
$$= 39.1 \text{ square feet} .$$

The periodic.growth is 39.1 − 25.0 or 14.1 square feet per acre, and the first cycle cut for sawtimber basal area is 75 percent of periodic growth: 10.6 square feet. Values for subsequent cutting cycles are determined in the same manner, and the following table may be constructed:

Table 6.-*Projected sawtimber basal area per acre for uneven-aged stands of shortleaf pine for different elapsed times, initial merchantable basal areas, and initial sawtimber basal areas*

Elapsed time	Initial merchantable basal area	Initial sawtimber basal area (ft^2)							
		20	30	40	50	60	70	80	90
years	ft^2				ft^2				
1	30	22	32						
	40	22	32	42					
	50	22	32	42	52				
	60	22	32	42	52	62			
	70	23	32	42	52	62	72		
	80	23	32	42	52	62	72	82	
	90	23	33	42	52	62	72	82	92
2	30	24	34						
	40	24	34	45					
	50	24	34	44	55				
	60	24	34	44	54	64			
	70	25	35	44	54	64	74		
	80	26	35	44	54	64	74	84	
	90	26	35	45	54	64	74	84	93
3	30	25	37						
	40	26	36	47					
	50	26	36	46	57				
	60	27	36	46	56	67			
	70	27	37	46	56	66	76		
	80	28	37	47	56	66	76	86	
	90	29	38	47	56	66	76	85	95
4	30	27	39						
	40	27	38	49					
	50	28	38	49	59				
	60	29	38	48	59	69			
	70	30	39	49	58	68	78		
	80	31	40	49	58	68	78	87	
	90	32	41	49	59	68	77	87	97
5	30	29	41						
	40	29	40	52					
	50	30	40	51	61				
	60	31	41	50	61	71			
	70	32	41	51	60	70	80		
	80	34	42	51	60	70	79	89	
	90	35	43	52	61	70	79	89	98
6	30	31	43						
	40	31	42	54					
	50	32	42	53	64				
	60	33	43	53	63	73			
	70	35	43	53	62	72	82		
	80	36	44	53	62	72	81	91	
	90	38	46	54	63	72	81	90	100
7	30	33	46						
	40	34	45	56					
	50	34	44	55	66				
	60	36	45	55	65	75			
	70	37	46	55	64	74	84		
	80	39	47	55	64	74	83	93	
	90	40	48	56	65	73	82	92	101
8	30	'35	48						
	40	36	47	58					
	50	37	47	57	68				
	60	38	47	57	67	77			
	70	40	48	57	66	76	86		

Table 6.—*Projected* sawtimber *basal* urea per *acre for* uneven-aged *stands* of *shortleaf pine for different elapsed* times, initial *merchantable* basal *areas,* and *initial* sawtimber basal *areas—(Continued)*

Elapsed time	Initial merchantable basal area	Initial sawtimber basal area (ft^2)							
		20	30	40	50	60	70	80	90
years	*ft^2*				*ft^2*				
	80	41	49	57	66	75	85	94	
	90	43	50	58	67	75	84	93	102
9	30	37	50						
	40	38	49	61					
	50	39	49	59	70				
	60	40	49	59	69	79			
	70	42	50	59	68	78	88		
	80	44	51	59	68	77	86	96	
	90	46	53	60	68	77	86	95	104
10	30	40	53						
	40	40	51	63					
	50	41	51	61	72				
	60	42	51	61	71	81			
	70	44	52	61	70	80	89		
	80	46	53	61	70	79	88	97	
	90	48	55	62	70	79	87	96	105

Sawtimber Basal Area

Time	Before cut	cut	After cut
years		ft^2	
0	25.0	...	25.0
7	39.1	10.6	28.5
14	42.9	10.8	32.1
21	46.6	10.9	35.7
28	50.3	11.0	39.3
35	53.9	11.0	42.9
42	57.6	12.6	45.0
49	59.7	14.7	45.0

The residual stocking goal for sawtimber basal area is reached in 42 years, two cutting cycles later than for merchantable basal area. The cyclic harvest for sawtimber basal area is 14.7 square feet after the stocking goal is reached.

Now that merchantable and sawtimber basal areas have been determined, volumes can be calculated. Merchantable volumes are calculated using equation (3). The initial volume is

$$V_m = 9.6209[45]^{1.1200}e^{0.0031623(70)},$$
$$= 853 \text{ cubic feet}.$$

The volume in 7 years is determined by substituting the projected merchantable basal area, which is 61.1 square feet,

$$V_m = 9.6209[61.1]^{1.1200}e^{0.0031623(70)},$$
$$= 1{,}201 \text{ cubic feet}.$$

The residual volume is found by using the residual merchantable basal area 49.0 square feet in equation (3),

$$V_m = 9.6209[49.0]^{1.1200}e^{0.0031623(70)},$$
$$= 938 \text{ cubic feet}.$$

The harvest is determined by subtracting after-cut from before-cut volumes. The remaining values are determined in like manner. The following tabulation can now be constructed for merchantable cubic-foot volume:

Merchantable Cubic-foot Volume

Time	Before cut	cut	After cut
years		ft^3, i.b.	
0	853	0	853
7	1,201	263	938
14	1,285	260	1,026
21	1,370	261	1,109
28	1,451	274	1,177
35	1,514	337	1,177

After the stocking goal is reached in year 28, the cyclic harvest for merchantable volume is 337 cubic feet. Periodic annual growth is 48 cubic feet.

Sawtimber cubic-foot volumes are determined by equation (4). The cubic volume for sawtimber at time zero is,

$$V_s = 10.030[25]^{1.1554}e^{0.0017908(70)},$$
$$= 469 \text{ cubic feet}$$

Subsequent volumes and cuts are determined by following the procedures already developed. When these have been calculated, the following table can be developed:

Sawtimber Cubic-foot Volume

Time	Before cut	cut	After cut
years		*ft³, i.b.*	
0	469	0	469
7	786	241	545
14	875	249	626
21	962	255	707
28	1,051	260	791
35	1,139	264	875
42	1,230	306	924
49	1,281	357	924

After the stocking goal for sawtimber is reached in year 42, the periodic cut will be 357 cubic feet for sawtimber. Periodic annual growth is 51 cubic feet.

Board-foot volumes for the Doyle rule are calculated using equation (5). The initial volume for the Doyle rule is

$$D = 16 \cdot 327(25)^{1.2944}e^{0.0070266(70)},$$
$$= 1{,}722 \text{ board feet}$$

After the volumes for the remainder of the planning period are calculated, the following tabulation can be assembled:

Doyle Board-foot Volume

Time	Before cut	Cut	After cut
years		*fbm*	
0	1,722	0	1,722
7	3,072	1,032	2,040
14	3,464	1,084	2,380
21	3,855	1,124	2,731
28	4,256	1,164	3,092
35	4,655	1,191	3,464
42	5,072	1,387	3,685
49	5,313	1,628	3,685

After the residual stocking goal is reached, the periodic cut is 1,628 board feet (Doyle rule), and the periodic annual growth is 233 board feet per acre. Notice that all the cuts are more than 1,000 board feet, so the harvesting constraint is satisfied by this management strategy.

The following harvest schedule for sawtimber volume for the Scribner was determined using equation (6):

Scribner Board-foot Volume

Time	Before cut	cut	After cut
years		*fbm*	
0	2,734	0	2,734
7	4,746	1,533	3,213
14	5,321	1,600	3,721
21	5,892	1,650	4,242
28	6,474	1,698	4,776
35	7,051	1,730	5,321
42	7,652	2,008	5,644
49	7,998	2,354	5,644

The periodic cut is 2,354 board feet (Scribner rule) after the residual stocking goal is reached, and periodic annual growth is 336 board feet.

Finally, the board-foot volumes, International 1/4-inch rule, were calculated using equation (7).

International 1/4-inch Board-foot Volume

Time	Before cut	Cut	After cut
years		*fbm*	
0	3,084	0	3,084
7	5,361	1,736	3,625
14	6,013	1,812	4,201
21	6,660	1,869	4,791
28	7,320	1,925	5,395
35	7,973	1,960	6,013
42	8,655	2,277	6,378
49	9,047	2,669	6,378

After the cyclic harvest levels are stabilized, the periodic cut is 2,669 board feet (International 1/4-inch rule). Periodic annual growth is 381 board feet.

A variety of other strategies could have been used to rehabilitate the stand. For example, half of growth could be cut provided that the harvest was at least 1,000 board feet (Doyle rule). If an operable volume was not present, the cycle cut could be deferred. The cutting cycle length in this case would be variable in length. The potential applicability of these growth and yield models is limited only by the imagination of the user.

A program to use these growth and yield models on the TI-59[2] programmable calculator is available from the authors upon request. Also available'is a listing of an example worksheet and ce11 contents for using the models in the electronic spreadsheet program SuperCalc.[2] The SuperCalc output for the foregoing example is shown in the appendix. Some corresponding values differ slightly due to differences in the text example and those used in the spreadsheet program.

CONCLUSIONS

Users should consider severa1 factors when using these models. First, the data carne from plots that exhibited a well-defined uneven-aged structure with shortleaf pine present as overstory, understory, and reproduction. Plots were also defined as being in the shortleaf pine forest type. Therefore, these equations should be restricted to these kinds of conditions.

Merchantable basal areas and volumes are for shortleaf pine trees 5.6 inches d.b.h. and larger; sawtimber basal areas and volumes, 8.6 inches d.b.h. and larger. Merchantability standards for both merchantable and sawtimber volumes should also be consulted.

Projected basal areas do include ingrowth, and the amount of ingrowth reflects average conditions represented in the data. Basal area predictions will be inaccurate in situations in which there is little ingrowth or where a large amount of shortleaf pines less than 5.6 inches d.b.h. are present and capable of contributing a large amount of ingrowth.

Projection periods should be limited to 10 years or less. Table 1 should also be consulted to see if your stand conditions are represented. If your stands are outside the range of the sample data or another geographic area is involved, the results should be treated with caution and validated if possible.

Shortleaf pine in the Interior Highlands is an important resource, and the results presented here provide data heretofore unavailable about the growth and yield of this species in uneven-aged stands. This information should be valuable to forest managers who would consider this management option but have been hanipered by the previous lack of growth and yield data.

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[2]TI-59 is a trademark of Texas Instruments, and SuperCalc is a trademark of Sorcim Corporation. The use of trade, firm, or corporation names is for the information and convenience of the reader. Such use does not constitute oficial endorsement or approval of the product or firm by the USDA to the exclusion of others which may be suitable.

Appendix-Shortleaf Pine Growth and Yield

U-A Bh1.: USh16YD

Tire	S.I.	BEFORE-CUT Be	MerCF	Bs	SawCF	Doyle	AFTER-CUT Be	MerCF	Bs	SawCF	Doyle
0	80	45.00	880	25.00	477	1847	45.00	880	25.00	471	1847
1	80	61.08	1240	39.08	199	3293	49.00	969	28.50	555	2189
14	80	64.93	1328	42.86	890	3711	53.00	1057	32.10	637	2553
21	Bo	68.71	1414	46.64	981	4141	56.90	1145	35.70	720	2930
28	80	12.32	1498	50.34	1071	4570	60.00	1215	39.30	805	3318
35	Bo	75.14	1563	53.95	1161	4999	60.00	1215	42.90	891	3716
42	Bo	75.14	1563	51.56	1251	5431	60.00	1215	45.00	941	3953
49	Bo	15.14	1563	59.68	1304	5698	60.00	1215	45.00	941	3953
56	80	75.14	1563	59.68	1304	5698					

Tire	S.I.		CUT Be	MerCF	Bs	SawCF	Doyle	P.A.I. Be	MerCF	BS	SawCF	Doyle
0	Bo		.00	0	.00	0	0					
7	Bo		12.08	211	10.58	244	1104	2.30	51.33	2.01	46.04	207
14	B0		11.93	210	10.76	253	1158	2.28	51.29	2.05	41.16	218
21	80		11.81	269	10.94	261	1212	2.24	50.96	2.08	49.13	221
28	B0		12.32	263	11.04	266	1253	2.20	50.39	2.09	50.14	234
35	Bo		15.14	348	11.05	270	1263	2.16	49.16	2.09	50.82	240
42	Bo		15.14	348	12.56	310	1483	2.16	49.16	2.09	51.45	246
49	Bo		15.14	348	14.68	363	1744	2.14	49.76	2.10	51.87	249
56	Bo							2.16	49.76	2.10	51.81	249
		CUT =	93.51	2138	81.61	1967	9238					
		CHANGE =	30.14	683	34.68	827	3850					
		YIELD =	123.72	2821	116.29	2794	13089					
		M.A.I. =	2.21	50.38	2.08	49.89	234					

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Equations are given to **estimate** current and projected **volumes** and projected basal **areas** of uneven-aged shortleafpine **(*Pinus echinata*** Mill.) **stands** managed under the selection system.