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Polymorphic Site Index Curves for White Pine in the Southern Appalachians

by

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POLYMORPHIC SITE INDEX CURVES FOR WHITE PINE IN THE SOUTHERN APPALACHIANS

by

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Site index — the height of the dominant stand at some particular age — is the commonly used yardstick by which relative productivity of forest sites is measured. In stands younger or older than index age, a family of height/age curves is required for projecting measured height to the height at index age. Most such curves now in use were constructed using the methods described by Bruce (1926). These methods involve measuring height and age of many stands at a single point in time, fitting an average curve of height on age to these data, and constructing a series of higher and lower curves with the same shape as the guide curve.

Evidence has accumulated in recent years that the curves constructed in this manner often do not represent accurately the growth of stands. In the first place, the guide curve is accurate only if the ranges of site indices are equally represented at all ages. Spurr (1955), King (1966), and Curtis (1964) report that unequal sampling frequently occurs because of the patterns of cutting and land abandonment in a particular region. For example, trees reach merchantable size faster and are often cut at younger ages on the higher quality sites. Consequently, a sample of stands drawn at one point in time would likely result in a lower average site index in older stands than in the young stands. A guide curve constructed from such a sample could not accurately depict actual growth trends. The second source of error is the assumption that the shape of the curve does not vary from site to site, i.e., the site curves are anamorphic. This assumption has been proved false for several species (Bull 1931; Spurr 1952, 1955; King 1966; Stage 1963; Brickell 1966, 1968).

The site index curves currently being used to evaluate site quality in natural stands of eastern white pine (*Pinus strobus* L.) in the Southern Appalachians were constructed by the anamorphic technique (Doolittle and Vimmerstedt 1960). These curves are subject to both sources of error mentioned above. A recent investigation[1]. showed that the published Doolittle-Vimmerstedt curves result in biased estimates of site index, particularly in young stands. Figure 1 compares actual growth to pattern of growth predicted by the published site index curves for representative stands of low, medium, and high site index. In all three circumstances, measured rate of growth below index age 50 was much slower than predicted by the site index curves. Beyond index age, measured rate of growth was slightly greater than predicted growth rate.

[1] Beck, Donald E. Height growth patterns in eastern white pine in the Southern Appalachians. 1969 (Unpubl. Ph.D. diss., N. C. State Univ., Raleigh.)

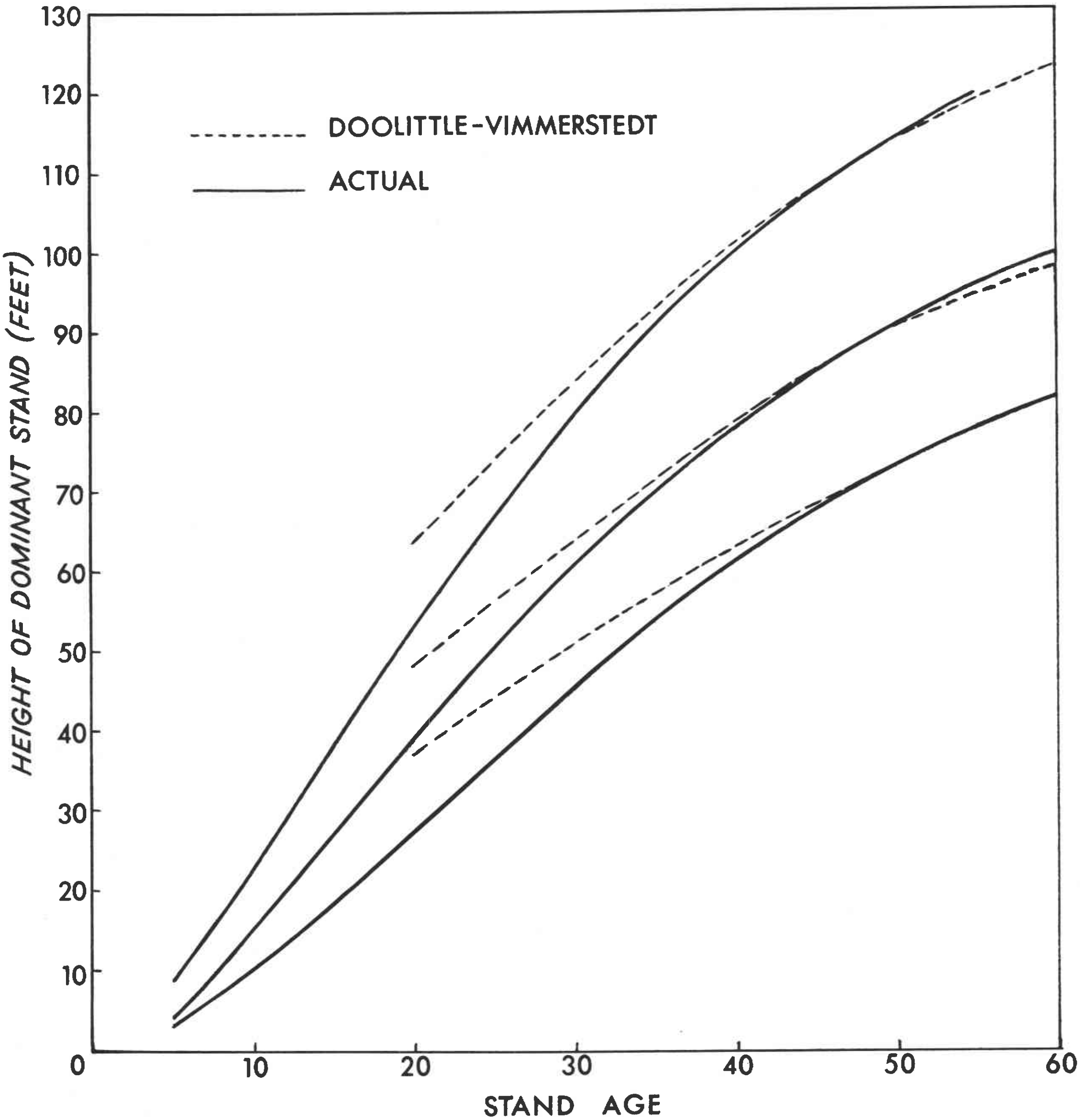

Figure 1.—Actual height-growth curves for three representative stands compared to the Doolittle-Vimmerstedt site curve predictions for similar sites.

If the published site index curves had been used to predict site index when these stands were 20 years old, actual site index would have been underestimated by 14 to 20 feet. As a result, board-foot yields at a rotation age of 70 would be underestimated by as much as 22 M bd. ft. per acre, or nearly 40 percent of actual yields. Consistent underestimation of this magnitude could cause serious underevaluation of forest land and mismanagement of forest stands.

Discrepancies shown in figure 1 suggest a sampling bias in the published curves in which a disproportionate number of high quality sites were represented in the young age classes. Though not as readily evident, failure to allow for polymorphic growth trends also may be contributing to the discrepancies between measured and predicted growth trends.

This paper presents new site index curves for natural, even-aged stands of white pine in the Southern Appalachians. These new curves are based on measured growth of stands and make allowances for polymorphic growth trends.

DATA COLLECTION

The study data were obtained from 42 even-aged stands of white pine of natural origin. At least 75 percent of the dominant and codominant trees in each stand were white pine. These stands were located in the mountains of southwestern Virginia, eastern Tennessee, western North Carolina, and northern Georgia—the southernmost range of eastern white pine—over an elevational range from 1,350 to 3,050 feet. Topography included slopes from 0 to 57 percent, many degrees of exposure, and positions ranging from stream bottoms to ridgetops. Site index (index age 50) ranged from 71 to 122. Stand age ranged from 44 to 70 so that measured height at index age was available for all but a few stands.

Three trees in each stand were selected from the dominant and codominant crown classes for measurement of the height/age relationship. These trees were free from injury and disease and had not been suppressed as evidenced by the annual rings. After felling, height at successive ages was determined by internode measurement. Nodes on the lower bole of the larger trees were frequently obscured by natural pruning and subsequent diameter growth. When the nodes were obscured, the lower 15 to 20 feet of the bole was split along the pith to avoid errors in determining heights. Ring counts were used also as a check on height determined by internode measurement.

For each sample tree the diameter of and distance to competing trees were measured for conversion to a measure of surrounding density. On each plot mean elevation above sea level, aspect in degrees, steepness of slope in percent, and position (stream bottom, lower, middle, or upper slope) were recorded.

SITE INDEX CURVES

The site index curves were developed in three steps:

1. Selecting a mathematical model that satisfactorily described the growth of individual stands.

2. Determining if and how the pattern of growth varied among stands of different site index and providing for change in shape of the curve in the mathematical model.

3. Testing to see if stand density and some topographic features of the site affected shape of the curve.

Each step is described more fully in the Appendix.

The resulting equation which expresses height as a function of age and site index is:

$$H = [63.06 + 0.67(S)]\left\{1 - e^{-[0.00985 + 0.00033(S)]t}\right\}^2 \quad (5)$$

where H is total height at age t, S is site index at index age 50, and e is base of the natural logarithms. This equation allows the pattern of height growth to vary with the level of site index. Figure 2 shows the family of curves generated by solution of the equation for selected values of site index.

APPLICATION OF THE CURVES

These site index curves apply to naturally established, even-aged stands of white pine in the Southern Appalachians. To estimate average site index in such stands, measure total height and total age of at least three dominant and codominant trees per plot or location of uniform site conditions. The trees selected should appear to have been in the dominant stand throughout their lives and show no evidence of damage, disease, or any condition which may have adversely affected height growth. The curves should be entered with the average height and age of the selected trees on each plot.

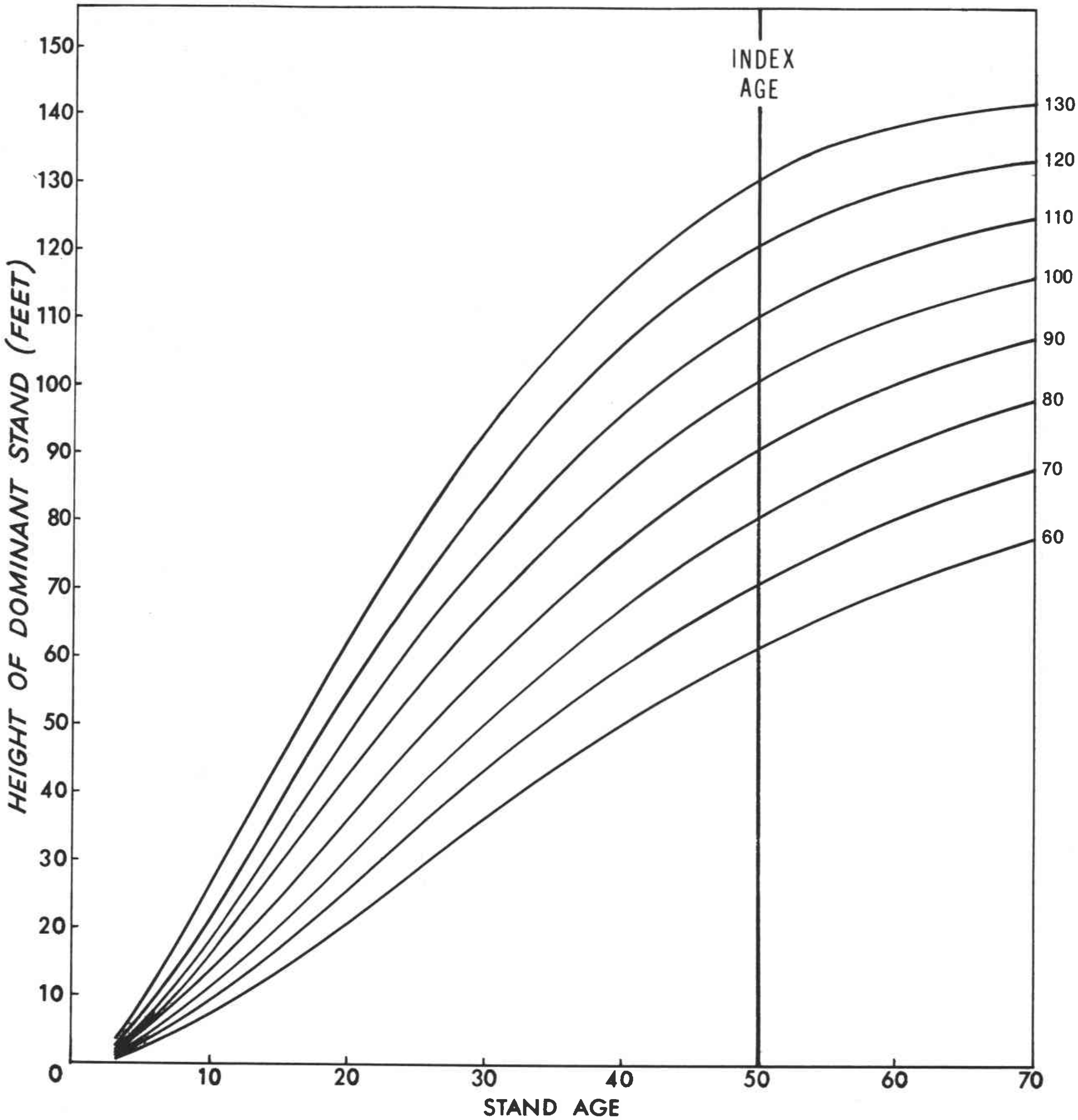

Figure 2.—Polymorphic site index curves for natural stands of eastern white pine in the Southern Appalachians.

APPENDIX

The Model

Graphs of height on age for individual trees and stands showed the characteristic sigmoid, or S-shaped, trend described by Spurr (1952). Investigation of a large number of sigmoid functions, including those discussed by Grosenbaugh (1965), led to Richards' (1959) modification of the Bertalanffy equation (Bertalanffy 1938, 1957). This model was suggested for use in describing growth of trees by Cooper (1961) and applied to height growth of Engelmann spruce and inland Douglas-fir by Brickell (1966, 1968).

The form of equation used in this study was:

$$H = A\left(1 - e^{-kt}\right)^{\frac{1}{1-m}} \qquad (1)$$

where H is height at age t, e is the base of natural logarithms, and A, k, and m are parameters to be estimated. This model has the properties, for certain values of the parameters, required to generate an asymmetrical S-shaped curve that passes through the origin and approaches a maximum height as age approaches infinity.

Deriving the Site Index Curves

Coefficients of the growth model (equation 1) were estimated for each of the 42 stands with an iterative least-squares fitting procedure. Then, to determine if the shape of the growth curve varied from one level of site index to another, the estimates of the coefficients A, k, and m (which determine curve shape) were examined in relation to site index by graphing and by regression analysis. It was found that both A and k were highly significantly related to site index by linear equations of the form:

$$A = b_0 + b_1(S) \qquad (2)$$

$$k = c_0 + c_1(S) \qquad (3)$$

where S is site index and b_0, b_1, and c_0, and c_1 are coefficients to be estimated. The coefficient m was apparently randomly distributed with respect to site index and average value of m was shown by t-test to be not significantly different from 0.5. Because it greatly simplified the growth equation, the coefficient m was allowed to take a value of 0.5.

Thus, allowance for varying shape of curve from site to site was made by expressing the coefficients A and k as functions of site index in the growth model (equation 1). The coefficient m was held constant at 0.5 making the quantity $\frac{1}{1-m}$ equal to 2. The function:

$$H = [b_0 + b_1(S)]\left\{1 - e^{-[c_0 + c_1(S)]t}\right\}^2 \qquad (4)$$

was fitted to the combined data for all stands by the iterative least squares procedure. The equation:

$$H = [63.06 + 0.67(S)]\left\{1 - e^{-[0.00985 + 0.00033(S)]t}\right\}^2 \qquad (5)$$

was used to calculate the polymorphic site index curves (index age 50) shown in figure 2.

Testing for Effect of Stand and Site Factors

To explain additional variation in curve form and to further refine estimates of site index, the parameters of growth curves for fixed levels of site index were examined in relation to stand density, geographic location, and a number of topographic features. Although the relationship between growth patterns and aspect, slope position, and steepness of slope were statistically significant, the improvement in estimation of site index was small. The additional accuracy did not warrant including these topographic features in a practical scheme for estimating site index. A particular effort was made to explore the effect of stand density on site index; however, no significant association between height growth and stand density was observed.

Precision of the Curves

To measure precision of the site index curves generated by equation 4, estimates of site index at age 10, 15, 20, and 30 years were made from total height of the 42 sample stands. Estimates were then compared to actual site index of the stands. These comparisons are summarized in table 1.

As is to be expected with any set of site curves, the estimates become more precise as stand age approaches index age. At age 30, all the estimates are within 10 feet of actual site index, but at 10 years 71 percent of the estimates are within 10 feet of actual site index and 8 percent err by 25 feet or more. There was no evidence of bias at any age, and the accuracy expected in even the 10-year-old stands is probably sufficient for most management purposes.

Table 1.—Deviations of estimated site index from actual site index

Deviations	10 years		15 years		20 years		30 years	
	Stands	Cumulative percentage	Stands	Cumulative percentage	Stands	Cumulative percentage	Stands	Cumulative percentage
	Number		*Number*		*Number*		*Number*	
Within ± 5 feet	16	38	19	45	27	64	37	88
Within ± 10 feet	14	71	14	78	12	93	5	100
Within ± 15 feet	6	85	7	95	2	97	----	----
Within ± 20 feet	3	92	1	98	1	100	----	----
Within ± 25 feet	3	100	1	100	----	----	----	----
Total	42		42		42		42	

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