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A Comparison of Height-Accumulation and Volume-Equation Methods for Estimating Tree and Stand Volumes

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SUMMARY

Estimating tree and stand volume in mature plantations is time consuming, involving much manpower and equipment; however, several sampling and volume-prediction techniques are available. This study showed that a well-constructed, volume-equation method yields estimates comparable to those of the often more time-consuming, height-accumulation method, even though the latter should be more accurate for any individual tree. Plot volumes were estimated by both methods in a remeasurement of trees in a 40-plot, planted slash pine thinning study. The mean percent age difference in total volume, inside bark, between the two methods ranged from 1 to 2.5 percent across all the plots; differences outside bark ranged from 7 to 10 percent. The results were similar when the effects of site, plot mean values, or tree-by-tree comparisons were incorporated.

Keywords: Growth and yield, *Pinus elliottii,* plantation management

INTRODUCTION

Accurately estimating the volume of stands of trees has long been a goal of foresters. One method used extensively in the South is the height-accumulation (HA) method, reported by Grosenbaugh (1954) and programmed for computer use by Lohrey and Dell (1969). This method is accurate because it accounts for taper in each tree that is measured; but, it has certain disadvantages. The required measurements on individual trees are time consuming, and for maximum accuracy, upper-stem diameters and heights should be measured directly-an almost impossible task when many large, old stands are involved. This disadvantage can be overcome by using reasonably accurate instruments such as relaskops or tele-dendrometers. However, the accuracy of these instruments is affected by the skill of the user, the height and visibility of the tree bole, weather conditions, and the condition of the instrument itself. Careful training of the personnel using the instruments, coupled with periodic calibration and maintenance, improves the accuracy of the measurements but does not overcome the problem of tree size nor does it greatly increase speed. Even with improved instruments and training, a typical field crew might be able to measure only 80 to 100 trees per day (Enghardt and Derr 1963).

The reduction in the number of workers, the aging of research plantations, and the desirability of measuring total height on all plot trees forced this research work unit to seek an alternative to the HA method that had been previously used in our research. The selected alternative was volume prediction equations developed for each species of pine from sample-tree measurements collected on plots with widely different site conditions, thinning intensities, age classes, and locations. Several of these equations had already been reported for each of the species being studied (Baldwin and Feduccia 1987; Baldwin and Saucier 1981; Clark and others 1991; Zarnoch and others 1991).

Some felt that changing the method of estimating such an important variable as volume would reduce the value of the data and invalidate year-to-year comparisons. Would the results be comparable, or would there be a significant difference between the two methods? To answer these questions, measurement results of the two methods were compared after conducting a specific case study.

Results of this research should be generally applicable to others because of the similarities between tree measurements on these research plots and those on standard inventory plots used by forest managers.

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METHODS

Volumes estimated by the HA method were compared with volumes estimated by the volume-equation (VE) method in a field study scheduled for periodic remeasurement in the winter of 1989–90. The study was a slash pine *(Pinus elliottii* var. *elliottii* Engelm.) test installed in two plantations near Peason, Louisiana. Plantation A was 39 years old with an average site index of 81 ft (base age 50), and plantation B was 41 years old with an average site index of 78 ft (base age 50). The study was begun in the winter of 1964-65 with eight treatments consisting of seven thinning intensities and an unthinned check. Forty 0.15-acre plots were measured.

Field crews first measured all plot trees for total height in feet using a relaskop and for diameter at breast height (d.b.h.) in tenths of an inch. Then they returned to each plot and selected sample trees following an established sampling procedure. The relaskop was then used to measure the height of these sample trees at each 2-inch change in bole diameter to determine taper-the information needed for the HA method of estimating volume. Individual tree cubic-foot volumes and finally total area volumes, expressed in cubic feet per plot outside and inside bark, were then computed using the height-accumulation and plot-summary computer programs developed by Lohrey and Dell (1969). These programs calculate individual sample tree volumes by height accumulation and then, given the basal area of each tree, estimate the total volume of each plot using the sample-tree mean-volume to mean-basal-area ratio estimator. Inside-bark measurements were estimated by applying a constant mean-inside-bark-diameter to outside-bark-diameter ratio at each measured taper point. The mean ratio selected, 0.87, had been previously determined from earlier measurements of bark thickness of plantation-grown slash pines from this and other slash pine growth-and-yield studies.[1]

The taper/volume functions of Thomas and Parresol (1991) were applied separately to estimate the total volume of individual trees. The Thomas and Parresol volume equations were derived using linear and nonlinear regression methods on felled-tree data from 199 sample trees representing a range of ages, diameters, stand densities, site indices, thinning treatments, and geographic locations.[2] The Thomas and Parresol equations were chosen for this comparison partly because some of the data used to provide parameters for the VE models came from trees felled from plots within this thinning study. The taper and corresponding volume functions are:

$$\hat{d}^2 / D^2 = b_1(x-1) + b_2 \sin(c\pi x) + b_3 \cot(\pi x/2) \quad (1)$$

$$\hat{v} = kD^2 H \int_{x_l}^{x_u} b_1(x-1) + b_2 \sin(c\pi x) + b_3 \cot(\pi x/2)dx \quad (2)$$

Where:

d = diameter at a given height
D = diameter-outside-bark at breast height (d.b.h.)
H = total height
x = height of observation/total height above ground
x_l, x_u = lower relative height (h_l/H) and upper relative height (h_u/H), respectively, where h_l is the lower height limit and h_u is the upper height limit
V = volume
π = 3.14159
k = $\pi/40{,}000$ if diameters are measured in centimeters and heights in meters or $\pi/576$ if diameters are measured in inches and heights in feet.
b_1, b_2, b_3, c = coefficients estimated from the data.

The estimated coefficient values for the slash pine data are presented in table 1. Note that: (1) because these equations involve predictions using relative heights and diameters, the coefficients are valid for measurements made in either Standard International (metric) or English units when the appropriate *k* is selected, and (2) there are separate sets of coefficients for trees from unthinned stands and for trees from stands thinned at least once.

In the alternative application, individual tree volumes were predicted using equation (2) and the corresponding coefficients from table 1, depending on whether the tree was in a thinned or an unthinned plot. These vol-

[1] Ferguson, R.B., Unpublished reports: "Thinning alternatives for planted slash pines," "The effect of age and residual basal area on growth and yield of planted slash pine *on a* medium to poor site." and "Timing of first thinning of planted slash pine;" (on file: USDA Forest Service, Southern Forest Experiment Station, RWU-4101, 2500 Shreveport Highway, Pineville, LA 71360).

[2] The Thomas and Parresol paper provides only coefficients for inside-bark taper or volume of planted slash pine. At our request, Mr. Parresol refit this model to the original data to also provide coefficients for outside-bark taper and volume. We gratefully acknowledge his contribution.

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Table 1.—*Coefficients for solving s/ash pine taper and volume equations by Thomas and Parresol (1991)*

Bole measurement	Coefficient values b_1	b_2	b_3	c
Unthinned plantations				
Inside bark	-0.66200	0.02780	0.00456	1.50
Outside bark	-1 .01750	.01789	.01009	1.50
Thinned plantations				
Inside bark	-0.74100	.03450	.00391	1.50
Outside bark	-1.02570	.01160	.00750	1.75

umes were then summed for each plot to obtain the predicted plot volumes.

The volume estimates from the two methods were then statistically compared on both an individual-tree and a plot basis. Differences were compared among all 40 plots and between plots on the 2 different plantation sites. Expected differences in volume due to the effect of thinning on taper (Baldwin and Feduccia 1991) were assumed to be accounted for by the use of the different coefficients for equations applied to trees from thinned or unthinned plots. The correlation coefficient (r) of the observed and predicted values was computed for each combination of volume estimates. In addition, the mean percent difference, $\overline{PDIFF} = (100/n) \sum (VE-HA)/HA$; the HA mean, $\overline{HA}$; the VE mean, $\overline{VE}$; and the mean difference, $\overline{DIFF} = (1/n) \sum (E-HA)$ statistics were computed.

RESULTS AND DISCUSSION

Generally, the VE method predicted higher individual tree and plot volumes than the HA method (table 2). The total mean predicted plot volume, inside bark, for all 40 plots differed by only 1.9 percent between the 2 methods. The mean percent age difference for the outside-bark volumes was higher, differing by 9.0 percent. These values remained about the same when age (site) and tree-to-tree differences were considered. Figures 1 and 2 show an extremely close correlation between the two volume-prediction procedures. The correlation would have been even closer if the five highest volume plots, which were unthinned, had been eliminated (fig. 1). However, all plots were included to generalize the comparisons. This does indicate, even though the taper-volume equations were developed with different coefficients for thinned and unthinned stands, that density still had a strong influence on the volume predictions. Better predictions might be obtained by accounting for stand density either as a predictor variable in volume-taper equations or by developing separate equations for different density classes. The latter method has been used with live crown ratio as the density surrogate (Dell 1979). The differences between the methods are similar for trees and plots (table 2), so it does not appear that the sampling procedure and the procedure for estimating the mean-volume to mean-basal-area ratio, when used with height accumulation to estimate plot volumes, contributed to the differences.

The 9.0-percent difference for outside-bark volumes was considered to be acceptable but large, which warranted additional study. A subset of 25 sample trees was selected that differed in outside bark volume by 6 ft^3 or more between the HA and the VE methods. Using the measured 2-inch bole diameter change points, their corresponding heights for each of these sample trees, and the compatible taper equations based on equation (1), the inside- and outside-bark diameters at each height were predicted. The height-versus-diameter points for the two methods were then plotted in an overlay fashion. A consistent trend was noted in each of these graphs. The bole taper of the lower portion of each sample tree was similar for the two methods, even though the diameters predicted by the taper equations were consistently larger. The upper portion of each bole, roughly corresponding to the live crown, tapered more sharply using the HA method. This difference indicates lower total outside-bark volume predictions by the HA method for the crown por-. tion of the bole (fig. 3).

Although the study reveals differences between the methods, it does not indicate which of the two methods is more accurate. Possibly, outside-bark measurements made on the population of trees in question should give more accurate results than those from an equation developed from measuring some other population. Moreover, given that the field measurement error is low, it is reasonable to assume that the HA method will be more accurate for any given tree, particularly if the height for each l-inch change in diameter rather than for each 2-inch change is measured. However, the value of that precision has to be weighed against the time needed to take those additional measurements.

The closeness of fit of these two methods may not hold for all plantations. Tests should be conducted on plantations with different site qualities, ages, densities,

Table 2.—*Closeness of fit statistics based on estimating total volume, inside and outside bark, by height accumulation (HA) procedures versus the Thomas and Parresol(1991) volume equations (VE)*

Treatment category for comparing methods of estimating total bole volume	Statistic'					
	n	*r*	*PDIFF*	$\overline{HA}$	$\overline{VE}$	*DIFF*
Full plot predictions						
				Ft³/plot (i.b.)		
All plots	40	0.991	1.92	450.75	455.61	4.86
Plantation A	16	.988	2.49	451.21	457.43	6.22
Plantation B	24	.993	1.57	450.45	454.40	3.95
				Ft³/plot (o.b.)		
All plots	40	0.995	9.03	603.24	658.00	54.76
Plantation A	16	.996	9.51	604.00	659.84	55.84
Plantation B	24	.995	8.72	602.74	656.78	54.04
Sum of sample trees only						
				Sample (i.b.)		
All sample trees	40	0.966	1.70	216.26	219.94	3.68
Plantation A	16	.954	2.90	227.74	234.54	6.80
Plantation B	24	.972	.90	208.60	210.20	1.60
				Sample (o.b.)		
All sample trees	40	0.978	8.64	289.75	315.09	25.34
Plantation A	16	.962	9.88	305.25	335.27	30.02
Plantation B	24	.982	7.82	279.42	301.63	22.21
Tree-by-tree comparisons						
				Ft³/tree (i.b.)		
All sample trees	520	0.990	1.27	16.67	16.92	0.25
Plantation A	206	.987	2.55	17.71	18.22	.51
Plantation B	314	.992	.42	16.00	16.07	.07
				Ft³/tree (o.b.)		
All sample trees	520	0.990	9.11	22.32	24.24	1.92
Plantation A	206	.988	10.35	23.71	26.04	2.33
Plantation B -	314	.992	8.30	21.41	23.05	1.64

● n = sample size of the comparison; *r* = correlation coefficient; *PDIFF* = mean percent age difference $= (100/n) \sum (VE-HA)/\overline{HA}$; $\overline{HA}$ = height accumulation mean; $\overline{VE}$ = Thomas and Parresol (1991) volume equation mean: $\overline{DIFF}$ = mean difference = $(1/n) \sum (VE-HA)$.

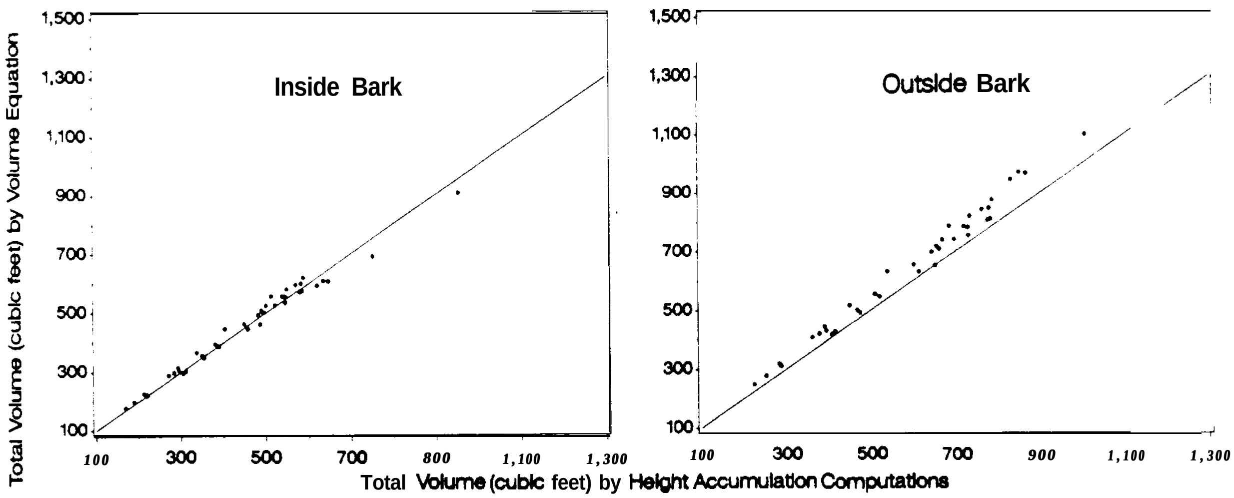

Figure 1 .-Comparison *of the height-accumulation (HA) method and the Thomas and Parresol(l99 1) volume-equation (VE) method for computing plot volumes, inside and outside bark.*

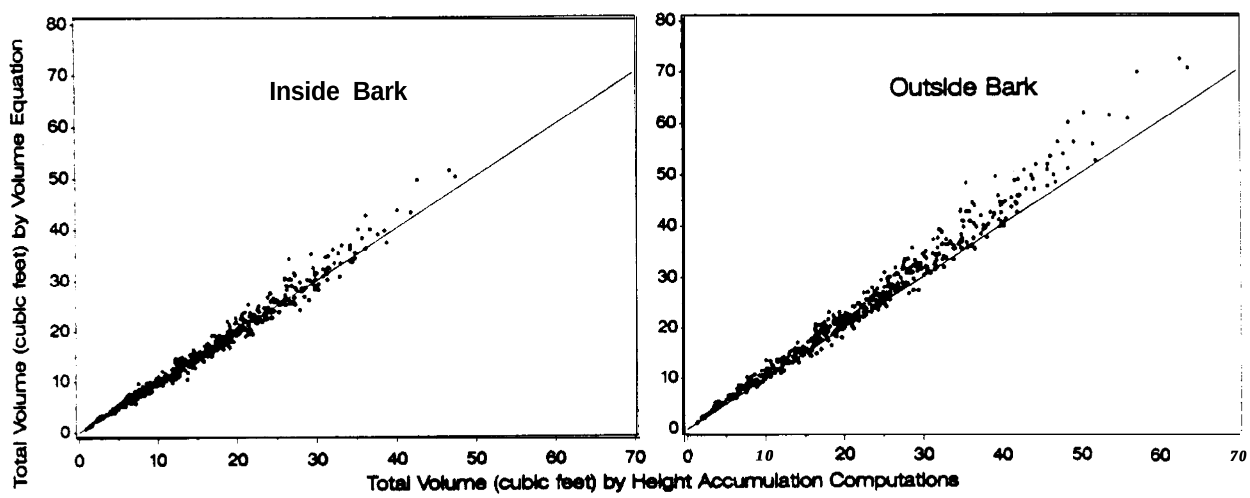

Figure 2.-Comparison *of the height-accumulation (HA) method and the Thomas and Parresol (1991) volume-equation (VE) method (VE) computing sample-tree volumes, inside and outside bark.*

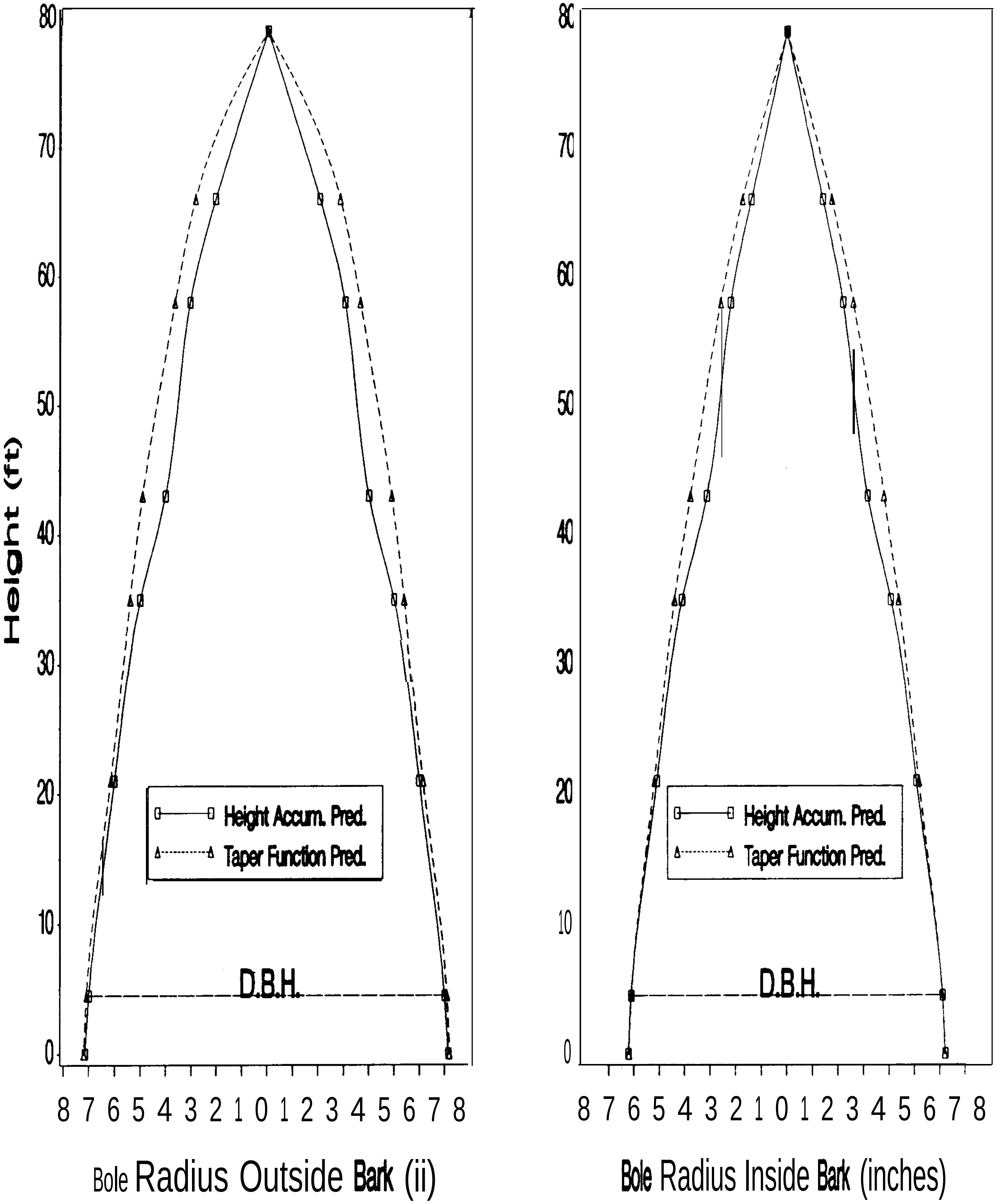

Figure 3.-*Comparison of bole taper, outside and inside bark, for a typical sample tree.*

and thinning treatments as well as different species to confirm these results. It would be better, but also more time consuming, to develop volume equations from trees sampled from each of the studies in question. Nevertheless, the predictions were close enough to justify the change to the VE method in this case, and it appears that this procedure is acceptable if time and field personnel are limited.

CONCLUSION

There was no significant loss of precision or year-to-year comparability when estimating stand volumes with volume equations instead of height accumulation. This result can largely be attributed to the quality of the database used to derive the equations. Thus, volume equations can be a viable alternative to the height-accumulation method for estimating stand volume.

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