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Volume Equations for Plantation Cottonwood Trees (*Populus deltoides*)

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SUMMARY

Equations and tables for cubic-foot volume of outside bark to the top of the tree and inside bark to a 3-inch top are given for plantation-grown cottonwoods (*Populus deltoides* Bartr. ex Marsh.) from 5 to 22 inches in d.b.h.

INTRODUCTION

A previous publication of volume tables for plantation-grown cottonwood (*Populus deltoides* Bartr. ex Marsh.) trees was based on 650 3- or 5-year-old trees sampled from 3 plantations near Greenville, Mississippi (Mohn and Krinard 1971). Diameters ranged from 2.3 to 9.9 inches, and heights ranged from about 25 to 65 feet. Volume equations have now been developed from planted cottonwoods near Fidler, Mississippi, that are up to 20 years old and more than 20 inches in d.b.h. This expands the range of tree sizes used in cottonwood volume determinations.

SAMPLE TREES

The sample trees came from a spacing study in the Mississippi River batture on Crown Zellerbach land (Krinard and Johnson 1984). Soil series were mainly Commerce and Convent silt loams, which have estimated site indices at age 30 of 105 to 125 feet for natural cottonwood stands (Broadfoot 1976). Cuttings used in the plantings were run-of-the-bar stock (nonselect material). Spacings were 4 by 9, 8 by 9, 12 by 12, and 16 by 18 feet. Several thinning treatments were applied, starting after the 4th year and ending after the 15th year.

Volumes were obtained both from trees cut in thinnings, with diameters measured at 4-foot intervals start-

ing with a 1-foot stump, and from standing trees, using a dendrometer and also starting with a 1-foot stump. Whether trees were cut or standing, volumes were determined by summing 1-foot frustrums (Grosenbaugh 1954), with the top calculated as a cone. Two bark thicknesses were measured at d.b.h. Inside bark diameters (d.i.b.) were calculated from outside bark diameters (d.o.b.) by $d.i.b. = d.o.b. (1 - (1 - d.b.h.i.b./d.b.h.o.b.)(1/(2 - d.o.b./d.b.h.o.b.)))$ (Mesavage 1969).

Distribution of sample trees by age, spacing, and whether cut or standing are given in table 1. Tree distribution by 1-inch diameter class and 1 0-foot height class is given in table 2 with the average measured total outside bark volume.

Tables for total stem cubic-foot volume outside bark from a 1-foot stump to the top of the tree (table 3) and inside bark stem volume to a 3-inch top (table 4) were constructed from the following regression equations:

$$\text{Volume outside bark} = 0.06 + 0.002221 D^2 H$$

$$r^2 = 0.987, S_e = 2.4 \text{ ft}^3, CV = 10.5\%, \bar{V} = 23.1 \text{ ft}^3$$

$$\text{Volume inside bark} = -0.86 + 0.001904 D^2 H$$

$$r^2 = 0.987, S_e = 2.1 \text{ ft}^3, CV = 10.9\%, \bar{V} = 18.9 \text{ ft}^3$$

where D = d.b.h. and H = total height.

Intercept and slope coefficients based on smaller, younger trees (Mohn and Krinard 1971) were 0.21 and 0.00221 for outside bark volumes and -0.62 and 0.00204 for inside bark volumes.

A weighted regression may be used to correct for nonhomogeneity of variance and may give a more precise estimate. Use of weights, whether $1/(D^2 H)$ or $1/(D^2 H)^2$, provided the same or very slightly larger values of S_e , CV, and fit index $(1 - \sum(Y - \hat{Y})^2 / \sum(Y - \bar{Y})^2)$ with this data. The index of fit (Furnival 1961) was slightly better for weighted regressions, with values of 1.6 and 1.3 compared to 2.4 for outside bark volumes and 1.4 and 1.2 compared to 2.1 for inside bark volumes. Weighted regressions, with

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weight $1/(D^2H)^2$, changed the coefficients slightly, as follows:

$$\text{Volume outside bark} = 0.09 + 0.002216 D^2H$$

$$\text{Volume inside bark} = -0.41 + 0.001852 D^2H$$

The volumes generated by these equations should be applicable to most commercial cottonwood plantation spacings. The equations represent trees from a mixture of four planting spacings, with 20, 26, 31, and 23 percent of sample trees coming from 4- by 9-, 8-by 9-, 12- by 12-, and 16- by 18-foot spacings, respectively. There were differences between spacings in volume equations, partly due to the number of trees measured and to differences in tree

sizes between spacings, where mean measured outside bark volumes were 19.0, 19.4, 23.6, and 30.2 cubic feet from closest to widest spacing. Spacing variables incorporated in the volume equations improved the fit statistics so very little that there was no useful benefit in doing so. Whether these equations would apply to cottonwood trees of genetically improved material-which would include most current and future plantations-is not known, but with slash (Tankersley and others 1983) and loblolly (Buford and Burkhart 1987) pine, improved and unimproved trees did not need separate equations.

Table 1 — Number of sample trees by age, spacing, and whether cut or standing

| Spacing Feet | Age of cut trees | | | | | Age of standing trees | | | | | Standing and cutoff total | |
|-----------------|------------------|----|----|----|----|-----------------------|-----|-----|----|----|------------------------------|-----|
| | 6 | 7 | 8 | 12 | 15 | 7 | 9 | 10 | 15 | 20 | | |
| 4 by 9 | 20 | | | 11 | 19 | 7 | 26 | 12 | | | 7 | 102 |
| 6 by 9 | 30 | 7 | 5 | 10 | 15 | 14 | 23 | 16 | 3 | 8 | | 131 |
| 12 by 12 | 32 | 10 | 20 | 6 | 10 | 11 | 23 | 35 | 3 | 10 | | 160 |
| 16 by 18 | | 10 | 10 | 1 | 10 | 4 | 37 | 38 | 2 | 7 | | 119 |
| Total | 82 | 27 | 35 | 28 | 54 | 36 | 109 | 101 | 8 | 32 | | 512 |

Table 2. -Number of trees and average measured cubic-foot volume outside bark from a 1-foot stump to the top of the tree by 10-foot total height class and 1-inch diameter class

| D.b.h. Inch | Height classes | | | | | | | | Total |
|----------------|-------------------|---------|---------|----------|----------|----------|----------|----------|-------|
| | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-99 | 100-109 | 110-119 | |
| | Cubic feet | | | | | | | | |
| 4 | 1.8(1) | 2.6(6) | 3.9(1) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | (8) |
| 5 | 0.0(0) | 3.6(16) | 4.8(6) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | (22) |
| 6 | 0.0(0) | 5.0(9) | 6.2(43) | 7.4(11) | 9.1(1) | 0.0(0) | 0.0(0) | 0.0(0) | (64) |
| 7 | 0.0(0) | 0.0(0) | 7.7(21) | 9.4(24) | 11.6(3) | 0.0(0) | 0.0(0) | 0.0(0) | (48) |
| 8 | 0.0(0) | 0.0(0) | 9.5(8) | 12.0(37) | 15.0(12) | 16.8(5) | 0.0(0) | 0.0(0) | (62) |
| 9 | 0.0(0) | 0.0(0) | 14.4(1) | 15.0(26) | 16.1(15) | 21.7(2) | 0.0(0) | 0.0(0) | (44) |
| 10 | 0.0(0) | 0.0(0) | 0.0(0) | 18.4(20) | 20.9(36) | 24.7(12) | 24.6(1) | 0.0(0) | (69) |
| 11 | 0.0(0) | 0.0(0) | 0.0(0) | 21.6(12) | 24.9(40) | 27.4(11) | 30.3(3) | 0.0(0) | (66) |
| 12 | 0.0(0) | 0.0(0) | 0.0(0) | 23.5(6) | 27.9(27) | 32.4(14) | 36.7(2) | 0.0(0) | (49) |
| 13 | 0.0(0) | 0.0(0) | 0.0(0) | 27.8(3) | 31.4(11) | 38.5(6) | 43.2(3) | 0.0(0) | (23) |
| 14 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 36.4(3) | 40.0(2) | 0.0(0) | 0.0(0) | (5) |
| 15 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 38.5(2) | 45.2(2) | 58.0(5) | 65.4(2) | (11) |
| 16 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 45.2(1) | 48.9(2) | 67.4(4) | 74.7(1) | (8) |
| 17 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 60.8(1) | 73.0(11) | 78.6(1) | (13) |
| 18 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 79.4(2) | 91.8(4) | (6) |
| 19 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 90.0(1) | 97.4(3) | (4) |
| 20 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 90.8(4) | 100.9(3) | (7) |
| 21 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 118.5(1) | (1) |
| 22 | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 0.0(0) | 121.9(2) | (2) |
| Total | (1) | (31) | (80) | (139) | (151) | (57) | (36) | (17) | (512) |

t Example: the 4-inch d.b.h. class represents trees whose d.b.h.'s range from 4.0 to 4.9 inches.

† Numeral in parentheses represents the number of trees in this height and d.b.h. class.

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Table 3 — Cubic-foot volumes outside bark from a 1-foot stump to the top of the tree

| D.b.h. inches | Total tree height (ft) | | | | | | | | |
|------------------|------------------------|------|------|------|------|-------|-------|-------|--|
| | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | |
| 5 | 2.8 | | 3.9 | 4.5 | | | | | |
| 6 | 4.1 | 4.9 | 5.7 | 6.5 | 7.3 | | | | |
| 7 | 5.5 | 6.6 | 7.7 | 8.8 | 9.9 | | | | |
| 8 | 7.2 | 8.6 | 10.0 | 11.4 | 12.9 | 18.0 | | | |
| 9 | 9.1 | 10.9 | 12.7 | 14.5 | 16.3 | 22.3 | 18.0 | | |
| 10 | | 13.4 | 15.6 | 17.8 | 20.0 | 22.3 | 24.5 | | |
| 11 | | 16.2 | 18.9 | 21.6 | 24.2 | 26.9 | 29.6 | | |
| 12 | | 19.2 | 22.4 | 25.6 | 28.8 | 32.0 | 35.2 | | |
| 13 | | 22.6 | 26.3 | 30.1 | 33.8 | 37.6 | 41.3 | | |
| 14 | | | 30.5 | 34.9 | 39.2 | 43.6 | 47.9 | | |
| 15 | | | 35.0 | 40.0 | 45.0 | 50.0 | 55.0 | 60.0 | |
| 16 | | | 39.9 | 45.5 | 51.2 | 56.9 | 62.6 | 68.3 | |
| 17 | | | | 51.4 | 57.8 | 64.2 | 70.7 | 77.1 | |
| 18 | | | | | 64.8 | 72.0 | 79.2 | 86.4 | |
| 19 | | | | | 72.2 | 80.2 | 88.3 | 96.3 | |
| 20 | | | | | 80.0 | 88.9 | 97.8 | 106.7 | |
| 21 | | | | | | 98.0 | 107.8 | 117.6 | |
| 22 | | | | | | 107.6 | 118.3 | 129.1 | |

† Area outlined represents range of measured trees.

Table 4 — Cubic-foot volumes inside bark from a 1-foot stump to a d-inch top

| D.b.h. Inches | Total tree height (ft) | | | | | | | | |
|------------------|------------------------|------|------|------|------|------|-------|-------|--|
| | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | |
| 5 | 1.5 | 2.0 | 2.5 | 2.9 | | | | | |
| 6 | 2.6 | 3.3 | 3.9 | 4.6 | 5.3 | | | | |
| 7 | 3.8 | 4.7 | 5.7 | 6.6 | 7.5 | | | | |
| 8 | 5.2 | 6.5 | 7.7 | 8.9 | 10.1 | 11.3 | | | |
| 9 | 6.9 | 8.4 | 9.9 | 11.5 | 13.0 | 14.6 | | | |
| 10 | | 10.6 | 12.5 | 14.4 | 16.3 | 18.2 | 20.1 | | |
| 11 | | 13.0 | 15.3 | 17.6 | 19.9 | 22.2 | 24.5 | | |
| 12 | | 15.6 | 18.3 | 21.1 | 23.8 | 26.6 | 29.3 | | |
| 13 | | 18.4 | 21.7 | 24.9 | 28.1 | 31.3 | 34.5 | | |
| 14 | | | 24.3 | 29.0 | 32.7 | 36.5 | 40.2 | | |
| 15 | | | 29.1 | 33.4 | 37.7 | 42.0 | 46.3 | 50.5 | |
| 16 | | | 33.3 | 38.1 | 43.0 | 47.9 | 52.8 | 57.6 | |
| 17 | | | | 43.2 | 48.7 | 54.2 | 59.7 | 65.2 | |
| 18 | | | | | 54.7 | 60.8 | 67.0 | 73.2 | |
| 19 | | | | | 61.0 | 67.9 | 74.7 | 81.6 | |
| 20 | | | | | 67.7 | 75.3 | 82.9 | 90.5 | |
| 21 | | | | | | 83.1 | 91.5 | 99.9 | |
| 22 | | | | | | 91.3 | 100.5 | 109.7 | |

† Area outlined represents range of measured trees.

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