OCCASIONAL PAPER 134 1954

NEW TREE-MEASUREMENT CONCEPTS: HEIGHT ACCUMULATION, GIANT TREE, TAPER AND SHAPE

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An entirely new concept of tree measurement was announced by the author in 1948 (11). Since the original theory and applications have subsequently been broadened considerably, it seems advisable to publish the entire development in readily usable form, along with other material helpful in tree measurement.

In its essence, the new concept consists of selecting tree diameters above d.b.h. in diminishing arithmetic progression (equivalent to saying that outside-bark diameter is treated as the independent variable); then tree height to each such diameter is estimated, recorded, and accumulated. This reverses the classical tree measurement concept which regarded height as the evenly-spaced independent variable, and which estimated and recorded diameter at regular intervals of height.

The greatest advantage of the new concept lies in the ease with which individual trees can be broken down into various classes of material and recombined with similar portions of other trees, using punched cards, business machines, or both. Sections varying in length and grade (from different trees) can be sorted by grade-diameter groups within which lengths will be additive. The ability of modern business machines to accumulate has been largely wasted in conventional procedures where height is treated as the independent variable. Their capability can be much more fully exploited in the new procedure using diameter as the independent variable.

Another advantage lies in the convenience of being able to use height, the sum of heights, and the sum of height-accumulations to calculate volume and surface of individual trees (or of classes containing portions of several trees). Final tabulations can be readily converted into surface, cubic volume, or volume by a number of log rules; no volume tables are required. Little extra work is involved in tabulating volumes to several different merchantable tops, or in segregating volume by grades.

There are already in existence dendrometers which are well adapted to locating diameters in diminishing progression along the bole of a standing tree--the Biltmore pachymeter (4), the Clark dendrometer (6), the Bitterlich relascope (1) and spiegel-relascope (2), and the Bruce slope-rotated wedge-prism (3), to mention a few. However, anyone currently content to use arbitrary volume tables not based on valid sampling of the given tree population, or to estimate butt-log form-class and accept arbitrary upper-log taper assumptions can use the new technique with the crudest instruments and still not exceed the error currently ignored or tolerated in commonly used volume-table techniques.

This paper not only describes the theory and application of the new concept but also makes available for the first time a giant-tree table, and discusses tree taper and shape.

HEIGHT ACCUMULATION--A NEW CONCEPT

If a peeled 16-foot log with a small-end d.i.b. (diameter inside bark) of 8 inches, and a large-end d.i.b. of 10 inches is stood upright on its large end, and if heights (H) are measured as the number of 4-foot sections occurring beneath diameters in the complete progressive series

2 inches, 4 inches, 6 inches, 8 inches, and 10 inches

the record will appear as follows:

d.i.b. (inches)	H Number
2	4
4	4
6	4
8	4
10	0

It will be noticed that all diameters in the series which are smaller than the small end of the log have the same height as the small end.

Now if these heights are differenced (upper minus lower) and if each difference in height is expressed as (L), the calculation will appear as follows:

d.i.b. (inches)	L	H
	Number	
2	0	4
4	0	4
6	0	4
8	4	4
10	0	0

Finally, if heights are accumulated so that each entry indicates the progressive total (H') of all heights recorded beneath it, and L, H, and H' are then summed, the calculation will appear as follows:

d.i.b. (inches)	L	H	H'
		Number	
2	0	4	16
4	0	4	12
6	0	4	8
8	4	$4 = \Sigma L$	4
10	0	0	0
	$4 = \Sigma L$	$16 = \Sigma H$	$40 = \Sigma H'$

Now these three sums can be converted to units of volume (or surface) by referring to Appendix A. Where taper-step is 2 inches, where height is expressed in 4-foot units, and where $\frac{\text{d.i.b.}}{\text{d.o.b.}} = 1$ (i.e., where it is not necessary to correct for bark included in diameter measurements), the board-foot volume by the Doyle log rule can be calculated as: $2\Sigma H' - 5\Sigma H + 4\Sigma L = 80 - 80 + 16 = 16$ bd. ft. This is identical with the volume obtained by the conventional Doyle rule for 16-foot logs: $(\text{d.i.b.} - 4)^2 = 16$. The theory behind all this is discussed in Appendix E.

Of course, the height-accumulation technique offers no advantage over the conventional method in the simple case above, but the new technique has real utility in more complicated timber-cruising or tree-measurement situations.

This technique can be applied when d.o.b. (diameter outside bark) is measured but volume inside bark is desired. For purposes of illustration, it will be postulated that bark constitutes 10 percent of d.o.b. (i.e., that mean $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio is .90), that all heights are measured above a 1/2-foot stump whose d.o.b. is assumed to be 2 inches greater than d.b.h., that it has been decided to adopt a taper-step of 2 inches, and that lengths, heights, and height-accumulations will be expressed as number of 4-foot sections.

The heights to regularly diminishing diameters of a 20-inch tree with five 16-foot logs (or twenty 4-foot sections) to the limit of merchantability might be estimated as in the boxed portion of the following tabulation:

	d.o.b. (inches)	L	H	H'
		Number of 4-foot sections		
	2	0	20	143
	4	0	20	123
	6	0	20	103
	8	0	20	83
merch. top	10	4	20 = ΣL	63
	12	2	16	43
	14	5	14	27
	16	6	9	13
	18	2	3	4
d.b.h.	20	1	1	1
		20 = ΣL:	143 = ΣH:	603 = ΣH':

Note that there must be an H and an H' for every 2-inch diameter step in a complete progression between (but excluding) stump and zero, even though no merchantable material is found outside the box beyond the merchantable top d.o.b. of 10-inches. Merchantable height must be repeated for each progressive diameter less than this merchantable top, however. Actually, if the complete H column (commencing with the H corresponding to d.b.h. and including repeated heights outside the box) is entered in a 2-register bookkeeping machine which can transfer subtotals from first to second register without clearing, then Σ H and Σ H' can be automatically and simultaneously computed, and Σ L will be the last entry in the H column. Some people may prefer to invert the column of diameters so that the corresponding H and H' columns progress downward as on the bookkeeping machine tape, but this is merely a matter of personal preference, as long as the arithmetic accumulation proceeds in a direction which is up the tree.

Thus, Σ H' = 603, Σ H = 143, and Σ L = 20 for a 20-inch five-log tree described in terms of H as 20: 1: 3: 9: 14: 16: 20, with 4 more 20's needed to complete the diminishing diameter progression. Appendix A gives coefficients A, B, C appropriate for these accumulations where unit-height is 4 feet, where taper-step is 2 inches, and where mean $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio is .90. The appropriate A, B, C coefficients convert the accumulations into surface or cubic volume (inside or outside bark) and into board-foot volume according to International 1/4-inch log rule, Scribner formula log rule, or Doyle log rule. This is demonstrated below for surface, cubic volume, and International 1/4-inch volume:

Surface inside bark = 1.88(143) + .942(20) = 288 sq. ft. i.b.

Cubic vol. inside bark = .141(603) + .0236(20) = 85.5 cu. ft. i.b.

Int. 1/4-inch vol. = 1.29(603) - 1.34(143) - .284(20) = 580 bd. ft.

Although the simple tally and bookkeeping-machine technique just outlined is well adapted to many situations, a more useful method involves punch-card breakdown of the tree into sections classified by grade (or utility) and d.o.b. Lengths of such sections can then be machine-sorted and accumulated by grade-diameter classes, and need not be converted to volume till the last step. If mark-sensing is employed, more than one mark-sensed card may be needed per tree, depending on how much information is desired. It is assumed that plot information will be recorded on a separate mark-sensed card. In addition to plot number, species, defect class, and priority class, only d.b.h. and the graded length of each taper-step need be recorded in the field on the mark-sensed tree-card. For the tree used in the preceding example, the latter information would be tallied as d.b.h. and successive L's with their grades A, B, C, D, etc., thus:

20: 1A: 2A: 6B: 5B: 2C: 4C

In actual practice, of course, grades would be coded numerically.

This would require only 14 of the 27 columns on a mark-sensed card. Occasionally it might be more convenient for the estimator to record H instead of L, but ordinarily this would waste mark-sensed columns and would require differencing later, if a breakdown by grades is desired. The 27-column mark-sensed cards will later be automatically punched, and in some cases may suffice for volume calculations, in conjunction with a 3-register bookkeeping machine. Generally, however, they will be used to reproduce a card for each tree section, so that these can easily be sorted and accumulated, after which ΣL, ΣH, and $\Sigma H'$ can be directly obtained by the progressive totalling process on some accounting machine such as the IBM 402 or 403.

• To illustrate quite simply how the mechanics of the sorting and recombining procedure might work, suppose that (in addition to the 20-inch tree described above) there were also an 18-inch tree, with d.b.h. and graded lengths (L) recorded as 18: 1A: 3B: 4B: 4C: 3D: Separate cards would be punched for each length in each tree, with a zero-length master card in each summarized d.o.b.-grade group to facilitate progressive totalling. A series of machine sorts and tabulations (by grade in each d.o.b. class) would break down the trees into their elements and would recombine and summarize them thus:

d.o.b. of sections

											:Sums:	: Sums of :1st and 2nd :progressive sums	
	20	18	16	14	12	10	8	6	4	2	ΣL	ΣH	$\Sigma H'$
Tree #1	1A	2A	6B	5B	2C	4C	0	0	0	0	20	143	603
Tree #2		1A	3B	4B	4C	3D	0	0	0	0	15	100	394
Total	1	3	9	9	6	7	0	0	0	0	35	243	997
A	1	3	0	0	0	0	0	0	0	0	4	37	190
B			9	9	0	0	0	0	0	0	18	135	576
C					6	4	0	0	0	0	10	56	186
D						3	0	0	0	0	3	15	45
Total	1	3	9	9	6	7	0	0	0	0	35	243	997

The individual tree sums and progressive sums are not needed except to illustrate that they agree with the later partition by grades.

The $\Sigma H'$, ΣH, and ΣL for each grade can be converted to volume by using the A, B, C coefficients in Appendix A, if an appropriate $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio is chosen. Although for most purposes this ratio is adequately estimated as the sample-based ratio $\sqrt{\frac{\text{volume i.b.}}{\text{volume o.b.}}}$ with linear interpolation between tabled values of A, B, C, occasionally it may be found desirable to sample both d.i.b. and d.o.b. at regular intervals of length and to derive two ratios: $\frac{\Sigma(\text{d.i.b.})^2}{\Sigma(\text{d.o.b.})^2}$ and $\frac{\Sigma(\text{d.i.b.})}{\Sigma(\text{d.o.b.})}$. These should be applied to outside-bark volume and surface formulae respectively, and the resultant formulae should be multiplied by the log rule formulae given in Appendix E to derive A, B, C coefficients more appropriate than those obtained by linear interpolation from Appendix A. Where volume or surface including bark is desired, the coefficients given for $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio = 1.00 are appropriate without any adjustment.

If it is desired to break down volume or surface summaries by d.o.b. class, first number each d.o.b. class in the complete series consecutively from the smallest to the largest (beginning with number 1), then accumulate the ordinals progressively from the beginning, then multiply each length (L) by its corresponding ordinal (I), or its ordinal accumulation (II), thus:

d.o.b. (inches)	(I) Numbered consecutively	(II) Accumulated progressively	L	(I)(L) H	(II)(L) H'
2	1	1	0		
4	2	3	0		
6	3	6	0		
8	4	10	0		
10	5	15	7	35	105
12	6	21	6	36	126
14	7	28	9	63	252
16	8	36	9	72	324
18	9	45	3	27	135
20	10	55	1	10	55
			$35 = \Sigma L$	$243 = \Sigma H$	$997 = \Sigma H'$

Each L, when multiplied by its corresponding entry in column I, will give the appropriate H for that d.o.b., and when multiplied by its corresponding entry in column II will give the appropriate H'. Of course, cumulative volumes of material larger than any specified d.o.b. could be obtained easily by merely assigning zeros in the L column for all smaller d.o.b.'s, and then getting ΣL, ΣH, $\Sigma H'$ in the usual fashion. Volumes or surfaces in any desired diameter class or group of diameter classes are then obtained by multiplying H', H, L or their desired sums by the A, B, C coefficients from Appendix A in the usual fashion.

One shortcut might be mentioned which frequently may furnish an acceptable approximation. Where the average $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio is exactly .932 (or where $\frac{\text{volume inside bark}}{\text{volume outside bark}}$ ratio is .869), if taper-step of 2 inches and unit-height of 4 feet have been adopted, the board-foot volume by the International 1/4-inch log rule can be easily calculated, thus:

$$\text{Board feet (Int. 1/4")} = (1.38)\left(\Sigma H' - \Sigma H - \frac{\Sigma L}{5}\right)$$

For the tree described on page 4, this would be:

$$(1.38)\left(603 - 143 - \frac{20}{5}\right) = (1.38)(456) = 629 \text{ bd. ft.}$$

There are three general situations in which slightly different variations of the height-accumulation technique may be found useful:

(1) Where it is desired to grade different portions of the sample tree, the entire job should be done on punched cards, preferably with the field tally made directly on mark-sensed cards. Trees should be graded and described in terms of d.b.h. and a series of graded L's (rather than H's), thus:

18: 1A: 3B: 4B: 4C: 3D

Separate cards should be reproduced for each d.o.b. starting with d.b.h. = 18 inches, and after desired sorts are made, an IBM 402 or 403 accounting machine will accumulate ΣL, ΣH, $\Sigma H'$ for each class whose volume is desired, provided that L = 0 is supplied for any d.o.b. missing from the complete progression.

(2) Where it is not desired to grade portions of a sample tree, an entire tree can be punched on an individual card, with each H (or L) recorded under its appropriate d.o.b. field starting with d.b.h. (e.g., starting with 18 inches for tree #2 in the tabulation on page 6), thus:

1: 4: 8: 12: 15: (with 15: 15: 15: 15 completing the H series)

or 1: 3: 4: 4: 3: (with 0: 0: 0: 0 completing the L series)

Each class for which a separate volume is desired will have H (or L) summarized for each d.o.b. field occurring in it. The H (or L) sequence (including repeat heights or zeros needed to complete the progression) for each class can then be entered in a 2- (or 3-) register bookkeeping machine able to transfer subtotals from register to register without clearing. Examples of suitable 2-register bookkeeping machines in which the H's can be entered are the Burroughs Sensimatic F-50, or the National Cash Register 30210 (1-6). Examples of suitable multiple register machines in which the L's can be entered are Burroughs Sensimatic F-200, or National Cash Register 30412 (17) or 3100, any of which have more than the 3 registers needed.

(3) Where it is not desired to use punched cards at all, tally for each sample tree will consist of its d.b.h. followed by a series of heights, thus:

18: 1: 4: 8: 12: 15

With a taper-step (T) = 2 inches and d.b.h. of 18 inches, there should be $\frac{\text{d.b.h.}}{T} = 9$ entries, so 1, 4, 8, 12, and 15, 15, 15, 15, 15 should be entered in a 2-register bookkeeping machine to obtain ΣH and $\Sigma H'$ for the individual tree; ΣL would, of course, be equal to the last entry (15).

In all three situations, it will usually be found desirable to employ the volume sampling techniques described above merely to get volume/basal area ratios which are applicable to a larger sample of basal area which has been stratified by species and height. A brief discussion of this system may be found on page 7 of Shortcuts for Cruisers and Scalers (12).

All the examples above have employed a taper-step of 2 inches and unit-height of 4 feet because such interval and unit are most convenient for United States measure, with stump height occurring 1 unit below breast height. However, Appendix A also gives A, B, C coefficients for taper-step of 1 inch and unit-height of 1 foot. Such interval and unit are recommended only with very accurate dendrometers or tree diagrams where unusually precise measures are desired; 4 rather than 1 unit-heights will occur between d.b.h. and stump-height. Also, Appendix A gives A, B, C coefficients for the most convenient interval and unit in the metric system: taper-step of 5 centimeters and unit-height of 1 meter. Assuming 1 unit-height below European breast-height of 1.3 meters would imply a stump height of about 30 centimeters. Of course, the retention of decimal fractions of a meter in height measurements would not change the coefficients. No board-foot volume coefficients were provided for metric intervals, since such units are not popular in countries using the metric system.

A, B, C coefficients for intervals and units other than those tabulated may be derived from the basic formulae given in Appendix E.

GIANT-TREE TABLE FOR SURFACE AND VOLUME

Existing tables of volume (cubic-foot or board-foot) are unsatisfactory in that none of them gives volumes for every combination of length and diameter (measured in fractional inches) apt to be encountered in practice. To overcome this drawback without inflating table size excessively, an accumulative table is needed. The author calculated values in Appendix B *ab initio*, retaining 6 to 9 significant digits until the final rounding to 5 digits. All columns have been checked by summation formulae involving the sums of numbers in arithmetic progression, the sums of their squares, and the sums of their cubes. Values for the International log rule with 1/4-inch kerf are based on the formula first published by H. H. Chapman (7), i.e., board feet = $.904762\ (.22D^2 - .71\ D)$ for a 4-foot section, with taper allowance of 1/2-inch for each 4 feet. To minimize accumulative and rounding error, however, the author expanded this to a form in which taper was an implicit joint function of length and diameter (12).

In effect, Appendix B is a huge upright peeled log or tree, standing on a 50-inch base and tapering upward for 400 feet at the rate of 1/8-inch per foot. To use it, enter at the desired small-end d.i.b., mark the surface or volume found opposite this diameter, move down the column a distance equal to the desired log length, mark the surface or volume found there, and subtract the marked values.

As an example, the board-foot volume of a log 38-1/4 inches in diameter and 81 feet long would be:

$$\begin{array}{r} 8396 \\ -\ 1447 \\ \hline 6949 \end{array} \text{ board feet (Int. 1/4-inch)}$$

Cubic volume of this log would be

$$\begin{array}{r} 1004.1 \\ -\ 171.6 \\ \hline 832.5 \end{array} \text{ cubic feet}$$

Surface of this log would be:

$$\begin{array}{r} 1085.9 \\ -\ 167.4 \\ \hline 918.5 \end{array} \text{ square feet}$$

Of course it is desirable to measure more than just one diameter in long logs or trees, but occasionally that is all that it is feasible to obtain, and a taper-assumption of 1/8-inch per foot may not be far amiss.

A particularly handy use of the giant-tree table is when tree d.i.b. at the top of the first 16-foot log (or other reference point) is measured or estimated. The extra column of tabular length 0 - 80 feet at the end of Appendix B can be cut out and used as a sliding scale with the table. If it is laid over the regular length column (positioned between the d.i.b. column and the surface column) with 16 feet on the sliding scale opposite the measured d.i.b., the volume between any specified heights above stump on the tree may be computed by subtracting the volumes which appear opposite these heights. As an example, if d.i.b. at the top of the first 16-foot log is 17-1/8 inches, the diameter, height, and cubic volume readings will look as follows after the sliding length (or height) scale is positioned:

	d.i.b. Inches	Sliding height scale Feet	Cubic volume Cubic feet
		etc.	etc.
		19	1 749.7
		18	1 748.2
	17	17	1 746.6
Set 16 feet at 17-1/8 inches	→	← 16	1 745.0
		15	1 743.4
		14	1 741.8
		13	1 740.1
		12	1 738.4
		11	1 736.7
		10	1 735.0
	18	9	1 733.2
		8	1 731.4
		7	1 729.6
		6	1 727.8
		5	1 726.0
		4	1 724.1
		3	1 722.2
		2	1 720.2
	19	1	1 718.3
		0	1 716.3

The volume in the 0 - 8 foot portion of the tree would be:

$$\begin{array}{r} 1\,731.4 \\ -\ 1\,716.3 \\ \hline 15.1 \end{array} \text{ cubic feet}$$

while the volume in the 8 - 18 foot portion of the tree would be:

$$\begin{array}{r} 1\,748.2 \\ -\ 1\,731.4 \\ \hline 16.8 \end{array} \text{ cubic feet}$$

Higher portions not shown could be handled similarly. Grades or use-classes could be assigned in the field to sections between various tree heights, and the actual differencing could be done later on bookkeeping machines or punched-card machines.

When used as above, the giant-tree table has most of the advantages of form-class volume tables with arbitrary upper-log tapers, yet only one table is needed, and that table is in a form facilitating breakdown of the tree into portions of varying lengths for each of which lengths both cubic and board foot volumes can be easily computed. If cubic volume, surface, and length of portions of a tree are known, they can be converted to International 1/4-inch, Scribner, or Doyle volumes by coefficients in Appendix E.

It is obvious that results will not be as accurate as those determined earlier by the new height-accumulation method, because the giant-tree table assumes taper of 1/8 inch per foot between diameter measurements, whereas the height-accumulation method utilizes actual taper. Appendix B is very handy, however, in that it is equally simple to get surface or volume for a log or tree of <u>any</u> length, by entering with a single diameter (to the nearlest 1/8 inch) at any specified point on the log or tree (small end, large end, middle, or any place else). A 1/8-inch (rather than a 1/10-inch) diameter interval is implicit in International 1/4-inch log rule assumptions where lengths are desired in whole feet.

TREE TAPER AND SHAPE

When viewed from the butt, the taper of a tree can be defined as its loss in diameter divided by the length affected. The shape of different portions of a tree can be visualized as a function of taper. Where taper tends to increase, tree shape resembles a paraboloid; where it tends to remain constant, tree shape resembles a conoid; where it tends to decrease, tree shape resembles a neiloid. The corresponding profiles of the surfaces of these tree shapes viewed from outside the tree would be convex, linear, and concave.

In general, trees tend to be neiloidal till butt swell disappears, then conoidal, and finally paraboloidal in the upper portions. This fact is helpful where d.b.h., merchantable height, and merchantable top (d.o.b.) have been measured, and where it is desired to distribute taper between d.b.h. and top d.o.b. by eye for use in the height-accumulation technique. Suppose a tree 18 inches in d.b.h. has 16 four-foot sections from a 1/2-foot stump to an 8-inch merchantable top d.o.b. It can usually be arbitrarily assumed that there is a two-inch diameter increase and one four-foot section below d.b.h. This would leave 15 sections above d.b.h. to be distributed among $\frac{18 - 8}{2} = 5$ two-inch taper-steps. An absolutely conoidal taper (except for the neiloidal butt swell) would be recorded as:

$$1 : \overbrace{3 : 3 : 3 : 3 : 3}^{\substack{\text{5 taper-steps above d.b.h.} \\ \text{containing 15 four-foot sections}}}$$

However, lacking any intermediate measures, the eye may still detect a convex (or paraboloidal) tendency near the top, and a better estimate of composite tree shape would be:

$$1 : \overbrace{4 : 4 : 3 : 2 : 2}^{\substack{\text{5 taper-steps above d.b.h.} \\ \text{containing 15 four-foot sections}}}$$

When only d.b.h. and merchantable height are estimated, and no information is available on top d.o.b. or shape above d.b.h., a crude conoidal assumption has often been used in volume-table work (especially with hardwoods) with reasonably satisfactory results. The International log rule and the giant-tree table discussed earlier are examples of such an assumption (2 inches of taper per 16 feet). If a taper assumption of slightly less than 2 inches per 16 feet were deemed adequate for the hypothetical tree above, the estimate of tree shape for use in the height-accumulation technique would be:

$$1 : \overbrace{4 : 4 : 4 : 3}^{\substack{\text{use four units of length per taper-step,} \\ \text{until total 15 units is exhausted.}}}$$

The recent popularity of butt-log form-class (i.e., the ratio $\frac{\text{d.i.b. at top of first 16 foot log}}{\text{d.b.h.}}$ as a variable in volume tables is based on the obvious fact that most volume and value in trees of moderate size is contained in the butt log. However, this butt-log form-class is usually either estimated or assumed, as is shape of the first log and taper above the first log. If d.o.b. of the butt log is measured in several places, and if $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratios for a species group do not vary excessively, the height-accumulation technique will permit more accurate description of both the lower and upper portions of trees than will the use of form-class volume tables which assume arbitrary butt-log shape and upper-log tapers. In addition, the height-accumulation technique is much better adapted to grading portions of the tree and to business-machine compilation.

If the effort involved in getting objective height-accumulation measurements is deemed excessive, the height-accumulation technique can be adapted to the same sort of eye-estimates and assumptions as are involved in the usual use of butt-log form-class volume tables. Below is an illustration showing how height-accumulation information about tree butts implicitly includes eye-estimated butt-log form-class information, plus additional shape and taper information. For convenience, a taper-step of 2 inches, a unit-height of 4 feet, and a $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio of .90 have been employed; a different $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ relationship would, of course, lead to different form-class values.

Lengths (number of 4-ft. sections per 2 inches of taper)	Taper from d.b.h. to d.o.b. at top of first 16-ft. log		Implied butt-log form-class of various d.b.h.'s			
			10 ins. d.b.h.	20 ins. d.b.h.	30 ins. d.b.h.	40 ins. d.b.h.
	- - Inches - -		- - - - - - Percent	- - - - -		
1:7	.9		82	86	87	88
1:6	1.0	(1)	81	86	87	88
1:5	1.2		79	85	86	87
1:4	1.5		76	83	86	87
1:3	2.0	(2)	72	81	84	86
1:2:5	2.4		68	79	83	85
1:2:4	2.5		68	79	83	84
1:2:3 or 1:1:6	2.7		66	78	82	84
1:1:5	2.8		65	77	82	84
1:2:2 or 1:1:4	3.0	(3)	63	76	81	83
1:1:3	3.3		60	75	80	82
1:2:1 or 1:1:2	4.0	(4)	54	72	78	81
1:1:1:5	4.4		50	70	77	80
1:1:1:4	4.5		50	70	76	80
1:1:1:3	4.7			69	76	80
1:1:1:2	5.0	(5)		68	75	79
1:1:1:1	6.0	(6)		63	72	76
1:0:1:1:2	7.0	(7)		58	69	74
1:0:1:1:1	8.0	(8)		54	66	72
1:0:0:1:1:2	9.0	(9)		50	63	70
1:0:0:1:1:1	10.0	(10)			60	68

Any forester who prides himself on being able to estimate butt-log form-class can readily adapt his talent to the height-accumulation techniques; he can also check his eye-estimates readily with one of the dendrometers discussed earlier. Pole calipers or tapes of various kinds (8)(9) are already in existence, and are well adapted to checking tapers on the most important portion of a tree--its butt log. Probably the most convenient hand-held dendrometer for use on upper as well as lower portions of the tree would be a slope-rotated wedge-prism (3) on which height or slope can be read. A horizontal target of adjustable width attached to d.b.h. will allow the observer to position himself or adjust his instrument so that the split-image of the target just fails to overlap. He can then slowly raise his line of sight up the tree trunk, and the height where split-image of the trunk just fails to overlap will be where d.o.b. equals the desired width for which the target has been adjusted.

It will be found convenient to have the hypsometer scale graduated to read height in number of four-foot sections with a 40-foot base-line assumed, and to have the rotating wedge-prism manufactured or adjusted to effect a maximum deviation of 143.25 minutes (equivalent to about 4.167 prism-diopters or enough to exactly juxtapose the direct and the split image of a 20-inch horizontal target at a distance of 40 feet).

In using such an instrument with a tree 20 inches in d.b.h. on flat terrain, the observer would successively occupy points 40, 36, 32, 28, 24, 20 feet, etc., distance from tree center and would ascertain at each point the height at which the split image of the upper bole pulled apart, or separated. In each case, this figure will be height read from the hypsometer scale multiplied by an appropriate factor such as 1.0, .9, .8, .7, .6, .5, etc. (computed as $\left(\frac{\text{actual distance to d.b.h.}}{\text{base for which hypsometer was graduated}}\right)$.

The reader can easily infer for himself modifications of this technique necessary where trees are more or less than 20 inches in d.b.h., where the terrain slopes, or where trees lean.

Technicians studying the effect of erroneous assumptions as to taper or shape may be interested in Appendix D, which helps in visualizing how scaled volume estimates employing erroneous log-rule assumptions may be improved by shortening the interval between measured diameters.

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APPENDIX A

Height-accumulation coefficients converting $A\Sigma H' + B\Sigma H + C\Sigma L$ to surface or volume under various assumptions as to taper-step, unit-height, and mean ratio: d.i.b./d.o.b.

Given:
Taper-step = 2 inches
Unit-height = 4 feet

Mean ratio: d.i.b./d.o.b.	Surface coefficients for square feet		
	A	B	C
1.00	0	2.09	1.047
.95	0	1.99	.995
.90	0	1.88	.942
.85	0	1.78	.890

Mean ratio: d.i.b./d.o.b.	Volume coefficients for cubic feet		
	A	B	C
1.00	.175	0	.0291
.95	.158	0	.0263
.90	.141	0	.0236
.85	.126	0	.0210

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (International 1/4")		
	A	B	C
1.00	1.59	-1.48	-.308
.95	1.44	-1.41	-.296
.90	1.29	-1.34	-.284
.85	1.15	-1.26	-.270

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (Scribner)		
	A	B	C
1.00	1.58	-1.78	-1.08
.95	1.43	-1.69	-1.06
.90	1.28	-1.61	-1.04
.85	1.14	-1.52	-1.02

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (Doyle)		
	A	B	C
1.00	2.00	-5.00	4.00
.95	1.80	-4.75	4.09
.90	1.62	-4.50	4.19
.85	1.44	-4.25	4.28

Given:
Taper-step = 5 centimeters
Unit-height = 1 meter

Mean ratio: d.i.b./d.o.b.	Surface coefficients for square meters		
	A	B	C
1.00	0	.157	.0785
.95	0	.149	.0746
.90	0	.141	.0707
.85	0	.134	.0668

Mean ratio: d.i.b./d.o.b.	Volume coefficients for cubic meters		
	A	B	C
1.00	.00393	0	.000654
.95	.00354	0	.000591
.90	.00318	0	.000530
.85	.00284	0	.000473

Metric equivalents:

1 centimeter = .393700 inches
1 meter = 3.28083 feet
1 sq. meter = 10.7639 sq. feet
1 cu. meter = 35.3145 cu. feet

N. B. Explanation of terms ("taper-step," "unit-height," "mean ratio $\frac{\text{d.i.b.}}{\text{d.o.b.}}$") and symbols ($\Sigma H'$, ΣH, ΣL) is given on page 4.

Other information on use of tables can be found on pages 6 and 9.

Given:
Taper-step = 1 inch
Unit-height = 1 foot

Mean ratio: d.i.b./d.o.b.	Surface coefficients for square feet		
	A	B	C
1.00	0	.262	.131
.95	0	.249	.124
.90	0	.236	.118
.85	0	.223	.111

Mean ratio: d.i.b./d.o.b.	Volume coefficients for cubic feet		
	A	B	C
1.00	.01091	0	.00182
.95	.00984	0	.00164
.90	.00884	0	.00147
.85	.00788	0	.00131

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (International 1/4")		
	A	B	C
1.00	.0995	-.185	-.0339
.95	.0898	-.176	-.0309
.90	.0806	-.167	-.0278
.85	.0719	-.158	-.0246

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (Scribner)		
	A	B	C
1.00	.0988	-.223	-.207
.95	.0892	-.212	-.203
.90	.0800	-.201	-.199
.85	.0714	-.190	-.195

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (Doyle)		
	A	B	C
1.00	.125	-.625	.125
.95	.113	-.594	.126
.90	.101	-.562	.128
.85	.090	-.531	.129

APPENDIX B

Giant-tree table: Diameter inside bark (taper: 1/8 inch per foot) with cumulative length, surface, and volume

D.I.B. (inches)	Length	Surface	Cubic volume	Int.1/4" volume
	Feet	Square feet	Cubic feet	Board feet
0	400	2 618.0	1 818.1	
1/8	399	2 618.0	1 818.1	
2/8	398	2 618.0	1 818.1	
3/8	397	2 617.9	1 818.1	
4/8	396	2 617.8	1 818.0	
5/8	395	2 617.6	1 818.0	
6/8	394	2 617.4	1 818.0	
7/8	393	2 617.2	1 818.0	
1	392	2 617.0	1 818.0	
	391	2 616.7	1 818.0	
	390	2 616.4	1 818.0	
	389	2 616.0	1 818.0	
	388	2 615.7	1 818.0	
	387	2 615.3	1 818.0	
	386	2 614.8	1 818.0	
	385	2 614.3	1 818.0	
2	384	2 613.8	1 817.9	
	383	2 613.3	1 817.9	
	382	2 612.7	1 817.9	
	381	2 612.1	1 817.9	
	380	2 611.5	1 817.8	
	379	2 610.8	1 817.8	
	378	2 610.1	1 817.7	
	377	2 609.4	1 817.7	
3	376	2 608.6	1 817.7	--
	375	2 607.8	1 817.6	--
	374	2 607.0	1 817.6	14 752
	373	2 606.1	1 817.5	14 752
	372	2 605.2	1 817.4	14 752
	371	2 604.3	1 817.4	14 752
	370	2 603.3	1 817.3	14 752
	369	2 602.3	1 817.2	14 752
4	368	2 601.3	1 817.1	14 751
	367	2 600.2	1 817.0	14 751
	366	2 599.1	1 816.9	14 751
	365	2 598.0	1 816.8	14 751
	364	2 596.8	1 816.7	14 751
	363	2 595.6	1 816.6	14 751
	362	2 594.4	1 816.5	14 750
	361	2 593.1	1 816.4	14 750
5	360	2 591.8	1 816.2	14 750
	359	2 590.5	1 816.1	14 749
	358	2 589.2	1 815.9	14 749
	357	2 587.8	1 815.8	14 748
	356	2 586.4	1 815.6	14 748
	355	2 584.9	1 815.5	14 747
	354	2 583.4	1 815.3	14 747
	353	2 581.9	1 815.1	14 746
6	352	2 580.3	1 814.9	14 745
	351	2 578.7	1 814.7	14 745
	350	2 577.1	1 814.5	14 744
	349	2 575.5	1 814.3	14 743
	348	2 573.8	1 814.1	14 742
	347	2 572.1	1 813.8	14 741
	346	2 570.3	1 813.6	14 740
	345	2 568.5	1 813.3	14 739
7	344	2 566.7	1 813.1	14 738
	343	2 564.9	1 812.8	14 737
	342	2 563.0	1 812.5	14 735
	341	2 561.1	1 812.2	14 734
	340	2 559.1	1 811.9	14 733
	339	2 557.1	1 811.6	14 731
	338	2 555.1	1 811.3	14 730
	337	2 553.1	1 810.9	14 728
8	336	2 551.0	1 810.6	14 726
	335	2 548.9	1 810.2	14 724
	334	2 546.8	1 809.9	14 723
	333	2 544.6	1 809.5	14 721
	332	2 542.4	1 809.1	14 719
	331	2 540.1	1 808.7	14 717
	330	2 537.9	1 808.3	14 714
	329	2 535.5	1 807.9	14 712
9	328	2 533.2	1 807.4	14 710
	327	2 530.8	1 807.0	14 707
	326	2 528.4	1 806.5	14 705
	325	2 526.0	1 806.1	14 702
	324	2 523.5	1 805.6	14 699
	323	2 521.0	1 805.1	14 697
	322	2 518.5	1 804.6	14 694
	321	2 515.9	1 804.0	14 691
10	320	2 513.3	1 803.5	14 688

D.I.B. (inches)	Length	Surface	Cubic volume	Int.1/4" volume
	Feet	Square feet	Cubic feet	Board feet
10				
1/8	319	2 510.7	1 803.0	14 684
2/8	318	2 508.0	1 802.4	14 681
3/8	317	2 505.3	1 801.8	14 678
4/8	316	2 502.6	1 801.2	14 674
5/8	315	2 499.8	1 800.6	14 670
6/8	314	2 497.0	1 800.0	14 667
7/8	313	2 494.2	1 799.3	14 663
11	312	2 491.3	1 798.7	14 659
	311	2 488.4	1 798.0	14 655
	310	2 485.5	1 797.3	14 651
	309	2 482.5	1 796.6	14 646
	308	2 479.5	1 795.9	14 642
	307	2 476.5	1 795.2	14 637
	306	2 473.4	1 794.5	14 633
	305	2 470.4	1 793.7	14 628
12	304	2 467.2	1 792.9	14 623
	303	2 464.1	1 792.1	14 618
	302	2 460.9	1 791.3	14 613
	301	2 457.7	1 790.5	14 607
	300	2 454.4	1 789.6	14 602
	299	2 451.1	1 788.8	14 596
	298	2 447.8	1 787.9	14 591
	297	2 444.4	1 787.0	14 585
13	296	2 441.1	1 786.1	14 579
	295	2 437.6	1 785.2	14 573
	294	2 434.2	1 784.2	14 567
	293	2 430.7	1 783.3	14 560
	292	2 427.2	1 782.3	14 554
	291	2 423.6	1 781.3	14 547
	290	2 420.0	1 780.2	14 540
	289	2 416.4	1 779.2	14 533
14	288	2 412.8	1 778.1	14 526
	287	2 409.1	1 777.1	14 519
	286	2 405.4	1 776.0	14 511
	285	2 401.6	1 774.8	14 504
	284	2 397.9	1 773.7	14 496
	283	2 394.0	1 772.6	14 488
	282	2 390.2	1 771.4	14 480
	281	2 386.3	1 770.2	14 472
15	280	2 382.4	1 769.0	14 463
	279	2 378.5	1 767.7	14 455
	278	2 374.5	1 766.5	14 446
	277	2 370.5	1 765.2	14 437
	276	2 366.4	1 763.9	14 428
	275	2 362.4	1 762.6	14 419
	274	2 358.3	1 761.2	14 410
	273	2 354.1	1 759.9	14 400
16	272	2 349.9	1 758.5	14 390
	271	2 345.7	1 757.1	14 381
	270	2 341.5	1 755.6	14 370
	269	2 337.2	1 754.2	14 360
	268	2 332.9	1 752.7	14 350
	267	2 328.6	1 751.2	14 339
	266	2 324.2	1 749.7	14 328
	265	2 319.8	1 748.2	14 317
17	264	2 315.4	1 746.6	14 306
	263	2 310.9	1 745.0	14 295
	262	2 306.4	1 743.4	14 283
	261	2 301.9	1 741.8	14 271
	260	2 297.3	1 740.1	14 260
	259	2 292.7	1 738.4	14 247
	258	2 288.1	1 736.7	14 235
	257	2 283.4	1 735.0	14 223
18	256	2 278.7	1 733.2	14 210
	255	2 274.0	1 731.4	14 197
	254	2 269.2	1 729.6	14 184
	253	2 264.4	1 727.8	14 170
	252	2 259.6	1 726.0	14 157
	251	2 254.8	1 724.1	14 143
	250	2 249.9	1 722.2	14 129
	249	2 244.9	1 720.2	14 115
19	248	2 240.0	1 718.3	14 101
	247	2 235.0	1 716.3	14 086
	246	2 230.0	1 714.3	14 071
	245	2 224.9	1 712.3	14 056
	244	2 219.8	1 710.2	14 041
	243	2 214.7	1 708.1	14 026
	242	2 209.6	1 706.0	14 010
	241	2 204.4	1 703.9	13 994
20	240	2 199.1	1 701.7	13 978

APPENDIX B (Continued)

Giant-tree table: Diameter inside bark (taper: 1/8 inch per foot) with cumulative length, surface, and volume (cont'd)

D.I.B. (inches)	Length	Surface	Cubic volume	Int. 1/4" volume
	Feet	Square feet	Cubic feet	Board feet
20				
1/8	239	2 193.9	1 699.5	13 961
2/8	238	2 188.6	1 697.3	13 945
3/8	237	2 183.3	1 695.0	13 928
4/8	236	2 177.9	1 692.7	13 911
5/8	235	2 172.6	1 690.4	13 894
6/8	234	2 167.1	1 688.1	13 876
7/8	233	2 161.7	1 685.7	13 859
21	232	2 156.2	1 683.4	13 841
	231	2 150.7	1 680.9	13 822
	230	2 145.1	1 678.5	13 804
	229	2 139.6	1 676.0	13 785
	228	2 134.0	1 673.5	13 766
	227	2 128.3	1 671.0	13 747
	226	2 122.6	1 668.4	13 728
	225	2 116.9	1 665.8	13 708
22	224	2 111.2	1 663.2	13 688
	223	2 105.4	1 660.5	13 668
	222	2 099.6	1 657.8	13 648
	221	2 093.8	1 655.1	13 627
	220	2 087.9	1 652.4	13 606
	219	2 082.0	1 649.6	13 585
	218	2 076.0	1 646.8	13 563
	217	2 070.1	1 644.0	13 542
23	216	2 064.1	1 641.1	13 520
	215	2 058.0	1 638.2	13 497
	214	2 051.9	1 635.3	13 475
	213	2 045.8	1 632.3	13 452
	212	2 039.7	1 629.3	13 429
	211	2 033.5	1 626.3	13 406
	210	2 027.3	1 623.2	13 382
	209	2 021.1	1 620.1	13 358
24	208	2 014.8	1 617.0	13 334
	207	2 008.5	1 613.8	13 310
	206	2 002.2	1 610.6	13 285
	205	1 995.8	1 607.4	13 260
	204	1 989.4	1 604.2	13 235
	203	1 983.0	1 600.9	13 209
	202	1 976.5	1 597.5	13 184
	201	1 970.1	1 594.2	13 158
25	200	1 963.5	1 590.8	13 131
	199	1 957.0	1 587.4	13 105
	198	1 950.4	1 583.9	13 078
	197	1 943.7	1 580.4	13 050
	196	1 937.1	1 576.9	13 023
	195	1 930.4	1 573.3	12 995
	194	1 923.7	1 569.7	12 967
	193	1 916.9	1 566.1	12 939
26	192	1 910.1	1 562.4	12 910
	191	1 903.3	1 558.7	12 881
	190	1 896.4	1 555.0	12 852
	189	1 889.5	1 551.2	12 822
	188	1 882.6	1 547.4	12 792
	187	1 875.7	1 543.5	12 762
	186	1 868.7	1 539.7	12 731
	185	1 861.7	1 535.7	12 700
27	184	1 854.6	1 531.8	12 669
	183	1 847.5	1 527.8	12 638
	182	1 840.4	1 523.7	12 606
	181	1 833.3	1 519.7	12 574
	180	1 826.1	1 515.6	12 541
	179	1 818.9	1 511.4	12 509
	178	1 811.6	1 507.2	12 476
	177	1 804.3	1 503.0	12 442
28	176	1 797.0	1 498.8	12 409
	175	1 789.7	1 494.5	12 375
	174	1 782.3	1 490.1	12 340
	173	1 774.9	1 485.8	12 306
	172	1 767.4	1 481.4	12 271
	171	1 760.0	1 476.9	12 235
	170	1 752.4	1 472.4	12 200
	169	1 744.9	1 467.9	12 164
29	168	1 737.3	1 463.3	12 127
	167	1 729.7	1 458.7	12 091
	166	1 722.1	1 454.1	12 053
	165	1 714.4	1 449.4	12 016
	164	1 706.7	1 444.7	11 978
	163	1 699.0	1 439.9	11 940
	162	1 691.2	1 435.1	11 902
	161	1 683.4	1 430.2	11 863
30	160	1 675.5	1 425.4	11 824

D.I.B. (inches)	Length	Surface	Cubic volume	Int. 1/4" volume
	Feet	Square feet	Cubic feet	Board feet
30				
1/8	159	1 667.7	1 420.4	11 785
2/8	158	1 659.8	1 415.5	11 745
3/8	157	1 651.8	1 410.4	11 705
4/8	156	1 643.9	1 405.4	11 664
5/8	155	1 635.9	1 400.3	11 623
6/8	154	1 627.8	1 395.2	11 582
7/8	153	1 619.8	1 390.0	11 541
31	152	1 611.7	1 384.8	11 499
	151	1 603.5	1 379.5	11 456
	150	1 595.4	1 374.2	11 414
	149	1 587.2	1 368.8	11 371
	148	1 578.9	1 363.5	11 327
	147	1 570.7	1 358.0	11 284
	146	1 562.4	1 352.5	11 239
	145	1 554.0	1 347.0	11 195
32	144	1 545.7	1 341.5	11 150
	143	1 537.3	1 335.9	11 105
	142	1 528.9	1 330.2	11 059
	141	1 520.4	1 324.5	11 013
	140	1 511.9	1 318.8	10 967
	139	1 503.4	1 313.0	10 920
	138	1 494.8	1 307.2	10 873
	137	1 486.2	1 301.3	10 825
33	136	1 477.6	1 295.4	10 777
	135	1 469.0	1 289.4	10 729
	134	1 460.3	1 283.4	10 680
	133	1 451.6	1 277.3	10 631
	132	1 442.8	1 271.2	10 582
	131	1 434.0	1 265.1	10 532
	130	1 425.2	1 258.9	10 482
	129	1 416.3	1 252.7	10 431
34	128	1 407.5	1 246.4	10 380
	127	1 398.5	1 240.1	10 329
	126	1 389.6	1 233.7	10 277
	125	1 380.6	1 227.3	10 224
	124	1 371.6	1 220.8	10 172
	123	1 362.5	1 214.3	10 119
	122	1 353.5	1 207.7	10 065
	121	1 344.3	1 201.1	10 011
35	120	1 335.2	1 194.5	9 957
	119	1 326.0	1 187.8	9 902
	118	1 316.8	1 181.0	9 847
	117	1 307.6	1 174.2	9 792
	116	1 298.3	1 167.4	9 736
	115	1 289.0	1 160.5	9 679
	114	1 279.6	1 153.5	9 622
	113	1 270.3	1 146.5	9 565
36	112	1 260.8	1 139.5	9 508
	111	1 251.4	1 132.4	9 450
	110	1 241.9	1 125.2	9 391
	109	1 232.4	1 118.0	9 332
	108	1 222.9	1 110.8	9 273
	107	1 213.3	1 103.5	9 213
	106	1 203.7	1 096.2	9 153
	105	1 194.1	1 088.8	9 092
37	104	1 184.4	1 081.3	9 031
	103	1 174.7	1 073.8	8 970
	102	1 165.0	1 066.3	8 908
	101	1 155.2	1 058.7	8 845
	100	1 145.4	1 051.1	8 782
	99	1 135.6	1 043.4	8 719
	98	1 125.7	1 035.6	8 655
	97	1 115.8	1 027.8	8 591
38	96	1 105.9	1 020.0	8 526
	95	1 095.9	1 012.1	8 461
	94	1 085.9	1 004.1	8 396
	93	1 075.9	996.1	8 330
	92	1 065.8	988.1	8 263
	91	1 055.7	979.9	8 197
	90	1 045.6	971.8	8 129
	89	1 035.4	963.6	8 061
39	88	1 025.2	955.3	7 993
	87	1 015.0	947.0	7 924
	86	1 004.7	938.6	7 855
	85	994.4	930.2	7 786
	84	984.1	921.7	7 715
	83	973.8	913.1	7 645
	82	963.4	904.6	7 574
	81	952.9	895.9	7 502
40	80	942.5	887.2	7 430

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APPENDIX B (Continued)

Giant-tree table: Diameter inside bark (taper: 1/8 inch per foot) with cumulative length, surface and volume (cont'd)

D.I.B (inches)	Length	Surface	Cubic volume	Int. 1/4" volume
	Feet	Square feet	Cubic feet	Board feet
40				
1/8	79	932.0	878.5	7 358
2/8	78	921.5	869.6	7 285
3/8	77	910.9	860.8	7 211
4/8	76	900.3	851.9	7 137
5/8	75	889.7	842.9	7 063
6/8	74	879.1	833.9	6 988
7/8	73	868.4	824.8	6 913
41	72	857.7	815.6	6 837
	71	846.9	806.4	6 761
	70	836.1	797.2	6 684
	69	825.3	787.9	6 607
	68	814.5	778.5	6 529
	67	803.6	769.1	6 450
	66	792.7	759.6	6 372
	65	781.7	750.1	6 292
42	64	770.7	740.5	6 213
	63	759.7	730.8	6 132
	62	748.7	721.1	6 052
	61	737.6	711.4	5 970
	60	726.5	701.5	5 888
	59	715.4	691.7	5 806
	58	704.2	681.7	5 723
	57	693.0	671.7	5 640
43	56	681.7	661.7	5 556
	55	670.5	651.6	5 472
	54	659.2	641.4	5 387
	53	647.8	631.2	5 302
	52	636.4	620.9	5 216
	51	625.0	610.5	5 129
	50	613.6	600.1	5 043
	49	602.1	589.6	4 955
44	48	590.6	579.1	4 867
	47	579.1	568.5	4 779
	46	567.5	557.9	4 690
	45	555.9	547.2	4 600
	44	544.3	536.4	4 510
	43	532.6	525.6	4 419
	42	520.9	514.7	4 328
	41	509.2	503.7	4 237
45	40	497.4	492.7	4 144
	39	485.6	481.6	4 052
	38	473.8	470.5	3 958
	37	461.9	459.3	3 865
	36	450.0	448.0	3 770
	35	438.1	436.7	3 675
	34	426.1	425.3	3 580
	33	414.2	413.9	3 484
46	32	402.1	402.4	3 387
	31	390.1	390.8	3 290
	30	378.0	379.1	3 193
	29	365.9	367.5	3 095
	28	353.7	355.7	2 996
	27	341.5	343.9	2 897
	26	329.3	332.0	2 797
	25	317.0	320.0	2 696
47	24	304.7	308.0	2 595
	23	292.4	295.9	2 494
	22	280.1	283.8	2 392
	21	267.7	271.6	2 289
	20	255.3	259.3	2 186
	19	242.8	247.0	2 082
	18	230.3	234.6	1 978
	17	217.8	222.1	1 873
48	16	205.3	209.6	1 767
	15	192.7	197.0	1 661
	14	180.1	184.3	1 554
	13	167.4	171.6	1 447
	12	154.7	158.8	1 339
	11	142.0	145.9	1 231
	10	129.3	133.0	1 122
	9	116.5	120.0	1 012
49	8	103.7	106.9	902
	7	90.8	93.8	792
	6	78.0	80.6	680
	5	65.0	67.3	568
	4	52.1	54.0	456
	3	39.1	40.6	343
	2	26.1	27.1	229
	1	13.1	13.6	115
50	0	.0	.0	0

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CUT OUT STRIP ALONG DOTTED LINES TO OBTAIN A SLIDING SCALE WHICH CAN BE POSITIONED IN BODY OF TABLE AS DESIRED

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APPENDIX C

Relative Mid-diameters and Volumes of Various Solids of Revolution For Given Ratios of Smallest Diameter (d) to Largest Diameter (D)

Ratio $\frac{d}{D}$ (Percent)	Ratio of $\frac{\text{Middle Diameter}}{D}$				Ratio of $\frac{\text{Frustum Volume}}{\text{Cylinder Volume}}$			
	Paraboloid	Conoid	Neiloid	Subneiloid*	Paraboloid	Conoid	Neiloid	Subneiloid*
	%	%	%	%	%	%	%	%
0	70.71	50.00	35.36	35.36	50.00	33.33	25.00	25.00
2.5	70.73	51.25	39.98	37.92	50.03	34.19	27.34	26.27
5	70.80	52.50	42.79	40.35	50.12	35.08	28.92	27.56
7.5	70.91	53.75	45.19	42.66	50.28	36.02	30.37	28.89
10	71.06	55.00	47.38	44.86	50.50	37.00	31.79	30.25
12.5	71.26	56.25	49.40	46.98	50.78	38.02	33.20	31.64
15	71.50	57.50	51.33	49.02	51.12	39.08	34.62	33.06
17.5	71.79	58.75	53.18	51.00	51.53	40.19	36.04	34.52
20	72.11	60.00	54.96	52.92	52.00	41.33	37.47	36.00
22.5	72.48	61.25	56.69	54.78	52.53	42.52	38.93	37.52
25	72.88	62.50	58.37	56.60	53.12	43.75	40.42	39.06
27.5	73.34	63.75	60.01	58.37	53.78	45.02	41.93	40.64
30	73.82	65.00	61.61	60.10	54.50	46.33	43.47	42.25
32.5	74.35	66.25	63.19	61.81	55.28	47.69	45.04	43.89
35	74.92	67.50	64.73	63.47	56.12	49.08	46.64	45.56
37.5	75.52	68.75	66.25	65.10	57.03	50.52	48.27	47.27
40	76.16	70.00	67.75	66.71	58.00	52.00	49.94	49.00
42.5	76.83	71.25	69.24	68.29	59.03	53.52	51.64	50.77
45	77.54	72.50	70.70	69.84	60.12	55.08	53.36	52.56
47.5	78.28	73.75	72.15	71.37	61.28	56.69	55.13	54.39
50	79.06	75.00	73.58	72.89	62.50	58.33	56.92	56.25
52.5	79.86	76.25	74.99	74.38	63.78	60.02	58.74	58.14
55	80.70	77.50	76.39	75.85	65.12	61.75	60.61	60.06
57.5	81.57	78.75	77.77	77.30	66.53	63.52	62.50	62.02
60	82.46	80.00	79.15	78.74	68.00	65.33	64.43	64.00
62.5	83.39	81.25	80.52	80.16	69.53	67.19	66.40	66.02
65	84.34	82.50	81.88	81.57	71.12	69.08	68.40	68.06
67.5	85.31	83.75	83.22	82.96	72.78	71.02	70.43	70.14
70	86.31	85.00	84.55	84.33	74.50	73.00	72.50	72.25
72.5	87.34	86.25	85.88	85.70	76.28	75.02	74.60	74.39
75	88.39	87.50	87.20	87.05	78.12	77.08	76.73	76.56
77.5	89.46	88.75	88.50	88.39	80.03	79.19	78.90	78.77
80	90.55	90.00	89.81	89.72	82.00	81.33	81.11	81.00
82.5	91.67	91.25	91.11	91.04	84.03	83.52	83.35	83.27
85	92.80	92.50	92.40	92.35	86.12	85.75	85.62	85.56
87.5	93.96	93.75	93.68	93.65	88.28	88.02	87.93	87.89
90	95.13	95.00	94.95	94.93	90.50	90.33	90.27	90.25
92.5	96.32	96.25	96.23	96.21	92.78	92.69	92.65	92.64
95	97.53	97.50	97.50	97.48	95.12	95.08	95.07	95.06
97.5	98.76	98.75	98.75	98.75	97.53	97.52	97.52	97.52
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

*A subneiloid is a fictitious solid of revolution whose volume equals that of a cylinder with diameter equal to $\frac{d + D}{2}$, and whose mid-diameter can be postulated on the assumption that Newton's prismatoidal formula holds true. Its volume is an easily calculated close approximation to that of a neiloid.