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HEIGHT ACCUMULATION, GIANT TREE, TAPER AND SHAPE

L. R. Grosenbaugh



SOUTHERN FOREST EXPERIMENT STATION

Philip A. Briegleb, Director

Forest Service, U.S. Department of Agriculture

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L. R. Grosenbaugh
Southern Forest Experiment Station

An entirely new concept of tree measurement was announced by the author in 1948 (11). Since the original theory and applications have subsequently been broadened considerably, it seems advisable to publish the entire development in readily usable form, along with other material helpful in tree measurement.

In its essence, the new concept consists of selecting tree diameters above d.b.h. in diminishing arithmetic progression (equivalent to saying that outside-bark diameter is treated as the independent variable); then tree height to each such diameter is estimated, recorded, and accumulated. This reverses the classical tree measurement concept which regarded height as the evenly-spaced independent variable, and which estimated and recorded diameter at regular intervals of height.

The greatest advantage of the new concept lies in the ease with which individual trees can be broken down into various classes of material and recombined with similar portions of other trees, using punched cards, business machines, or both. Sections varying in length and grade (from different trees) can be sorted by grade-diameter groups within which lengths will be additive. The ability of modern business machines to accumulate has been largely wasted in conventional procedures where height is treated as the independent variable. Their capability can be much more fully exploited in the new procedure using diameter as the independent variable.

Another advantage lies in the convenience of being able to use height, the sum of heights, and the sum of height-accumulations to calculate volume and surface of individual trees (or of classes **containing** portions of several trees). Final tabulations can be readily converted into surface, cubic volume, or volume by a number of log rules; no volume tables are required. Little extra work is involved in tabulating volumes to several different merchantable tops, or in segregating volume by grades.

There are already in existence dendrometers which are well adapted to locating diameters in diminishing progression along the bole of a standing tree--the Biltmore pachymeter (4), the Clark dendrometer (6), the Bitterlich relascope (1) and Spiegel-relascope (2), and the Bruce slope-rotated wedge-prism (3), to mention a few. However, anyone currently content to use arbitrary volume tables not based on valid

sampling of the given tree population, or to estimate butt-log form-class and accept arbitrary upper-log taper assumptions can use the new technique with the crudest instruments and still not exceed the error currently ignored or tolerated in commonly used volume-table techniques.

This paper not only describes the theory and application of the new concept but also makes available for the first time a giant-tree table, and discusses tree taper and shape.

HEIGHT ACCUMULATION--A NEW CONCEPT

If a peeled 16-foot log with a small-end d.i.b. (diameter inside bark) of 8 inches, and a large-end d.i.b. of 10 inches is stood upright on its large end, and if heights (H) are measured as the number of 4-foot sections occurring beneath diameters in the complete progressive series

2 inches, 4 inches, 6 inches, 8 inches, and 10 inches

the record will appear as follows:

d.i.b. (inches)	H Number
2	4
4	4
6	4
8	4
10	0

It will be noticed that all diameters in the series which are smaller than the small end of the log have the same height as the small end.

Now if these heights are differenced (upper minus lower) and if each difference in height is expressed as (L), the calculation will appear as follows:

d.i.b. (inches)	L Number	H
2	0	4
4	0	4
6	0	4
8	4	4
10	0	0

Finally, if heights are accumulated so that each entry indicates the progressive total (H') of all heights recorded beneath it, and L, H, and H' are then summed, the calculation will appear as follows:

d.i.b. (inches)	L	H Number	H'
2	0	4	16
4	0	4	12
6	0	4	8
8	4	4 = Σ L	4
10	0	0	0
	4 = Σ L	16 = Σ H	40 = Σ H'

Now these three sums can be converted to units of volume (or surface) by referring to Appendix A, Where taper-step is 2 inches, where height is expressed in 4-foot units, and where $\frac{d.i.b.}{d.o.b.} = 1$ (i.e., where it is not necessary to correct for bark included in diameter measurements), the board-foot volume by the Doyle log rule can be calculated as: $2\sum H' - 5\sum H + 4\sum L = 80 - 80 + 16 = 16$ bd. ft. This is identical with the volume obtained by the conventional Doyle rule for 16-foot logs: $(d.i.b. - 4)^2 = 16$. The theory behind all this is discussed in Appendix E.

Of course, the height-accumulation technique offers no advantage over the conventional method in the simple case above, but the new technique has real utility in more complicated timber-cruising or tree-measurement situations.

This technique can be applied when d.o.b. (diameter outside bark) is measured but volume inside bark is desired. For purposes of illustration, it will be postulated that bark constitutes 10 percent of d.o.b. (i.e., that mean $\frac{d.i.b.}{d.o.}$ ratio is .90), that all heights are measured above a 1/2-foot stump whose d.o.b. is assumed to be 2 inches greater than d.b.h., that it has been decided to adopt a taper-step of 2 inches, and that lengths, heights, and height-accumulations will be expressed as number of 4-foot sections.

The heights to regularly diminishing diameters of a 20-inch tree with five 16-foot logs (or twenty 4-foot sections) to the limit of merchantability might be estimated as in the boxed portion of the following tabulation:

	d.o.b. (inches)	L Number of	H 4-foot sections	H'
	2			
	4	00	20	143
	6			123
	8	0	20	103
	0			83
merch. top	10	4	20 = $\sum L$	63
	12	2	16	43
	14	5	14	27
	16	6	9	13
	18	2	3	4
d.b.h.	20	1	1	1
		20 = $\sum L$:	143 = $\sum H$:	603 = $\sum H'$:

Note that there must be an H and an H' for every 2-inch diameter step in a complete progression between (but excluding) stump and zero, even though no merchantable material is found outside the box beyond the merchantable top d.o.b. of 10-inches. Merchantable height must be repeated for each progressive diameter less than this merchantable top,

however. Actually, if the complete H column (commencing with the H corresponding to d.b.h. and including repeated heights outside the box) is entered in a 2-register bookkeeping machine which can transfer subtotals from first to second register without clearing, then $\sum H$ and $\sum H'$ can be automatically and simultaneously computed, and $\sum L$ will be the last entry in the H column. Some people may prefer to invert the column of diameters so that the corresponding H and H' columns progress downward as on the bookkeeping machine tape, but this is merely a matter of personal preference, as long as the arithmetic accumulation proceeds in a direction which is up the tree.

Thus, $\sum H' = 603$, $\sum H = 143$, and $\sum L = 20$ for a 20-inch five-log tree described in terms of H as 20: 1: 3: 9: 14: 16: 20, with 4 more 20's needed to complete the diminishing diameter progression. Appendix A gives coefficients A, B, C appropriate for these accumulations where unit-height is 4 feet, where taper-step is 2 inches, and where mean d.i.b. ratio is .90. The appropriate A, B, C coefficients convert the d.o.b. accumulations into surface or cubic volume (inside or outside bark) and into board-foot volume according to International 1/4-inch log rule, Scribner formula log rule, or Doyle log rule. This is demonstrated below for surface, cubic volume, and International 1/4-inch volume:

$$\text{Surface inside bark} = 1.88(143) + .942(20) = 288 \text{ sq. ft. i.b.}$$

$$\text{Cubic vol. inside bark} = .141(603) - .0236(20) = 85.5 \text{ cu. ft. i.b.}$$

$$\text{Int. 1/4-inch vol.} = 1.29(603) - 1.34(143) - .284(20) = 580 \text{ bd. ft.}$$

Although the simple tally and bookkeeping-machine technique just outlined is well adapted to many situations, a more useful method involves punch-card breakdown of the tree into sections classified by grade (or utility) and d.o.b.. Lengths of such sections can then be machine-sorted and accumulated by grade-diameter classes, and need not be converted to volume till the last step. If mark-sensing is employed, more than one mark-sensed card may be needed per tree, depending on how much information is desired. It is assumed that plot information will be recorded on a separate mark-sensed card. In addition to plot number, species, defect class, and priority class, only d.b.h. and the graded length of each taper-step need be recorded in the field on the mark-sensed tree-card. For the tree used in the preceding example, the latter information would be tallied as d.b.h. and successive L's with their grades A, B, C, D, etc., thus:

20: 1A: 2A: 6B: 5B: 2C: 4C

In actual practice, of course, grades would be coded numerically.

This would require only 14 of the 27 columns on a mark-sensed card. Occasionally it might be more convenient for the estimator to record H instead of L, but ordinarily this would waste mark-sensed columns and would require differencing later, if a breakdown by grades

is desired. The 27-column mark-sensed cards will later be automatically punched, and in some cases may suffice for volume calculations, in conjunction with a 3-register bookkeeping machine. Generally, however, they will be used to reproduce a card for each tree section, so that these can easily be sorted and accumulated, after which $\sum L$, $\sum H$, and $\sum H'$ can be directly obtained by the progressive totalling process on some accounting machine such as the IBM 402 or 403.

To illustrate quite simply how the mechanics of the sorting and recombining procedure might work, suppose that (in addition to the 20-inch tree described above) there were also an 18-inch tree, with d.b.h. and graded lengths (L) recorded as. 18: 1A: 3B: 4B: 4C: 3D: Separate cards would be punched for each length in each tree, with a zero-length master card in each summarized d.o.b.-grade group to facilitate progressive totalling. A series of machine sorts and tabulations (by grade in each d.o.b. class) would break down the trees into their elements and would recombine and summarize them thus:

d.o.b. of sections

: Sums of
: 1st and 2nd
: progressive
: Sums: SUMS

	20	18	16	14	12	10	8	6	4	2	$\sum L$	$\sum H$	$\sum H'$
Tree #1	1A	2A	6B	5B	2C	4C	0	0	0	0	20	143	603
Tree #2		1A	3B	4B	4C	3D	0	0	0	0	15	100	394
Total	1	3	9	9	6	7	0	0	0	0	35	243	997
A	1	3	0	0	0	0	0	0	0	0	4	37	190
B			9	9	0	0	0	0	0	0	18	135	576
C					6	4	0	0	0	0	10	56	186
D						3	0	0	0	0	3	15	45
Total	1	3	9	9	6	7	0	0	0	0	35	243	997

The individual tree sums and progressive sums are not needed, except to illustrate that they agree with the later partition by grades.

The $\sum H'$, $\sum H$, and $\sum L$ for each grade can be converted to volume by using the A, B, C coefficients in Appendix A, if an appropriate $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio is chosen. Although for most purposes this ratio is adequately estimated as the sample-based ratio $\sqrt{\frac{\text{volume i.b.}}{\text{volume o.b.}}}$ with linear interpolation between tabled values of A, B, C, occasionally it may be found desirable to sample both d.i.b. and d.o.b. at regular intervals of length and to derive two ratios: $\frac{\sum(\text{d.i.b.})^2}{\sum(\text{d.o.b.})^2}$ and $\frac{\sum(\text{d.i.b.})}{\sum(\text{d.o.b.})}$. These should be

applied to outside-bark volume and surface formulae respectively, and the resultant formulae should be multiplied by the log rule formulae given in Appendix E to derive A, B, C coefficients more appropriate than those obtained by linear interpolation from Appendix A. Where volume or surface including bark is desired, the coefficients given for $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio = 1.00 are appropriate without any adjustment.

If it is desired to break down volume or surface summaries by d.o.b. class, first number each d.o.b. class in the complete series consecutively from the smallest to the largest (beginning with number 1), then accumulate the ordinals progressively from the beginning, then multiply each length (L) by its corresponding ordinal (I), or its ordinal accumulation (II), thus:

d.o.b. (inches)	(I) Numbered consecutively	(II) Accumulated successively	L	(I)(L) H	(II)(L) H'
2	1	1	0		
4	2	3	0		
6	3	6	0		
8	4	10	0	35	
10	5	15	7	36	105
12	6	21	6		126
14	7	28	9		
16	8	36	9	72	252
18	9	45	3	27	135
20	10	55	1	10	55
			$\frac{1}{35} = \sum L$	$\frac{10}{243} = \sum H$	$\frac{55}{997} = \sum H'$

Each L, when multiplied by its corresponding entry in column I, will give the appropriate H for that d.o.b., and when multiplied by its corresponding entry in column II will give the appropriate H'. Of course, cumulative volumes of material larger than any specified d.o.b. could be obtained easily by merely assigning zeros in the L column for all smaller d.o.b.'s, and then getting $\sum L$, $\sum H$, $\sum H'$ in the usual fashion. Volumes or surfaces in any desired diameter class or group of diameter classes are then obtained by multiplying H', H, L or their desired sums by the A, B, C coefficients from Appendix A in the usual fashion.

One shortcut might be mentioned which frequently may furnish an acceptable approximation. Where the average $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio is exactly .932 (or where $\frac{\text{volume inside bark}}{\text{volume outside bark}}$ ratio is .869), if taper-step of 2 inches and unit-height of 4 feet have been adopted, the board-foot volume by the International 1/4-inch log rule can be easily calculated, thus:

$$\text{Board feet (Int. 1/4")} = (1.38) \left(\sum H' - \sum H - \frac{\sum L}{5} \right)$$

For the tree described on page 4, this would be:

$$(1.38)\left(603 - 143 - \frac{20}{5}\right) = (1.38)(456) = 629 \text{ bd. ft.}$$

There are three general situations in which slightly different variations of the height-accumulation technique may be found useful:

(1) Where it is desired to grade different portions of the sample tree, the entire job should be done on punched cards, preferably with the field tally made directly on mark-sensed cards. Trees should be graded and described in terms of d.b.h. and a series of graded L's (rather than H's), thus:

18: 1A: 3B: 4B: 4C: 3D

Separate cards should be reproduced for each d.o.b. starting with d.b.h. = 18 inches, and after desired sorts are made, an IBM 402 or 403 **accounting** machine will accumulate $\sum L$, $\sum H$, $\sum H'$ for each class whose volume is desired, provided that L = 0 is supplied for any d.o.b. missing from the complete progression.

(2) Where it is not desired to grade portions of a sample tree, an entire tree can be punched on an individual card, with each H (or L) recorded under its appropriate d.o.b. field starting with d.b.h. (e.g., starting with 18 inches for tree #2 in the tabulation on page 6), thus:

1: 4:8: 12:15: (with 15: 15: 15: 15 completing the H series)

or 1:3: 4: 4: 3: (with 0: 0: 0: 0 completing the L series)

Each class for which a separate volume is desired will **have** H (or L) summarized for each d.o.b. field occurring in it. The H (or L) sequence (including repeat heights or zeros needed to complete the -progression) for each class can then be entered in a 2- (or 3-) **register bookkeeping** machine able to transfer subtotals from register to register without clearing. Examples of suitable 2-register bookkeeping machines in which the H's can be entered are the Burroughs Sensimatic F-50, or the **National** Cash -Register 30210 (1-6). Examples of suitable multiple register machines in which the L's can be entered are Burroughs Sensimatic F-200, or National Cash Register 30412 (17) or 3100, any of which have more than the 3 registers needed.

(3) Where it is not desired to use punched cards at all, tally for each sample tree will consist of its d.b.h. followed by a series of heights, thus:

18: 1: 4: 8: 12: 15

With a taper-step (T) = 2 inches and d.b.h. of 18 inches, there should be d.b.h. = 9 entries, so 1, 4, 8, 12, and 15, 15, 15, 15, 15 should be

entered in a 2-register bookkeeping machine to obtain $\sum H$ and $\sum H'$ for the individual tree; $\sum L$ would, of course, be equal to the last entry (15).

In all three situations, it will usually be found desirable to employ the volume sampling techniques described above merely to get volume/basal area ratios which are applicable to a larger sample of basal area which has been stratified by species and height. A brief discussion of this system may be found on page 7 of Shortcuts for Cruisers and Scalers (12).

All the examples above have employed a taper-step of 2 inches and unit-height of 4 feet because such interval and unit are most convenient for United States measure, with stump height occurring 1 unit below breast height. However, Appendix A also gives A, B, C coefficients for taper-step of 1 inch and unit-height of 1 foot. Such interval and unit are recommended only with very accurate dendrometers or tree diagrams where unusually precise measures are desired; 4 rather than 1 unit-heights will occur between d.b.h. and stump-height. Also, Appendix A gives A, B, C coefficients for the most convenient interval and unit in the metric system: taper-step of 5 centimeters and unit-height of 1 meter.. Assuming 1 unit-height below European breast-height of 1.3 meters would imply a stump height of about 30 centimeters. Of course, the retention of decimal fractions of a meter in height measurements would not change the coefficients. No board-foot volume coefficients were provided for metric intervals, since such units are not popular in countries using the metric system.

A, B, C coefficients for intervals and **units** other than those tabulated may be derived from the basic *formulae* given in Appendix E.

GIANT-TREE TABLE FOR SURFACE AND VOLUME

Existing tables of volume (cubic-foot or board-foot) are unsatisfactory in that none of them gives volumes for every combination of length and diameter (measured in fractional inches) apt to be encountered in practice. To overcome this drawback without inflating table size excessively, an accumulative table is needed. The author calculated values in Appendix B ab initio, retaining 6 to 9 significant digits until the final rounding to 5 digits. All columns have been checked by summation formulae involving the sums of numbers in arithmetic progression, the sums of their squares, and the sums of their cubes. Values for the International log rule with 1/4-inch kerf are based on the formula first published by H. H. Chapman (7), i.e., board feet = $.904762 (.22D^2 - .71D)$ for a 4-foot section, with taper allowance of 1/2-inch for each 4 feet. To minimize accumulative and rounding error, however, the author expanded this to a form in which taper was an implicit joint function of length and diameter (12).

In effect, Appendix B is a huge upright peeled log or tree, standing on a 50-inch base and tapering upward for 400 feet at the rate of 1/8-inch per foot. To use it, enter at the desired small-end d.i.b., mark the surface or volume found opposite this diameter, move down the column a distance equal to the desired log length, mark the surface or volume found there, and subtract the marked values.

As an example, the board-foot volume of a log 38-1/4 inches in diameter and 81 feet long would be:

$$\begin{array}{r} 8396 \\ - 1447 \\ \hline 6949 \text{ board feet (Int. } 1/8\text{-inch)} \end{array}$$

Cubic volume of this log would be

$$\begin{array}{r} 1004.1 \\ - 171.6 \\ \hline 832.5 \text{ cubic feet} \end{array}$$

Surface of this log would be:

$$\begin{array}{r} 1085.9 \\ - 167.4 \\ \hline 918.5 \text{ square feet} \end{array}$$

Of course it is desirable to measure more than just one diameter in long logs or trees, but occasionally that is all that it is feasible to obtain, and a taper-assumption of 1/8-inch per foot may not be far amiss.

A particularly handy use of the giant-tree table is when tree d.i.b. at the top of the first 16-foot log (or other reference point) is

measured or estimated. The extra column of tabular length 0 - 80 feet at the end of Appendix B can be cut out and used as a sliding scale with the table. If it is laid over the regular length column (positioned between the d.i.b. column and the surface column) with 16 feet on the sliding scale opposite the measured d.i.b., the volume between any specified heights above stump on the tree may be computed by subtracting the volumes which appear opposite these heights. As an example, if d.i.b. at the top of the first 16-foot log is 17-1/8 inches, the diameter, height, and cubic volume readings will look as follows after the sliding length (or height) scale is positioned:

	<u>d.i.b.</u> <u>Inches</u>	Sliding height scale <u>F e e t</u>	<u>Cubic volume</u> <u>Cubic feet</u>
		etc.	etc.
		19	1 749.7
		18	1 748.2
		17	1 766.0
Set 16 feet at 17-1/8 inches	17 →	← 16	1745.0
		15	1743.4
		14	1741.8
		13	1740.1
		12	1738.4
		11	1736.7
		10	1735.0
	18	9	1733.2
		8	1731.4
		7	1729.6
		6	1727.8
		5	
		4	1 726.0 724.1
		3	1 722.2
		2	1 720.2
	19	1	1 718.3
		0	1 716.3

The volume in the 0 - 8 foot portion of the tree would be:

$$\begin{array}{r}
 1731.4 \\
 - 1716.3 \\
 \hline
 15.1 \text{ cubic feet}
 \end{array}$$

while the volume in the 8 - 18 foot portion of the tree would be:

$$\begin{array}{r}
 1748.2 \\
 - 1731.4 \\
 \hline
 16.8 \text{ cubic feet}
 \end{array}$$

Higher portions not shown could be handled similarly. Grades or use-classes could be assigned in the field to sections between various tree heights, and the actual differencing could be done later on bookkeeping machines or punched-card machines.

When used as above, the giant-tree table has most of the advantages of form-class volume **tables** with arbitrary upper-log tapers, yet only one table is needed, and that table is **in** a form facilitating breakdown of the tree into portions of varying lengths for each of which lengths both cubic and board foot volumes can be easily computed. If cubic volume, surface, and length of portions of a tree are known, they can be converted to International **1/4-inch, Scribner,** or Doyle volumes by coefficients in Appendix E.

It is obvious that results will not be as accurate as those determined earlier by the new height-accumulation method, because the giant-tree table assumes taper of **1/8** inch per foot between diameter measurements, whereas the height-accumulation method utilizes actual taper. Appendix B is very handy, however, in that it is equally simple to get surface or volume for a log or tree of **any** length, by entering with a single diameter (to the **nearest 1/8 inch**) at any specified point on the log or tree (small end, large end, middle, or any place else). A **1/8-inch** (rather than **a 1/10-inch**) diameter interval is implicit in International 1/k-inch log rule assumptions where lengths are desired in whole feet.

TREE TAPER AND SHAPE

When viewed from the butt, the taper of a tree can be defined as its loss in diameter divided by the length affected. The shape of different portions of a tree can be visualized as a function of taper. Where taper tends to increase, tree shape resembles a paraboloid; where it tends to remain constant, tree shape resembles a conoid; where it tends to decrease, tree shape resembles a neiloid. The corresponding profiles of the surfaces of these tree shapes viewed from outside the tree would be convex, linear, and concave.

In general, trees tend to be neiloidal till butt swell dis-appears, then conoidal, and finally paraboloidal in the upper portions. This fact is helpful where d.b.h., merchantable height, and merchantable top (d.o.b.) have been measured, and where it is desired to distribute taper between d.b.h. and top d.o.b. by eye for use in the height-accumulation technique. Suppose a tree 18 inches in d.b.h. has 16 four-foot sections from a 1/2-foot stump to an 8-inch merchantable top d.o.b. It can usually be arbitrarily assumed that there is a two-inch diameter increase and one four-foot section below d.b.h. This would leave 15 sections above d.b.h. to be distributed among $\frac{18 - 8}{2} = 5$ two-inch taper-steps. An absolutely conoidal taper (except for the neiloidal butt swell) would be recorded as:

5 taper-steps above d.b.h.
containing 15 four-foot sections
 $\overbrace{1 : 3 : 3 : 3 : 3 : 3}$

However, lacking any intermediate measures, the eye may still detect a convex (or paraboloidal) tendency near the top, and a better estimate of composite tree shape would be:

5 taper-steps above d.b.h.
containing 15 four-foot sections
 $\overbrace{1 : 4 : 4 : 3 : 2 : 2}$

When only d.b.h. and merchantable height are estimated, and no information is available on top d.o.b. or shape above d.b.h., a crude conoidal assumption has often been used in volume-table work (especially with hardwoods) with reasonably satisfactory results. The International log rule and the giant-tree table discussed earlier are examples of such an assumption (2 inches of taper per 16 feet). If a taper assumption of slightly less than 2 inches per 16 feet were deemed adequate for the hypothetical tree above, the estimate of tree shape for use in the height-accumulation technique would be:

use four units of length per taper-step,
until total 15 units is exhausted.
 $\overbrace{1 : 4 : 4 : 4 : 3}$

The recent popularity of butt-log form-class (i.e., the ratio $\frac{\text{d.i.b. at top of first 16 foot log}}{\text{d.b.h.}}$ as a variable in volume tables is based on the obvious fact that most volume and value in trees of moderate size is contained in the butt log. However, this butt-log form-class is usually either estimated or assumed, as is shape of the first log and taper above the first log. If d.o.b. of the butt log is measured in several places, and if $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratios for a species group do not vary excessively, the height-accumulation technique will permit more accurate description of both the lower and upper portions of trees than will the use of form-class volume tables which assume arbitrary butt-log shape and upper-log tapers. In addition, the height-accumulation technique is much better adapted to grading portions of the tree and to business-machine compilation.

If the effort involved in getting objective height-accumulation measurements is deemed excessive, the height-accumulation technique can be adapted to the same sort of eye-estimates and assumptions as are involved in the usual use of butt-log form-class volume tables. Below is an illustration showing how height-accumulation information about tree butts implicitly includes eye-estimated butt-log form-class information, plus additional shape and taper information. For convenience, a taper-step of 2 inches, a ~~unit-height~~ height of 4 feet, and a $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ ratio of .90 have been employed; a different $\frac{\text{d.i.b.}}{\text{d.o.b.}}$ relationship would, of course, lead to different form-class values.

Lengths (number of 4-f-t. sections per 2 inches of taper)	Taper from d.b.h. to d.o.b. at top of first 16-ft. log	Implied butt-log form-class of various d.b.h.'s			
		110 ins. d.b.h.	20 ins. d.b.h.	30 ins. d.b.h.	40 ins. d.b.h.
	Inches	Percent			
1:7	.9	82	86	87	88
1:6	1.0 (1)	81	86	87	88
1:5	1.2	79	85	86	87
1:4	1.5	76	83	86	87
1:3	2.0 (2)	72	81	84	86
1:2:5	2.4	68	79	83	85
1:2:4	2.5	68	79	83	84
1:2:3	2.7	66	78	82	84
	1:1:6				
	1:1:5	65	77	82	84
1:2:2	3.0 (3)	63	76	81	83
	1:1:4				
	1:1:3	60	75	80	82
1:2:1	4.0 (4)	54	72	78	81
	1:1:2				
1:1:1:5	4.4	50	70	77	80
1:1:1:4	4.5	50	70	76	80
1:1:1:3	4.7		69	76	80
1:1:1:2	5.0 (5)		68	75	79
1:1:1:1	6.0 (6)		63	72	76
1:0:1:1:2	7.0 (7)		58	69	74
1:0:1:1:1	8.0 (8)		54	66	72
1:0:0:1:1:2	9.0 (9)		50	63	70
1:0:0:1:1:1	10.0 (10)			60	68

Any forester who prides himself on being able to estimate butt-log form-class can readily adapt his talent to the height-accumulation techniques; he can also check his eye-estimates readily with one of the dendrometers discussed earlier. Pole calipers or tapes of various kinds (8)(P) are already in existence, and are well adapted to checking tapers on the most important portion of a tree--its butt log. Probably the most convenient hand-held dendrometer for use on upper as well as lower portions of the tree would be a slope-rotated wedge-prism (3) on which height or slope can be read. A horizontal target of adjustable width attached to d.b.h. will allow the observer to position himself or adjust his instrument so that the split-image of the target just fails to overlap. He can then slowly raise his line of sight up the tree trunk, and the height where split-image of the trunk just fails to overlap will be where d.o.b. equals the desired width for which the target has been adjusted.

It will be found convenient to have the hypsometer scale graduated to read height in number of four-foot sections with a 40-foot base-line assumed, and to have the rotating wedge-prism manufactured or adjusted to effect a maximum deviation of 143.25 minutes (equivalent to about 4.167 prism-diopters or enough to exactly juxtapose the direct and the split image of a 20-inch horizontal target at a distance of 40 feet).

In using such an instrument with a tree 20 inches in d.b.h. on flat terrain, the observer would successively occupy points 40, 36, 32, 28, 24, 20 feet, etc., distance from tree center and would ascertain at each point the height at which the split image of the upper bole pulled apart, or separated. In each case, this figure will be height read from the hypsometer scale multiplied by an appropriate factor such as 1.0, .9, .8, .7, .6, .5, etc. (computed as $\frac{\text{actual distance to d.b.h.}}{\text{base for which hypsometer was graduated}}$).

The reader can easily infer for himself modifications of **this** technique necessary where trees are more or less than 20 inches in d.b.h., where the terrain slopes, or where trees lean.

Technicians studying the effect of erroneous assumptions **as** to taper or shape may be interested in Appendix D, which helps in visualizing how scaled volume estimates employing erroneous log-rule **assump-tions** may be improved by shortening the interval between measured diameters.

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APPENDIX A

Height-accumulation coefficients converting $A\epsilon H' + B\epsilon H + C\epsilon L$ to surface or volume
under various assumptions as to taper-step, unit-height, and mean ratio: d.i.b./d.o.b.

Given:
 Taper-step = 2 inches
 Unit-height = 4 feet

Mean ratio: d.i.b./d.o.b.	Surface coefficients for square feet		
	A	B	C
1.00	0	2.09	1.047
.95	0	1.99	.995
.90	0	1.88	.942
.85	0	1.78	.890

Mean ratio: d.i.b./d.o.b.	Volume coefficients for cubic feet		
	A	B	C
1.00	.175	0	.0291
.95	.158	0	.0263
.90	.141	0	.0236
.85	.126	0	.0210

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (International 1/4")		
	A	B	C
1.00	1.59	-1.48	-.308
.95	1.44	-1.41	-.296
.90	1.29	-1.34	-.284
.85	1.15	-1.26	-.270

Mean ratio: d.o.b.	Volume coefficients for board feet (Scribner)		
	A	B	C
1.00	1.58	-1.78	-1.08
.95	1.43	-1.69	-1.06
.90	1.28	-1.61	-1.04
.85	1.14	-1.52	-1.02

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (Doyle)		
	A	B	C
1.00	2.00	-5.00	4.00
.95	1.80	-4.75	4.09
.90	1.62	-4.50	4.19
.85	1.44	-4.25	4.28

Given:
 Taper-step = 5 centimeters
 Unit-height = 1 meter

Mean ratio: d.i.b./d.o.b.	Surface coefficients for square meters		
	A	B	C
1.00	0	.157	.0785
.95	0	.149	.0746
.90	0	.141	.0707
.85	0	.134	.0668

Mean ratio: d.i.b./d.o.b.	Volume coefficients for cubic meters		
	A	B	C
1.00	.00393	0	.000654
.95	.00354	0	.000591
.90	.00318	0	.000530
.85	.00284	0	.000473

Metric equivalents:

1 centimeter = .393700 inches
 1 meter = 3.28083 feet
 1 sq. meter = 10.7639 sq. feet
 1 cu. meter = 35.3145 cu. feet

Given:
 Taper-step = 1 inch
 Unit-height = 1 foot

Mean ratio: d.i.b./d.o.b.	Surface coefficients for square feet		
	A	B	C
1.00	0	.262	.131
.95	0	.249	.124
.90	0	.236	.118
.85	0	.223	.111

Mean ratio: d.i.b./d.o.b.	Volume coefficients for cubic feet		
	A	B	C
1.00	.01091	0	.00182
.95	.00984	0	.00164
.90	.00884	0	.00147
.85	.00788	0	.00131

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (International 1/4")		
	A	B	C
1.00	.0995	-.185	-.0339
.95	.0898	-.176	-.0309
.90	.0806	-.167	-.0278
.85	.0719	-.158	-.0246

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (Scribner)		
	A	B	C
1.00	.0988	-.223	-.207
.95	.0892	-.212	-.203
.90			
.85	.0714	-.190	-.195

Mean ratio: d.i.b./d.o.b.	Volume coefficients for board feet (Doyle)		
	A	B	C
1.00	.125	-.225	.125
.95	.113	-.204	.126
.90	.101	-.182	.128
.85	.090	-.161	.129

N. B. Explanation of terms ("taper-step," "unit-height," "mean ratio $\frac{d.i.b.}{d.o.b.}$ ") and symbols ($\epsilon H'$, ϵA , ϵL) is given on page 4.

Other information on use of tables can be found on pages 6 and 9.

APPENDIX B

Giant-tree table: Diameter inside bark (taper: 1/8 inch per foot) with cumulative length, surface, and volume

D.I.B. (inches)	Length		Surface	Cubic volume	Int.1/4" volu	D.I.B. (inches)	Length		Surface	Cubic volume	Int.1/4" volu
	Feet	Square feet	Cubic feet	Board feet	Feet		Square feet	Cubic feet	Board feet		
0	400	2 618.0	1 818.1			10	319				
1/8	399	2 618.0	1 818.1			2/8	318	508.0	1 803.0	14 684	
2/8	398	2 618.0	1 818.1			3/8	317		1 802.4		
3/8	397	2 617.9	1 818.1			4/8	315	2 2 505.3	1 801.8	1800.2	14 14 678 674
4/8	396	2 617.8	1 818.0			5/8	314	2 499.8	800.6		14 670
5/8	395	2 617.6	1 818.0			6/8	312	2 497.0	1 799.3		14 667
6/8	394	2 617.4	1 818.0			7/8	313	2 494.2	1 798.7		14 663
7/8	393	2 617.2	1 818.0			11	312	2 491.3			14 659
	392	2 617.0	1 818.0				311				
	391	2 616.7	1 818.0			12	309	488.4	1777.98.0	11655.651	
	390	2 616.4	1 818.0			308	2 482.5	1 796.6		14 6646	
	389	2 616.0	1 818.0			307	479.5	1 795.9			
	388	2 615.7	1 818.0			306	476.4	1 794.2		14 14 633 637	
	387	2 615.3	1 818.0				305				14 628
	386	2 614.8	1 818.0			12	304	2 2 470.4	1 791.792		14 623
	385	2 614.3	1 818.0				303	2 464.1			
2	384	2 613.8	1 817.9			302	2 460.9	1 79792.1		1161613	
	383	2 613.3	1 817.9			301	2 457.7			14 607	
	382	2 612.7	1817.9			300	2 454.4	1 79015	789.6	14 602	
	381	2 612.1	1 817.9			299	2 451.1	1 788.8		14 596	
	380	2 611.5	1 817.8			298	2 447.8	1 787.9		14 591	
	379	2 610.8	1 817.8			297	2 444.4	1 787.0		14 585	
	378	2 610.1	1 817.7			13	296	2 441.1	1 786.1		14 579
	377	2 609.4	1 817.7				295	2 437.6	1 785.2		14 573
3	376	2 608.6	1 817.7	--		294	2 434.2	1 784.2		14 567	
	375	2 607.8	1817.6			293	2 430.7	1 783.3		141455560	
	374	2 607.0	1 817.6	14 752		292	2 427.2	1 782.3			
	373	2 606.1	1 817.5	14 752			291	2 423.6	1 781.3		14 570
	372	2 605.2	1 817.4	14 752		14	289	2 419.9	1 780.2		14 564
	371	2 604.3	1 817.4	14 752			288	2 416.4	1 779.2		14 558
	370	2 603.3	1 817.3	14 752		14	287	2 412.8	1 777.0		14 552
	369	2 602.3	1 817.2	14 752			286				14 546
4	368	2 601.3	1 817.1	14 751		14	285	2 409.1	1 776.0		14 540
	367	2 600.2	1 817.0	14 751			284	2 405.4	1 774.8		14 534
	366	2 599.1	1 816.9	14 751			283	2 401.7	1 773.7		14 528
	365	2 598.0	1 816.8	14 751			282	2 398.0	1 772.6		14 522
	364	2 596.8	1816.7	14 751			281	2 394.0	1 771.4	770.2	14 516
	363	2 595.6	1816.6	14 751			280	2 390.2	1 770.2		14 510
	362	2 594.4	1 816.5	14 750		15	279	2 386.3	769		14 504
	361	2 593.1	1 816.4	750			278	2 382.4	1 769.0		14 498
5	359	2 591.8	1 816.2	14 750			277	2 378.5	1 767.7		14 492
	358	2 590.5	1 815.9	14 749			276	2 374.6	1 766.5		14 486
	357	2 587.8	1 815.8	14 748			275	2 370.7	1 765.2		14 480
	356	2 586.4	1 815.6	14 748			274	2 366.8	1 764.0		14 474
	355	2 585.0	1 815.5	14 748			273	2 362.9	1 762.8		14 468
	354	2 583.6	1815.3	14 747			272	2 359.0	1 761.6		14 462
	353	2 582.1	1 815.1	14 746			271	2 355.1	1 760.4		14 456
	352	2 580.3	1 814.9	14 745		16	270	2 351.2	1 759.2		14 450
	351	2 578.7	1814.7	14 745			269	2 347.3	1 758.0		14 444
	350	2 577.1	1 814.3	14 744			268	2 343.4	1 756.8		14 438
	349	2 575.5	1 814.1	14 743			267	2 339.5	1 755.6		14 432
	348	2 573.8	1813.9	14 742			266	2 335.6	1 754.4		14 426
	347	2 572.1	1 813.8	14 741			265	2 331.7	1 753.2		14 420
	346	2 570.3	1813.6	14 740			264	2 327.8	1 752.0		14 414
	345	2 568.5	1 813.5	14 739			263	2 323.9	1 750.8		14 408
	344	2 566.9	1 813.2	14 738		7	262	2 320.0	1 749.6		14 402
	343	2 565.0	1 812.9	14 737			261	2 316.1	1 748.4		14 396
	342	2 563.6	1 812.5	14 735			260	2 312.2	1 747.2		14 390
	341	2 561.1	1812.2	14 734			259	2 308.3	1 746.0		14 384
	339	2 559.1	1 811.9	14 733			258	2 304.4	1 744.8		14 378
	338	2 557.1	1811.3	14 731			257	2 300.5	1 743.6		14 372
	337	2 555.1	1 811.0	14 730			256	2 296.6	1 742.4		14 366
	336	2 553.1	1 810.9	14 728			255	2 292.7	1 741.2		14 360
	335	2 551.0	1 810.6	14 726			254	2 288.8	1 740.0		14 354
	334	2 548.9	1 810.2	14 724			253	2 284.9	1 738.8		14 348
	333	2 546.6	1 809.9	14 723			252	2 281.0	1 737.6		14 342
	332	2 544.4	1 809.5	14 721			251	2 277.1	1 736.4		14 336
	331	2 542.4	1 809.1	14 719			250	2 273.2	1 735.2		14 330
	330	2 540.1	1 808.7	14 717			249	2 269.3	1 734.0		14 324
	329	2 537.8	1 808.3	14 714			248	2 265.4	1 732.8		14 318
	328	2 535.5	1 807.9	14 712			247	2 261.5	1 731.6		14 312
	327	2 533.2	1 807.4	14 710			246	2 257.6	1 730.4		14 306
	326	2 530.0	1 807.0	14 707		9	245	2 253.7	1 729.2		14 300
	325	2 528.4	1 806.5	14 705			244	2 249.8	1 728.0		14 294
	324	2 523.5	1 806.1	14 702			243	2 245.9	1 726.8		14 288
	323	2 521.0	1805.1	14 699			242	2 242.0	1 725.6		14 282
	322	2 518.5	1 804.6	14 697			241	2 238.1	1 724.4		14 276
	321	2 516.1	1 804.1	14 694			240	2 234.2	1 723.2		14 270
	320	2 513.3	1 803.5	14 688		0	239	2 230.3	1 722.0		14 264
							238	2 226.4	1 720.8		14 258
							237	2 222.5	1 719.6		14 252
							236	2 218.6	1 718.4		14 246
							235	2 214.7	1 717.2		14 240
							234	2 210.8	1 716.0		14 234
							233	2 206.9	1 714.8		14 228
							232	2 203.0	1 713.6		14 222
							231	2 199.1	1 712.4		14 216
							230	2 195.2	1 711.2		14 210
							229	2 191.3	1 710.0		14 204
							228	2 187.4	1 708.8		14 198
							227	2 183.5	1 707.6		14 192
							226	2 179.6	1 706.4		14 186
							225	2 175.7	1 705.2		14 180
							224	2 171.8	1 704.0		14 174
							223	2 167.9	1 702.8		14 168
							222	2 164.0	1 701.6		14 162
							221	2 160.1	1 700.4		14 156
							220	2 156.2	1 699.2		14 150
							219	2 152.3	1 698.0		14 144
							218	2 148.4	1 696.8		14 138
							217	2 144.5	1 695.6		14 132
							216	2 140.6	1 694.4		14 126
							215	2 136.7	1 693.2		14 120
							214	2 132.8	1 692.0		14 114
							213	2 128.9	1 690.8		14 108
							212	2 125.0	1 689.6		14 102
							211	2 121.1	1 688.4		14 96
							210				

APPENDIX B (Continued)

Giant-tree table: Diameter inside bark (taper: 1/8 inch per foot) with cumulative length, surface, and volume (cont'd)

D.I.B. (inches)	Length		Surface	Cubic volume	Int.1/4" volum	D.I.B. (inches)	Length		Surface	Cubic volume	Int.1/4" volum
	Feet	Square feet	Cubic feet	Board feet	Feet		Square feet	Cubic feet	Board feet		
20	1/8	239	2 193.9	1 699.5	13 961	30	1/8	159	1 667.7	1 420.4	11 785
	2/8	238	2 188.6	1 697.3	13 945		2/8	158	1 659.0	1 415.5	11 745
	3/8	237	2 183.3	1 695.0	13 928		3/8	157	1 651.9	1 405.4	11 705
	4/8	236	2 177.9	1 690.4	13 911		4/8	155	1 635.9	1 400.3	11 664
	5/8	235	2 172.6	1 688.1	13 894		5/8	154	1 627.8	1 395.2	11 623
	6/8	234	2 167.1	1 685.7	13 876		6/8	153	1 619.8	1 390.0	11 582
	7/8	233	2 161.7	1 683.4	13 859		7/8	152	1 611.7	1 384.8	11 541
21	232	2 156.2	1 681.0	13 841	31	151	1 603.5	1 379.5	11 499		
	230	2 150.7	1 678.5	13 822		150	1 595.4	1 374.2	11 456		
	229	2 145.1	1 676.0	13 804		149	1 587.2	1 368.8	11 414		
	228	2 139.6	1 673.5	13 785		148	1 578.9	1 363.5	11 371		
	227	2 134.0	1 671.0	13 766		147	1 570.7	1 358.0	11 327		
	226	2 128.3	1 668.4	13 747		146	1 562.4	1 352.5	11 284		
	225	2 122.6	1 665.8	13 728		145	1 554.0	1 347.0	11 239		
	224	2 116.9	1 663.2	13 708		144	1 545.7	1 341.5	11 195		
	223	2 111.2	1 660.5	13 688		143	1 537.3	1 335.9	11 150		
	222	2 105.4	1 657.8	13 668		142	1 528.9	1 330.2	11 105		
22	221	2 099.6	1 655.1	13 648	141	1 520.4	1 324.5	11 059			
	220	2 093.8	1 652.4	13 627	140	1 511.9	1 318.8	11 013			
	219	2 087.9	1 649.6	13 606	139	1 503.4	1 313.0	10 967			
	218	2 082.0	1 646.8	13 585	137	1 486.2	1 307.2	10 920			
	217	2 076.0	1 644.0	13 563	136	1 477.6	1 301.2	10 875			
	216	2 070.1	1 641.1	13 542	135	1 469.0	1 295.4	10 829			
	215	2 064.1	1 638.2	13 520	134	1 460.3	1 289.4	10 782			
	214	2 058.0	1 635.3	13 497	133	1 451.6	1 283.3	10 735			
	213	2 051.9	1 632.3	13 475	132	1 442.8	1 277.3	10 688			
	212	2 045.8	1 629.3	13 452	131	1 434.0	1 271.2	10 641			
23	211	2 039.7	1 626.3	13 429	130	1 425.2	1 265.1	10 592			
	210	2 033.5	1 623.2	13 406	129	1 416.4	1 258.9	10 542			
	209	2 027.3	1 620.1	13 382	128	1 407.5	1 252.7	10 492			
	208	2 021.1	1 617.0	13 358	127	1 398.6	1 246.4	10 441			
	207	2 014.9	1 613.8	13 334	126	1 389.6	1 240.1	10 389			
	206	2 008.5	1 610.6	13 310	125	1 380.6	1 233.7	10 337			
	205	2 002.2	1 607.4	13 285	124	1 371.6	1 227.3	10 284			
	204	1 995.8	1 604.2	13 260	123	1 362.5	1 220.9	10 231			
	203	1 989.5	1 600.9	13 235	122	1 353.5	1 214.3	10 177			
	202	1 983.0	1 597.5	13 209	121	1 344.3	1 207.7	10 123			
24	201	1 976.5	1 594.2	13 184	120	1 335.2	1 194.5	10 065			
	200	1 970.1	1 590.8	13 158	119	1 326.0	1 187.8	10 007			
	199	1 963.5	1 587.4	13 131	117	1 316.6	1 174.2	9 947			
	198	1 957.0	1 583.9	13 105	116	1 295.3	1 167.4	9 886			
	197	1 950.4	1 580.4	13 078	115	1 289.0	1 160.5	9 824			
	196	1 943.7	1 576.7	13 050	114	1 279.6	1 153.5	9 761			
	195	1 937.1	1 573.3	13 023	113	1 270.3	1 146.5	9 697			
	194	1 930.4	1 569.7	12 995	112	1 260.8	1 139.5	9 632			
	193	1 923.7	1 566.1	12 967	111	1 251.4	1 132.4	9 567			
	192	1 916.9	1 562.4	12 939	110	1 241.9	1 125.2	9 501			
25	191	1 910.1	1 558.7	12 910	109	1 232.4	1 118.0	9 435			
	190	1 903.3	1 555.0	12 881	108	1 222.9	1 110.5	9 368			
	189	1 896.5	1 551.3	12 852	107	1 213.3	1 102.6	9 301			
	188	1 889.5	1 547.5	12 822	106	1 203.7	1 094.6	9 234			
	187	1 882.6	1 543.7	12 792	105	1 194.1	1 086.8	9 167			
	186	1 875.7	1 539.7	12 762	104	1 184.4	1 078.3	9 100			
	185	1 868.7	1 535.7	12 731	103	1 174.7	1 069.8	9 032			
	184	1 861.7	1 531.8	12 700	102	1 165.0	1 061.3	8 964			
	183	1 854.6	1 527.8	12 669	101	1 155.2	1 052.7	8 896			
	182	1 847.5	1 523.7	12 638	100	1 145.4	1 044.1	8 827			
26	181	1 840.4	1 519.7	12 606	99	1 135.6	1 035.6	8 758			
	180	1 833.3	1 515.6	12 574	98	1 125.7	1 027.0	8 689			
	179	1 826.1	1 511.5	12 541	97	1 115.8	1 018.4	8 620			
	178	1 818.9	1 507.2	12 509	96	1 105.9	1 009.8	8 551			
	177	1 811.6	1 503.0	12 476	95	1 095.9	1 001.1	8 482			
	176	1 804.3	1 498.8	12 442	94	1 085.9	992.4	8 413			
	175	1 797.0	1 494.5	12 409	93	1 075.9	983.7	8 344			
	174	1 789.7	1 490.1	12 375	92	1 065.9	975.0	8 275			
	173	1 782.3	1 485.8	12 340	91	1 055.7	966.3	8 206			
	172	1 774.9	1 481.4	12 306	90	1 045.4	957.6	8 137			
27	171	1 767.4	1 476.9	12 271	89	1 035.2	948.9	8 068			
	170	1 760.0	1 472.4	12 235	88	1 025.0	940.2	8 000			
	169	1 752.4	1 467.9	12 200	87	1 014.8	931.5	7 931			
	168	1 744.9	1 463.3	12 164	86	1 004.7	922.8	7 862			
	167	1 737.3	1 458.7	12 127	85	994.5	914.1	7 793			
	166	1 729.7	1 454.1	12 091	84	984.3	905.4	7 724			
	165	1 722.1	1 449.4	12 053	83	974.1	896.7	7 655			
	164	1 714.4	1 444.7	12 016	82	963.9	888.0	7 586			
	163	1 706.7	1 439.9	11 978	81	953.7	879.3	7 517			
	162	1 699.0	1 435.1	11 940	80	943.5	870.6	7 448			
28	161	1 691.2	1 430.2	11 902	79	933.3	861.9	7 379			
	160	1 683.4	1 425.3	11 864	78	923.1	853.2	7 310			
	159	1 675.7	1 420.4	11 824	77	912.9	844.5	7 241			
	158	1 667.9	1 415.5	11 784	76	902.7	835.8	7 172			
	157	1 660.1	1 410.6	11 744	75	892.5	827.1	7 103			
	156	1 652.3	1 405.7	11 704	74	882.3	818.4	7 034			
	155	1 644.5	1 400.8	11 664	73	872.1	809.7	6 965			
	154	1 636.7	1 395.9	11 624	72	861.9	801.0	6 896			
	153	1 628.9	1 391.0	11 584	71	851.7	792.3	6 827			
	152	1 621.1	1 386.1	11 544	70	841.5	783.6	6 758			
29	151	1 613.3	1 381.2	11 504	69	831.3	774.9	6 689			
	150	1 605.5	1 376.3	11 464	68	821.1	766.2	6 620			
	149	1 597.7	1 371.4	11 424	67	810.9	757.5	6 551			
	148	1 589.9	1 366.5	11 384	66	800.7	748.8	6 482			
	147	1 582.1	1 361.6	11 344	65	790.5	740.1	6 413			
	146	1 574.3	1 356.7	11 304	64	780.3	731.4	6 344			
	145	1 566.5	1 351.8	11 264	63	770.1	722.7	6 275			
	144	1 558.7	1 346.9	11 224	62	759.9	714.0	6 206			
	143	1 550.9	1 342.0	11 184	61	749.7	705.3	6 137			
	142	1 543.1	1 337.1	11 144	60	739.5	696.6	6 068			
30	141	1 535.3	1 332.2	11 104	59	729.3	687.9	5 999			
	140	1 527.5	1 327.3	11 064	58	719.1	679.2	5 930			
	139	1 519.7	1 322.4	11 024	57	708.9	670.5	5 861			
	138	1 511.9	1 317.5	10 984	56	698.7	661.8	5 792			
	137	1 504.1	1 312.6	10 944	55	688.5	653.1	5 723			
	136	1 496.3	1 307.7	10 904	54	678.3	644.4	5 654			
	135	1 488.5	1 302.8	10 864	53	668.1	635.7	5 585			
	134	1 480.7	1 297.9	10 824	52	657.9	627.0	5 516			
	133	1 472.9	1 293.0	10 784	51	647.7	618.3	5 447			
	132	1 465.1	1 288.1	10 744	50	637.5	609.6	5 378			

(BLANK)

The formulae from which previously given functions were derived are:

$$\text{Volume (cubic feet)} = (.005454 L) \left(d + \frac{T}{2} \right)^2 + S$$

where S is $\frac{\pi^2}{4}$ for paraboloid, $\frac{T^2}{12}$ for conoid, and 0 for subneiloid.

$$\text{Surface (square feet)} = (.2618L) \left(d + \frac{T}{2} \right) \text{ for conoid.}$$

The increase in precision attributable to breaking up long logs into several shorter portions each having a single measured diameter is well known in log scaling. It restricts log-rule taper and shape assumptions to shorter lengths and this prevents bias from building up when multiplied by length. The author has derived a function expressing the scaled volume of a long conoidal log as a joint function of log-rule assumptions and scaling interval, together with log diameter at small end, log length, and actual log taper,

Let long-log parameters be denoted as follows (for a conoidal log):

d = diameter in inches at small end

L = total length in feet

T = actual constant rate of taper, in inches per foot

Let scaling interval be denoted as follows:

K = scaling interval in feet (a constant)

Let log rule assumptions be denoted as follows:

\textcircled{T} = fictitious constant rate of taper assumed-by log rule (in inches per foot: $1/8$ for International, 0 for Scribner and Doyle)

M = minimum length to which taper assumed by log rule will apply
(4 ft. in case of International, irrelevant in case of Scribner and Doyle)

$N = d \cdot 1.6136$ for Int. $1/4$ (milling frustum diameter)

$d \cdot 1.255$ for Scribner

$d - 4$ for Doyle

$C = .0498 L$ for Int. $1/4$ in. (kerf, length, and scale factors)

$.0494 L$ for Scribner

$.0625 L$ for Doyle

$V = C(N^2 - 2.60)$ for Int. $1/4$ in. (volume of minimum length)

$C(N^2 - 7.01)$ for Scribner

$C(N^2)$ for Doyle

Let certain joint functions be denoted as follows:

$$P = (L - K)(T)$$

$$\textcircled{Q} = (K - M)(\textcircled{T})$$

$$R = LT$$

$$\textcircled{S} = K \textcircled{T}$$

Then the log-rule board foot volume of a long log with small end diameter (d), length (L), actual rate of taper (T), and with diameters taken at constant interval (K) will be:

$$V + (C) \left(\frac{(P + \textcircled{Q})(P + \textcircled{Q}) + R + 6N) - \textcircled{S}(K - M)(T - \textcircled{T})}{6} \right)$$

If, instead of the constant interval K above, diameters had been measured at a longer constant interval K', and if the board foot volume obtained by using interval K were subtracted from that obtained by using interval K', the difference in board foot volume would be:

$$(C)(K - K')(T - \textcircled{T}) \left(\frac{P + \textcircled{Q} + P' + \textcircled{Q}' + R + 6N + \textcircled{T}(K + K' - M)}{6} \right)$$

Primed symbols denote quantities containing K', and circled symbols denote quantities containing taper assumed by log rule (zero in case of Scribner and Doyle).

As an example, consider a 20-inch log of 80-foot length tapering at a constant rate of 1 inch per 16 feet. If actual diameters were measured at 16-foot intervals, the basic formula above would compute board foot volume as 1,778 board feet by International 1/k-inch log rule and 1,680 board feet by Scribner formula log rule. If, however, the scaling interval were lengthened and actual diameters **were only** measured at 20-foot intervals instead of at 16-foot intervals, the International 1/k-inch volume of the same log would scale 22 board feet higher than previously, while the Scribner formula volume of the **same** log would scale 21 feet lower than previously.

The above calculations can be accomplished or verified by tabular methods, but the two previous formulae give some insight into the mechanism through which **erroneous taper** assumptions in log rules and interval between actual scaling diameters affect log scale.

APPENDIX E

Theory of Height Accumulation

The bole of a tree can be considered as the sum of several frusta of solids of revolution, each generated by a different function having the generalized form $D^2 = BL^{K-1}$, where:

B = an appropriate constant.

D = d.o.b. (diameter outside bark) at one end of frustum.

T = taper-step or difference in diameter between small and large ends of frustum.

H = height above stump at which D is measured.

L = difference in height between small and large ends of frustum.

K = shape divisor (2 if paraboloid, 3 if conoid, 4 if neiloid or subneiloid, which last term is defined in the footnote to Appendix C).

If d.o.b. of the bole of a tree be measured at two points, and if the height above stump of those two points also be measured, then the author has shown (10) that the volume of any frustum cut by planes normal to the tree axis and passing through the two points can be expressed as:

$$\frac{\pi T^2}{4} \left[L \left(\frac{D}{T} \right)^2 + L \left(\frac{D}{T} \right) + \frac{L}{K} \right]$$

The surface of any such conoidal frustum can be expressed as:

$$\pi T \left[L \left(\frac{D}{T} \right) + \frac{L}{2} \right] \left[\sqrt{1 + \left(\frac{T}{2L} \right)^2} \right]$$

where D, T, and L are all measured in the same scale units (i.e., all in feet, or all in inches, or all in meters, etc.).

Now it is apparent that if taper-step (T) is kept constant, and if stump diameter is chosen so that it is a multiple of T, and if $X = \frac{D}{T}$, then X's will occur in diminishing arithmetic progression as successive D's ascend the tree.

The entire tree volume above stump then can be calculated as:

$$\frac{\pi T^2}{4} \left[\sum X^2 L + \sum XL + \frac{\sum L}{K} \right]$$

and the entire tree surface can be calculated as:

$$\pi T \left[\sum XL + \frac{\sum L}{2} \right] \left[\sqrt{1 + \left(\frac{T}{2L} \right)^2} \right]$$

Since X occurs in an arithmetic progression diminishing up the tree bole till it reaches 1, appropriate combinations of the sum of first upward progressive totals ($\sum L'$) and the sum of second upward progressive totals ($\sum L''$) can be substituted for $\sum XL$ and $\sum X^2 L$. The tree volume can then be calculated as:

$$\frac{\pi T^2}{2} \left[\sum L'' + \frac{\sum L}{2L} \right]$$

and the tree surface as:

$$\pi T \left[\sum L' + \frac{\sum L}{2} \right] \left[\sqrt{1 + \left(\frac{T}{2L} \right)^2} \right]$$

It is obvious that the radical constituting the last factor above will be negligible (1.00001356) where tree taper averaging about 2 inches in 16 feet is involved. It is also obvious that if H = height of each D above stump, then $\sum H = \sum L'$, and $\sum H = \sum L''$. Thus, assuming an average taper of 2 inches per 16 feet (merely to allow evaluation of the radical), the above expressions can be written as:

$$\text{Tree volume} = \frac{\pi T^2}{2} \left[\sum H' + \frac{\sum L}{2K} \right]$$

$$\text{Tree surface} = \pi T \left[\sum H + \frac{\sum L}{2} \right] [1.00001356]$$

Height accumulation theory stemmed from the author's derivation of these formulae in 1948 (10). As long as a reasonable taper-step (T) is selected, the conoidal assumption (with K = 3) will give rise to negligible error in either formula, as can be seen from Appendices C and D.

J. F. Clark (who devised the International 1/8-inch rule) originally conceived of his board-foot rule as a function of log length, surface, and cubic volume (5). However, derivation of exact functions in the case of the International 1/4-inch log rule, the Scribner formula log rule, and the Doyle log rule required explicit volume and surface formulae not published till much later (12). On pages 10 and 11 of the cited reference (Shortcuts for Cruisers and Scalers), the author gave explicit functions leading to the deduction that board-foot volume in the case of 16-foot logs can be expressed as:

Board-foot volume (International 1/8-inch kerf) =
 9.1236 (cu.ft. vol.) - $.70846$ (sq.ft. surface) + $.042222$ (ft. length)

Board-foot volume (Scribner log rule) =
 9.057 (cu.ft. vol.) - $.852$ (sq.ft. surface) - $.112$ (ft. length)

Board-foot volume (Doyle log rule) =
 11.459 (cu.ft. vol.) - 2.387 (sq.ft. surface) + 1.542 (ft. length)

Although there is no change in the International coefficients as log length changes, there is change in the Scribner and Doyle coefficients. For a 12-foot log, the Scribner $.852$ would become $.756$, and $.112$ would become $.158$; the Doyle 2.387 would become 2.269 , and 1.542 would become 1.399 .

The various board-foot volume coefficients A, B, C given in Appendix A are based on coupling these formulae for 16-foot log-lengths with the basic height-accumulation formulae derived earlier. No tabulations of A, B, C coefficients have been made in Appendix A for the situation where 12-foot logs are commonly cut and Scribner or Doyle scale is used. However, such coefficients may be easily calculated from formulae given above.