CHAPTER 2

PERSPECTIVES AND METHODS OF SCALING

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2.1 INTRODUCTION

Transferring information between or across scales or organizational levels is inevitable in both basic research and its applications, a process generally known as "*scaling*" (Wu and Li, Chapter 1). Scaling is the essence of prediction and understanding both of which require cross-scale translation of information, and is at the core of ecological theory and application (Levin 1992, Levin and Pacala 1997, Wu 1999). While the importance of scaling in ecology has been acutely recognized in recent decades, how to conduct scaling across heterogeneous ecosystems remains a grand challenge (Turner et al. 1989, Wu and Hobbs 2002).

A number of scaling approaches and methods have been developed and applied in different disciplines ranging from physics, engineering, biology, to social sciences. Two general scaling approaches can be distinguished: similarity-based scaling and dynamic model-based scaling methods (Blöschl and Sivapalan 1995). *Similarity-based scaling methods* are rooted in the concepts and principles of similarity and self-similarity and often characterized by relatively simple mathematical or statistical scaling functions, even though the underlying ecological processes of a phenomenon may be extremely complex. In contrast, *dynamic model-based scaling methods* use deterministic or stochastic models to simulate the processes of interest, and to transfer information across scales by either modifying the parameters and input variables of the same model or developing multiple-scaled models. In this case, information transfer between different scales is accomplished through manipulating the inputs, outputs, and formulations of dynamic models. In both approaches, it is important to properly identify scaling thresholds at which scaling relations often change abruptly, reflecting fundamental shifts in underlying processes or controlling factors and defining the domains of applicability of specific scaling methods.

While the previous chapter (Wu and Li, Chapter 1) discussed various concepts of scale and scaling, in this chapter we focus on the major characteristics of the two scaling approaches and several more specific upscaling and downscaling methods

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within each approach. The purpose of this chapter is not to provide a recipe for scaling. Rather, we shall review scaling perspectives and methods in different disciplines, and provide a synthesis based on a common conceptual framework. By so doing, we expect that a more comprehensive and cohesive understanding of ecological scaling will emerge.

2.2 SIMILARITY-BASED SCALING METHODS

2.2.1 Concepts of Similarity

The concepts of *similarity* traditionally have been important in scaling-related studies. In general, similarity exists between two systems whenever they share some properties that can be related across the systems by a simple conversion factor (Blöschl and Sivapalan 1995). LaBarbera (1989) summarized three types of similarity concepts applied in body size-oriented studies: geometric, physical, and functional similarities (also see Gunther 1975). *Geometric similarity* is characterized by the constancy in shape with changing size. In other words, geometric similarity assumes that "geometry and shape are size-independent properties" (Niklas 1994). For example, for different sized objects of the same shape and geometry, $S \propto L^2$, and $S \propto V^{2/3}$, where L, S, and V are the linear dimension, surface area, and volume of the objects. *Physical similarity* is defined based on the constancy of the ratios of different forces (also called *dynamic similarity*; see Blöschl and Sivapalan1995). For example, two systems are said to have *hydrodynamic similarity* if they have the same Reynolds number (i.e., the ratio of inertial to viscous forces). Barenblatt (1996) stated that the concept of physical similarity is a natural generalization of that of geometric similarity in that two similar triangles differ only in the numerical values of side lengths, whereas two similar physical phenomena differ only in the numerical values of the dimensional governing parameters. *Functional similarity* refers to the constancy in the changes in functional variables over a range of system sizes. For example, animal metabolic rates (R) change with body size or mass (M) following a power law (i.e., $R \propto M^b$). If the primary production of an ecosystem (P) changes with the size of the ecosystem (A) in a similar fashion (i.e., $P \propto A^b$), then the ecosystem may be said to have functional similarity.

In recent decades, the concept of *self-similarity* has become a cornerstone of similarity-based studies. It refers to the phenomenon that the whole is composed of smaller parts that resemble the whole itself or that patterns remain similar at different scales. Self-similarity is the key idea in fractal geometry (Mandelbrot 1982, Hastings and Sugihara 1993), and is considered to be the unifying concept underlying fractals, chaos, and power laws (Schroeder 1991). While admitting that the terms, *fractal* and *multifractal*, still lack an agreed mathematical definition, Mandelbrot (1999) offered an informal definition of *fractal geometry* as "the systematic study of certain very irregular shapes, in either mathematics or nature, wherein each small part is very much like a reduced size image of the whole." Such irregular shapes, or fractals, exhibit properties of self-similarity which entails scale-invariance (i.e., patterns or relationships remain unchanged over a range of scales).

Commonly cited examples of fractals include coastlines, clouds, snowflakes, branching trees, and vegetation patches. However, not all self-similar objects are fractals because self-similarity is also found in Euclidean geometry.

Simple fractals exhibit scale-invariant patterns that can be characterized with only one scaling exponent, which is often interpreted as implying one single generating process. However, many fractal-like structures in nature are generated by a number of generating processes that operate at different scales. These are called *generalized fractals*, or *multifractals*, which are characterized by a spectrum of fractal dimensions that vary with scale. It has been suggested that additive processes tend to create *monofractals* (simple fractals), whereas multiplicative random processes generate multifractal structures (Stanley and Meakin 1988, Schroeder 1991). Multifractals have been used to describe the spatial distribution of people and minerals, energy dissipation in turbulence, and many other patterns and processes in nature. It is now widely recognized that many if not most fractal patterns and processes in nature show scale-invariance only over a limited range of scales. Hastings and Sugihara (1993) suggested that linear regression methods be used to distinguish between patterns with one scaling region (a single power law) and those with multiple scaling regions (separate power laws over separate regions). These authors asserted that multiscaling is detected if the slope of the regression line changes significantly over adjacent regions.

2.2.2 Dimensional Analysis and Similarity Analysis

The concepts of similarity are the foundation of dimensional analysis (Blöschl and Sivapalan 1995), and have long been used in engineering and physical sciences. Barenblatt (1996) indicated that the main idea behind dimensional analysis is that "physical laws do not depend on arbitrarily chosen basic units of measurement," and thus the functions expressing physical laws must possess some fundamental property (mathematically termed generalized homogeneity or symmetry) that allows the number of arguments in these functions to be reduced. *Dimensional analysis* aims to produce dimensionless ratio-based equations that can be applied at different scales for a phenomenon under study. In practice, dimensional analysis only applies in the framework of Euclidean geometry and Newtonian dynamics (Scheurer et al. 2001). Dimensional techniques have long been used to derive similarity relationships, establish scaling laws, reduce data volume, and help elucidate processes and mechanisms in physical and biological sciences (Gunther 1975, Blöschl and Sivapalan 1995). *Similarity analysis* is similar to dimensional analysis in that it is also a simplification procedure to replace dimensional quantities required for describing a phenomenon with fewer dimensionless quantities; but unlike dimensional analysis, it requires the governing equations of the phenomenon to be known (Blöschl and Sivapalan 1995).

Examples of similarity analysis are abundant in physical sciences. For example, similarity analysis in soil physics and hydrology started in the 1950s with the concept of *Miller-Miller similitude*, an intuitive depiction of structural similarities in porous media at fine spatial scales (Miller and Miller 1956). Miller-similar porous media have microscopic structures that look similar in the same way as triangles in

Euclidean geometry (Sposito 1998). Similarity analysis, then, involves the derivation of scale factors for soil-water transport coefficients on the basis of the fine-scale similar-media concept. Later studies extended the concept of the Miller-Miller similitude from microscopic to macroscopic scales using the idea of functional normalization (related to functional similarity) rather than dimensional techniques (Haverkamp et al. 1998). In recent decades, fractal and multifractal models of soil structure have been increasingly used in similarity analysis of hydrological processes and beyond. As Sposito (1998) noted, "fractal geometry has become the signature approach to both spatial-scale invariance and temporal-scale invariance, as epitomized by self-similarity in the patterns of hydrologic and other geophysical processes."

One of the most successful examples of using similarity analysis to deal with complex physical processes is *Monin-Obukhov similarity theory*. Atmospheric boundary-layer flows, though mostly turbulent, can be viewed as being dynamically similar, such that the concepts of similarity can provide a powerful framework for analyzing empirical data and parameterizing models to represent these complex processes. In particular, Monin-Obukhov theory assumes that surface layers with the same ratio of the aerodynamic roughness length (z_0) to the Obukhov length (L) are dynamically similar, with z_0/L being considered as a dimensionless similarity parameter. In other words, the theory is based on the assumption of complete similarity of fluxes in terms of Reynolds number (Barenblatt 1996). The development of Monin-Obukhov theory follows the general procedures of similarity analysis: (1) identifying the atmospheric processes that conform the dynamic similarity principle, (2) characterizing these processes with dimensionless similarity parameters (e.g., Reynolds number), (3) determining a set of scaling parameters (e.g., scaling wind velocity, scaling temperature, scaling humidity) and non-dimensionalized dependent and independent variables, and (4) deriving a set of similarity laws that are valid over a broad range of scales (Barenblatt 1996). By so doing, Monin-Obukhov theory relates turbulent fluxes in the surface layer to mean vertical gradients of wind, temperature, and specific humidity (Wu 1990).

As an important part of similarity analysis, *renormalization group methods* (Wilson 1975) have been used for studying scaling behavior associated with critical phenomena and phase transitions in physical sciences, including turbulence, flows in porous media, fracture mechanics, flame propagation, atmospheric and oceanic processes (Binney et al. 1993, Barenblatt 1996). The general idea of renormalization groups is to simplify mathematically complex models that contain much fine-scale detail into simpler models and to develop scaling laws using similarity principles and techniques. The simpler models (or equations) consist only of essential information of the phenomenon under study, and are able to describe and predict coarse-scale patterns with explicit scaling relations. Renormalization group methods represent a fundamental concept and powerful technique in theoretical physics (Barenblatt 1996), which "make rigorous the scaling process through the derivation of equations for blocks of cells in terms of the units that make them up" (Levin and Pacala 1997). Critical phenomena and phase transitions are common in ecology, particularly with spatial problems (Gardner et al. 1987, Milne 1998), but only until

recently have renormalization group methods been applied in ecological studies (e.g., Levin and Pacala 1997, Milne 1998).

Gunther (1975) pointed out that "Structures and functions of all living beings, irrespective of their size, can be studied by means of some basic physical methods, viz., dimensional analysis and theories of similarity." Although it is unlikely that all structures and functions of the biological world (even at the organism level) can be effectively studied by using dimensional analysis and similarity analysis alone, there is little doubt that they will continue to play an important role in biological and ecological scaling. A great number of allometric studies in biology and ecology have further demonstrated the power and elegance of similarity-based methods. However, the applicability and accuracy of these methods may depend on the levels of biological organization and the variability of processes with scale. In the following, we turn our attention to some of the major issues in allometric scaling.

2.2.3 Biological Allometry

Gould (1966) defined *allometry* as "the study of size and its consequences." Similarly, Niklas (1994) described allometry as "the study of size-correlated variations in organic form and process." Among other definitions of allometry is any "departure from geometric similarity" (LaBarbera 1989). For several decades allometry has focused primarily on the body size (or mass) of organisms as the fundamental variable (Calder 1983, Peters 1983, Schmidt-Nielsen 1984). Niklas (1994) summarized three meanings of allometry: (1) a relationship between the growth of a part of an organism and the growth of the whole organism (or some part of it), (2) a relationship between organic size and biological form and process, and (3) a size-correlated relationship deviating from geometric similarity that is exhibited by objects of varying sizes with the same geometry and shape. Brown et al. (2000) noted that allometric studies in biology have been carried out at three levels of biological organizations: within individual organisms (e.g., animal circulatory networks and tree branching architecture), among individual organisms of different sizes (e.g., body-size related variations in biological pattern and process), and within populations or communities (e.g., allometric scaling of population density and community biomass).

Allometric scaling is rooted in the concepts of similarity and, as in physical sciences, allometric relations in biology usually take the form of a power law:

$$Y = Y_0 M^b \tag{2.1}$$

$$\text{or, } \log Y = \log Y_0 + b \log M \tag{2.2}$$

where Y is some biological variable, Y_0 is a normalization (or scaling) constant, M is some size-related variable (usually body mass), and b is the *scaling exponent.*

In Equation 2.1, if $b = 1$, the relationship becomes linear, and is called *isometric scaling*; if $b \neq 1$, then the relationship is either *geometric scaling* or *allometric*

scaling (including fractal scaling). Geometric (or Euclidean) scaling is based on *complete similarity*, whereas allometric scaling is based on *incomplete similarity* or self-similarity (Barenblatt 1996, Schneider 2001a). For example, based on the geometric similarity of Euclidean objects we can analytically derive the following relationships among volume (V), area (A), the length dimension (l), and mass (M): $A \propto l^2$, $V \propto l^3$, $M \propto V$, $l \propto M^{1/3}$, and $A \propto M^{2/3}$. These simple geometric scaling rules mean that, if objects of different sizes are completely similar, their liner dimensions and surface areas should be proportional to the 1/3 and 2/3 powers of their mass (assuming a constant density). In other words, if $b = 1/3$, Equation 2.1 suggests that a property of an object (Y) is dependent on the length dimension of the object (M); if $b = 2/3$, then Equation 2.1 suggests that Y is dependent on the surface area of the object. However, Brown et al. (2000, 2002), among others, argued that organisms do not seem to follow such simple geometric scaling rules; rather, they commonly exhibit "quarter-power scaling" relationships – i.e., the scaling exponent takes the value of simple multiples of 1/4. For example, $b = 3/4$ for the whole-organism metabolic rates of a variety of animals ranging from mice to elephants; $b = 1/4$ for the heart rates of animals; $b = -1/4$ for the life span of animal species; $b = 3/8$ for the radius of the aorta of animals and the trunks of trees; and $b = -3/4$ for the population density of animals (Brown et al. 2000, Schmid et al. 2000, Carbone and Gittleman 2002). While these scaling relations are general, variability can be substantial even for the same biological process. For instance, LaBarbera (1989) reported that, for scaling of home range area with body size of terrestrial mammals, $b = 1.18$ for herbivores, $b = 1.51$ for carnivores, $b = 0.97$ for omnivores, and $b =$ 0.74, 1.39, or 1.65 for all mammals depending on data sets used for calculation.

One of the best-known examples of allometric scaling in plant ecology is the self-thinning law in plants. In even-aged plant communities, the average biomass of individual plants (W) scales with plant density (D) following a power law: $W = cD^{-3/2}$, or $B = cD^{-1/2}$, where c is a scaling constant and B ($=WD$) is the stand biomass density. This means that plant population density scales with plant weight with a scaling exponent of –2/3 (i.e., $D \propto W^{-2/3}$) rather than –3/4 as in animals. This scaling relation was obtained from regression analysis based on empirical data as well as analytical studies based on geometric similarity – the so-called the "surface area law" ($S \propto V^{2/3}$, where V is the volume and S is the surface area; Niklas 1994). While this biomass-density relation has been held as a "law" for decades, recent studies have found little empirical evidence to support its universality and consistency (Weller 1987, Zeide 1987, Lonsdale 1990). In particular, the scaling exponent is not a constant, but rather a variable that is influenced by the shade tolerance of plants under study and taxonomic groups of choice. Zeide (1987) concluded that "the law is neither precise nor accurate," and Lonsdale (1990) stated that, in the log-log plot of stand biomass vs. plant density, "straight lines are the exception rather than the rule."

Enquist et al. (1998) showed that whole-plant resource use scales as $W^{3/4}$ and that, accordingly, the scaling exponent for the biomass-density relation or the self-

thinning law is −3/4 (i.e., $D \propto W^{-3/4}$), not −2/3 as previously reported. Thus, they concluded that plants do not differ from animals in terms of scaling of population density with respect to body mass, confirming the prediction of their general mechanistic model of resource use in fractal-like branching networks (West et al. 1997). This model, however, has met an increasing number of criticisms claiming that it is mathematically flawed and empirically unwarranted (e.g., Magnani 1999, Bokma 2004, Cyr and Walker 2004, Kozlowski and Konarzewski 2004). Nevertheless, allometric scaling, as a general approach, remains useful, and its rule in spatial scaling is discussed below.

2.2.4 Spatial Allometry

While sharing common features of similarity-based scaling methods, biological allometry has focused primarily on body size. Most of the allometric equations do not directly address the problem of spatial scaling. However, allometry as a general method can be applied to spatial scaling when the independent variable is spatial scale instead of body mass. Such studies have been termed *spatial allometry* (Schneider 2001a, b). In this case, the similarity principles pertain to the spatially extended systems (e.g., habitats, landscapes) rather than the individual organisms. A general spatial allometric scaling relation can be written as follows:

$$Q(S) = Q(S_0)\left(\frac{S}{S_0}\right)^{\beta} \tag{2.3}$$

$$\text{or, } Q(S) = kS^{\beta}, \text{ with } k = Q(S_0)S_0^{-\beta} \tag{2.4}$$

where $Q(S)$ and $Q(S_0)$ are the values of an ecological variable Q at spatial scales of S_0 and S, respectively, and β is the scaling exponent.

In Equations 2.3 and 2.4, S and S_0 may be expressed as extent or grain size. If S is extent and S_0 is grain size, then the ratio, S/S_0, defines the spatial (or temporal) scope (*sensu* Schneider 2001a), which is useful for comparing scaling studies with different systems. As with Equation 2.1, Equation 2.3 indicates *isometric scaling* when $\beta = 1$, and *geometric* (Euclidean) or *fractal scaling* when $\beta \neq 1$. Schneider (2001b) pointed out that geometric scaling results when β is "an integer or ratio of integers," whereas fractal scaling is indicated by a value of β that is "not an integer." In practice, however, it is not a trivial matter to distinguish between a "ratio of integers" and a "fractal" dimension. Thus, inferring the nature of similarity based merely on regression results, as often done in biological allometry, is not warranted.

Some allometric relations at the levels of populations and communities may be directly related to spatial scaling. For example, if population density scales with body mass as $D = D_0 M^{-0.75}$, one can derive a scaling relation between the total

number of animals (N) and habitat area (A): $N = D_0 A M^{-0.75}$ or between the total biomass (B) of the animal species and habitat area: $B = DAM = D_0 A M^{0.25}$. If home range scales with body mass as $H \propto M^b$, then population density can be directly related to the size of home range: $D \propto HM^{-(0.75+b)}$. The best known example of spatial allometry, however, may well be the *species-area relationship* (SAR). SAR is commonly described by a power-law function: $S = cA^z$, where c is a constant influenced by the effect of geographical variations on S, and z is the scaling exponent with a value close to 0.25. SAR has been regarded as "ecology's most general, yet protean pattern" (Lomolino 2000) and one of the few widely accepted laws in ecology (Schoener et al. 2001).

Some recent studies suggested that the species-area relationship is an example of scale invariance that reflects self-similarity in species abundance and distribution (e.g., Harte et al. 1999, Kunin 1999). However, many others have indicated that the value of the scaling exponent of SAR may vary widely and that the power-law scaling only holds over a finite range of spatial scales in real landscapes (Crawley and Harral 2001, Schoener et al. 2001). While scale invariant pattern is often believed to imply a single underlying process, the species-area relationship may have multiple scaling domains if examined over many orders of magnitude in space. This observation favors the explanation that different factors determine species diversity at different ranges of scales (Shmida and Wilson 1985, Crawley and Harral 2001, Whittaker et al. 2001). For example, Lomolino (2000) argued that, for isolated ecosystems, SAR has three fundamentally different realms: (1) erratic changes influenced by idiosyncratic difference among islands and random catastrophic disturbance events for small islands, (2) a monotonic deterministic pattern determined by island area and associated ecological factors for intermediate-sized islands, and (3) again a monotonically increasing pattern for islands large enough for *in situ* speciation. Nevertheless, as with the self-thinning law, the debate and controversies on the universality, scale invariance, and ecological interpretation of SAR do not necessarily invalidate the use of the allometric scaling approach; it actually demonstrates its usefulness as a research tool.

In landscape geomorphology, it has long been noted that landform attributes exhibit allometric relationships (Woldenberg 1969, Bull 1975, Church and Mark 1980). For example, Hood (2002) identified several allometric scaling relations between slough attributes (e.g., area, outlet width, perimeter, length) for rivers in the Pacific Northwest of the United States, and showed that detrital insect flotsam density was also allometrically related to slough perimeter. In a recent study of the landscape dynamics of over 640 peatland bog pools in northern Scotland, Belyea and Lancaster (2002) found that the pools became deeper and more convoluted in shape with increasing size, and that the relationships between the area, depth, width, and length of the bog pools showed allometric (rather than geometric) scaling. Schneider (2001a,b) provided a number of examples of spatial allometry for lake ecosystems and aquatic mesocosms in terms of the geometric attributes of the systems (e.g., the volume, area, perimeter, and depth of lakes or mesocosms) and biological properties (e.g., fish catch, primary production). In recent decades, the

allometric study of landform, or *landscape allometry*, has been elevated to a new level of enthusiasm and insight by applying the concepts of fractals and self-organization (Mandelbrot 1982, Turcotte 1995, Rodriguez-Iturbe and Rinaldo 1997, Phillips 1999, Schneider 2001a, b).

In landscape ecology, there have been many examples of spatial patterns exhibiting allometric or fractal scaling relations (e.g., Milne 1991, Nikora et al. 1999, Wu 2004). Although some authors attempt to associate power scaling relations to underlying "universal" laws or scale invariance theories, such scaling relations usually only hold for limited ranges of scale (Milne 1991, Berntson and Stoll 1997, Wu 2004). Without resorting to any such grandiose assumptions, however, spatial allometry can still be used as a valuable empirical scaling method to summarize and extrapolate observed patterns over a range of scales, and to provide clues about the underlying processes, using a "scalogram approach" (Wu 2004).

2.3 DYNAMIC MODEL-BASED SCALING METHODS

2.3.1 Some Concepts of Scaling with Dynamic Models

In contrast with similarity-based scaling methods that deal with complex phenomena in a relatively simple manner, dynamic model-based scaling methods focus more on the processes and mechanisms of the phenomena under study. They may incorporate, but do not rely on, similarity concepts in theory and dimensional techniques in practice. *Dynamic models* are composed of state variables, rate variables, input variables, output variables, parameters, and constants. Parameters and constants help define rate variables and relate input and output variables to state variables. Because these terms are defined differently in the literature, some clarifications are needed here to avoid confusion. Following Bierkens et al. (2000), parameters may change in space, but not in time; constants are the only part of a model that does not change in space and time (i.e., scale-invariant); and all other model components may change in both space and time. Dynamic models can be implemented in mathematically explicit forms (e.g., differential or difference equations) or mathematically implicit forms (e.g., mathematical relation-based or rule-based simulation algorithms written in computer languages).

To illustrate different scaling methods clearly and precisely, let's assume that a dynamic model at a local scale s_1 is:

$$y(s_1) = f(v,\theta,i) \tag{2.5}$$

where $y(s_1)$, v, θ, and i are the output variables, state variables, parameters, and input variables at scale s_1, respectively.

Also, let's assume that a model can be developed at a broader scale s_2 as:

$$Y(s_2) = F(V,\Theta,I) \tag{2.6}$$

where $Y(s_2)$, V, Θ, and I are the output variables, state variables, parameters, and input variables at scale s_2, respectively.

Note that all the model arguments can be vectors. Then, transferring information from s_1 to s_2 usually involves one or more of the following transformations: $v \leftrightarrow V$, $\theta \leftrightarrow \Theta$, $i \leftrightarrow I$, and $f(v,\theta,i) \leftrightarrow F(V,\Theta,I)$, depending on how the model arguments and relationships at the two scales are linked (Blöschl and Sivapalan 1995, Wu 1999, Bierkens et al. 2000). Thus, the transfer of information between scales using dynamic models is done through rescaling or other kinds of alterations of inputs, parameters, state variables, and model conceptualizations.

Scaling with dynamic models typically consists of two major steps. For *upscaling*, the two steps are characterizing heterogeneity and aggregating information (Figure 2.1). First, characterizing spatial heterogeneity involves the classification and quantification of spatial patterns (e.g., the number, size, and spatial configuration of different types of patches in a landscape), which influence model inputs and parameters. Spatial heterogeneity can be characterized either in a spatially explicit way (i.e., in the form of maps) or in statistical terms (e.g., statistical moments, probability density functions, or pattern indices). In cases where data do not cover the entire study area, interpolation is often needed. Many spatial interpolation methods exist (Lam 1983, Goovaerts 1997), and geostatistical methods such as kriging are particularly useful. In all these cases, geographic information systems (GIS) and remote sensing have proven extremely useful (Quattrochi and Goodchild 1997, Marceau 1999, Hay et al. 2001).

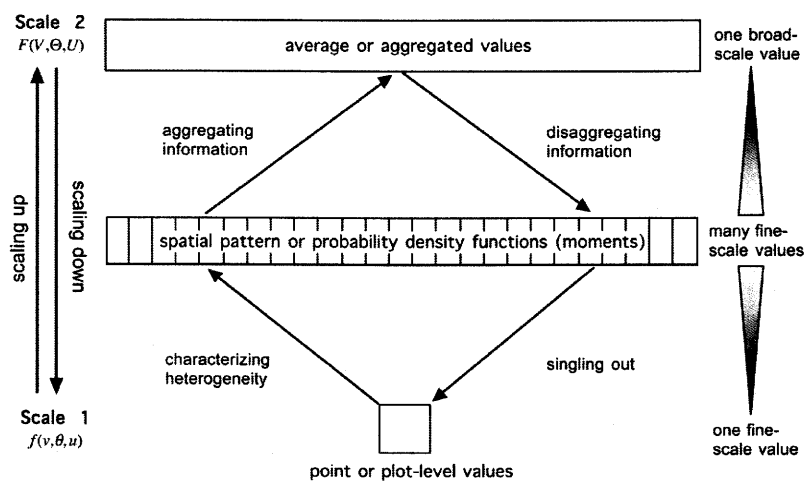

Figure 2.1. *Upscaling and downscaling as a two-step process when a dynamic model-based approach is used (modified from Blöschl and Sivapalan 1995).*

Second, aggregating information is to incorporate the quantitative description of spatial heterogeneity into local models to obtain predictions at a broader scale (or the target scale), be it a larger grain (coarse-graining) or a larger extent (extrapolation). Again, a variety of methods may be used in this step depending on what to be aggregated and how, as discussed below. In general, aggregating state variables and inputs can be readily done following such first principles as the laws of mass and energy conservation, but aggregating model parameters can be quite challenging (Blöschl and Sivapalan 1995).

For *downscaling*, the two steps are disaggregating information and singling out (Figure 2.1). Disaggregating coarse-grained information is to derive the detailed pattern within a spatial domain (fine-graining) with auxiliary data. Because of the lack of within-grain (or within-pixel) information, this often requires stochastic or probabilistic methods. Singling out is simply to find the location of the disaggregated pattern that corresponds to the site of interest, which is usually a trivial matter. Note that scaling relations (power laws) derived from similarity-based methods are supposed to work for both scaling up and scaling down. In contrast, scaling with dynamic models employs both deterministic and stochastic formulations, which may differ significantly for upscaling versus downscaling. In the following, we discuss several dynamic model-based scaling methods in detail.

2.3.2 Upscaling Methods

The literature on upscaling methods is both abundant and confusing because of the diversity in disciplines and approaches as well as the idiosyncrasy in terminologies and traditions. From a landscape modeling perspective, King (1991) distinguished four extrapolation methods: extrapolation by lumping, direct extrapolation, extrapolation by expected value, and extrapolation by explicit integration. Blöschl and Sivapalan (1995), Becker and Braun (1999), and Bierkens et al. (2000) discussed a number of scaling methods in the context of hydrological modeling and soil physics. Based on these and other studies, we compare and contrast several model-oriented upscaling methods. We focus more on the second step of upscaling – aggregating information. It must be emphasized, however, that adequately characterizing spatial heterogeneity is a crucial and necessary first step for upscaling with dynamic models, because the accuracy in representing spatial pattern may not only affect the accuracy of scaling results (see Li and Wu, Chapter 3), but also the model conceptualization and simulation scheme.

2.3.2.1. Extrapolation by lumping

One of the simplest ways to transfer information between two scales is to obtain the target-scale estimate as the output of the local-scale model with the mean values of parameters and inputs averaged over the study area – a method called *extrapolation by lumping* (King 1991) or *simple averaging* (Bierkens et al. 2000). If the local model is deterministic, only one model run is needed. This method can be used for extrapolation with increasing extent as well as for coarse-graining with increasing

grain size (see Wu and Li, Chapter 1). In this case, the local model $f()$ is assumed to remain valid at the target scale. That is, $f() = F()$. The method also assumes that $f()$ is linear such that changes in output variables are proportional to changes in input variables and parameters. In addition, horizontal interactions and feedbacks are negligible or at steady state. Under these assumptions, the treatment of the spatial heterogeneity of the study system is extremely simplistic: spatial variability is all squeezed into the averages of model arguments. Mathematically, the lumping method can be expressed as:

$$Y(s_2) = f(< v >,< \theta >,< i >) \tag{2.7}$$

where <*variable*> denotes the average of a variable, and all other terms are the same as before.

Equation 2.7 holds when y and Y represent a density measure (e.g., population density, flux density, carbon storage per unit area). If y and Y represent some cumulative or additive variable (e.g., population size, total flux, biomass), then the target-scale estimate becomes the product of the local-scale average multiplied by the total area, i.e.:

$$Y(s_2) = Af(< v >,< \theta >,< i >) \tag{2.8}$$

where A is the size of the target grain size in the case of coarse-graining, or the spatial extent in the case of extrapolation.

Thus, extrapolation by lumping uses the same local model at the target scale, with highly aggregated values of parameters and inputs. Simply put, lumping is about averaging over space. If grain elements are of the same size, arithmetic averaging is usually used. But if grain elements are of different sizes, area-weighted averaging should be used, instead. The local model is defined at the scale of a grain element or patch in the case of increasing extent, and at the size of the smaller grain element whose aggregates form the larger grain element in the case of coarse-graining. As a consequence of the oversimplifying assumptions, extrapolation by lumping is expected to produce large scaling errors when the model is nonlinear, when the local model formulation is no longer applicable at the target scale, or when horizontal interactions between grain elements (or patches) are strong and asymmetric (King 1991, Bierkens et al. 2000). For example, if the target scale is a landscape consisting of a number of different interacting ecosystems, the simple lumping method is more than likely to fail.

2.3.2.2 Extrapolation by effective parameters

Similar to the simple lumping method, *extrapolation by effective parameters* assumes that the local model applies to the target scale such that upscaling can be done by manipulating its parameters and inputs. However, instead of simply averaging parameters and inputs over space, the method uses "effective" (also called "equivalent" or "representative") parameters and inputs to produce the target scale

estimates (L'homme et al. 1994, Blöschl and Sivapalan 1995, Bierkens et al. 2000). That is, the estimated value of a variable at the target scale, $Y(s_2)$, is the output of the local model with a set of effective parameters and inputs:

$$Y(s_2) = f(\Theta_r, I_r) \tag{2.9}$$

where Θ_r and I_r are the effective or representative parameters and inputs.

The effective parameter approach has been widely used in soil physics, hydrology, and micrometeorology (L'homme et al. 1996, Bierkens et al. 2000). A prototypical example of using this method is to find the effective hydraulic conductivity for models of groundwater or soil water dynamics (Blöschl and Sivapalan 1995). For uniform steady saturated flow through a soil block that is made up of smaller blocks of different hydraulic conductivities, the effective conductivity equals the arithmetic mean of the small-block conductivities when they are arranged in parallel, and the harmonic mean when the small blocks of soil are arranged in series. For unsaturated flow, infiltration, and overland flow, such general effective conductivity does not exists, and a number of factors other than the porous medium itself may affect hydraulic conductivity, even though the geometric mean is found to be "effective" in some situations. Micrometeorologists have long used the "flux matching" technique in modeling fluxes over heterogeneous landscapes (L'homme et al. 1996, Wood 1998). That is, to upscale a certain surface flux over a patchy geographic area, one assumes that the plot-scale model, $f(\theta(x), i(x,t))$, still holds at the landscape scale, and then seeks the representative parameters $\Theta(x)$ and inputs $I(x,t)$ that produce the same estimated landscape-scale flux as the summation of fluxes predicted by the local model using heterogeneous parameters and inputs, i.e.,

$$F(\Theta_r, I_r) = f(\Theta_r, I_r) = \int_A f[\theta(x), i(x,t)]dA(x) \tag{2.10}$$

where A is the area of a region over which the integration is performed.

Thus, extrapolation by effective parameters essentially is to run "micro-scale equations" using "macro-scale parameters" whose values are assigned to points within the study area to assure that the "uniform parameter field" produces the same model output as the "heterogeneous parameter field" (Blöschl and Sivapalan 1995). The key to this method is to successfully derive a set of representative parameters and inputs for scale s_2 from parameters and inputs at scale s_1 through synoptic descriptions of the fine-scale heterogeneous patterns. This is relatively easy for linear models where the representative parameters and inputs can be found by simply averaging over space. In this case, it becomes the simple lumping method again. For nonlinear models, however, finding effective parameters can be a difficult task. The values of representative parameters and inputs are determined by several factors: the values of parameters and inputs of scale s_1, the detailed formulation of the local model, and the difference between the two scales as measured by their ratio, s_2/s_1 (Bierkens et al. 2000). Effective parameters are rarely unique, and fine-

scale parameters used to estimate the target-scale effective parameters are themselves mostly approximations (Bierkens et al. 2000). A number of methods have been used for deriving representative parameters and inputs, including analytical approaches, Monte Carlo simulations, inverse modeling, and direct measurements (Blöschl and Sivapalan 1995, Dolman and Blyth 1997, Bierkens 2000). In general, the accuracy of extrapolation by effective parameters is dependent upon whether the local model is valid at the target scale and whether representative parameters and inputs can be found that adequately account for the spatial heterogeneity of the system under study.

2.3.2.3 Direct extrapolation

Different from the lumping and effective parameter methods, *direct extrapolation* (*sensu* King 1991) does not assume that the local model apply at the target scale. Instead of averaging parameters and inputs before running the model as in the lumping method, direct extrapolation obtains the target-scale estimates by first running the local model for each grain element (or patch) of the system with geospatially corresponding parameters and inputs, and then averaging (or summing up) the model outputs for all the grain elements. Mathematically, the relationship between the two scales can be described as:

$$Y(s_2) = \langle y(s_1) \rangle = \langle f(v,\theta,i) \rangle \tag{2.11}$$

$$\text{or, } Y(s_2) = \sum_{k=1}^{N} y(s_1)_k = \sum_{k=1}^{N} f(v,\theta,i)_k \tag{2.12}$$

where N is the total number of areal units at scale s_1. Again, Equation 2.11 is applicable when y and Y represent a density measure, while Equation 2.12 is used if y and Y represent an additive variable.

In direct extrapolation, therefore, spatial heterogeneity is treated explicitly in terms of model parameters and inputs. This spatial explicitness can be retained in the target-scale estimates as well because the model outputs can also be presented in the form of maps in the case of coarse-graining (see Wu and Li, Chapter 1). More importantly, if the local model is nonlinear, running it first with spatial data and then averaging the outputs can reduce the errors due to model nonlinearity that are associated with the lumping method. If the local model is linear (rarely true in reality), however, direct extrapolation should produce the same results as the simple lumping method, but with higher demands for data preparation and computation.

Direct extrapolation is conceptually intuitive and technically straightforward, and it has been widely used in ecology, hydrology, and soil science. Examples include numerous spatially explicit or spatially distributed ecosystem and landscape models. Nevertheless, because it requires running the local model at all grain elements, direct extrapolation may suffer from excessive computational demand and redundancy when the total number of grain elements is great. This can be a real problem even with the most advanced computing facility because the computational demand for

such simulations increases exponentially with the number of grain elements. This problem can be alleviated by running the local model for each patch type (the aggregate of grain elements of the same type) instead of each grain element or individual patch because direct extrapolation is spatially "*explicit*," but not spatially "*interactive*." In other words, grain elements or patches are treated individually without considering the influences among them (see Peters et al., Chapter 7). The scaling results in the two cases should be similar as long as the variability of inputs and parameters within each patch type is insignificant.

Moreover, it is important to note that the assumption behind direct extrapolation that horizontal interactions and feedbacks are negligible or at steady state may not be valid in many situations, especially when lateral hydrological flows and exchanges of energy, materials and biological organisms in landscapes are significant and asymmetric. In other words, only one of the two components of spatial pattern, compositional heterogeneity (the diversity and relative abundance of patches), is recognized in this approach, leaving configurational heterogeneity (the geometric and spatial arrangement of patches) and its functional consequences unaccounted. Another implicit assumption is that there are no new patterns and processes emerging as the spatial extent increases. This can be a severe problem when direct extrapolation is practiced over a broad range of scale. Finally, spatial data on many processes often are not available for all the grain elements, resulting in obstacles for the effective application of this method.

2.3.2.4 Extrapolation by expected value

Similar to direct extrapolation, *extrapolation by expected value* (*sensu* King 1991) does not require the local-scale model to apply at the target scale. The expected value method treats model arguments as random variables, quantifies spatial heterogeneity by the joint probability distribution of model arguments, and uses the expected value of the local model outputs (also random variables) as the target-scale estimate (King 1991), i.e.,

$$Y(s_2) = E[y(s_1)] = E[f(v,\theta,i)] \tag{2.13}$$

$$\text{or, } Y(s_2) = A\,E[y(s_1)] = A\,E[f(v,\theta,i)] \tag{2.14}$$

where $E[]$ is the mathematical expectation operator.

For discrete variables, the mathematical expectation of model outputs can be expressed as summations, i.e.,

$$E[y(s_1)] = \sum_{j=1}^{q}\sum_{k=1}^{r}\sum_{l=1}^{s} f(v_j,\theta_k,i_l)\,\phi(v_j,\theta_k,i_l) \tag{2.15}$$

where $\phi()$ is the joint probability distribution functions of the model arguments that describe the spatial heterogeneity of the model domain. For continuous variables, integrals are used in the places of summations.

Apparently, the most critical step in upscaling with the expected value method is to compute the mathematical expectation of the outputs of the local model, which requires an accurate description of the fine-scale spatial heterogeneity or variability. If the joint probability density functions, $\phi()$, can be estimated explicitly, the expected value of model outputs can be derived directly using analytical or numerical techniques (King 1991). However, in many cases the explicit forms of the joint probability density functions cannot be found, and thus a sampling approach (e.g., Monte Carlo simulation) may be used to compute the expected value of the outputs of the local model. While direct extrapolation requires a full representation of the heterogeneity in a spatially explicit manner, the expected value method defines the fine-scale spatial heterogeneity only in statistical terms. However, both methods share two key issues: accurately describing spatial heterogeneity in terms of model arguments at the fine scale, and properly aggregating outputs from the local model to derive the estimate at the broader scale.

Because the local model does not have to apply to the target scale, extrapolation by expected value can be a rather general upscaling approach. It also overcomes some of the problems encountered in the previous methods: for example, the problem of oversimplifying spatial heterogeneity in the lumping method, difficulties in deriving effective parameters, and excessive data and computational demands with direct extrapolation when the target scale is too broad in extent. The method of expected value is quite amenable to uncertainty analysis (Li and Wu, Chapter 3) and, particularly, the results of Monte Carlo simulation may be used to estimate an unbiased confidence interval for the extrapolated value at the target scale (King 1991). This approach has been applied to a number of important ecological phenomena, including extrapolating information on primary production, trace gas fluxes, and other biophysical properties from local plots to landscapes or regions (e.g., King 1991, King et al. 1991).

A major potential problem with the expected value method is that, as in direct extrapolation, neither the geometric attributes and spatial arrangement of patches nor the interactions and feedbacks among grain elements (or patches) are explicitly considered. Therefore, scaling errors with this method are expected to increase when lateral flows and feedbacks are strong and significantly asymmetric or far from a steady state. Also, as with direct extrapolation, this method does not account for any new patterns and processes that emerge with increasing extent. In real landscapes, however, new processes and controls do emerge at progressively broader scales. If these "new" attributes have significant nonlinear effects, a method completely ignoring them is not only theoretically improper but practically inaccurate as well.

2.3.2.5 Explicit integration

If the spatial variations of all the arguments of the local-scale model can be adequately and explicitly represented as functions of space (x, y) in closed form, and

if the indefinite integral of the local model with respect to space exists (and can be found), upscaling between two scales can be accomplished by directly integrating the local-scale model. This method was termed *explicit integration* (King 1991), i.e.,

$$Y(s_2) = \iint_R f\big(v(x,y),\theta(x,y),i(x,y)\big)dA \tag{2.16}$$

where x and y are spatial coordinates, and A is the area of the region R over which the integration is performed.

The local model only needs to be evaluated once, and the prediction can be made precisely at any spatial scale within the defined region, R. In contrast with the lumping, effective parameters, direct, and expected value methods discussed earlier, the structure of the local-scale model now changes as a function of space during extrapolation by explicit integration. Explicit integration is elegant, efficient, and accurate when all of its requirements are met. However, because of the prevalence of nonlinear relationships in ecological models and complex spatial structures of model arguments, the applicability of explicit integration as an upscaling method is rather limited in practice. First of all, it is difficult or even impossible to represent spatial heterogeneity with closed-form mathematical functions of model arguments with acceptable accuracy. Second, even if this can be done, finding the indefinite integrals of nonlinear models, in general, is a formidable task. If the closed-form indefinite integrals cannot be found, approximating the double definite integral by numerical methods is equivalent to either direct extrapolation or extrapolation by expected value (King 1991).

2.3.2.6 Spatially interactive modeling

Not only do many processes in landscapes vary in their characteristics from one place to another, but also they often interact in space to generate feedbacks and emergent properties (Raupach et al. 1999, Wu 1999, Peterson 2000). Such examples include population dynamics in patchy environment, hydrological and biogeochemical dynamics in complex landscapes, and land-water and land-atmosphere interactions on various scales. To adequately understand and predict such phenomena across scales, models must explicitly consider the horizontal interactions of the processes under study.

Spatially interactive modeling integrates the two aspects of spatial scaling – characterizing heterogeneity and aggregating information between scales – into the dynamic models themselves. In this case, local-scale models or submodels are usually embedded in the larger-scale model (e.g., a metapopulation model consisting of many interacting local population models or a landscape model composed of multiple ecosystem models). Spatially interactive modeling deals explicitly with not only spatial variations in model arguments, but also the interactions among grain elements or patches. In other words, the values of the arguments of a local-scale model in one grain element or patch not only differ from, but also are functions of, the attributes of other (often neighboring) patches and the landscape matrix. Spatially interactive modeling is able to incorporate feedbacks, time delays, and new

features on larger scales. Different types of models emphasize different aspects of between-patch interactions (e.g., dispersal and species interactions in population and community models; hydrological and material exchanges in ecosystem models; spreading disturbances like fires and pests in many landscape models). In this case, the model conceptualization at the local scale (s_1) may be described as follows:

$$y(s_1) = f\left(v_k, \theta_k, i_k\right) + \delta p_k + \delta m_k \tag{2.17}$$

where $y(s_1)$ is the model output at the local scale s_1, v_k is a state variable for patch k, θ_k and i_k are parameters and input variables for the same patch k, δp_k is the net exchange between patch k and all other patches, δm_k is the net exchange between patch k and the landscape matrix, and $f()$ defines the local model in terms of within-patch dynamics. Then, the prediction at the target scale at any point of time t, $Y(s_2,t)$, can be described as:

$$Y(s_2,t) = \sum_{k=1}^{N} y_k(s_1,t) = \sum_{k=1}^{N}\left(f\left(v_k, \theta_k, i_k\right) + \delta p_k + \delta m_k\right) \tag{2.18}$$

where N is the total number of grain elements or patches at scale s_1.

For ecological processes in a shifting landscape mosaic, the dynamics of both landscape pattern and ecological processes, as well as their interactions, have to be modeled explicitly. For example, the overall population density in a dynamic landscape may be predicted by coupling the patch dynamics of the biological population of interest and the changing habitat patches (Levin and Paine 1974, Levin 1976):

$$\begin{aligned}\bar{n}_j(t) = \frac{1}{A}\{ [A - \int_0^\infty\int_0^\infty \xi\, \rho(t,\alpha,\xi)\, d\alpha d\xi]\, \varphi_j^o(t) \\ + \int_0^\infty\int_0^\infty \xi\, \rho(t,\alpha,\xi)\, \varphi_j(t,\alpha,\xi)\, d\alpha d\xi \}\end{aligned} \tag{2.19}$$

where $\bar{n}_j(t)$ is the overall population density of species j in the landscape, $\rho(t,\alpha,\xi)$ is the probability density function describing the frequency distribution of patches of age α and size ξ at time t, $\varphi_j(t,\alpha,\xi)$ is the population density of species j within a patch of age α and size ξ at time t, $\varphi_j^0(t)$ is the population density of the same species in the non-patch area, and A is the total area of the landscape.

Specifying such models in closed forms and solving them analytically are difficult or impossible for real landscapes. Spatially explicit simulation modeling

with Monte Carlo integration provides a general approach to dealing with such complex patch dynamics problems (Wu and Levin 1997).

In the past two decades, a great number of spatially interactive models of different kinds have been developed in ecology and earth sciences, examining such topics as metapopulation dynamics (Hanski 1999), landscape dynamics (e.g., Li et al. 1993, Wu and Levin 1997), hydrological and biogeochemical processes (e.g., Tenhunen and Kabat 1999, Beven 2000), and socioeconomic processes (Schweitzer 1997). Many modeling approaches have been used, including various grid-based models, cellular automata, and individual-based and agent-based models. Most, if not all, spatially interactive models are multiscaled or hierarchical. For scaling across broader spatial scales such as landscapes and regions, GIS and remote sensing techniques have increasingly been used in such models. The main sources of error in upscaling with spatially interactive modeling vary from case to case, but are due primarily to the characterization of spatial heterogeneity, formulation of spatial interactions, interface of multiple scales, and computational algorithms. In addition, such models can be quite demanding in data and computational requirements.

2.3.2.7 Extrapolation along a scaling ladder

The upscaling methods discussed above are in principle only applicable to situations where there are neither significant asymmetric between-patch interactions nor scale-dependent or emergent patterns and processes. These are typically "short-range" scaling methods because the assumptions behind them are less likely to be satisfied over a broad range of scale. When scaling involves multiple scale domains or levels of organization, new patterns and processes at different scales as well as vertical linkages need to be taken into account. How can these "short-range" upscaling methods be used for transferring information over a "long range" of scales that have multiple scaling domains or organizational levels?

To address this question, Wu (1999) proposed a "scaling ladder" approach based on the *hierarchical patch dynamics* (HPD) paradigm, which integrates hierarchy theory and patch dynamics (Wu and Loucks 1995). The *scaling ladder approach* facilitates the understanding and scaling of patterns and processes in different kinds of heterogeneous landscapes (Hay et al. 2001, Poole 2002, Wu and David 2002, Burnett and Blaschke 2003). The first step in this approach is to construct a spatially nested hierarchical system with distinctive scaling domains or levels of organization. Top-down (partitioning) or bottom-up (aggregation) schemes can be used in this step (Wu 1999). A top-down approach identifies the levels of a hierarchy by progressively partitioning the entire system downscale, whereas a bottom-up scheme involves successively aggregating or grouping similar entities upscale. *A priori* spatial hierarchies based on empirical observations and natural biophysical boundaries may be used as long as they are spatially nested and relevant to processes of interest. Such empirical hierarchies are commonly found in all fields of study, including the individual-population-community-biome or plot-ecosystem-landscape-region hierarchies (e.g., Urban et al. 1987, Jarvis 1995), soil-type hierarchies (Woodmansee 1990), hydrologic unit hierarchies (e.g., site-drainage-subwatershed-

watershed-subbasin-basin-subregion-region; Griffith et al. 1999), and landscape and geomorphological hierarchies (e.g., Reynolds and Wu 1999).

It is important to realize that the appropriateness of a given hierarchy varies with the process under study and research questions to be addressed (Wu and Loucks 1995, Wu 1999, Omernik 2003). Quantitative methods, including landscape metrics, spatial statistics, and object-oriented approaches can be used for identifying patch hierarchies (e.g., O'Neill et al. 1991, Li and Wu 2004, Hay et al. 2001, 2003, Hall et al. 2004, Wu 2004). As emphasized in Wu and Loucks (1995) and Wu (1999), these spatial hierarchies should be constructed with consideration of both pattern and process, such that they are neither completely arbitrary, nor simply products of spatial analyses. Also, they are dynamic, not static, as indicated by the phrase "hierarchical patch dynamics." Once an appropriate patch hierarchy, or a scaling ladder, is established, the short-range scaling methods discussed above can be applied within each scale domain or between two adjacent hierarchical levels (Figure 2.2). This can be accomplished by changing grain, extent, or both.

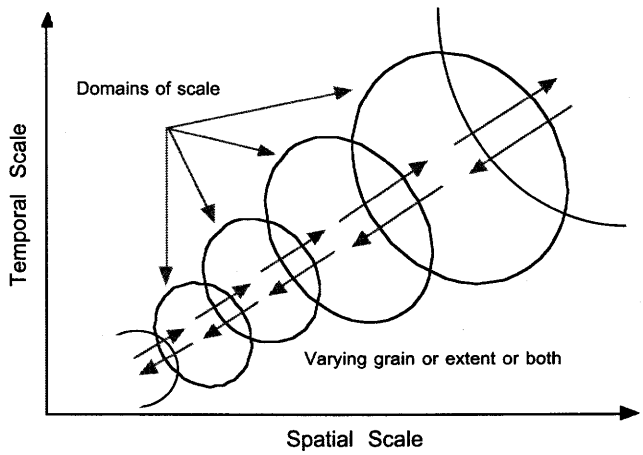

Figure 2.2. *Illustration of the scaling ladder approach in which scaling up (or down) is implemented by changing model grain size, extent, or both across successive domains of scale (redrawn from Wu 1999).*

2.3.3 Downscaling Methods

The central question of *downscaling* is: given the aggregated values, the probability distributions, or the functional relationships of variables at a particular scale, how can they be derived at a smaller scale (Bierkens et al. 2000)? In many applications, the objective of downscaling is, "given the average value over a certain domain, to derive the detailed pattern within that domain" (Blöschl and Sivapalan 1995). Downscaling methods usually share the assumption that the variable to be downscaled varies according to some function of space within the support units, and

thus downscaling the variable is essentially to seek the parameters of this function such that the average value of the support unit or its probability distribution is maintained (Bierkens et al. 2000). In principle, downscaling does not produce unique solutions because the values of a variable at scale s_1 may vary within a grain unit in an infinite number of ways without changing the average value at s_2. For example, for a grain unit at scale s_2 composed of 4 smaller grain units at scale s_1, there are an infinite number of combinations of the values that the four s_1 grain units may take to produce the average value of 1.

Bierkens et al. (2000) discussed three kinds of downscaling problems: (1) deterministic downscaling, (2) conditional stochastic downscaling, and (3) unconditional stochastic downscaling. In deterministic downscaling, the average property at scale s_2 is known exactly, and the objective is to find a single deterministic function to describe the spatial or temporal variation of values at scale s_1, such that the average of disaggregated values matches the known average value at scale s_2. In conditional stochastic downscaling, the average property at scale s_2 is also known exactly, but the objective is to find a set of equally probable functions that can predict the disaggregated values at s_1 while maintaining the same known average value at s_2. Monte Carlo analysis can be used in this case. Unconditional stochastic downscaling occurs when only the probability density function of the average property at scale s_2 is known. The objective is to find a family of equally probable functions of the spatial or temporal variation at s_1 that produce the known probability density function of the average property at s_2. Blöschl and Sivapalan (1995) and Bierkens et al. (2000) provided several examples in the context of soil and hydrological sciences (e.g., disaggregating information on soil properties in a soil profile or over a geographic area, and downscaling hydrological time series or spatial pattern of rainfall). These three kinds of downscaling problems can be studied with either empirical functions or mechanistic models (Bierkens et al. 2000).

While there are various needs for downscaling over relatively fine scales, the current literature on downscaling is clearly dominated by climate studies on much broader scales. Because General Circulation Models (GCMs) operate at spatial resolutions (usually >2° in both latitude and longitude) that are too coarse for understanding the regional and local impacts of global climate change, there has been a great deal of research in climate downscaling in the past few decades. Specifically, climate downscaling refers to "a set of procedures by which we attempt to take information available at the relatively poor spatial resolution of the GCMs, and derive regional-scale data that can be used for ecosystems modeling, climate impact assessment, and other tasks that require higher resolution climate data" (Crane et al. 2002). Thus, the primary objective of climate downscaling is disaggregating (i.e., fine-graining) GCM outputs to produce patterns of surface climatic conditions (e.g., temperature, precipitation, wind velocity) at regional (and eventually local) scales on which most ecological and socioeconomic processes operate. Parallel to the two general approaches to scaling, the methods of

downscaling are also commonly classified into two main approaches: empirically-based statistical downscaling and process model-based downscaling (Hewitson and Crane 1996, Crane et al. 2002).

The *empirically-based statistical downscaling approach* derives quantitative relationships between large-scale synoptic circulation features (e.g., upper level winds, geopotential heights, and sea level pressure) predicted by GCMs and regional climate conditions (e.g., temperature, precipitation, wind velocity) based on observations. These relations, in some form of $y = f(x)$, are often referred to as "transfer functions," which are obtained through multiple linear regression, artificial neural networks (ANNs), classification and regression trees (CART), and other statistical methods (Hewitson and Crane 1996, Wilby et al. 1998, Sailor et al. 2000, Crane et al. 2002). Note that, in contrast with the empirical methods in allometric and similarity analysis discussed earlier, empirical downscaling in climate studies rarely invokes similarity assumptions of any kind. The feasibility and the validity of the empirical approach hinge on the fundamental assumption that "stable empirical relationships can be established between atmospheric processes occurring at disparate temporal and/or spatial scales" (Wilby et al. 1998). Thus, such empirical or statistical relationships do not have the capacity to explain how circulation-related processes affect regional/local climate, and their predictive ability is undermined by the fact that transfer functions are often varying in time. Empirical techniques in climate downscaling have been widely used because they are operationally simpler and computationally much less demanding than the process modeling approach.

The *process-based downscaling approach*, also called *nested modeling*, embeds a higher-resolution regional climate model within a global GCM so that the coarse-grained predictions from GCM are dynamically translated into fine-grained outputs of the regional model. In most cases, the output of GCM from a large grid cell is used to provide boundary conditions for the regional climate model inside the grid cell in one-way nesting schemes. There are also two-way nesting schemes in which GCM and the embedded regional climate model run simultaneously and interact with each other across scales. Because the land surface characteristics may significantly affect local, regional, and even global climatic processes (Pielke and Avissar 1990, Raupach et al. 1999), nested modeling provides a necessary and promising approach to climate downscaling and to coupling geophysical and ecological processes across scales in general. However, because of the excessive computational demands and the lack of understanding of interface mechanisms of climatic processes at different scales, two-way nesting has not yet been commonly used in climate downscaling with GCMs.

2.4 DISCUSSION AND CONCLUSIONS

We have classified numerous scaling methods into two general approaches: similarity-based versus dynamic model-based. The first approach itself has a diversity of methods, including those relying on empirical, statistical methods and those based on first principles and analytical methods such as dimensional and similarity techniques. The dynamic model-based scaling approach, in contrast, puts

more emphasis on the processes and mechanisms of interest, and employs a variety of methods for modeling (deterministic and stochastic), quantifying heterogeneity, and aggregating information across scales. Similarity-based scaling techniques also have been used in dynamic model-based scaling methods. Thus, the two general approaches are not mutually exclusive; on the contrary, they should be considered complementary.

The similarity-based scaling approach is elegant and powerful when it is found applicable to the problem at hand. If the scaling relations are valid, using them for extrapolation is simple and bi-directional (both up and down). However, caution must be taken when underlying processes and mechanisms are inferred from such scaling relations. Brown et al. (2000) asserted that "most biological scaling relationships are manifestations of a single underlying scaling process, which appears to be based on quarter powers and to be unique to living things." However, because ecological systems are mixtures of physical, chemical, biological, and socioeconomic processes, it is unlikely that a single similarity criterion applies to all or most of them (Prothero 1986). The empirical determination of a power scaling relationship cannot be simply taken as *prima facie* evidence for similarity (i.e., objects of different sizes are similar in geometry and shape) because "pseudosimilarity" may also result in straight lines in a log-log regression plot (Prothero 1986). Thus, geometric similarity will for sure result in power laws of simple multiples of 1/3, but the reverse is not guaranteed (Niklas 1994). The same can be said about the relationship between power laws and other kinds of similarity. One needs to bear in mind that statistical equations merely express correlations, any of which can be spurious (Prothero 1986).

There are several reasons why dynamic-model scaling methods ought to be used in many situations. First of all, not all ecological patterns and processes adequately meet the criteria of complete similarity (e.g., geometric similarity) or incomplete similarity (e.g., self-similarity or self-affinity). Therefore, alternative methods are needed for translating information from one scale to another for those patterns and processes. Second, the goals of scaling may be not only to describe and predict across scales, but also to understand patterns and processes at multiple scales. However, most similarity-based methods are empirical, relying primarily on statistical techniques, and do not deal directly with dynamic processes. Empirical equations provide useful information on quantitative relations among variables, but tell us little beyond the quantitative relations themselves (Prothero 1986). On the other hand, analytical similarity-based methods either demand explicit mathematical expressions of processes under study or start with well-established first principles. These requirements can rarely be met for most complex ecological problems. Third, ecologists are often interested in quantifying how spatial heterogeneity interacts with ecological processes in their efforts to scale across space. This requires an approach that deals with space explicitly and processes directly. Apparently, similarity-based methods that rely on regression or differential equations are inadequate to achieve this objective.

Dynamic model-based scaling methods can be used to overcome these shortcomings because (1) they are not constrained by similarity assumptions, (2) they can explicitly consider dynamic processes and their interactions, and (3) they

can couple patterns and process in spatially explicit fashion. Spatial scaling with dynamic models has two critical steps: accurately defining and quantifying the fine-scale heterogeneity and properly aggregating or integrating this heterogeneity through model arguments or outputs to derive broad-scale projections (King 1991, Blöschl and Sivapalan 1995, Wu 1999). Besides the methods discussed here, there are other dynamic modeling techniques also relevant to spatial upscaling. For examples, the methods of model simplification or model aggregation (e.g., O'Neill and Rust 1979, Iwasa et al. 1989, Cale 1995) are useful for upscaling especially when they directly address the problem of spatial aggregation. Metamodeling – developing coarse-scale models based on fine-scale models – is directly relevant to upscaling (e.g., de Vries et al. 1998, Urban et al. 1999, Bierkens et al. 2000).

To develop a science of scaling, a pluralistic strategy is necessary. The pluralism should not only be reflected in the views and theories of scaling, but also need to be implemented in the methods and applications of transferring information across scales. Pluralism does not mean arbitrary division and diversification; rather, it provides a realistic basis for enlightened scaling. It would be nice if the systems of all kinds in the universe behaved like a sandpile, so that simple power laws could adequately describe "how nature works." Although some physical, ecological, and socioeconomic systems may indeed exhibit scale-invariant behavior within certain temporal and spatial scale domains (e.g., Chave and Levin 2003, Wu 2004), scaling is certainly more than just deriving power laws. In particular, the progress in ecological scaling depends on how well we can integrate the different scaling approaches and methods, and use them appropriately for the intended problems.

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