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Estimating parameters for tree basal area growth with a system of equations and seemingly unrelated regressions

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Abstract

A method was developed for estimating parameters in an individual tree basal area growth model using a system of equations based on dbh rank classes. The estimation method developed is a compromise between an individual tree and a stand level basal area growth model that accounts for the correlation between trees within a plot by using seemingly unrelated regression (SUR) to estimate the restricted parameters. Previously, basal area growth has been modeled on either the stand or the individual tree level. Individual tree models have usually disregarded the regression assumption of independent error terms. Violation of the regression independence assumption may lead to serious underestimation of the mean square error (MSE) and standard error(s) of the parameter estimate(s). The SUR parameter estimation technique has been shown to provide a gain in efficiency for parameter estimation when the error terms for a system of equations are correlated. The data are from an ongoing natural even-aged shortleaf pine growth and yield study being conducted by the USDA Forest Service and Oklahoma State University Department of Forestry for the Ouachita and Ozark National Forests. The basal area growth model based on SUR estimation using a system of four equations (Model 2) corresponding to four dbh rank classes within a plot was compared with a basal area growth model (Model 1) using ordinary least squares (OLS) parameter estimation. The calibration, validation, and complete data set results reveal that Model 2 has a better fit index (FI) and MSE, but that Model 1 has a smaller absolute average error. Model 2 accounts for partial tree interdependency within a plot and consequently should more accurately estimate the parameter standard errors. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Shortleaf pine; Interdependency; Seemingly unrelated regression

1. Introduction

Growth and yield studies often use a system of equations to describe stand development. Early applications of systems of equations in forestry fitted the parameters of each equation independently using ordinary least squares (OLS) (e.g., Moser, 1972). Furnival and Wilson (1971) suggested that fitting the parameters for each equation in a system independently was not satisfactory because a variable may be dependent in one equation and independent in another equation. Therefore, coefficients of one equation may be functionally related to coefficients in another equation and the residuals of the equations may be correlated. They proposed that parameters for a system of equations describing forest growth and yield could be fitted simultaneously using known

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econometric techniques, which may provide an increase in parameter estimation efficiency and consistency. The general simultaneous parameter estimation technique for a system of equations may be applied to linear or nonlinear systems with small, large, or unequal sample sizes and with or without imposing constraints between parameters of the system (Reed, 1987).

Systems of equations have been used extensively in recent years to model individual tree attributes. Dyer and Burkhart (1987) developed compatible crown ratio and crown height models. The parameters were estimated using OLS for an equation by equation fit and by seemingly unrelated regression (SUR) to estimate the restricted parameters. The results illustrated a gain in parameter estimation efficiency when using SUR with parameter restrictions versus fitting each equation separately using OLS. Lynch and Murphy (1995) developed a compatible height prediction and projection system for natural even-aged shortleaf pine (*Pinus echinata* Mill.) in the Ouachita Highlands that used SUR for parameter estimation. Studies have indicated that parameter variances estimated from large samples using SUR may be less than parameter variances obtained using OLS (Judge et al., 1988). The SUR parameter estimation technique has been proven to provide a gain in parameter estimation efficiency when no endogenous (dependent) variables appear on the right side and the error terms of equations in the system are correlated (Zellner, 1962).

Individual tree basal area increment or diameter growth is often modeled using either a composite model or a potential/modifier model. Composite models usually predict individual tree growth as a function of individual tree size, competition measures, and site attributes (Wykoff et al., 1982; Wykoff, 1990; Monserud and Sterba, 1996). Individual tree growth models using a potential/modifier type function may fit the potential growth function to a data subset to estimate the maximum potential growth. The modifier function then modifies the potential growth based upon an individual tree, stand attributes, and competition measures. A variety of potential and competition modifier functions have been developed (e.g., Hahn and Leary, 1979; Leary and Holdaway, 1979; Belcher et al., 1982; Shifley and Brand, 1984; Shifley, 1987). For both the composite models and the potential/modifier models, trees within a plot are assumed to be independent.

Hasenauer et al. (1998) suggested that fitting the parameters for each equation in a system of individual tree growth models separately, which usually consists of a basal area or diameter increment model, height prediction model, and a crown ratio model, is not entirely satisfactory. It has become common to fit compatible equations using econometric techniques (e.g., height prediction and height projection equations), but the relationship among error terms in a system of growth models describing individual tree growth has been largely ignored (e.g., height prediction and basal area growth prediction equations). Hasenauer et al. (1998) fitted a system of three individual tree growth models separately using OLS, and then fitted the same equations simultaneously using two- and three-stage least squares. The three models were a basal area growth model, height increment model, and crown ratio model. Since the system of growth models had endogenous variables appearing on the right side and cross-equation correlation existed, the three-stage least squares was the most efficient technique.

Among the major linear regression assumptions is the one that the errors are independent and identically normally distributed. Nonlinear and linear regression models differ in that the nonlinear model least squares estimators of their parameters are not unbiased, normally distributed, or minimum variance estimators (Ratkowsky, 1990). Properties of nonlinear regression models tend to conform to the linear regression properties (unbiased, normally distributed parameter estimates) asymptotically as the sample size approaches infinity. Generally, the regression assumptions for linear or nonlinear regression need only be correct approximately because the least squares criterion tends to be robust in minor departures from the assumptions. However, major departures from regression assumptions, such as violating the independence assumption, may lead to serious underestimation of the mean square error (MSE) and standard error(s) of the parameter estimate(s) when calculated according to the OLS procedures (Ratkowsky, 1990; Neter et al., 1996). Gregoire et al. (1995) and Gregoire and Schabenberger (1996) used a mixed model approach to account for correlation of errors among observations for some common stand level forestry applications and standing tree cumulative bole volume.

In forestry applications, models describing individual tree basal area or diameter growth have usually disregarded the regression independence assumption. It is reasonable to assume that trees located within a plot have some interdependency because of the competition for resources on the plot. Trees within a plot share a similar microenvironment, which may be above or below average for tree growth. The purpose of this study was to develop an individual tree basal area growth model using a system of equations to account for tree interdependency within a plot. The system of equations basal area growth model will be based on a density-independent even-aged shortleaf pine basal area growth model similar to that currently used in a growth and yield prediction system for shortleaf pine (Lynch et al., 1999). The performance of the basal area growth model fitted by SUR using a system of equations will be compared with the same basal area growth model form fitted to individual trees by OLS.

2. Data

The data are from a cooperative study being conducted by the USDA Forest Service and Oklahoma State University Department of Forestry to develop growth and yield models for natural even-aged shortleaf pine (*P. echinata* Mill.) stands of the Ouachita and Ozark National Forests of southeastern Oklahoma and western Arkansas. Original plot installation was during the dormant seasons of 1985–1987 when basic forest measurements were recorded; subsequent remeasurements were recorded on a 4- or 5-year interval for each plot. A detailed discussion of plot reconnaissance, installation, and location is given by Murphy (1988).

A total of 191 plots located in the Ouachita and Ozark National Forests were installed covering a wide spectrum of site index (height in meters for base age 25 years), age (years), and density (basal area per hectare) classes. There are four site index (<17, 18, 21, and >23 m), age (20, 40 60, and 80 years), and density (7, 14, 21, and 28 m^2/ha) classes for a total of 64 combinations. Each combination of the site–age–density classes originally was to have three replicates for a total of 192 plots. However, only two plots were located for the age 20 years, site index >23 m, and residual basal area 7 m^2/ha combination. Therefore, only 191 plots were actually installed.

The study design called for 0.04592 ha (0.2 acre) circular plots, surrounded by a 3.0657 m (33 ft) isolation buffer for each site–age–density combination. Chemical herbicide Tordon 101R or Weedon CB was injected to control any existing hardwoods greater than or equal to 2.54 cm (1.0 in.) diameter at ground level. The shortleaf pines were thinned from below when necessary to maintain or achieve the basal area class for both the plot and the buffer. The residual shortleaf pines on the 0.04592 ha plot were numbered, measured, located from plot center, and tallied for all trees greater than or equal to 2.54 cm (1.0 in.) dbh. At each plot, representative dominant and codominant shortleaf pines were selected for each 5 cm dbh class by the following method for determining total height, height to live crown, and age. If there were only one or two trees, then both trees were sampled; if there were less than 25 trees, then three sample trees were selected, or if there were more than 25 trees, then five trees were selected. The plot site index was

Table 1
Summary statistics of the OSU/USDA cooperative Ouachita Highlands natural even-aged shortleaf pine study for developing a basal area growth model

Attribute	*N*	Minimum	Maximum	Mean	Standard error
Plot age (years)	183	21.0	96.0	42.4	19.09
Stand basal area (m^2/ha)	183	5.17	32.72	23.50	6.97
dbh (cm)	7444	2.92	63.26	19.15	9.70
Average annual individual tree basal area growth (m^2/year)	7444	0.00	0.006667	0.001221	0.0009783
Individual tree basal area (m^2)	7444	0.0006714	0.3143	0.03620	0.03622
Ratio of quadratic mean diameter to individual tree dbh	7444	0.4424	4.2410	1.0971	0.3352

calculated using a site index equation developed by Graney and Burkhart (1973) for shortleaf pine of the Ouachita Highlands based on the average total height and age of the representative dominant and codominant trees. The dbh was measured to the nearest 0.254 cm (0.1 in.) for all shortleaf pine greater than or equal to 2.54 cm. Eight plots from the USDA/OSU cooperative growth and yield study were decommissioned because of stand damage or failure to establish thinning treatments. The remaining 183 plots contain 7444 individual tree observations that are available for developing growth and yield models for natural even-aged shortleaf pine of the Ouachita Highlands. The summary statistics for the 183 plots are given in Table 1.

3. Model development

A system of four equations using dbh rank classes within a plot was developed based on the functional form of the distance-independent basal area growth model that is currently being used in a growth and yield prediction system for natural even-aged shortleaf pine (Lynch et al., 1999). The basal area growth model is of the potential/modifier form in which the potential function (numerator) is a modified Chapman–Richards (Richards, 1959; Chapman, 1961) function that was developed by Shifley and Brand (1984) to constrain the maximum growth of the potential function. The potential function is constrained for a biologically reasonable maximum tree size, which for shortleaf pine of the Ouachita highlands was estimated as having a 95.94 cm dbh, and the equivalent basal area is 0.7229 m^2. The biologically reasonable maximum tree size was derived by averaging the estimated maximum diameter found from local records and the largest shortleaf pine recorded in the National Records of Big Trees (American Forests, 2000). The modifier (denominator) is a modified logistic function similar to that used by Murphy and Shelton (1993, 1996) that is constrained between 0 and 1. The competition modifier reduces potential growth based upon variables representing stand conditions, tree attributes, and competition measures. The potential/modifier function is of the same basic form at that used in TWIGS (Miner et al., 1989) for the north central states, although TWIGS uses a different modifier. Model 1 is currently used in the shortleaf pine simulator to predict individual tree basal area growth in natural stands of even-aged shortleaf pine and has the following form:

$$\mathrm{AABAG}_i = \frac{\beta_1 \mathrm{BA}_i^{\beta_2} - (\beta_1 \mathrm{BA}_i / M^{(1-\beta_2)}) + e_i}{1 + \mathrm{e}^{(\beta_3 + \beta_4 \mathrm{SBA} + \beta_5 \mathrm{AGE} + \beta_6 \mathrm{DD}_i + \beta_7 \mathrm{BA}_i)}} \quad (1)$$

where AABAG_i is the average annual growth (m^2/year) for the individual shortleaf pine i, BA_i m^2 of basal area for the individual tree i, M the maximum basal area (M=0.7229 m^2), SBA the stand basal area (m^2/ha), DD_i the ratio of quadratic mean diameter to individual tree diameter for tree i, AGE the stand age in years, e_i is the random error and where β_k for $k=1,2,\ldots,7$ are parameters.

Model 2 uses an individual tree equation that is mathematically identical to Model 1, but the data set was revised for modeling basal area growth using a system of four equations that correspond to their respective diameter rank class within each plot. Diameter rank classes were used rather than traditional dbh classes because parameter estimation with SUR requires the same number of equations, with corresponding diameter classes, for each plot. Using traditional 1 or 2 in. dbh classes resulted in an unequal number of dbh classes for many plots. Therefore, many plots would be dropped during the SUR parameter estimation procedure. The use of diameter rank classes can be used to obtain an equal number of classes for each plot. Four diameter rank classes allow for fitting the model over a range of dbh classes. The original data set consists of one record for each individual tree. It was revised to create four dbh rank classes corresponding to the individual tree diameters within each plot.

The revised data set for modeling basal area growth using a system of four equations was developed through the following steps. Individual tree diameters within each plot were ranked in ascending order. The four dbh rank classes were computed by dividing the ranked dbh tree list by 4 for each plot. If the ranked dbh tree list was evenly divisible by 4, then each dbh rank class had an equal number of trees. The dbh rank class 1 corresponds to the dbh of the smallest tree(s) for a given plot. A coded routine was written to place trees in the correct dbh rank classes if the number of trees on a plot was not evenly divisible by 4. For

example, if a plot has 10 trees, then the first and second dbh rank classes would have three trees and the remaining two dbh rank classes would have two trees.

Model 2 used the revised data set containing the dbh rank classes and has four equations that correspond to their respective dbh rank class. Model 2 was designed as a compromise between fitting parameters at the individual tree level and fitting parameters at the stand level. Diameter class mean growth is fitted to the mean functional form for the four dbh rank classes, with common equation parameters, as follows:

$$\overline{\mathrm{AABAG}_{1m}} = \frac{1}{n_1}\sum_{j=1}^{n_1}\frac{\beta_1 \mathrm{BA}_j^{\beta_2} - (\beta_1 \mathrm{BA}_j/M^{(1-\beta_2)})}{1+\mathrm{e}^{(\beta_3+\beta_4\mathrm{SBA}+\beta_5\mathrm{AGE}+\beta_6\mathrm{DD}_j+\beta_7\mathrm{BA}_j)}} + e_1$$

$$\overline{\mathrm{AABAG}_{2m}} = \frac{1}{n_2}\sum_{j=1}^{n_2}\frac{\beta_1 \mathrm{BA}_j^{\beta_2} - (\beta_1 \mathrm{BA}_j/M^{(1-\beta_2)})}{1+\mathrm{e}^{(\beta_3+\beta_4\mathrm{SBA}+\beta_5\mathrm{AGE}+\beta_6\mathrm{DD}_j+\beta_7\mathrm{BA}_j)}} + e_2$$

$$\overline{\mathrm{AABAG}_{3m}} = \frac{1}{n_3}\sum_{j=1}^{n_3}\frac{\beta_1 \mathrm{BA}_j^{\beta_2} - (\beta_1 \mathrm{BA}_j/M^{(1-\beta_2)})}{1+\mathrm{e}^{(\beta_3+\beta_4\mathrm{SBA}+\beta_5\mathrm{AGE}+\beta_6\mathrm{DD}_j+\beta_7\mathrm{BA}_j)}} + e_3$$

$$\overline{\mathrm{AABAG}_{4m}} = \frac{1}{n_4}\sum_{j=1}^{n_4}\frac{\beta_1 \mathrm{BA}_j^{\beta_2} - (\beta_1 \mathrm{BA}_j/M^{(1-\beta_2)})}{1+\mathrm{e}^{(\beta_3+\beta_4\mathrm{SBA}+\beta_5\mathrm{AGE}+\beta_6\mathrm{DD}_j+\beta_7\mathrm{BA}_j)}} + e_4 \tag{2}$$

where, $\overline{\mathrm{AABAG}_{1m}}$ is the plot m mean average annual basal area growth for dbh rank class l, j the individual tree observation within a dbh rank class (class=1, 2, 3, 4) on a plot, n_l the individual tree observation(s) in dbh rank class l on a plot, e_j the error term with mean 0 for dbh rank class j, and where BA, M, SBA, AGE, β_k, and DD are as defined previously.

4. Parameter estimation

Model 1 parameters were estimated using nonlinear regression and the following algorithm. The estimated parameters for the potential function were initially fitted separately using the 5% fastest growing shortleaf pine by the 5 cm diameter class. If a diameter class had less than 21 observations, then all observations for that diameter class were used to estimate the initial potential function parameters. The modifier parameters were estimated while holding the potential function estimated parameters constant. These initial parameter estimates were then used as starting values for fitting all parameters simultaneously.

Model 2 consists of four equations that can be classified as seemingly unrelated equations because these equations have no endogenous variables appearing on the right-hand side and are linked because the error terms across equations are correlated (Pindyck and Rubinfeld, 1981). If the $\mathrm{cov}(e_i, e_j)=0$, where e_i and e_j are error terms for equations i and j, for all combinations of i and j, then SUR is not appropriate, but OLS would be appropriate. If the $\mathrm{cov}(e_i, e_j)\neq 0$, then a correlation between the errors of the equations exists and SUR is an appropriate parameter estimation technique. Zellner (1962) proved that the SUR parameter estimation technique provides a gain in efficiency when the error components of a system of seemingly unrelated equations are correlated. Model 2 used SUR to estimate the parameters for the system of equations because a correlation is expected between the error components within each plot for the four dbh rank classes. The correlation is expected because it is reasonable to assume that trees within a plot are ecologically interdependent and competing for finite resources.

The potential and modifier parameters for Model 2 were estimated using the technique described for Model 1 and the ITSUR option of the SAS/ETS MODEL (SAS Institute Inc., 1993) procedure with parameter restrictions placed across the system of equations. The parameters for Model 2 were estimated using the means of the basal area growth function for each dbh rank class within a plot. Because the function evaluated at the mean does not equal the mean of the function for nonlinear equations, a coded routine was employed to estimate the mean of the basal area growth equations. The means for the basal area growth equations were computed before each iteration by dbh rank class for each plot using the estimated parameters for the respective iteration until the convergence criteria were achieved.

5. Results

The data set was divided into calibration and validation data sets to calibrate and evaluate the individual tree basal area growth models. The plots were stratified by site indices, stand ages, and basal area per hectare combinations. Approximately one-third of the plots within each combination of age, site index, and basal area classes were selected randomly to form a validation data set. The calibration and validation data

sets contain 128 and 55 plots, respectively. The summary statistics for the calibration and validation data sets are similar with no substantial differences.

The calibration data set was used to estimate the initial parameters for both models. The calibration data set residuals for both models exhibited heteroscedasticity, primarily with respect to the dbh classes. The parameters were re-estimated using the reciprocal of individual tree basal area and reciprocal of plot basal area as weights for Models 1 and 2, respectively. The resulting residuals for both models using their respective weights exhibit no major departures from homoscedasticity, and therefore, no further weighting or transformations were necessary. Model 1 has a fit index[2] (FI) of 0.6343 and an RMSE of 0.0005942, while Model 2 has an FI of 0.6394 and an RMSE of 0.0005901. The validation data set used the calibration data set parameters to evaluate the performance of the models on an independent data set. Results from the validation data set for Models 1 and 2 are presented in Table 2. These results indicate that Model 2 has a higher FI while Model 1 has a smaller absolute average error.

The pairwise correlation between dbh rank class residuals was examined for Model 2 to evaluate the appropriateness of the SUR methodology. The residual correlation matrix is

$$\begin{bmatrix} 1.0 & 0.4683 & 0.6196 & 0.4426 \\ & 1.0 & 0.5850 & 0.3618 \\ & & 1.0 & 0.5928 \\ & & & 1.0 \end{bmatrix}$$

for the dbh rank classes 1–4 (left to right, top to bottom). These are the correlation coefficients for residuals corresponding to the four dbh rank classes in the system of equations. A likelihood ratio test described in Morrison (1976) was performed to determine if $H_0{:}\sigma_{ij}=0$, $\forall\ i\neq j$ of the residual dbh rank class covariance matrix, indicating insignificant correlation. The calculated χ^2 ($\cong$216) was compared with a $\chi^2_{(6,0.05)}$ critical value, and hence, there is evidence of a correlation between the dbh rank classes (P<0.000001). Consequently, SUR should provide a gain in parameter estimation efficiency as compared to the OLS fit of the system. Model 2 was fitted using OLS with no parameter restrictions placed across the equations for comparison with the ITSUR fit with parameter restrictions. Reduction of standard errors for estimates from ITSUR indicated a large gain in parameter estimation efficiency when Model 2 was fitted using ITSUR with parameter restrictions placed across equations. The pairwise correlation for dbh rank classes may be positive due to the fact that the study entails thinning from below and most suppressed trees were removed. Since the microenvironment on a plot is similar for all dbh rank classes, if growth in the first rank class is above the mean, growth in the second class also tends to be above the mean. Therefore, the residuals tend to be positively correlated.

Both models were fitted to the complete data set for further evaluation. Plot 261 was removed from the data set for fitting the parameters for Model 2. There were only two trees on plot 261, and therefore, it could not be used for estimating the parameters for a system of four equations when using the dbh rank classification system. The parameter estimates and standard errors for both models are presented in Table 3. The

[2] Fit index$=1-\sum(Y_i-\hat{Y}_i)^2/\sum(Y_i-\overline{Y})^2$.

Table 2
Models 1 and 2 fit index (FI), mean square error (MSE), root mean square error (RMSE), mean error using the validation data set (N=2328)

Model	FI[a]	MSE[b]	RMSE[c]	Mean error[d]
1	0.6467	0.0000003317	0.0005759	0.00002724
2	0.6490	0.0000003296	0.0005741	0.00006876

[a] FI$=1-\sum(Y_i-\hat{Y}_i)^2/\sum(Y_i-\overline{Y})^2$.
[b] MSE$=\sum(Y_i-\hat{Y}_i)^2/(n-p)$.
[c] RMSE$=\sqrt{\text{MSE}}$.
[d] Mean error$=\sum(\hat{Y}_i-Y_i)/n$.

Table 3
Parameter estimates and standard errors for Models 1 (N=7444) and 2 (N=182) when fitted to the complete data set

Parameter	Model 1		Model 2	
	Estimate	Standard error	Estimate	Standard error
β_1	0.076527	0.009221	0.020183	0.006535
β_2	0.710360	0.018820	0.442285	0.108450
β_3	−2.829696	0.229556	−1.933852	0.614670
β_4	0.059874	0.002728	0.052266	0.006893
β_5	0.025990	0.001326	0.022179	0.004430
β_6	1.260140	0.060193	0.723326	0.215280
β_7	−6.593778	0.611129	−11.716314	2.211250

Table 4
Models 1 and 2 fit index (FI), mean square error (MSE), root mean square error (RMSE), and mean error using the complete data set (N=7444)

Model	FI	MSE	RMSE	Mean error
1	0.6376	0.0000003468	0.0005889	−0.00001224
2	0.6461	0.0000003387	0.0005820	0.00003579

parameter estimates for both models exhibit the same overall trends with respect to the signs and magnitude. The standard errors for Model 1 are more favorable than for Model 2; however, the standard errors of parameter estimates in Model 1 may be underestimated due to violation of the assumption of independent errors. In addition, the standard errors for Model 1 are based on 7444 individual tree observations, whereas the standard errors for Model 2 are based on 182 plot observations.

The FI, MSE, RMSE, and mean error for Models 1 and 2 when calculated using the complete data set are presented in Table 4. Model 2 has a more favorable FI (0.6461) and RMSE (0.00005820) than Model 1 (FI=0.6376 and RMSE=0.00005889). However,

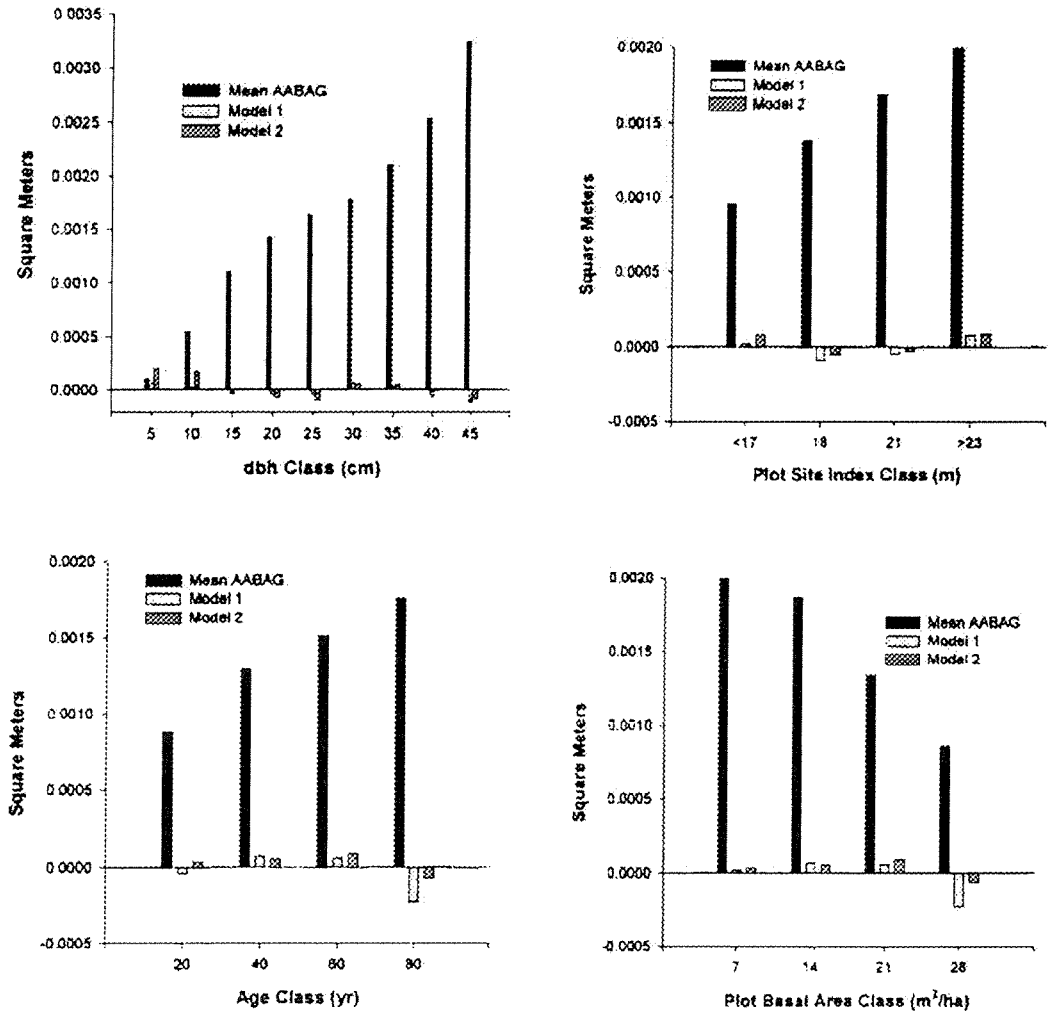

Fig. 1. Bar charts for average deviation by dbh, age, site index, and plot basal area class and mean average annual basal area growth for Models 1 and 2 using the complete data set.

Model 1 (−0.00001224) has a smaller absolute average error than Model 2 (0.00003579).

The Model 1 and 2 average deviation by attribute classes and mean AABAG are presented in Fig. 1. The graphs reveal that both models follow the same general trend with respect to bias. The bar chart by dbh class illustrates that Model 2 performs poorly for the 5 and 10 cm dbh class with an average deviation larger than the mean AABAG for the 5 cm dbh class. Model 1 performs more favorably for the 5 and 10 cm dbh class, but performs poorly versus Model 1 for the 15 and 40 cm dbh classes. The bar chart for average deviation by site index class and mean AABAG illustrates that Model 1 performs more favorably for site index class 17, while Model 2 performs more favorably for the plot site index classes 18 and 21. Model 1 performs more favorably for the plot basal area class 21 but performs poorly for plot basal area

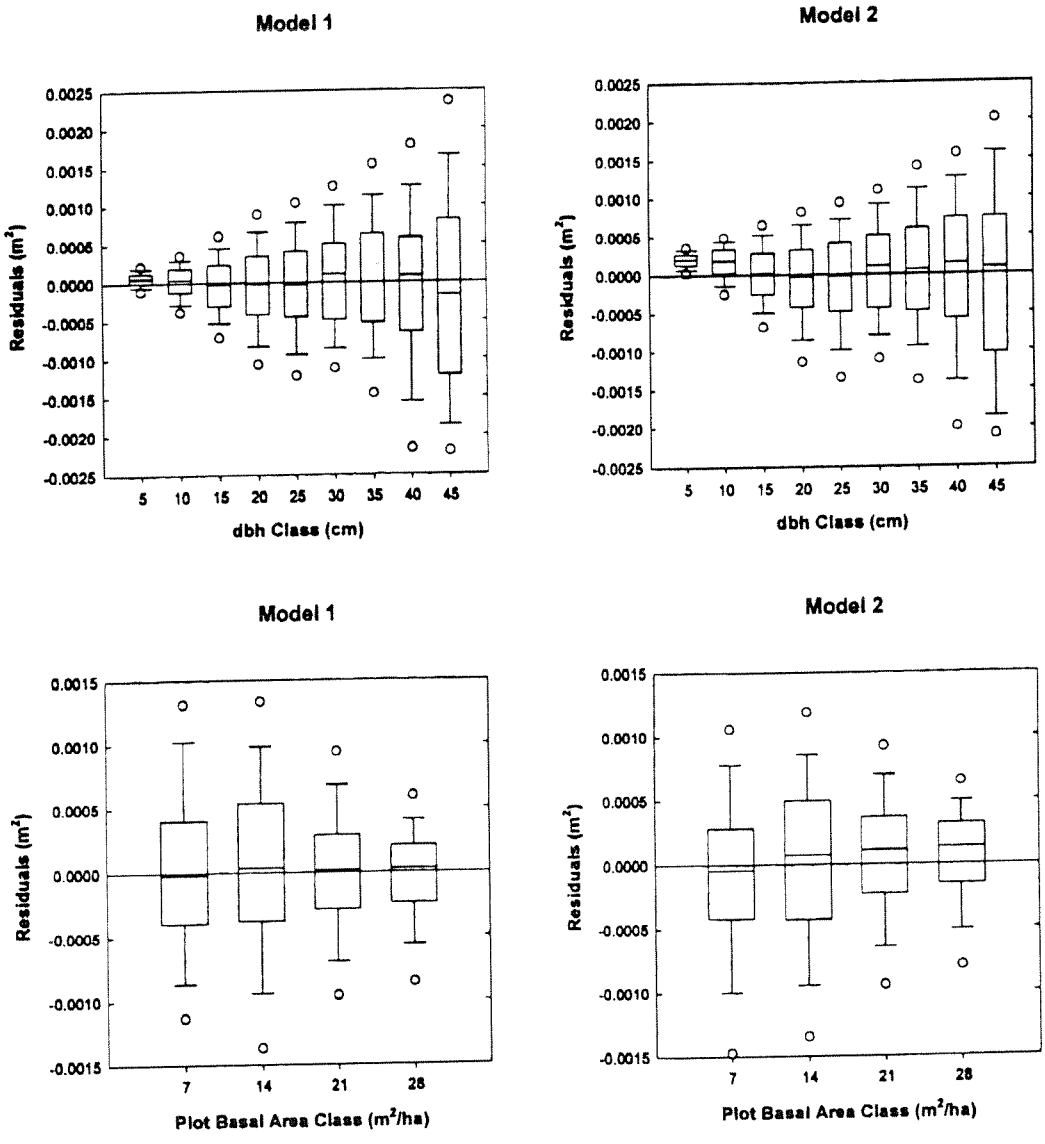

Fig. 2. Box plots of the residuals by dbh and plot basal area class for Models 1 and 2 using the complete data set.

6. Conclusion

The calibration, validation, and complete data set results reveal that Model 2 performs more favorably for FI and MSE, whereas Model 1 has a smaller absolute average error. Neither model performs substantially better across the range of attribute classes with respect to bias. However, Model 2 does exhibit less variability across the spectrum of attribute classes than Model 1. Using SUR to estimate the parameters should provide a gain in parameter estimation efficiency while providing consistency across the system of equations. A formal test for differences between the standard errors for the estimated parameters for Models 1 and 2 would present difficulties. The estimated parameters for Model 1 are based on 7444 individual tree observations, whereas Model 2 parameter estimates are based on 182 plot observations. Although Model 1 has smaller standard errors for the parameter estimates, this is likely due to the substantial number of individual tree observations used in the parameter estimation. As discussed earlier, major departures from the assumption of uncorrelated errors such as for Model 1 may result in a significant underestimation for the standard error(s) of the estimated parameter(s) and MSE. If the MSE is underestimated, then the t-value for $H_0{:}\beta_i{=}0$ will be overestimated, and consequently, H_0 may be rejected when it should not be rejected. Hence, the model covariate selection process is biased. In addition, the confidence and prediction intervals which depend on the standard errors may be substantially underestimated. Although SUR provided gains in efficiency over the equation by equation fit using OLS in Model 2, it cannot be definitely concluded that Model 2 provides a gain in parameter estimation efficiency when compared with Model 1. Model 2 partially accounts for tree interdependency within a plot and should more accurately estimate the standard errors for parameter estimates. Therefore, the confidence and prediction intervals based on Model 2 should be more accurate than intervals based on Model 1.

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