The Components of Change for an Annual Forest Inventory Design

Francis A. Roesch

Abstract: The sample design of the USDA Forest Service's Forest Inventory and Analysis Program (FIA) with respect to a three-dimensional population (forest area × time) of tree attributes is formally defined and evaluated. The definitions for both the traditional components of growth, as presented by Meyer (1953, *Forest Mensuration*), and a discrete analog to the time invariant redefinition of the components of change given by Eriksson (1995, *Forest Sci.* 41(4):796–822), are compared and contrasted. Special problems in the application of the traditional definitions due to the continuous and overlapping temporal intervals featured in the sample design are explored. This exploration supports a contention that the traditional definitions are at a theoretical disadvantage because they are not based purely on the population(s) of interest, while the redefinitions dubbed the components of change by Eriksson (1995) were based solely on population attributes. The temporally discrete analog to the Eriksson definitions are used in this article to define the three-dimensional populations in terms of a set of mutually exclusive component matrices that can be summed to represent the entire population, independently of the sample design. FOR. SCI. 53(3):406–413.

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THE USDA FOREST SERVICE'S Forest Inventory and Analysis Units (FIA) report on the condition of forests within the United States and its territories. To this end, the Forest Service uses a temporally rotating, panelized forest inventory sampling design to estimate both the current resource inventory and changes in the resource. The sample plots are located relative to a systematic triangular grid consisting of g mutually exclusive interpenetrating panels. If the number of sample plots equals n, then each panel consists of approximately n/g plots. One panel is measured per year for g consecutive years, after which the panel measurement sequence reinitiates. That is, if panel 1 was measured in 1997, it will also be measured in 1997 + g, 1997 + $2g$, and so on. Panel 2 would then be measured in 1998, 1998 + g, 1998 + $2g$, etc. (Figure 1.) The panels are assigned to plots in a spatially systematic manner. Since the initiation of the rotating panel design for FIA, there have been quite a few articles focusing on the combination of panels for the purpose of improving value per area estimates. Fewer focus on the improvement of value estimation temporally. Some exceptions have been Van Deusen (1996, 1999) and Roesch et al. (2003).

For simplicity, I will restrict this discussion to the case of five annual panels ($g = 5$) sampling a fixed area through time. Considering a fixed area allows us temporarily to ignore land entering and exiting the area of interest, as may occur in actual inventories. This can be a practical problem; however, it is not a theoretical problem. A practitioner must simply keep in mind that if land is allowed to enter and exit the population of interest, then either the conditional inclusion probabilities must be treated as temporally specific or those areas that were not in the population during the entire period of interest must be treated as subpopulations. As stated above, it is intended that a panel be measured each year. Ideally, starting with year 6, 20% of the plots (i.e., one panel) will be remeasured each year. Roesch and Reams (1999) give one explanation of the current sample frame, as it applies to implementation of the design by the Southern Research Station (SRS) for the interested reader.

In this article assume that the entire sample frame is a three-dimensional volume, two dimensions constitute the land area and the third dimension is time. One could consider the sample unit to be a series of line segments, linear in time. That is, when the time dimension is collapsed onto the area dimensions, each series of line segments collectively appear as a single point on the area. Each line segment within a series is of an appropriate length to the question being addressed. For discussion, I will begin with a line segment of length 1 year. Individual segments occur every g years within each series. In the case we consider, the entire series of segments constitutes the sample unit. The temporal slices of interest could be of any height; however, the thinner the slice, the smaller the sample per land area of interest and the taller the slice, the greater the number of sample subpopulations due to potential changes in both the land area of interest and the biological subpopulations of trees within the land area. In this case, the entire series of line segments within the panel (one segment for each time the panel is measured within a period of interest) constitutes the sample unit within an infinite population. The role of this linear sample unit in the three-dimensional population is analogous to the role of the sample point in the two-dimensional populations usually considered in the forestry literature. Likewise, a three-dimensional finite population sampling unit can be defined by collecting sets of these series of line segments in the same way that Roesch et al. (1993b) defined a two-dimensional (areal) finite unit by collecting sets of points. For most forestry purposes, annual

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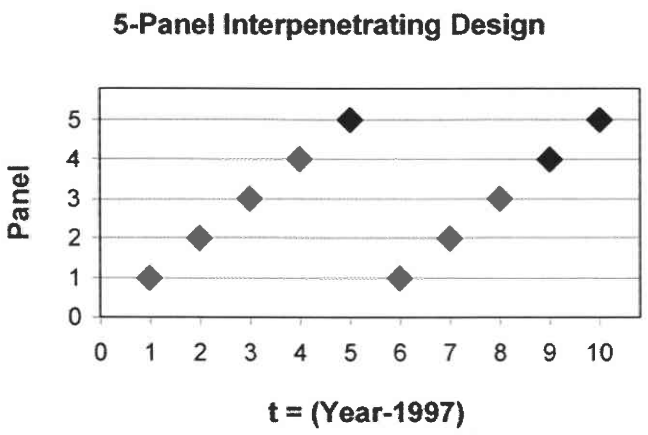

Figure 1. A temporal rotating panel design with five panels.

slices will constitute the minimum height for the line segments to achieve a reasonable compromise between temporal specificity and land area generality.

FIA's bi-areal plot design (Bechtold and Patterson 2005, p. 29) consists of fixed area plots of two sizes resulting in a reduced but similar problem for estimating the components of growth to those that occur on variable-radius plots. The latter have received much discussion in the literature, owing to the phenomenon that trees cross thresholds of inclusion probability as they grow, e.g., Flewelling (1981), Martin (1982), Flewelling and Thomas (1984), Van Deusen et al. (1986), Roesch (1988, 1989), Roesch et al. (1989, 1991, 1993a), Thomas and Roesch (1990), Roesch and Van Deusen (1993). This is not a concern here because we consider only the population of trees above the merchantability limit of 12.7 cm dbh and leave the obvious extensions to the cases of other (or multiple) merchantability limits to the reader. Specifically, only the population being sampled by the larger of the two plots in the bi-areal design is considered here.

The remainder of the article is arranged as follows. First, I show the traditional components of growth as presented by Meyer (1953), and show estimators that have been used with analogous designs to estimate those components for this design. Not considered here are the many potential subcategories of these components, such as those owing to changes of tree class over time. Next, I show the intuitively appealing redefinition of the components of change given by Eriksson (1995), and explain how a temporally discrete analog to those components is related to an annual inventory design.

The Traditional Components of Growth

Traditionally, the components of forest growth have been expressed as:

$$V_2 - V_1 = S + I - M - C, \qquad (1)$$

where V_i = the total value at time i, $i = 1, 2$; S = survivor growth; I = ingrowth; M = mortality; and C = cut.

Estimators for the components of growth are considered compatible if they can replace the population parameters in Equation 1 without destroying the equality. Equation 1 was presented by Meyer (1953), and used in related contexts by Beers and Miller (1964), Flewelling (1981), Martin (1982), Flewelling and Thomas (1984), Van Deusen et al. (1986), Roesch (1988, 1989), Roesch et al. (1989, 1991, 1993b), Thomas and Roesch (1990), and Roesch and Van Deusen (1993), to name a few.

Survivor growth is the growth in value that occurs on trees that are above some minimum merchantability limit at both measurement times 1 and 2, while ingrowth is the time 2 value of trees that are below the merchantability limit at time 1 and above the limit by time 2. Mortality is the time 1 value of trees that die during the measurement interval. Similarly, cut is the time 1 value of trees that are harvested during the measurement interval.

Define the indicator column vector for ingrowth trees, $\mathbf{i}_I$, of length equal to the number of trees alive at either time 1 or 2. Each row corresponds to an individual tree in the population and contains a 1 if the corresponding tree is an ingrowth tree and a zero otherwise. Similarly define the analogous indicator vectors for survivor trees, $\mathbf{i}_S$, mortality trees, $\mathbf{i}_M$, and harvest trees, $\mathbf{i}_C$, utilizing the same indices as $\mathbf{i}_I$.

The four indicator vectors are of equal length and can be concatenated into the population indicator matrix of four columns, $\mathbf{P} = \mathbf{i}_I : \mathbf{i}_M : \mathbf{i}_C : \mathbf{i}_S$. Each row of $\mathbf{P}$ sums to 1 because, by definition, all trees in the population belong to one, and only one, of these four categories. Likewise, define the parameter matrix for the values of the population of trees at times 2 and 1, again using the same row indices as $\mathbf{i}_I$. That is, row i represents tree i, and column j represents time $j = 2, 1$:

$$\mathbf{V}_{2,1} = \begin{bmatrix} v_{1,2} & v_{1,1} \\ v_{2,2} & v_{2,1} \\ \vdots & \vdots \\ \vdots & \vdots \\ v_{N,2} & v_{N,1} \end{bmatrix}$$

Let $\mathbf{v}_2$ represent the first column of $\mathbf{V}_{2,1}$, and $\mathbf{v}_1$ represent the second column of $\mathbf{V}_{2,1}$. Also, let $\mathbf{i}_1 = \mathbf{i}_S + \mathbf{i}_M + \mathbf{i}_C$, and $\mathbf{i}_2 = \mathbf{i}_S + \mathbf{i}_I$ represent the merchantable population members at times 1 and 2, respectively. Additionally, we define the difference vector with one column and two rows:

$$\mathbf{d} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

The change components to be estimated are:

Ingrowth: $I = \mathbf{i}_I'\mathbf{v}_2$;
Survivor growth: $S = \mathbf{i}_S'(\mathbf{V}_{2,1}\mathbf{d})$;
Merchantable mortality: $M = \mathbf{i}_M'\mathbf{v}_1$;
Merchantable cut: $C = \mathbf{i}_C'\mathbf{v}_1$.

The instantaneous volumes to be estimated are

Merchantable volume at time 1: $V_1 = \mathbf{i}_1'\mathbf{v}_1$;
Merchantable volume at time 2: $V_2 = \mathbf{i}_2'\mathbf{v}_2$.

For simplicity, we will assume a population of N trees, indexed with $k = 1$ to N, with each tree having a known probability of inclusion for each random point, $\pi_k = a_k/\mathrm{A}$, where a_k is the selection area of tree k, and A is the total area of the population. These assumptions disallow any potential

for edge bias because exact inclusion probabilities are known. Although the consideration of edge bias is beyond the scope of this article, a quite thorough treatment of the topic can be found in Ducey et al. (2004). The sample is represented by a diagonal matrix, $\boldsymbol{\Psi}$, indexed as above for each tree, $k = 1, \ldots, N$, such that row k, column k contains a 1 if tree k is selected for the sample, and a zero otherwise.

Let the expansion constant for tree k be $C_k = m_k(n\pi_k)^{-1}$, where n is the number of points and m_k is number of points from which tree k is selected for the sample. Note that in the FIA design m_k may only take on the values of 0 or 1. Collect the expansion constants into an $N \times N$ diagonal matrix, $\boldsymbol{\Pi}$, indexed as above for each tree in the population, $i = 1, \ldots, N$, such that row k, column k contains C_k.

Survivor Growth

The sample of survivor trees can be represented by the vector $\boldsymbol{\psi}_S = \boldsymbol{\Psi}\mathbf{i}_S$. The estimator for a sample of size n plots for the total value of survivor trees at time 1 can be expressed as

$$\hat{S}_1 = (\boldsymbol{\Pi}\boldsymbol{\psi}_S)'\mathbf{v}_1.$$

Likewise, the value of survivor trees at time 2 can be expressed as

$$\hat{S}_2 = (\boldsymbol{\Pi}\boldsymbol{\psi}_S)'\mathbf{v}_2.$$

An estimator for the traditional definition of total survivor growth is simply the difference between $\hat{S}_2$ and $\hat{S}_1$:

$$\hat{S} = \hat{S}_2 - \hat{S}_1 = (\boldsymbol{\Pi}\boldsymbol{\psi}_S)'(\mathbf{V}_{2,1}\mathbf{d}).$$

Ingrowth

The traditional definition of ingrowth is the value at time 2 of trees that were below the merchantability limit at time 1 and above the limit by time 2. The sample of ingrowth trees can be represented by the vector $\boldsymbol{\psi}_I = \boldsymbol{\Psi}\mathbf{i}_I$.

Several partitions of this sample category may be recognized, however we will not discuss them here because we do not need them for the most common estimator of ingrowth, which ignores any observations that were made on ingrowth at time 1. The estimator for a sample of size n plots for the total value of ingrowth trees at time 2 can be expressed as

$$\hat{I} = (\boldsymbol{\Pi}\boldsymbol{\psi}_1)'\mathbf{v}_2.$$

Mortality

Trees that were alive and above minimum dbh at the first measurement but die before the second measurement belong to the mortality population. The sample of this population is indicated by $\boldsymbol{\psi}_M = \boldsymbol{\Psi}\mathbf{i}_M$.

The growth on trees that were not sampled and below the minimum dbh at the first measurement, become eligible but die before the second measurement, regardless of their final diameter, will be ignored (even if, in fact, there is some evidence of the existence of these trees). Growth on mortality is sometimes estimated through application of a growth model. Assuming this is not done, the sample-based estimator would be

$$\hat{M} = (\boldsymbol{\Pi}\boldsymbol{\psi}_M)'\mathbf{v}_1.$$

Cut

Trees that were alive and above minimum dbh at the first measurement but are harvested before the second measurement belong to the cut population. The sample of this population is represented as $\boldsymbol{\psi}_C = \boldsymbol{\Psi}\mathbf{i}_C$.

As with mortality, trees that were not sampled and below the minimum dbh at the first measurement but are cut before the second measurement will be treated as unobserved (even if, in fact, there is some evidence of their existence). As with mortality, growth on cut is sometimes estimated with the use of a model. When this is not the case, the sample-based estimator of cut would be

$$\hat{C} = (\boldsymbol{\Pi}\boldsymbol{\psi}_C)'\mathbf{v}_1.$$

Instantaneous Estimates of Volume

The sample of trees at time 1 can be represented by the matrix $\boldsymbol{\psi}_1 = \boldsymbol{\Psi}\mathbf{i}_1$. An estimator for a sample of size n plots for the total value at time 1 can be expressed as

$$\hat{V}_1 = (\boldsymbol{\Pi}\boldsymbol{\psi}_1)'\mathbf{v}_1.$$

Likewise, the estimator for value at time 2 can be expressed as

$$\hat{V}_2 = (\boldsymbol{\Pi}\boldsymbol{\psi}_2)'\mathbf{v}_2, \quad \text{where} \quad \boldsymbol{\psi}_2 = \boldsymbol{\Psi}\mathbf{i}_2.$$

Compatibility

The estimator for the merchantable value at time 1 can be re-expressed as $\hat{V}_1 = \hat{S}_1 + \hat{M} + \hat{C}$, while that for the merchantable value at time 2 is re-expressed as $\hat{V}_2 = \hat{S}_2 + \hat{I}$. This leads to the compatible grouping, $\hat{V}_2 - \hat{V}_1 = \hat{S} + \hat{I} - \hat{M} - \hat{C}$.

Above, we gave the simple sample based estimators for the components of growth by the traditional definitions of those components. The weakness of the traditional definitions of the components of growth lies in their inherent dependence on the length of the measurement interval. That is, the definition of the components of growth for the population of trees is dependent on the time that the relative sample stages are executed. Therefore, by this definition, these components are not strictly population parameters to be estimated. Rather, they are a convenient marriage of a population and its sample. Unfortunately, the marriage only remains convenient as long as the sample design does not change in any significant way. To relate the data to the population, the following two criteria must be met: (1) the time interval must be fixed, and (2) the duration of each measurement period must be sufficiently small so that growth within the measurement period can be ignored. Unfortunately, forest inventories often result in significant variance in the length of measurement interval and therefore growth data are usually "annualized" by simply dividing by the measurement interval for each plot.

Previously, during the historic periodic FIA inventories,

the average annual growth was estimated over a relatively long remeasurement interval. In the southern United States it was usually between 7 and 12 years. The estimate of average annual growth calculated over a shorter growth interval is actually a measure of a different population parameter, because tree growth is nonlinear with respect to time. Therefore, estimates from different measurement intervals are not readily comparable. For example, let us look from the population perspective and consider at first a single condition class. We will define a condition class as a group of stands similar enough to be modeled by a common yield curve. For discussion purposes, I will use a "typical" yield curve for cubic volume, specifically the one given in Avery and Burkhart (1983, p. 277), converted to cubic meters per hectare:

$$Y = 0.0699726 * e^{10-32A^{-1}} \quad (2)$$

where A = age of the stand. The graph for Equation 2 is given in Figure 2.

When we calculate the average annual cubic meter accretion from a series of t-year lags, (where t = 1, 2, 5, 10, and 20), by differencing the model in Equation 2 from itself, we obtain the five curves in Figure 3. That is, the curve for the t-year average accretion (R_t) is

$$R_t = 0.0699726 * \frac{e^{10-32A^{-1}} - e^{10-32(A-t)^{-1}}}{t}.$$

If our sample design had a 2-year measurement interval (a 2-year lag), we would be selecting our estimates of average annual cubic meter accretion from the curve weighted the most toward the actual annual cubic meter accretion curve (the 1-year lag curve). If we used a 5-year measurement interval, we would be selecting our annual cubic meter accretion estimates from the curve weighted more toward the right. The 10- and 20-year curves are still further toward the right, and therefore "smoother." These curves are the sampling distributions from which we sample growth given a particular measurement interval. Given this, coupled with the fact that we sample many condition classes concurrently, all following different initial growth curves, the usual indifference assumption with respect to measurement interval length is ill advised. These observations suggest that the traditional definitions are inadequate for continuous forest inventories, such as the USDA Forest Service's annual inventory design.

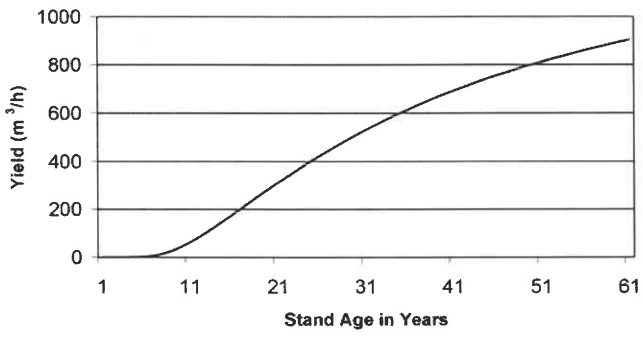

Figure 2. The graph of a "typical" yield curve ($Y = 0.0699726 * e^{10-32A^{-1}}$), converted to cubic meters per hectare from Avery and Burkhart (1983, p. 277).

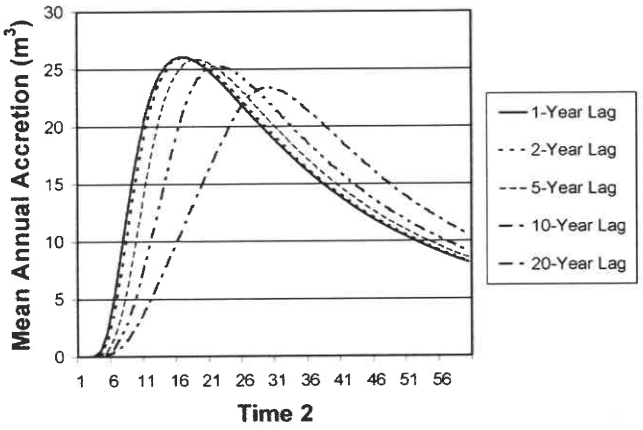

Figure 3. The average annual cubic meter accretion from a 1-year lag, a 2-year lag, a 5-year lag, a 10-year lag, and a 20-year lag, using the model in Figure 2.

A Discrete Analog to the Components of Change

Eriksson (1995) recommended a new set of definitions and labeled them the components of change, applicable over a temporal continuum as opposed to the traditional sample-based definitions defined over discrete periods. Eriksson (1995) took the position that the traditional growth component definitions do not allow estimators that are time-additive over multiple period lengths. For example, if a 10-year period was of interest and all plots were measured in years 0, 5, and 10, then the sums of the expected values of the estimators of each component over the two intervals (years 0–5 and 5–10) would not equal the expected value of the estimators sans the year 5 measurement. The components of change given by Eriksson (1995) are defined by population attributes and are therefore not sample-dependent. Nonadditivity is a valid concern due to a fundamental flaw in the original definition of the components of growth. Additionally, the redefinitions become extremely compelling in the realm of annual inventories. A mild criticism of Eriksson's work that could be raised by adherents to design-based estimation is that for the time additivity argument to be true, one must somehow have knowledge of unobserved events. Specifically, the series of events in which a tree becomes eligible to be measured and subsequently dies or is harvested between observation instances must be known. Eriksson (1995) points out this problem and offers a partial solution. A full solution is not possible. However, I might remind the reader that sampling is, by definition, a subset of the potential observations on a natural system. The discussion above at least suggests that the traditional definitions are inadequate for time-interpenetrating sample designs, such as FIA's rotating panel design.

The traditional component of ingrowth consists of (1) the value of the ingrowth trees when they reach the minimum merchantability limit, and (2) the value growth subsequent to attaining the minimum merchantability limit. Eriksson

(1995) identified this latter component as a component of survivor growth, using the argument that the tree is a survivor once it has passed the minimum merchantability threshold (or "entered" the population), regardless of whether or not the entry was observed.

To apply the time invariant definition of the components of change and to properly partition the population, the traditional ideas of ingrowth and survivor growth become less useful, and all growth occurring after a tree's entry into the merchantable size class would more properly be referred to as live merchantable tree growth. Following the work in Eriksson (1995), live tree growth is the growth in value that occurs on trees after the minimum merchantability limit has been achieved. Entry is the value of trees as they attain the minimum merchantability limit. Mortality is the value of trees as they die, while Cut is the value of trees as they are harvested. Eriksson's (1995) definitions were purposefully continuous; however, I will use a discrete analog with a small (1-year) interval length. I am doing this for two reasons: (1) I am making an assumption that 1 year is about the minimum interval length, in most forest conditions, required for each of the respective change signals to overpower measurement error; and (2) to allow some reasonable temporal partitioning of the observations. The discrete intervals allow the definition of a set of indicator matrices, one for each component, having one row for each tree in the population during the forest inventory. To estimate the components of change for intervals of any length, let N = the total number of live merchantable trees that are in the population during a specific temporal period, h = the initial year in the period of interest, t = the number of years of interest, and P = the number of years in the population ($t \leq P$).

Define the ($N \times P$) indicator matrix for trees as they enter the merchantable population during the forest inventory:

$$\begin{array}{c} \text{time} = \quad P \quad P-1 \quad \cdots \quad 1 \quad \text{tree} \\ \mathbf{I}_E = \begin{bmatrix} 0 & 0 & \cdots & 1 \\ 0 & 0 & \ddots & 0 \\ 0 & 1 & \ddots & 0 \\ 1 & 0 & \cdots & 0 \end{bmatrix} \begin{matrix} 1 \\ \vdots \\ \vdots \\ N \end{matrix} \end{array}.$$

In $\mathbf{I}_E$ and all subsequent population matrices, each column represents a year, the first column being the most recent year of the inventory (or the present, P), and each successive column 1 year before the previous column. Each row corresponds to an individual tree in the population. A 1 in row i and column j indicates that tree i entered the population in year j. All other entries in row i are zeros. Similarly define the analogous indicator matrices for tree mortality year, $\mathbf{I}_M$, and tree harvest year, $\mathbf{I}_H$. The indicator matrix for the live category also has one row for each tree and contains a 1 in the column for each year that a tree is alive and in the population of interest subsequent to the entry year and before its year of harvest or death, and a zero otherwise:

$$\begin{array}{c} \text{time} = \quad P \quad P-1 \quad \cdots \quad 1 \quad \text{tree} \\ \mathbf{I}_L = \begin{bmatrix} 1 & 1 & \cdots & 0 \\ 1 & 1 & \ddots & 1 \\ 1 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \begin{matrix} 1 \\ \vdots \\ \vdots \\ N \end{matrix} \end{array}.$$

The four indicator matrices are of equal dimension and sum to the population indicator matrix, $\mathbf{I}_P$. Observe that, by definition, the elements of $\mathbf{I}_P$ consist solely of ones and zeros. The value matrix is analogously structured in that row i represents tree i, and column j represents time j in reverse annual order from the present year back:

$$\mathbf{V} = \begin{bmatrix} v_{1,P} & v_{1,P-1} & \cdots & v_{1,1} \\ v_{2,P} & v_{2,P-1} & & \\ \vdots & & \ddots & \vdots \\ v_{N,P} & & \cdots & v_{N,1} \end{bmatrix}.$$

Entry value can be represented in the column vector

$$\mathbf{v}^{\mathrm{E}} = \begin{bmatrix} v_{E,1} \\ v_{E,2} \\ \vdots \\ \vdots \\ v_{E,N} \end{bmatrix}$$

In general, when used independently, we will index column vectors of $\mathbf{V}$ with the year represented, that is the column of $\mathbf{V}$ corresponding to year h will appear $\mathbf{v}_h$. Usually a specific period of length (t) beginning in year h within the length of the continuous inventory (years 1 through P) will be of interest. To effect the partitioning of $\mathbf{V}$ and derived matrices, we form the column vector $\mathbf{y}_{h+t,h}$ with a row for each year in reverse annual order from the last year to the first year of the inventory. A position contains a 1 for all years of interest ($h + t$ through h) and a zero otherwise.

The temporal selection operator $\mathbf{f}_{h+t,h}(\cdot)$ selects the columns from a row vector or matrix that falls in the range $h + t$ to t. For instance, the value matrix of interest may actually be a temporal partition of $\mathbf{V}$:

$$\mathbf{V}_{h+t,h} = \mathbf{f}_{h+t,h}(\mathbf{V}) = \begin{bmatrix} v_{1,h+t} & v_{1,h+t-1} & \cdots & v_{1,h} \\ v_{2,h+t} & v_{2,h+t-1} & & \\ \vdots & & \ddots & \vdots \\ v_{N,h+t} & & \cdots & v_{N,h} \end{bmatrix}.$$

We should mention that by only partitioning $\mathbf{V}$ temporally, there will be some rows of $\mathbf{V}_{h+t,h}$ in which all values are zero because of trees not being in the merchantable population during the period of interest.

We define a temporally inferior selection matrix, $\check{\mathbf{S}}$, having P columns and P rows, such that in the last $P - 1$ columns, i, row $i + 1$ contains a one and all other entries are zero. The first row and last column contain all zeros.

We also define the first difference matrix, $\mathbf{D}$, with P columns and P rows, such that in the first $P - 1$ columns, i, row i contains a one and row $i + 1$ contains a negative

one, and all other entries are zero. The final column contains all zeros.

We might be interested in estimators of the sum of the temporal vectors of population change components:

Entry: $E_{h,h+1} = (\mathbf{I}'_{\mathrm{E}}\mathbf{v}^{\mathrm{E}})'\mathbf{y}_{h+t,h+1}$,

Live growth:

$$L_{h,h+t} = \{[([\mathbf{I}_{\mathrm{L}} \cdot * (\mathbf{VD})] + [\mathbf{I}_{\mathrm{E}} \cdot * (\mathbf{V} - \mathbf{v}^{\mathrm{E}})])]'\mathbf{1}\}'y_{h+t,h+1},$$

where $\cdot *$ represents element-by-element matrix multiplication and $\mathbf{1}$ is an $(N \times 1)$ summation vector of ones. When the negation operator $(-)$ occurs between a matrix and a column vector, as in the above equation $(\mathbf{V} - \mathbf{v}^{E})$, and later in this article, it indicates subtraction of the column vector from each column of the matrix. Additionally, we have

Mortality: $M_{h,h+t} = [(\mathbf{I}_M \cdot * (\mathbf{V\check{S}}))'\mathbf{1}]'\mathbf{y}_{h+t,h+1}$,

and Harvest: $H_{h,h+t} = [(\mathbf{I}_H \cdot * (\mathbf{V\check{S}}))'\mathbf{1}]'\mathbf{y}_{h+t,h+1}$.

Alternatively, we might be interested in estimators of the temporal vectors of population change components:

Entry vector: $\mathbf{e}_{h+t,h+1} = (\mathbf{I}'_{\mathrm{E}}\mathbf{v}^{\mathrm{E}}) \cdot * [\mathbf{y}_{h+t,h+1}]$,

Live growth vector:

$$\mathbf{1}_{h+t,h+1} = \{[([\mathbf{I}_L \cdot * (\mathbf{VD})] + [(\mathbf{I}_E \cdot * (\mathbf{V} - \mathbf{v}^{E})])]'\mathbf{1}\} \cdot * [\mathbf{y}_{h+t,h+1}],$$

Mortality vector: $\mathbf{m}_{h+t,h+1} = [(\mathbf{I}_{\mathrm{M}} \cdot * (\mathbf{V\check{S}}))'\mathbf{1}] \cdot * [\mathbf{y}_{h+t,h+1}]$,

and Harvest vector:

$$\mathbf{h}_{h+t,h+1} = [(\mathbf{I}_{\mathrm{H}} \cdot * (\mathbf{V\check{S}}))'\mathbf{1}] \cdot * [\mathbf{y}_{h+t,h+1}],$$

We also seek estimators of the total value vector at times $h + t$ to h:

$$\mathbf{y}_{h+t,h} = [(\mathbf{I}_{\mathrm{P}} \cdot * \mathbf{V})'\mathbf{1}] \cdot * [\mathbf{y}_{h+t,h}].$$

Sample-Based Estimators

Again, we will assume a population of N trees, indexed with $k = 1$ to N, with each tree having a known probability of inclusion for each point, $\pi_k = a_k/\mathrm{A}$, where a_k is the selection area of tree k and A is the total area of the population. Again, these assumptions disallow any potential for edge bias because exact inclusion probabilities are known. Also, without loss of generality, assume that a tree's assignment to temporal panel p is random, with probability equal to $1/g$. In this case, the joint probability of selection of tree k in panel p from a single point is then $\pi_{kp} = \pi_k/g$.

Let the sample panel be represented by a diagonal matrix, $\mathbf{\Psi}$, indexed as above for each tree in the population during the inventory, $k = 1, \ldots, N$, such that row k, column k contains a one if tree k is selected for the sample panel, and a zero otherwise. The time dependant representation of the sample $\mathbf{\Psi}_P = \mathbf{\Psi I}_P$.

Let the expansion factors of the panel sample be represented by an $N \times N$ diagonal matrix, $\mathbf{\Pi}_p$, indexed as above for each tree in the population, $i = 1, \ldots, N$, such that row k, column k contains $m_{kp}(n\pi_{kp})^{-1}$, where m_{kp} is the number of times tree k is selected for sample panel p from the n points. Again m_{kp}, like m_k above, will only take on the values of 0 or 1 in the FIA design.

The panel sample of merchantable trees living until the next year can be represented by the matrix $\mathbf{\Psi}_L = \mathbf{\Psi I}_L$, while the panel sample of entry trees since the previous year is represented by $\mathbf{\Psi}_E = \mathbf{\Psi I}_E$. Following suite, represent the panel sample of trees that will die before the next year as $\mathbf{\Psi}_M = \mathbf{\Psi I}_M$, and the panel sample of trees that will be harvested before the next year as $\mathbf{\Psi}_H = \mathbf{\Psi I}_H$. Unfortunately, the panel design does not allow $\mathbf{\Psi}_L$, $\mathbf{\Psi}_E$, $\mathbf{\Psi}_M$, or $\mathbf{\Psi}_H$ to be uniquely observed because of the remeasurement interval of t years. Rather, what we observe are the vectors $\boldsymbol{\psi}'_E = \mathbf{\Psi I}_E\mathbf{y}_{h+t,h+1}$, $\boldsymbol{\psi}'_M = \mathbf{\Psi I}_M\mathbf{y}_{h+t,h+1}$, and $\boldsymbol{\psi}'_H = \mathbf{\Psi I}_H\mathbf{y}_{h+t,h+1}$ for entry, mortality, and harvest, respectively. For live growth let $\mathbf{f} = \mathbf{\Psi I}_L\mathbf{y}_{h+t,h+1}$, where $\mathbf{f}$ is a column vector of length N. Form another column vector of length N, $\boldsymbol{\psi}'_L$, the observation vector due to the sample. For $i = 1$ to N, let $\boldsymbol{\psi}'_L[i] = 1$ if $\mathbf{f}[i] = t$.

We will use the assumption that no growth occurred on mortality and cut trees during the year of death or harvest. During the time interval of interest, a tree can contribute to multiple components of change. For example, an individual may enter the population, live for 2 years, and then die between observation instances.

With an overlapping panel design, not all of the events are observable because the measurement interval is t years and the minimum growth period that we are recognizing is 1 year. However, it is important to distinguish clearly the attributes that truly belong to the population from those that are artifacts of the sample design. Any suggestion for estimation must recognize and efficiently use (1) a measurement interval length that is longer than the minimum growth interval of interest, and (2) annually overlapping measurement intervals that result from the rotating panel design.

Within Panels—The New in Terms of the Old

The relationship between the period of interest and the measured data can be quite complex under the new definitions of the components of change. The traditional definitions conveniently assumed that the period of interest corresponded to the times of measurement. I will first consider the simple case in which the number of years of interest (t) equals the measurement interval for a single panel. This simple case illustrates the relationship between the traditional estimators and the new estimators.

Entry

The traditional ingrowth component consists of both the entry value of the ingrowth trees between years h and $h + t$, $E_{h+t,h+1}$, plus the growth on the ingrowth trees ($I^g_{h+t,h+1}$) subsequent to entry. Subsequent to the first cycle of measurements, each year's measurements provides change estimates over the interval t. The strictly sample-based per-panel estimator for $E_{h+t,h+1}$ would be

$$\hat{E}_{h+t,h+1} = (\mathbf{\Pi}_p\boldsymbol{\psi}'_E)'\mathbf{v}_E.$$

The per-panel estimator for growth on ingrowth

($I^g_{h+t,h+1}$) can then be expressed as $\hat{I}^g_{h+t,h+1} = \hat{I} - \hat{E}_{h+t,h+1}$. Note that $\hat{E}_{h+t,h+1}$ is an unavoidably biased estimator of $E_{h+t,h+1}$, as mentioned above, because it cannot include trees that became eligible and subsequently died or were harvested between observation instances. The longer the interval between observations relative to the expected rotation age of the forest type (or life expectancy of the forest stand), the more significant this bias becomes.

Mortality

The mortality component ($M_{h+t,h+1}$) consists of the traditional mortality component plus the growth on the mortality trees ($M^g_{h+t,h+1}$). The strictly sample-based per-panel estimator for $M_{h+t,h+1}$, $h > t$, could be expressed as $\tilde{M}_{h+t,h+1} = (\boldsymbol{\Pi}_p \boldsymbol{\psi}^t_M)' \mathbf{v}_i = \hat{M}$. As mentioned earlier, the value is unobserved for mortality trees past year h and the value at the year of death ($h + d$) for tree i ($v_{h+d,i}$) would have to be modeled conditional on the value at year h ($v_{h,i}$). For a model-based estimator, form the column vector $\mathbf{v}^g_d$, in which element i for tree i is equal to $v_{h+d,i} = v_{h,i}$, if the row i in $\boldsymbol{\psi}^t_M$ contains a 1. Assuming an unbiased estimator of $\mathbf{v}^g_d$, $\hat{M}_{h+t,h+1}$ could be formed by $\hat{M}_{h+t,h+1} = \tilde{M}_{h+t,h+1} + \hat{M}^g_{h+t,h+1}$, where $\hat{M}^g_{h+t,h+1} = (\boldsymbol{\Pi}_p \boldsymbol{\psi}^t_M)' \mathbf{v}^g_d$. Even if an unbiased estimator exists for $M_{h+t,h+1}$, like $\hat{E}_{h+t,h+1}$, $\hat{M}_{h+t,h+1}$ is a biased estimator because it cannot include the unobserved subpopulation of trees that became eligible and subsequently died between observation instances.

Harvest

As with mortality, the harvest component ($H_{h+t,h+1}$) consists of both the traditional cut component plus the growth on the harvested trees ($H^g_{h+t,h+1}$). The strictly sample-based per-panel estimator for $H_{h+t,h+1}$ could be expressed as $\tilde{H}_{h+t,h+1} = (\boldsymbol{\Pi}_p \boldsymbol{\psi}^t_H)' \mathbf{v}_h = \hat{C}$. Also, as with mortality, the value at the year of harvest ($h + c$), for tree i ($v_{h+c,i}$) is unobserved and would have to be modeled conditional on the value at year h ($v_{h,i}$). Form the column vector $\mathbf{v}^g_H$, in which element i for tree i is equal to $v_{h+c,i} - v_{h,i}$ if the corresponding row in $\boldsymbol{\psi}^t_H$ contains a 1. Assuming an unbiased estimator of $\mathbf{v}^g_H$, $\hat{H}_{h+t,h+1}$ could be partitioned into $\hat{H}_{h+t,h+1} = \tilde{H}_{h+t,h+1} + \hat{H}^g_{h+t,h+1}$, where $\hat{H}^g_{h+t,h+1} = (\boldsymbol{\Pi}_p \boldsymbol{\psi}^t_H)' \mathbf{v}^g_H$. As with the previous two estimators, even if an unbiased estimator exists for $H^g_{h+t,h+1}$, $\hat{H}_{h+t,h+1}$ is a biased estimator because it cannot include trees that became eligible, and were subsequently harvested, between observation instances.

Live Growth

Define the true value matrix for the values of the population of trees at times $h + t$ and h by selecting the corresponding columns from $\mathbf{V}$. That is, row i represents tree i, and column j represents time $j = h + t, h$,

$$\mathbf{V}_{h+t,h} = \begin{bmatrix} v_{1,h+t} & v_{1,h} \\ v_{2,h+t} & v_{2,h} \\ \vdots & \vdots \\ \vdots & \vdots \\ v_{N,h+t} & v_{N,h} \end{bmatrix}.$$

Additionally, we define the panel difference vector with one column and two rows,

$$\mathbf{d}^P = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

A simple estimator for live growth per panel would be equivalent to the traditional survivor growth estimator, where time 1 equals h and time 2 equals $h + t$, plus the estimators for growth on ingrowth, mortality, and cut:

$$\hat{L}_{h+t,h+1} = (\mathbf{V}_{h+t,h}\mathbf{d}^P)'(\boldsymbol{\Pi}_p \boldsymbol{\psi}^t_L) + \hat{I}^g_{h+t,h+1} + \hat{M}^g_{h+t,h+1} + \hat{H}^g_{h+t,h+1}.$$

Again, as with the previous estimators, $\hat{L}_{h+t,h+1}$ will be biased by the volume of growth on trees that became eligible and then died or were harvested between observation instances.

Instantaneous Estimates of Volume

Let $\mathbf{v}_{h+t}$ be the first column of $\mathbf{V}_{h+t,t}$ and $\mathbf{v}_h$ the second column of $\mathbf{V}_{h+t,t}$. For this simple case, the merchantable value at the beginning of the panel (time h), may still be expressed much as it was previously in context for the traditional components of growth,

$$\hat{V}_h = (\boldsymbol{\Pi}_p \boldsymbol{\psi}_P)' \mathbf{v}_h = \hat{S}_h + \hat{M} + \hat{C}. \tag{3}$$

The estimator for the merchantable value at the end of period of interest (time $h + t$):

$$\hat{V}_{h+t} = (\boldsymbol{\Pi}_p \boldsymbol{\psi}_P)' \mathbf{v}_{h+t} \tag{4}$$
$$= \hat{V}_h + \hat{L}_{h+t,h+1} + \hat{E}_{h+t,h+1} - \hat{M}_{h+t,h+1} - \hat{H}_{h+t,h+1}.$$

Compatibility

Compatibility has been achieved because the sample-based estimate of the merchantable value at time $h + t$ equals the sum of the estimates of each component plus the merchantable value at time h. By construct, Equation 4 is immediately analogous to the compatible grouping for the traditional definitions of the components of growth, only because the period of interest equals the measurement interval:

$$\begin{aligned} \hat{V}_{h+t} &= \hat{V}_h + \hat{L}_{h+t,h+1} + \hat{E}_{h+t,h+1} - \hat{M}_{h+t,h+1} - \hat{H}_{h+t,h+1} \\ &= (\hat{S}_h + \hat{M} + \hat{C}) + (\hat{S} + \hat{I}^g_{h+t,h+1} + \hat{M}^g_{h+t,h+1} + \hat{H}^g_{h+t,h+1}) \\ &\quad + \hat{E}_{h+t,h+1} - (\hat{M} + \hat{M}^g_{h+t,h+1}) - (\hat{C} + \hat{H}^g_{h+t,h+1}) \\ &= \hat{S}_{h+t} + \hat{I}. \end{aligned}$$

Although the change component estimators in (4) are all biased, the grouping is nevertheless compatible because the bias in $\hat{L}_{h+t,h+1}$ and $\hat{E}_{h+t,h+1}$ is complemented exactly by the bias in $\hat{M}_{h+t,h+1}$ and $\hat{H}_{h+t,h+1}$. Additionally, the bias in each of these components will often be extremely small for two reasons: (1) the contribution to the bias comes from relatively small trees, and (2) trees within ~5 years of having reached eligibility that die or are harvested are a relatively small component of many forest populations. The smaller the investigator's area of interest and the stronger the market forces to harvest merchantable trees in that area of interest, the greater will be the risk in ignoring the bias in these components.

Conclusion

If we subscribed to the traditional definitions of the components of growth, there would be no logical argument for combining panels because the panels represent overlapping but different temporal intervals. That is, by the traditional definitions of the components of growth, alternative intervals are not of interest because a specific interval, demarked by the times of measurement, is an explicitly defined part of the component.

In this article we observed and supported a contention that the traditional definitions of the components of growth in the sense of Meyer (1953) were not based purely on the population(s) of interest, while the redefinitions dubbed the components of change by Eriksson (1995) were based strictly on population attributes. The dependency of those original definitions on the temporal scale of observation confounded understanding of the measurement of basic forest dynamics under alternative sample designs. If we adopt these new definitions of the components of change, and if we can successfully estimate each of the annual components L_j, E_j, M_j, and H_j, then we would have estimators for any interval of interest. Recall that the discussion above referencing Figures 2 and 3 demonstrates the ill-advisedness of simply assuming linearity within the panel and applying an annual mean to each component for each year in the period. In addition, sample sizes within panels are too small to provide good estimates within many geographic subareas of interest. This leads to the question of how we might combine panels to estimate the components of change. It is obvious from the data structure that there are many potential weighting schemes to combine the information from temporally overlapping panels to add strength to the interval estimates. Van Deusen (1996, 1999) and Roesch et al. (2003) discuss this for general value estimation but neither applies those results to the components of change. This article establishes purely population-based definitions for the components of change over discrete temporal intervals.

Adoption of the discrete analog presented here to the more recent definitions of Eriksson allows a clear distinction between the effect of scale in the definition of the components of change from the effect on our ability to estimate the components of change at different scales caused by the sample design. Once this distinction has been made, it is clear that the annual estimates of the components of change can only be obtained through the use of models applied to the sample design. There are no strictly design-unbiased estimators for the annual components of change available for this rotating panel design. Although not demonstrated here for the components of change, the mixed estimation technique (Van Deusen 1996, 1999) was designed to use models to make improved estimates for small temporally specific samples by drawing strength from measurements made on temporal "neighbors." Mixed estimation is therefore a viable approach that bears investigation for the estimation of the components of change.

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