CHAPTER 3

NATURAL DISTURBANCE PRODUCTION FUNCTIONS

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1. INTRODUCTION

Natural disturbances in forests are driven by physical and biological processes. Large, landscape scale disturbances derive primarily from weather (droughts, winds, ice storms, and floods), geophysical activities (earthquakes, volcanic eruptions, even asteroid strikes), fires, insects, and diseases. Humans have always been affected by these processes and have invented ways to harness such processes or manipulate vegetation to enhance the values obtained from nature or reduce their negative impacts on human societies. For example, humans have cleared brush using fire to reduce pest[1] populations and encourage forage for animals (Pyne 1995). Historically, humans have relied on traditions, rules of thumb, and trial and error to predict how their actions may affect disturbance probabilities and characteristics. More recently, economic assessment tools have helped gauge the consequences of natural disturbances on forests.

As the availability of science, technology, and environmental data have improved, scientists and economists have been able to quantify disturbances as production processes that emanate from a combination of biological, physical, and (or) human-initiated inputs. Ecologists have long recognized that disturbances lead to changes in ecological communities, which subsequently affect human societies. Economists, on the other hand, have been focused on understanding how humans can intervene to alter both the frequency and severity of natural disturbances. Improving scientific and economic assessment tools, and experience using them, have in turn helped us to appreciate the many consequences of natural disturbances. The objectives of this chapter are to (1) define disturbances and their stages, (2) discuss how mathematical expressions of disturbance processes, disturbance production functions, may differ from the production functions defined in neoclassical economics, (3) identify the stages of disturbances, (4) provide a typology of production functions relevant to forest

[1] We define a "pest" in this chapter as a plant, animal (especially an insect), or disease that potentially causes damages to, or reduces output of, a valued good or service.

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disturbances, and (5) conclude with a discussion of management and science implications of recent research. Our focus is to understand how disturbances are produced and how they may be affected by intentional managerial actions. We show that quantitative characterization of disturbance processes is required to understand how management interventions into disturbances can lead to net societal gains. Throughout the chapter, we provide examples of how information about disturbances can be used to better achieve management and policy goals.

2. DEFINITIONS OF DISTURBANCES

2.1 Natural Disturbances as Production Processes

A natural disturbance is a process that results in significant changes in ecosystem structure, leading to alterations in function and the goods and services that humans derive from nature. Disturbances, or their outcomes, may be affected by human-mediated inputs. Forest disturbances can be small or large—e.g., affecting a few plants in the forest or areas the size of a continent. Disturbances often have multidimensional implications for ecosystems and society. For example, fires can be described by the area that they burn, the quantity or value of the timber that is damaged, or the heat that they release. The disturbance process is also multi-staged. It proceeds from introduction to establishment, spread, and an endpoint. Although disturbances require non-human mediated natural inputs at every stage, human-mediated inputs can affect any or all stages. For example, a pest can be spread by people but requires suitable weather and hosts to survive and reproduce. As well, fire can be started by a match but driven by wind and fueled by native vegetation.

Human and natural inputs into disturbance processes may also be temporally defined and sequence–dependent. For example, above normal rain last year followed by a drought this year would produce a different wildfire output this year than if the sequence of rain and drought were reversed. Disturbances are stochastic—their outputs are in part randomly determined, even given the same level and temporal sequence of all inputs. In mathematical notation, $h = H(Y,Z)+\varepsilon$, where h is a disturbance output such as acres burned, H is a function describing how the variables Y and Z affect the disturbance output, and ε is a random shock added to $H(\bullet)$. Finally, disturbances may have short- and long-run consequences for ecosystems and the societies that depend on them. For example, a large wildfire in one location consumes fuels and vegetation today, altering how future fires in the same and neighboring locations may develop.

Economists have developed models that account for disturbances when making production and investment decisions. The models incorporate natural disturbances in two ways: (1) in a commodity production objective function and (2) in a management objective function. In the commodity production approach, disturbances have been viewed as either an exogenous (nuisance) or an endogenous process.

When viewing disturbances as nuisances, management decisions do not affect the probability that a disturbance will occur. Disturbances such as ice storms and hurricanes are suited to the nuisance approach in forestry, for example, because their occurrence is not affected by how land is managed. The nuisance approach in forestry was first described by Martell (1980) and Routledge (1980) and then by Reed (1984) and extended by several others (Yin and Newman 1996). In this approach, the likelihood of a nuisance disturbance leads to lower optimal stand densities and shorter optimal rotation lengths. In agricultural and resource economics, economists have long recognized that exposure to a production hazard of any sort lowers optimal investment levels (Just 1975, Pope and Kramer 1979), especially when decision makers are risk averse (Friedman and Savage 1948, Babcock and Shogren 1995).

More recent research has recognized that human actions can affect the probability, extent, duration, or severity of many disturbance events. For example, Shogren (1991) described an economic model that included disturbances as "endogenous risks" in the production process. Here, actions that individuals take can affect the probability of the disturbance and therefore individual utility. Shogren and Crocker (1991), recognizing work by Erlich and Becker (1972), describe the problem as a joint decision on how much effort to expend in self protection and reducing the probability of loss. In general, then, an aggregate objective function could be described that maximizes welfare by allocating spending across efforts that reduce the value lost if a disturbance occurs and the probability that a disturbance related loss occurs.

At its simplest, disturbance enters an endogenous risk objective function as a probability of occurrence, expressed as a function of a single action taken by a manager. An example is construction of a firebreak to reduce wildfire arrival rates. More complex are actions that can affect multiple features of the disturbance. In this case, human interventions affect not only the probability of occurrence but also qualitative features (e.g., severity) of the disturbance affecting the commodity in question.

When the time and space dimensions of disturbances are important considerations in production of desired goods and services from nature, then decisions on how to intervene in the disturbance process may increase in complexity. For example, actions taken today to reduce damages from a pest invasion in one forest may affect the future risks faced by other forests (Gumpertz et al. 2000). In wildfire management, reducing fuels levels in one location can affect fire arrival rates in other locations and may have effects that last several years. These spatio-temporal effects of management can sometimes limit the scope of action for managers: management decisions in location *A* are subject to the conditions in locations *B*, *C*, and *D* and to the decisions made in the past in location *A*.

Additionally, human attempts to reduce the probability of occurrence or damages resulting from a disturbance may affect the probabilities of other forest disturbances occurring in the same location (Meyers and van Lear 1998). For

example, forest thinning to reduce fuels for wildfires might increase the probability of insect or disease infestation; and salvaging burned timber to reduce net economic damages can raise the probability of other disturbances such as exotic species invasions (McIver and Starr 2000, 2001).

Because disturbances themselves can affect many economic sectors (Butry et al. 2001, Kent et al. 2003), it is possible that actions in one sector have spillover consequences for other sectors. For example, forest thinning to reduce damages to timber from a potential forest fire may affect features important to recreators in that same forest. Somewhat more complicated still is when an intervention helps improve one value obtained from a forest but worsens other values. For example, prescribed burning can reduce wildfire severity and extent but can also worsen air quality.

In contrast to the endogenous risk approach, some economists have placed the disturbance production process at the center of economic decision making, especially when values produced are dispersed, public, or multi-sectoral, and one example is the "cost plus loss model" (Headly 1916, Sparhawk 1925). This model describes the wildfire suppression resource allocation decision as choosing the quantities of wildfire intervention inputs that minimize the sum of expected net damages from wildfire (the losses) and expenditures on the intervention inputs (the costs). Davis (1965) outlines a method of minimizing the sum of costs and expected losses from wildfires occurring over a wildfire season by manipulating the amounts of fuels and other inputs into wildfire management in the management unit. The cost plus loss framework is not the only one available for managing disturbances directly. For example, the optimal set of inputs to wildfire management can be chosen so as to maximize averted damages minus input costs (see chapter 18). Sharov and Liebhold (1998) describe how to optimally slow the spread of an exotic forest insect by identifying the best width and location of buffer zones. The disturbance-centered approach requires understanding not only of the disturbance production process but also how the disturbance creates losses. In other words, the nature of the loss function must be known. In the case of wildfire, the loss function's value must be identified for every possible or feasible combination of disturbance management inputs.

2.2 Disturbances as Damage Processes

Research has shown that managerial actions can influence the scale of losses from disturbances (Holmes 1991, Butry et al., Kent et al. 2003, Prestemon and Holmes 2004). One way to capture how a disturbance causes economic losses is to define a damage function, a mathematical expression that quantifies how variables influencing a disturbance result in damages to valued goods and services.

In agricultural economics, much research has focused on understanding how to optimally use pesticides to reduce the damages to agricultural commodities (Lichtenberg and Zilberman 1986, Carpentier and Weaver 1997, Kuosmanen et al. 2006). Mathematically, a damage function may be described as $G(X,Y,Z)$,

where X are inputs intended to increase the output of the desired good (i.e., the purchased inputs into good production), Y are inputs intended to decrease the damages caused by the disturbance process (i.e., the purchased inputs into the damage process), and Z are the natural inputs into the damage and good production processes (i.e., the free inputs). In terms of forests or landscapes, the desired good Q could be the flow of goods and services provided by an "undamaged" forest or landscape in a given time period. In terms of a country, the desired good Q could be total economic welfare produced by the economy of the country in a given time period. Q is directly affected by X and Z but potentially also by Y, and it is reduced by the damage process, G: $Q=f[X,Y,Z,G(X,Y,Z)]$. For example, inputs applied to change the amount of fine fuels on a landscape (part of Y) can also lead to changes in the growing conditions faced by trees whose timber may be the desired output, Q. Free inputs, such as rain, contained in Z, can affect the productivity of fuels management efforts and the growth rate of trees.

If we define a disturbance process as the more general description of a phenomenon that can damage a commodity or reduce the quality or quantity of a value produced by, say, a forest, then the damage function is a transformation of the disturbance process: $G(X,Y,Z) = g[H(X,Y,Z)]$. The function g is a transformation of a disturbance function, $H(\bullet)$. In the case of wildfire, this transformation could be a summation of the amount of area burned by multiple wildfires in a specific region in a given fire season, divided by the total area of the region (Davis and Cooper 1963, Prestemon et al. 2002). H could also combine two kinds of disturbance functions, one describing the aggregate area affected by a pest in a given year in a specific landscape, and another defining the degree (severity) of damage by that pest within the area affected.

2.3 Disturbances as Probability Distributions

Disturbances can be defined in various forms, and each form has its own uses in for addressing questions in science and strategies for management. Disturbances can be discrete events or collections of events; they can be qualitative measures; or they can be ordered aggregations, or size-frequency distributions, of events produced in a landscape during a specified period of time. In other words, disturbance processes operate at multiple spatial and temporal scales, and recognition of such scaling issues can inform how to intervene in the process to achieve a desired outcome. For example, wildfires ignite at specific points in a landscape, and their timing and locations in that landscape can be measured as counts of events and related statistically to hypothesized driving factors.

Disturbances also often produce multiple outputs, creating scientific and statistical challenges for capturing the effects of inputs on each of their outputs. For example, wildfire output can be measured as area burned, the number of structures lost or threatened, or the average intensity of fire over time. Inputs such as suppression resources and fuels management can simultaneously affect many outputs—in this case, all of the three listed measures.

The stochastic nature of disturbance processes has implications for predicting and managing disturbances across landscapes and over time. Davis (1965) recognized that disturbance management meant managing the landscape to shift the probability distribution of future disturbance outcomes. For example, constructing firebreaks across a large management unit may reduce the expected total area of fire observed during the fire season in the management unit by altering fire spread and affecting suppression input productivities. Although other factors besides firebreaks would also affect the expected total area of wildfire, building more firebreaks in the landscape could shift the probability distribution of the total area of wildfire observed in a fire season. Figure 3.1 illustrates how alternative probability distributions (disturbance probability density functions) may be affected by a change in an input. Part A of figure 3.1 shows how a Normally distributed measure could exhibit a reduction in variance without a change in the mean, or a reduction in the mean without a change in variance, in response to a change in an input to the disturbance process. Of course, probability distributions do not have to be statistically Normal. Parts B, C, and D of figure 3.1 show the effects of input changes on the positions and shapes of Poisson- Exponential- and Gamma-distributed measures.

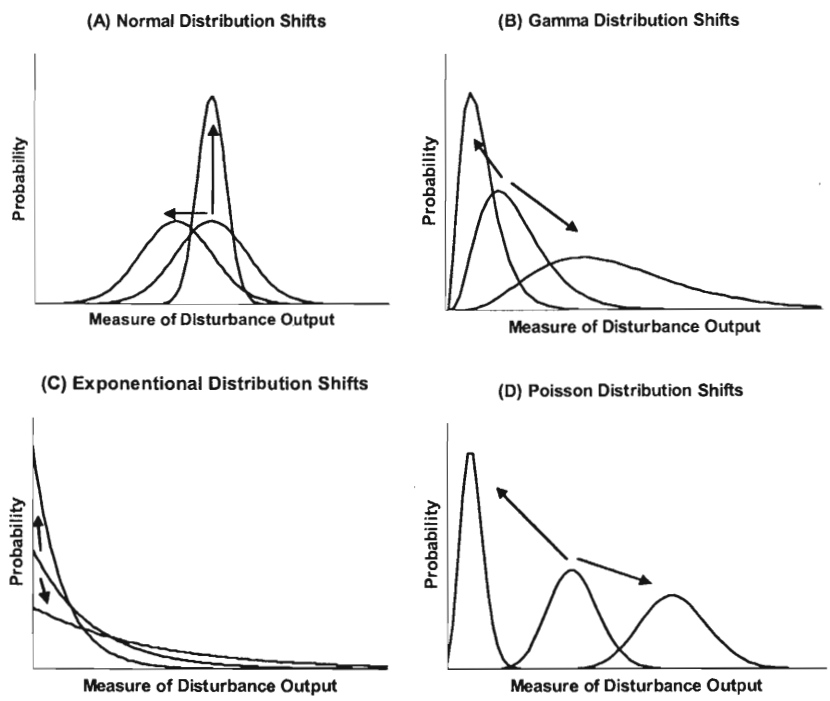

Figure 3.1. Hypothetical probability distribution shifts under alternative distributional assumptions for a measure of a disturbance process, as affected by an input that is intended to affect the disturbance process.

2.4 Disturbances as Production Processes in Economics

Disturbances are far more complicated than those implied by the classical production function (Chambers 1991). Below, we describe how disturbance production functions may differ from classical production functions, as defined in economics. Appendix table 4.1 provides a concise listing of these differences.

Although a principal characteristic of a classical production function is that output increases with increased amounts of a purchased input (monotonicity), this may not apply to disturbances. Inputs to disturbance functions may be intended to decrease some negative aspect of the disturbance. Free inputs, such as those associated with rain or human activities not intended to affect the disturbance, may have any direction of effect. Lastly, the effect of additional inputs may not be describable as an "increase," such as when an ecosystem changes from state A to state B.

A second characteristic of classical production functions is that each additional unit of purchased input should produce no more than the previous unit of additional input (quasi-concavity). In disturbances, concavity may not be relevant, as in the case of discrete or qualitative output measures. Alternatively, it may be true only in the negative sense, such as where each additional unit of input yields an equal amount or smaller reduction in output than the previous input; in essence, disturbance production functions may be quasi-convex.

A third characteristic of classical production is that if any or all purchased input quantities are zero, then output is zero (essentiality or weak essentiality). Many disturbance outputs occur without active intervention by humans. That is, they can operate with free inputs provided by nature or society. Thus withholding purchased inputs does not set outputs to zero.

A fourth characteristic of classical production is that the set of possible outputs is closed for all levels of output. In other words, it is feasible to produce any desired level of output. In the case of disturbances, if the process is defined as a collection of discrete events, then production is discontinuous and therefore not a closed set. This is especially true when disturbance production can be measured qualitatively.

A fifth characteristic of classical production is its nonstochasticity—a specific quantity of input always yields the same quantity of output. With natural disturbances, randomness can yield a different quantity of output for the same quantity of input.

Lastly, classical production functions are continuous and twice-differentiable (Chambers 1991, p. 9). In other words, to identify optimal input amounts, it is necessary for production functions to be increasing at a decreasing rate across some region of economical output. Because disturbance production can be discrete, qualitative, or discontinuous, it is clear that disturbance functions can sometimes not be continuous or twice-differentiable. As we shall see later, however, there are ways to identify optimal inputs into disturbance production functions that yield desired outputs, even while the disturbance process itself

may not conform to all the classical assumptions of production. Nonetheless, a primary implication of the discontinuities, discreteness, and other features of disturbances is that it may not be economically optimal to intervene. In other words, the best choice may be to set purchased input levels to zero.

3. STAGES OF DISTURBANCE PRODUCTION FUNCTIONS

Accurately modeling disturbances and their damages requires understanding how physical, biological, and human mediated inputs affect key processes. Typical forest disturbances proceed in four stages (Williamson 1996): introduction, establishment, spread, and post-disturbance. Between spread and post-disturbance is a point called extinction or outbreak cessation. Humans can intervene productively in some or all stages. Figure 3.2 traces out these stages and indicates where interventions may be possible. In the case of insects, diseases, and wildfires, the first stage is the introduction or the ignition. The second stage, establishment, occurs when introduction is successful—that is, the disturbance takes hold or survives. In the case of pests, establishment means that the pest invader carries out a life cycle and reproduces. In the third stage, spread, the disturbance spreads spatially

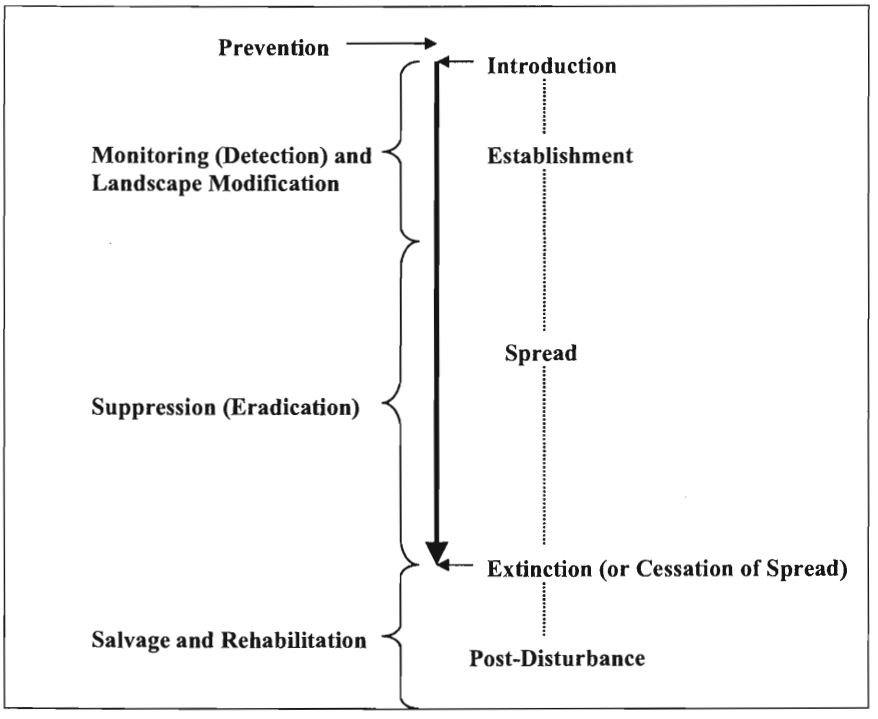

Figure 3.2. Stages of disturbances and intervention points.

until extinction or, in the case of some pests, returns to innocuous or endemic population levels (i.e., outbreak cessation occurs). Finally, "post-disturbance" follows extinction or outbreak cessation, which lasts indefinitely and may be characterized by ecosystem changes from the disturbance.

We define occurrence as the appearance of a new instance of a disturbance, possibly deriving from a distant or exogenous source but not as a result of a spatially connected spread process. Introduction and establishment can therefore be combined into one stage called "occurrence." The distinction between stages is often indefinite or fuzzy. For example, spreading to a neighboring point is the same as occurrence at that neighboring point.

3.1 The Introduction Stage

Introduction is the placement, through some process, of the disturbance into the landscape. An introduction could be an ignition of a wildfire by escape from a campfire ring or the appearance of an exotic pest in a new landscape by release from a shipping container. Introductions can be prevented by many kinds of actions. For wildfire, these can be banning of campfires or open debris fires, which are typical sources of accidental wildfire ignitions. In the case of pests, humans introduce exotic plants and animals intentionally and unintentionally through international trade or through (unintentional) long distance transport (di Castri 1989, Mack et al. 2000). Sometimes, these exotics become invasive pests. Prevention measures for exotic pest introductions, then, could include the banning of trade in certain, potentially infested commodities or shipping containers, or it could mean inspection of recreational boats for pests before they are moved between lakes. In wildfire, law enforcement efforts have been linked to reduced wildland arson ignitions (Prestemon and Butry 2005) (see chapter 7 for additional details and support). Prevention is not currently possible, of course, for many kinds of natural disturbances affecting forests—e.g., volcanic eruptions, hurricanes, and ice storms.

3.2 The Establishment Stage

Establishment of a natural disturbance means that the disturbance has moved past mere introduction. In terms of insects and diseases, establishment could be defined as the successful reproduction *in situ*. A wildfire is "established" when an ignition is sustained long enough so that further spread is possible. (This stage may only be brief and defined only *ex post*, if spread actually occurs.) For many disturbances, establishment depends on the collocation of sufficient quantities and qualities of host materials (or fuel) and favorable weather or other site conditions. Because establishment requires favorable conditions for propagation or survival, managers can alter the probabilities of successful establishment by modifying the landscape. A pest whose potential host is not present cannot become established, even if introduced. Research shows that non-establishment

is the most frequent outcome following introduction. Pest managers have estimated that introductions average five to twenty times the rate of establishment (Williamson 1996). Successful establishment may be defined as an "event" in empirical analyses and then related to measures designed to limit introductions or establishment.

3.3 The Spread Stage

Widespread ecologically and economically significant changes are produced during the spread stage. Some disturbances, such as ice storms and hurricanes, are exogenous and rapid, so that features of a forest, for example, may not significantly affect its overall extent. In these cases, actions taken to reduce losses of valued goods or services are applied either *ex ante*, by removing or reducing values at risk in anticipation of a potential disturbance, or *ex post*, in the post-disturbance stage. Note that *ex post* interventions are possible for all disturbance processes, not just fast ones. For slower spread processes, such as those of insects, diseases, and fires, limiting spread is often possible.

Variables affecting the rate and ultimate extent of spread of slower disturbance processes such as fire and pests also often affect establishment: the quantity of available host material in a landscape, weather, climate, geographical features, and the amounts and timing of efforts to suppress the disturbance. Manipulation of potential host material and placement of suppression inputs are *ex ante* actions that can be taken to reduce the spread of a disturbance. During active spread, suppression primarily involves manipulating (wetting, burning) or removing host material.

Once a disturbance is established and detected, the final extent of disturbance spread may depend on the speed of application of suppression resources (Butry 2006). For example, in wildland fire management in the United States, the so-called "10 a.m. policy" focuses on extinguishing fires as quickly as possible following detection of an ignition. This kind of suppression guideline is based on the notion that fire area can increase exponentially (Donovan and Rideout 2003a), and this exponential rate of spread is often higher later in the day, after temperatures rise and humidity falls. Fire managers often credit the policy with the successful suppression within 24 hours of 98 percent of all wildfires on federal lands. For insects, efforts to control or slow the spread (Sharov et al. 1998) involve taking quick action to suppress establishments occurring beyond the advancing front of a spreading pest. Managers therefore exploit the Allee effect (Leung et al. 2004), which involves keeping insect populations low on the spreading front, which reduces the reproduction rate of the invasive insect.

The economics of spread management (or suppression in wildfire terminology) is the subject of extensive theoretical development and modeling. Elaborate strategies and infrastructures have been developed to manage the spread of insects and diseases (Sharov and Liebhold 1998b, Mack et al. 2000) and wildfire (Sparhawk 1925, Donovan and Rideout 2003b).

3.4 The Post-Disturbance Stage

The post-disturbance stage is defined spatially as the area of influence of the disturbance, which can extend beyond the boundaries of the actual area directly affected. Although the length of time of the post-disturbance stage is indefinite, the timing of human actions may be important in determining the short- and long-run implications of the disturbance. In post-disturbance, landowners and managers often quickly assess the effects of the disturbance, sometimes salvage part of the affected timber or other valued products, take actions that reduce long-run negative side effects of the disturbance, and often work to restore some of the features of the ecosystem present before the disturbance. Human actions taken following the disturbance are often termed "rehabilitation and recovery." Rapid assessment of the effects of a disturbance is important for planning further actions. One action, timber salvage, has been shown to yield significant economic returns and be time sensitive (Prestemon et al. 2006). Removal of some of the killed timber and erosion control following a disturbance may alter risks of additional damage (McIver and Starr 2000, 2001, Kent et al. 2003). Although the specification of a meta-model that describes these types of feedbacks is beyond the scope of this chapter, a disturbance production function for one type of disturbance might include a set of variables that derive from other disturbance types. This approach would allow for joint modeling of production functions for a variety of disturbances (Hyde et al. 2006).

4. TYPES OF DISTURBANCE FUNCTIONS AND FUNCTIONAL FORMS

Disturbance functions can be classified into at least the following five broad classes: (1) event, (2) individual extent, (3) aggregate extent, (4) effect, and (5) joint (combinations of the other classes). Each class describes the stages of the disturbance across varying spatial and temporal scales or aggregates, and each may be useful in economic analysis. The five classes of disturbance production functions are briefly discussed below.[2] Also offered are examples or guidance on the statistical methods that could be used to identify the relative economic importance and direction of influence of free and purchased inputs to the disturbance processes defined in each class of model. We also suggest how simulation methods can be used to identify these influences, especially in cases where information about disturbance inputs are not available or are available at a different spatial or temporal scale than the output variable of interest.

[2] Mercer and Prestemon (2005) discuss a similar typology for wildfire production and provide empirical examples.

4.1 Event Models

Disturbance events can be modeled in at least three ways: (a) discrete event models that explain whether the disturbance occurred or the number of occurrences of the disturbance; (b) point process models, which describe the spatial and temporal distribution of occurrences; and (c) continuous models, which describe the rate of arrival or elapsed time between occurrences. An example of a discrete event approach is a binary choice (logit, probit) model that predicts the occurrence of a disturbance with particular characteristics. For example, in a wildfire event model, each point on a landscape each day might have a certain ignition probability, hypothesized to be a function of weather variables, vegetation features, and terrain. A logit or probit model could be used to estimate the probability that a fire would occur, given the measured levels of these causal variables. Data required to estimate the model would include occurrence data in many locations across a landscape as the dependent variable, coded to indicate whether a fire occurs at a given location during a specified time period, along with measures of the hypothesized causal variables for each location. Scales of analysis should be fine grained enough that more than one event does not occur in the same time and place. An example of this kind of modeling is found in Pye et al. (2003).

Count data models are extensions of the binary choice event models. In count models, the measure of observation is a count of the occurrences within a given time period and spatial unit. For example the unit of observation in a count model might be the number of fire starts in a county in a year, rather than the probability of a single ignition at a specific time and location. Poisson-type models are a common choice for relating the count to hypothesized causal variables (Martell et al. 1987, Gill et al. 1987, Vega Garcia et al. 1995, Prestemon and Butry 2005, Lee et al. 2006).

Point process models (Ripley 1976) are used to describe the spatial or temporal dispersion of events observed across a landscape within a given time period—for example, whether or not the pattern is random or non-random. The degree of randomness could inform the analyst about the effectiveness of spatially targeted interventions. An example is an analysis by Genton et al. (2006), who evaluate the clustering of wildfire ignitions in Florida.

Duration or survival and hazard type models relate hypothesized explanatory variables to the amount of time elapsed until an event occurs (Cox and Oakes 1984, Collett 1994). Duration modeling could use time series data on individual fire starts to relate the amount of time between fire starts to a variety of weather, ecosystem, management, and socio-economic variables. Survival models are common in analyses of treatment efficacy to reduce mortality from pest attacks (Woodall et al. 2005) and could also be used to evaluate time to events or occurrence probabilities of disturbances.

4.2 Individual Extent and Spread Models

Individual extent models relate explanatory variables to the amount of a resource or commodity affected by a single event. Many of the variables influencing establishment also help explain the extent of a particular disturbance, although an additional set of variables to include would be those associated with suppression or cessation of spread. Individual extent models that include suppression strategies can aid in tactical decision making aimed at slowing or stopping the spread of the disturbance.

Spread models focus on the spatial and temporal dynamics of an individual disturbance process after establishment but before cessation. Spread models may or may not include variables related to suppression efforts. Spread models may describe the arrival rate and direction of spread, and they are often used to compare the effects of alternative suppression tactics. Wildfire spread models have been embedded in fire simulation tools used by wildfire managers (Andrews and Bevins 1999). Tools such as FARSITE (Finney 1998, Finney and Andrews 1999) allow simulation of the effects that simple suppression strategies have on fire spread. Repeated runs of wildfire spread simulation models can show how a particular strategy affects the probability distribution of burned areas under operational or experimental conditions. Pest management makes similar use of simulated spread processes to compare the effects of alternative control strategies. Such experimentation can help managers and policy makers understand the trade-offs and economic returns of alternative suppression strategies (Sharov and Liebhold 1998a,b,c, Sharov et al. 1998).

Sharov and Liebhold (1998a,b,c) illustrate how spread models can answer important economic and management questions about barrier zone suppression strategies. The European gypsy moth (and many other pests) spreads in a stratified dispersal process (Liebhold 1998c), where spot outbreaks appear randomly or chaotically some distance beyond the zone of infestation. Spots continue to grow until they coalesce with other spots, merge with the infested zone, or are eradicated. Control actions consist of using aerial surveillance or pheromone traps to monitor the transition zone, an area of land surrounding the completely infested zone that encompasses the range of potential spread. Spot eradication measures are applied when a colony spreads into the transition zone.

The spread process described in these studies of the gypsy moth can be defined mathematically as a traveling wave equation for every cell (spatial unit) in the actual or potentially invaded range. Once a cell's population reaches a carrying capacity, the cell is considered a part of the colony in the infested zone. The population of any particular cell is determined by the probability of a new spot invading the cell and the population in the colony. Invasion probability for any cell is a negative function of distance from the infested zone. The colony's population is a positive function of the colony's age. The spread rate slows as the number of spots in the transition zone is reduced. However, spread can continue in a wave even without any successful spotting. In this case, slowing the spread

rate requires the eradication of all individuals in the transition zone. Because spots spread at a rate that increases with spot age, more intense monitoring of transition zones and quicker response times once a spot is identified typically produce greater control benefits. As such, monitoring and eradication are production substitutes under most conditions.

Calibrating a model of pest spread as a function of monitoring effort and eradication efforts requires data on spread rates with and without eradication efforts and how time since initiation of eradication affects its success. In empirical analysis, the success of barrier zone management can be quantified and potentially compared to a "no-action" alternative by simulating how the average spot size changes in response to differing levels of pest monitoring or lags before initiation of eradication.

4.3 Aggregate Extent Models

Aggregate extent models relate the amount of a resource or commodity affected by disturbance events occurring over a defined area and time. Statistical models of aggregate extent often rely heavily on long run and spatially aggregated measures of weather, climate, host materials, and suppression. An example is a model of the likelihood of beetle outbreak in a county, as related to the amount of host forest in the county, seasonal average precipitation and temperature levels in the county, the amount of National Forest lands in the county, and measures of spatial autocorrelation (Gumpertz et al. 2000).

The increased spatial and temporal aggregation of these models allows analysis of large and long scale disturbance patterns and dynamics. Because natural disturbances are stochastic in both location and timing, this broader scale analysis can help reveal the overall effects of management and suppression strategies across wider scales. Such broad analyses may also more effectively capture the underlying effects of free inputs to disturbance processes, especially when these other inputs may vary little within a small location or a short time period but more widely when viewed across broad landscapes and long time horizons. For example, the area burned in a county in a year could be expressed as a function of areas burned in that county in previous years, aggregate amounts of fuel treatments in the county in the current and previous years, county level annual measures of socioeconomic variables, and broad scale weather patterns such as a measure of ocean temperature oscillations. Barnett and Brenner (1992), Keeley et al. (1999), Prestemon et al. (2002), Westerling et al. (2002), Norman and Taylor (2003), and others have developed empirical aggregate extent models of wildfire in different parts of the United States.

Statistical methods are not always available for quantifying the impacts of disturbances at broad spatial and temporal scales. In these cases, it still may be possible to quantify their impacts by using simulation approaches. For example, the aggregate amount of wildfire in a landscape in a given fire season could be simulated using statistical models of individual fire occurrence (event models)

and spread, simulated weather, and imputation of known vegetation and landscape features. If the fire occurrence and spread models are specified as functions of fuels, weather, and suppression variables, then repeated simulations can reveal the effects of altering assumed levels of each of these, producing a picture of the broad spatial and temporal effectiveness of fuels management and fire suppression efforts.

An example from wildfire illustrates how the wildfire disturbance process exhibited at broad spatial and temporal scales can be used to identify the effects of free and purchased inputs into wildfire management. Prestemon et al. (2002) develop a model relating wildfire probability in a county in a year as a function of both non-purchased inputs (climate measures and historical wildfire) and purchased inputs (prescribed fire and small diameter timber removals). Using a cross-sectional time series empirical model, the area of wildfire (W_{it}) relative to the area of county i's forest (f_{it}) in year t, $(W_{it}/f_{it}) = \pi_{it}$, is specified as a function of prescribed fire area (x_i) relative to forest area, $(x_i/f_i)=y_{it}$, in that same year and one previous year $(y_{it}, y_{it-1}) = \mathbf{y}_i$, small diameter timber removals in that county in the three previous years $(h_{it-1}, h_{it-2}, h_{it-3})=\mathbf{h}_{it}$, historical proportions burned by wildfire in that county for the previous twelve years $(\pi_{it-1}, \pi_{it-2},\ldots, \pi_{it-12})= \pi_{it}$, the El Niño-Southern Oscillation Niño-3 sea surface temperature anomaly in degrees centigrade (E_t), a dummy measuring a Super El Niño cycle (D_t) in 1998, and the county's housing density (U_{it}). The proportion of forest area burned is assumed stochastic, such that

$$\ln(\pi_{it}) = \alpha_i + \boldsymbol{\beta}'\ln(\boldsymbol{\pi}_{it}) + \boldsymbol{\gamma}'\ln(\mathbf{y}_{it}) + \boldsymbol{\delta}'\ln(\mathbf{h}_{it}) + \mu_1 E_t + \mu_2 D_t + \mu_3 \ln(U_{it}) + \varepsilon_{it} \quad 3.1$$

Equation (2) is estimated with weighted least squares and a heteroscedasticity correction, using a short panel (1994-1999) and 37 cross-sections. Mercer and Prestemon (2005) and Mercer et al. (2007) estimate similar models with longer and wider panels of data. Prestemon et al. (2002) found that prescribed fire can have an effect on wildfire activity, but that its effect is not large relative to long run climatic patterns and historical wildfire activity.

4.4 Effects Models

Effects models describe how independent variables influence the characteristics of a particular event. For example, the species diversity of a forest might be altered as a result of successful invasion of an exotic species. The effect could be measured in terms of changed species diversity levels observed following an invasion. Another example is timber quality changes following a storm. Because damages to timber quality might take years to manifest following a storm, an effects model would relate the presence or absence of storm damage in each forest stand some number of years following the storm to features of the storm in that location, site conditions, and vegetation conditions before the storm.

For a wildfire example, the proportion of fire-killed timber per unit area or the soil temperatures observed during a wildfire in each location might be related to

wind, humidity, temperature, and the amounts of fuels of different sizes in each location. If forest fuels can be manipulated by a land manager and are known to affect the intensity of wildfires that burn in the forest, then a statistical model relating the degree of wildfire-related losses of goods or services provided per unit area of wildfire area burned would describe how purchased inputs into fuels management would directly affect these losses.

4.5 Combined Models

Any version of at least two of the above models can be combined to yield another class of disturbance model. For example, size-frequency distribution models, which quantify the parameters of a statistical distribution of wildfire across size classes, summarize disturbance activity across broad landscapes and long time scales. Research has shown that size-frequencies of many natural phenomena including disturbances are distributed in log-linear fashion (Strauss et al. 1989, Li et al. 1999, Holmes et al. 2004). Extreme value functions (Moritz 1997) are models increasingly used in insurance applications, can describe how the number of events of different ordered classes are distributed in probability (see chapter 4). As with aggregate extent models, size-frequency distribution and extreme value models could be used to identify the effects of long-run or large-scale changes in free and purchased inputs. For example, estimates of the parameters of size-frequency distributions of wildfires occurring in simulated or otherwise identical landscapes with and without fuels management could reveal the effect of efforts to reduce negative outcomes of wildfires in the landscape.

In another wildfire example, a measure of overall damages by wildfire in a season across a landscape can be constructed by combining both the intensity and the aggregate extent of wildfires in a landscape over a fire season. This measure of damages can then be related to variables hypothesized to influence the effect and the aggregate extent of damages. For example, Mercer et al. (2007) relate an aggregate of the product of wildfire intensity (an effect) and area burned by all the fires occurring in one year in one county (aggregate extent) to several hypothesized explanatory variables, including prescribed fire and relate historical data on intensity-weighted area burned to the economic damages associated with wildfire in the State of Florida. In their economics application, the benefits of wildfire economic damages averted by intense wildfires trade-off with the costs of to identify economically preferable fuels management rates. A variation on the Mercer et al. (2007) and Holmes et al. (2004) approaches would be to identify a family of wildfire size-frequency distributions, a distribution for each fire intensity level. Similarly, one might use combination models to analyze whether spot sizes of southern pine beetle infestations possess the kinds of spatial dynamics identified by Gumpertz et al. (2000).

Another kind of combined model is of spatio-temporal point processes (STPP). These models describe how a collection of events is distributed across space and time. The empirical manifestation of a STPP is a spatio-temporal point pattern.

A primary focus of STPP analyses is to evaluate whether the pattern observed differs significantly from a random distribution of events across space and time. Examples of such patterns might be the occurrences of disease outbreaks, wildfire ignitions, and pest infestations. STPP's could be of use to wildland managers if analysts were able to link the patterns to variables that managers can affect, or if optimal planning for a disturbance depends on the amount of clustering of events. For example, wildfire managers might want to understand the STPP's to understand wildfire suppression resource needs. Examples of published research include Podur et al. (2003), who use STPP's to analyze lightning fires in Canada, and Genton et al. (2006), who apply STPP's to analyze wildfires produced by all major ignition categories in the United States.

5. IMPLICATIONS FOR MANAGEMENT, POLICY, AND SCIENCE

This chapter has sought to explain what disturbance production processes are, describe how they differ from classical economic production processes, characterize the various forms of disturbance processes, and briefly describe how analysts have modeled them. The availability of large and long term datasets on natural disturbances and improvements in software and computing power have led to advances in science and management. These advances include a better understanding of the long-run, broad scale effects of human interventions and free inputs into disturbance processes (e.g., societal variables not intended to affect the process but nevertheless do affect it, climate, weather), quantification of the long-run economic net benefits and effects of various kinds of interventions into these processes, and revelations about previously unidentified spatial and temporal patterns in disturbances. We anticipate that application of the kinds of modeling approaches outlined here could lead to advances in questions of current and future importance to society, including those associated with large scale spending on fuels management to reduce the net economic damages from wildfire.

An avenue for further study involves examining how agents of disturbances respond to actions to limit the agents' effectiveness. Research into agent-based disturbance modeling would focus on how humans and pests respond to interventions to mitigate the effectiveness of the interventions. For example, little is known about how arsonists might change their behavior in response to stepped up law enforcement (Prestemon and Butry 2005). Research should focus on how greater enforcement in one area could lead to simple shifts of arson activities in space and time. Similarly, controls on the importation of invasive species could create averting actions by importers to get around rules and regulations. In terms of invasive species spatial processes, barrier zone management might induce changes in the aggregate spread behavior of populations. Alternatively, pesticide use may, in the long-run, lead to increased pesticide resistance in the population, requiring more complex models of pest spread and control (Carpentier and Weaver 1997).

A better understanding of these kinds of feedbacks may reveal important limitations and open up new approaches to forest and landscape management with disturbances.

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Appendix Table 3-1. Comparisons between classical economic production functions and disturbance production functions.

Characteristics of Classical, Single-Output Production Functions (Chambers 1991, p. 9)	Disturbance Production Function	Notes
1a. Monotonicity: if $x'>x$ then $f(x')>f(x)$	Negative monotonicity for purchased inputs (x) of single output disturbance functions: if $x'>x$ then $f(x', \mathbf{z})\leq f(x,z)$; non-monotonic for free inputs ($\mathbf{z}$)	(Negative) Monotonicity not required for individual outputs respect to purchased individual outputs of multioutput functions
1b. Strict monotonicity	Negative strict monotonicity for purchased inputs of single output disturbance functions: if $x'>x$ then $f(x',\mathbf{z})<f(x,\mathbf{z})$	Non-monotonic when purchased inputs are zero: $f(0,z)\geq 0$
2a. Quasi-concavity	Quasi-concavity possible in purchased inputs; no assumption for free inputs	Quasi-concavity assumption undermined by possible sequencing in temporally defined input sets, interaction of purchased and free inputs in production
2b. Concavity	Concavity is possible in purchased inputs; no assumption for free inputs	Concavity assumption undermined by possible sequencing in temporally defined input sets, interaction of purchased and free inputs in production
3a. Weak essentiality: $f(0)=0$	No essentiality for purchased inputs: $f(0,\mathbf{z})\geq 0$	Purchased inputs are not required for nonzero output, due to free inputs

(continued)

Appendix Table 3-1. Comparisons between classical economic production functions and disturbance production functions. *(continued)*

Characteristics of Classical, Single-Output Production Functions (Chambers 1991, p. 9)	Disturbance Production Function	Notes
3b. Strict essentiality: $f(x_1,x_2,\ldots x_{i-1},0,x_{i+1},\ldots,x_n)=0$	No strict essentiality for purchased inputs: $f(x_1,x_2,\ldots x_{i-1},0,x_{i+1},\ldots,x_n,z) \geq 0$	No single purchased inputs is required for nonzero output, due to free inputs
4. V(y) is a nonempty and closed set for all y>0	V(y) is nonempty, but discontinuous production is possible, implying a non-closed set	Discontinuity is possible when disturbance is a discrete process (e.g., an event occurrence or count process) or is categorical or qualitative
5. f(x) = y is finite, nonnegative, real valued, and single-valued for a finite x	Because of stochasticity, f(x) may not be single-valued	
6a. f(x) is continuous	Not assumed	
6b. f(x) is twice-differentiable	Not assumed	
Other Characteristics		
No free inputs	Free inputs exist (z)	
Non-stochastic	Stochastic	
Single-output	Sometimes multioutput	

(continued)

Appendix Table 3-1. Comparisons between classical economic production functions and disturbance production functions. *(continued)*

Characteristics of Classical, Single-Output Production Functions (Chambers 1991, p. 9)	Disturbance Production Function	Notes
Previous inputs are nonessential: for any constant, a, $f_t(x_t,x_{t-k}) = f_t(x_t,ax_{t-k})$ for all x_{t-k} and all $k\neq 0$	Possibly temporally defined input set: for any constant, $f(x_{1,t},\ldots, x_{i-1,t}, x_{i,t},x_{i+1,t},\ldots,x_{n,t},x_{1,t-1},\ldots, x_{i-1,t-1}, x_{i,t-1},x_{i+1,t-1},\ldots x_{n,t-1},\ldots,\mathbf{z}) \leq \geq f(x_{1,t},\ldots,x_{i-1,t}, x_{i,t},x_{i+1,t},\ldots, x_{n,t},x_{1,t-1},\ldots,x_{i-1,t-1},ax_{i,t-1}, x_{i+1,t-1},\ldots x_{n,t-1},\ldots,\mathbf{z})$	Temporally defined input set for any purchased or free input
Not sequence-dependent with respect to inputs (guaranteed by an assumption of static with respect to inputs)	Possibly sequence-dependent with respect to inputs: for any nonzero and non-unitary constants a and c, $f(x_{1,t}=c,x_{2,t}, \ldots x_{n,t},x_{1,t-1}=ac,x_{2,t-1},\ldots x_{n,t-1}, \ldots ,\mathbf{z}) \leq \geq f(x_{1,t}=ac,x_{2,t},\ldots x_{n,t}, x_{1,t-1}=c,x_{2,t-1},\ldots x_{n,t-1},\ldots,\mathbf{z})$	Potentially sequence-dependent with respect to inputs for any purchased or free input
Previous outputs are nonessential: for any constant, a: $f_t(x_t,y_{t-k}) = f_t(x_t,ay_{t-k})$ for all $k\neq 0$	Potentially dynamic production: for any constant, a, $f_t(x_t,y_{t-k},\mathbf{z}) \leq \geq f_t(x_t,ay_{t-k},\mathbf{z})$ for any $k\neq 0$	