WATER RESOURCES RESEARCH, VOL. 29, NO. 8, PAGES 2637–2649, AUGUST 1993

How Much Complexity Is Warranted in a Rainfall-Runoff Model?

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Development of mathematical models relating the precipitation incident upon a catchment to the streamflow emanating from the catchment has been a major focus of surface water hydrology for decades. Generally, values for parameters in such models must be selected so that runoff calculated from the model "matches" recorded runoff from some historical period. Despite the fact that the physics governing the path of a drop of water through a catchment to the stream involves complex relationships, evidence indicates that the information content in a rainfall-runoff record is sufficient to support models of only very limited complexity. This begs the question of what limits the observed data place on the allowable complexity of rainfall-runoff models. Time series techniques are applied for estimating transfer functions to determine how many parameters are appropriate to describe the relationship between precipitation and streamflow in the case where data on only precipitation, air temperature, and streamflow are available. Statistics from an "information matrix" provide the clues necessary for determining allowable model complexity. Time series models are developed for seven catchments with widely varying physical characteristics in different temperate climatic regimes to demonstrate the method. It is found that after modulating the measured rainfall using a nonlinear loss function, the rainfall-runoff response of all catchments is well represented using a linear model. Also, for all catchments a two-component linear model with four parameters is the model of choice. The two components can be interpreted as defining a "quick flow" and "slow flow" response of the given catchment. The method therefore provides a statistically rigorous way to separate hydrographs and parameterize their response behavior. The ability to construct reliable transfer function models for describing the rainfall-runoff process offers a new approach to investigate empirically the controls of physical catchment descriptors, land use change, climate change, etc., on the dynamic response of catchments through the extensive analysis of historical data sets.

1. INTRODUCTION

The construction and application of watershed models describing precipitation to streamflow processes has been a prime focus of hydrological research and investigations for many decades. Both the amount of effort and the complexity of models seem to have increased continually with the expansion in available computing power. Most attention has been given to catchments subject to basically temperate climatology where hydrological responses tend to be simpler or involve a subset of the processes which occur in other climatic regimes. Despite the activity in modeling the rainfall-runoff process and the concentration on temperate catchments, hydrologists have noted the lack of real progress being made in watershed modeling generally and the problems of developing the process knowledge derived at small scales for use at larger scales [e.g., *Beven*, 1987]. *Philip* [1975, p. 23] saw the need "to identify and to recognise frankly the limits of what natural science and the 'scientific method' can bring to the tasks of catchment prediction." More specifically and recently, *van Genuchten* [1991, p. 190] summarized that

> future research in catchment modeling must address the problem of permissible system and model complexity, the scales over which model components are valid, and the integration of model components into an overall balanced framework.

Paper number 93WR00877.
0043-1397/93/93WR-00877$05.00

One of the major problems in rainfall-runoff modeling is dealing with overparameterization. *Loague and Freeze* [1985, p. 245] applied several models of varying complexity to a number of catchments and concluded that the "fact that simpler, less data intensive models . . . provided as good or better predictions than a more physically based model is food for thought." *Hornberger et al.* [1985] incorporated parameterizations into a version of TOPMODEL for processes observed to be occurring in the field but found that the 13 parameters could not be reliably estimated using rainfall-runoff data; they found that four parameters seemed to suffice to represent the transformation of rainfall to streamflow. *Hooper et al.* [1988] examined a very simple hydrological model with six parameters and still found it to be overparameterized. *Beven* [1989, p. 159] comments,

> There is a great danger of overparameterization if it is attempted to simulate all hydrological processes thought to be relevant and fit those parameters by optimisation against an observed discharge record. . . . It appears that 3 to 5 parameters should be sufficient to reproduce most of the information in a hydrological record.

Hydrologists are faced with something of a dilemma. The most frequent application of rainfall-runoff models is in cases where the only data available are precipitation, temperature, and streamflow. Models that seek to incorporate processes known to be important hydrologically (at small scales) are likely to contain a rather large number of parameters, many of which will be correlated with other parameters [e.g., *Clarke*, 1973]. How are these observations to be reconciled with the contentions that only a model with a few parameters can be supported by rainfall-runoff data? It appears that there are challenges that face those concerned with rainfall-runoff modeling. How much information is contained in records of precipitation and streamflow? How complex a parameterization is warranted in a rainfall-runoff model? Does adding spatially distributed data on physical catchment descriptors, such as on terrain, hydrologic soil, and vegetation properties, permit a more detailed parameterization?

A response to these challenges is attempted here first by presenting a framework to answer the question of what reliable information may reside in concurrent precipitation-streamflow measurements for assessing the dynamic characteristics of catchment response and for prediction of streamflow. In particular, the paper addresses the limitations of precipitation-streamflow modeling when measurements of other dynamic flow or concentration variables are not used as additional prior information in model construction. The framework allows inference of the number of streamflow components that can be identified from given precipitation and stream discharge observations. Its use is illustrated for catchments spanning a range of scales and basically temperate hydroclimatological regimes. Mainly daily data are used, and the specific results are applicable to analyses involving observational time series whose length is of the order of 100 times the quick flow response time constant. In the case of daily data this is of the order of 1 year.

The outcome of applying the framework is a hypothesis that, after allowing for antecedent conditions, the response of a catchment is predominantly linear over a wide range of temperate climatological regimes and down to small catchment scale. In response to *van Genuchten* [1991], the "permissible model complexity" seems to be generally low, containing around half a dozen parameters, and "the scales over which the model components are valid" are very wide. With much longer observational time series, it may be possible to identify the values of additional parameters. However, as pointed out by *Sorooshian et al.* [1983], rather than length, it is the quality of information contained in the data, which is important; data sequences which contain larger hydrologic variability are more likely to result in reliable parameter estimates.

2. METHODS

2.1. *The Model*

There are many formal ways to assess the information content in data with respect to some model M. In a stochastic setting the model M and its parameters are most completely specified by its probability distribution $p(M)$. Given an evenly spaced time sequence of N rainfall-runoff samples

$$\{\mathbf{r}_N, \mathbf{q}_N\}, (\mathbf{r}_N = r_1, r_2, \cdots, r_N; \mathbf{q}_N = q_1, q_2, \cdots, q_N)$$

where r_k is observed rainfall and q_k is observed streamflow at time step k, the conditional distribution $p(M/\{\mathbf{r}_N, \mathbf{q}_N\})$ can be called the information about M contained in $\{\mathbf{r}_N, \mathbf{q}_N\}$. In the next section, it will be argued how the covariance matrix of our model parameters can be invoked to determine the information about M in the time series samples.

The model used here to extract the information in rainfall-streamflow time series data consists of one nonlinear and one linear module. The nonlinear or rainfall loss module represents the transformation of rainfall $\mathbf{r}_N$ to "excess" rainfall $\mathbf{u}_N$. At each time step k a catchment wetness index, s_k, or antecedent precipitation index is calculated by a weighting of the rainfall time series, the weights decaying exponentially backward in time from step k, namely,

$$\begin{aligned} s_k &= cr_k + (1 - \tau_w^{-1})s_{k-1} \\ &= c[r_k + (1-\tau_w^{-1})r_{k-1} + (1-\tau_w^{-1})^2 r_{k-2} + \cdots] \end{aligned} \quad (1)$$

The parameter τ_w is approximately the time constant, or inversely, the rate at which the catchment wetness declines in the absence of rainfall. Hence a larger value of τ_w gives more weight to the effect of antecedent rainfall on catchment wetness than a smaller one. The excess or effective rainfall is computed using

$$u_k = r_k s_k \quad (2)$$

The parameter c in (1) is chosen so that the volume of excess rainfall is equal to the total streamflow volume over the calibration period, after adjustment for change in catchment storage between the beginning and end of the period. It is the increase in storage index per unit rainfall in the absence of evapotranspiration. It is not really a free parameter but merely a normalizing one.

To account for fluctuations in evapotranspiration, a simple function of temperature can be used to modulate the rate at which the catchment dries out. Then τ_w in (1) is replaced with the function

$$\tau_w(t_k) = \tau_w \exp [(20 - t_k)f] \quad (3)$$

where t_k is the temperature in degrees Celsius at time step k. In this way, τ_w is inversely related to the rate at which catchment wetness declines at 20°C. The parameter f is a temperature modulation factor which determines how $\tau_w(t_k)$ changes with temperature.

The general conclusions of this paper are independent of this nonlinear loss module. The structure of the accompanying linear module identified from any $\{\mathbf{u}_N, \mathbf{q}_N\}$ is independent of the values of the parameters τ_w, c, and f, whereas the parameter values in the identified linear components are dependent. An explanation for this is that the nonlinearity between rainfall and streamflow has not been strong enough to impede the identification.

The linear module of the model converts excess rainfall u_k at time step k to streamflow q_k. It is a transfer function of the form

$$x_k = -a_1 x_{k-1} - \cdots - a_n x_{k-n} + b_0 u_k + \cdots + b_m u_{k-m} \quad (4)$$

$$q_k = x_k + \xi_k \quad (5)$$

wherein ξ_k represents the addition of all data and model errors and x_k is a hypothetical error-free streamflow variable. The $n + m + 1$ elements of the vector $\mathbf{a} = (a_1, \cdots, a_n, b_0, \cdots, b_m)^T$ are the parameters to be optimized in the linear module.

There are several reasons that this model (1)–(5) is a natural one to use for extracting the information in time series $\{\mathbf{u}_N, \mathbf{q}_N\}$:

1. The model and a similar version with a simpler nonlinear loss module are good predictors of streamflow and

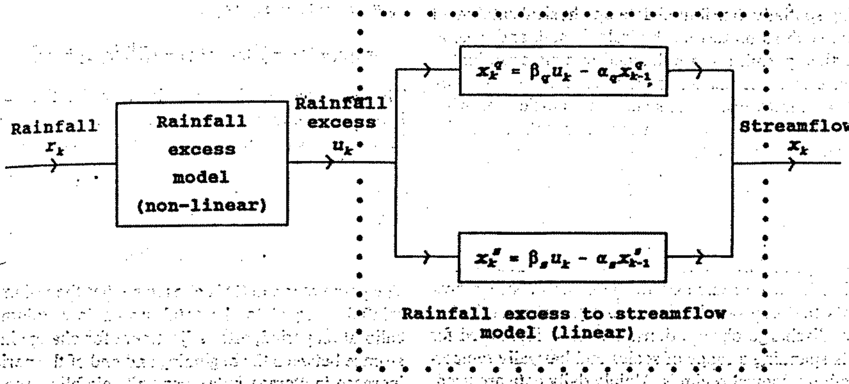

Fig. 1. Systems diagram of the most common model configuration identified.

"validate" well on streamflow records not used for calibration [*Jakeman et al.*, 1990, 1991, 1993*a*, *b*; *Littlewood and Jakeman*, 1992]. They fit streamflow records from calibration periods quite well, having been found more often than not to account for greater than 80% of streamflow variance. The fits for the catchments analyzed in this paper are representative of the range experienced with application of the model to around 50 catchments to date.

2. The transfer function representation (4), including excess rainfall inputs and streamflow outputs, is a discrete-time equivalent of an approximation of the convolution integral relating excess rainfall and streamflow through a unit hydrograph. It uses a series expansion of the unit hydrograph in terms of a linear combination of exponential terms of order n. The associated approximation of the unit hydrograph is an efficient one parametrically, corresponding to a rational function representation of a polynomial function which eventually decays. As will be seen, when used in conjunction with an appropriate parameter estimation technique, this structure allows one to appeal to simple criteria to discriminate among alternative model orders n. It is mathematically attractive to solve an ill-posed inverse problem (such as that of unit hydrograph estimation) by expanding the solution in terms of smooth slowly varying functions, and then estimating the dominant terms in that expansion to determine the amount of reliable information in the data [*Newsam*, 1984]. This discrimination process is objective in the sense that it is user-independent, particularly the method of parameter estimation.

3. In physical terms, the transfer function representation corresponds to a flexible configuration of linear storages connected in parallel and/or series paths for the transit of excess rainfall to the stream. This is consistent with the general design of conceptual watershed models that possess lumped parameterizations. The main difference between the transfer function and more standard conceptual models is that the nonlinearities in the former are all dealt with in the first module, whereas thresholds and other nonlinearities can be found throughout the traditional conceptual models.

4. More physically based models can also be regarded as a configuration of conceptual storages. Data on physical catchment descriptors help to define the number and configuration of storages. These and most models, including the model (1)–(5), also tend to require the calibration of parameters related to flow rates and volumetric throughputs. Therefore the implications of the results obtained from applying this paper's framework can be qualitatively deduced for physically based and conceptual models.

In using model (1)–(5), our explicit purpose is to infer what conceptual pathways and lumped stores are unambiguously manifest in time series $\{\mathbf{u}_N, \mathbf{q}_N\}$, taking into account the noise levels in the time series and model. The properties of these stores and their configuration are determined by the values in $\mathbf{a}$ and the orders (n, m), respectively. Note that each storage $i (i = 1, 2, \cdots, n)$ can be parameterized completely by the volumetric throughput v_i, relative to other storages, and its time constant τ_i, in response to a pulse input of excess rainfall. The time constant can be defined as the time taken for the peak in output of a storage to recess to exp (-1) of that peak value. In terms of an equivalent continuous time formulation, the characteristic response of each storage can be considered to be defined by a unit hydrograph (component) of form $I_i \exp(-t/\tau_i)$, where I_i is the relative peak of the hydrograph response and v_i is the integral of or area under this hydrograph response component. We define $\boldsymbol{\tau} = (\tau_1, \cdots, \tau_n, v_1, \cdots, v_n)^T$ and the parameters τ_w, f, and c of the nonlinear module as the dynamic response characteristics (DRCs) of the catchment. The quantities I_i can also be regarded as DRCs, but the elements of the vector $\boldsymbol{\tau}$ are sufficient to completely define the exponential response.

When $n = 2$ and $m = 1$ in (1), excess rainfall can be considered to travel through a configuration of two parallel storages as illustrated in Figure 1. In this case, $I_1 + I_2 = v_1 + v_2 = 1$, and the storage with smaller time constant represents the quick flow component while that with the larger the slow flow component. In this case, $\boldsymbol{\tau}$ can be written as $(\tau_q, \tau_s, v_q, v_s)^T$. When $n = 2$ and $m = 0$, the storages in Figure 1 are in series. The simple relationships between the parameters in $\boldsymbol{\tau}$ and $\mathbf{a}$ for the two parallel storage configuration in Figure 1 are

$$\tau_q = -\Delta/\ln(-\alpha_q) \qquad (6)$$

$$\tau_s = -\Delta/\ln(-\alpha_s) \qquad (7)$$

$$v_q = \beta_q/(1 + \alpha_q) \qquad (8)$$

$$v_s = \beta_s/(1 + \alpha_s) \tag{9}$$

where Δ is the sampling interval for the precipitation and streamflow time series and the α and β parameters are obtained from the decomposition of the polynomial transfer function in the backward shift operator B ($Bu_k = u_{k-1}$) according to

$$\frac{b_0 + b_1B}{1 + a_1B + a_2B^2} = \frac{\beta_q}{1 + \alpha_qB} + \frac{\beta_s}{1 + \alpha_sB}$$

The sampling interval selected or available for the time series data clearly has an effect on the information that can be extracted about a model M. Too coarse a sampling interval will result in a loss of information about response dynamics. Too fine an interval can result in numerical instabilities [e.g., *Jakeman and Young*, 1980]. For our model, the appropriate sampling interval to select is one that is of the order of, but preferably less than, the time constant of the quickest identifiable response. This selection can make identification of slower components numerically difficult. A particular algorithm was applied by *Jakeman et al.* [1990] to obviate this problem and is again used here to extract the different components.

Note that with a two-parallel storage configuration, our overall model has seven parameters, six of which must be estimated. Use of the parameter c in (1) constrains the volumetric gain between excess rainfall and streamflow, $(b_0 + b_1)/(1 + a_1 + a_2)$, to be unity, so that only three of the linear module parameters need be independently estimated.

2.2. *Parameter Estimation and the Form of the Covariance Matrix*

The main interest of the paper is on the complexity of the linear part of the model. Estimation is therefore focused on the mean and covariance of the parameters in the linear module given values of the parameters τ_w, f, and c in the nonlinear module. The parameter c is obtained simply from the data and τ_w and f using (2). The latter parameters can be optimized by trial and error using a simple search technique. The search can be applied to select those values which, when used in conjunction with an automatic algorithm for estimating the parameters **a**, satisfy an objective function in fitting streamflow.

There are many algorithms available for estimating the parameters in transfer function models of the form (4). Instrumental variable techniques are preferred here because they lead to simpler algorithms with good properties if one is mainly interested in the system dynamics and one cannot model or is not interested in the precise nature of the errors. They yield consistent estimates provided the errors ξ_k are uncorrelated with the input u_k. Therefore they do not require the errors to be Gaussian or even independent and identically distributed. They can yield asymptotically efficient estimates if the errors are stationary. A covariance matrix is a by-product of the algorithms. The algorithmic details will not be reported here. *Jakeman et al.* [1989, 1990] summarize the properties of various instrumental variable algorithms and cite the major literature. The simple refined version is the one preferred because it performs well in the most difficult cases, in particular the case of estimating two hydrological stores or components where one component is small in volume and decays very slowly relative to the other component.

An information matrix **I** for any input-output time series $\{\mathbf{u}_N, \mathbf{q}_N\}$ and transfer function model of the form (4)–(5) with parameters **a** can be estimated [e.g., *Pierce*, 1972] as

$$\mathbf{I} = (N/\hat{\sigma}^2)E[\hat{\mathbf{x}}_k\hat{\mathbf{x}}_k^T] \tag{10}$$

where $\hat{\mathbf{x}}_k^T = [-\hat{x}_{k-1}\ -\hat{x}_{k-2} \cdots -\hat{x}_{k-n}\ u_k\ u_{k-1} \cdots u_{k-m}]$, $\hat{x}_k$ is an estimate of the noise-free streamflow x_k, and $\hat{\sigma}^2 = \mathrm{Var}\,(\hat{\xi}_k) = \mathrm{Var}\,[(q_k - \hat{x}_k)]$ is the model error or residual variance. As described below, the information matrix and the residual variance allow one to determine how many model parameters are supported by the data. The specific approach was first used by *Young et al.* [1980] and is in the spirit of the philosophy of model parsimony espoused most notably by *Box and Jenkins* [1976].

If a simple refined instrumental variable (SRIV) algorithm is used to estimate **a** then an analogous matrix $\mathbf{I}^*$ is the information matrix. This corresponds to **I** above except that each element is replaced by an analogous asterisked variable which is filtered according to

$$\hat{x}_k^* = -\hat{a}_1\hat{x}_{k-1}^* - \cdots - \hat{a}_n\hat{x}_{k-n}^* + \hat{x}_k$$

$$u_k^* = -\hat{a}_1u_{k-1}^* - \cdots - \hat{a}_nu_{k-n}^* + u_k$$

Here $(\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_n)$ are the SRIV estimates of the first n elements of **a**.

The covariance matrix **P** of the parameters in **a** is estimated as

$$\mathbf{P} = (\mathbf{I}^*)^{-1} \tag{11}$$

Under certain conditions [e.g., *Pierce*, 1972], the parameters estimated in **a** are asymptotically normally distributed with covariance **P**. Thus for any model M, **a** and **P** theoretically are total extracts of the information in the data $\{\mathbf{u}_N, \mathbf{q}_N\}$. They contain the probability distribution of the linear response characteristics of the associated catchment. In the context of the model (4)–(5), either the information or covariance matrix permits specification of which configuration (or model orders (n, m)) can be identified unambiguously (or with specified uncertainty) from the data. Overspecification of either model order as n' or m' will result in an **I** matrix which is not well conditioned because of a lack of cross correlation between $\hat{x}_{k-n'}$ and lagged values of u_k or between $u_{k-m'}$ and lagged values of $\hat{x}_k$. These cross correlations form some of the off-diagonal elements in (10). The greater the overspecification of n and m, the worse the conditioning of the information matrix or tendency to singularity, and hence the larger the elements in the covariance matrix. Underspecification of either model order will result in a substantially higher value of $\hat{\sigma}^2$ than for any model orders $n' \geq n$, and $m' \geq m$. In practice, the values of $\hat{\sigma}^2$ (or the coefficient of determination $D = 1 - \hat{\sigma}^2/\mathrm{Var}\,(q_k)$) can be expected to decline (rise) to a plateau as the model order increases while standard measurement norms of the covariance matrix become unacceptably large. A measure of the passage of **I** from being well conditioned for small parameterizations to being ill conditioned for models that are overparameterized is the average relative parameter error (ARPE). This quantity is the average of diagonal entries in the covariance matrix **P**, each entry normalized by the

TABLE 1. Hydrometeorological Characteristics of Catchments for Years Analyzed in This Paper

Catchment	Area, km^2	Precipitation, mm/yr	Average Daily Maximum Temperature, °C	Annual Yield, %
Orroral Valley (Australian Capital Territory)	89.6	1101	18.9	27
Licking Hole (Australian Capital Territory)	20.6	1426	18.9	53
Monachyle (Balquhidder, Scotland)	7.7	2953	5.4	93
Kirkton (Balquhidder, Scotland)	6.9	2795	5.4	88
Watershed 36 (Coweeta, North Carolina)	0.49	2012	19.6	64
Watershed 34 (Coweeta, North Carolina)	0.33	2012	19.6	46
Hydrohill (Nanjing)	0.00049	950	19.5	na

Here na denotes not available.

square of its estimated mean value. It has been used by *Jakeman et al.* [1989, 1990], for example.

The covariance matrix in (11) is dependent on (1) the input sequence of excess rainfall u_k, (2) the underlying response (model) parameters (because $\hat{x}_k$ is the output of the model and $\hat{x}_k^*$ and u_k^* are transformed from x_k and u_k using $\hat{a}_1, \cdots, \hat{a}_n$), (3) the variance of the combined data and model errors, $\hat{\sigma}^2$, and (4) the sample size, N. For any temporal pattern of excess rainfall, response parameter values, and error level (or alternatively sample size), P can be evaluated using (10) and (11) to determine the absolute minimum sample size (error variance) required to achieve some predetermined accuracy in the parameter estimates a and hence in the linear dynamic response characteristics τ.

2.3. *Example Catchments*

To answer our fundamental question, How complex a parameterization is warranted in a rainfall-runoff model?, the modeling framework was applied to a selection of catchments covering a range of scales and climatic conditions (Table 1). Except for the experimental Hydrohill catchment, pairs of catchments were chosen in close proximity but with differing responses so that model performance could be examined under similar climatic conditions (almost identical calibration periods) but different catchment descriptors.

The largest pair of catchments selected for analysis consisted of the upland subcatchments of the Murrumbidgee and Cotter rivers in the Australian Capital Territory about 50 km southwest of Canberra. Their centers are about 10 km apart. The stream gauge on the larger Orroral Valley catchment is at an elevation of 870 m, while that on the Licking Hole catchment is at 1090 m. Land falls steeply in both catchments from high ridges. Both contain a large diversity of vegetation, mainly native eucalypts, but the cover at Licking Hole was completely burned by brushfire just prior to the period of analysis. Soils are deep in the two, but quick flow in the Orroral Valley is considered to be mainly interflow above a semipermeable layer (R. Knee, personal communication, 1992).

The pair of intermediate sized catchments selected contain the Kirkton and Monachyle streams, situated near Balquhidder, Scotland, about 60 km north of Glasgow. The catchment centers are less than 5 km apart. Both catchments have steep slopes, flat valley bottoms, and generally thin soils, but the Monachyle has more extensive peat areas in its upper parts. The Kirkton has forest and grass vegetation, while the Monachyle is covered with heather and grass. *Jakeman et al.* [1993*b*] have analyzed precipitation-streamflow data from these catchments before and after experimental land use changes were effected to examine the associated changes in the hydrological response. Detailed descriptions of the catchments and the experimental Balquhidder program are given by *Blackie* [1987] and *Johnson* [1988].

The smallest pair of catchments selected was from the Coweeta Hydrological Laboratory in the United States. Coweeta is located in the Nantahala Mountains of western North Carolina. Watershed 36 is a high-elevation, steeply sloping catchment with shallow soils, and a high annual yield and a large proportion of quick flow [*Swift et al.*, 1988]. Watershed 34 is a midelevation catchment with somewhat deeper soils and, consequently, substantially more delayed flow. Details of the physical characteristics of the Coweeta catchments are given by *Swank and Crossley* [1988].

Hydrohill is a small experimental catchment of 490 m^2 near Nanjing, China. It has been used to investigate isotopic heterogeneity in subsurface waters by *Kendall and Gu* [1991]. According to them, the catchment

> was constructed with a concrete aquiclude consisting of two intersecting slopes with 14° gradients overlaying bedrock. Impermeable walls enclose the catchment on the top and sides. The aquiclude was covered with 1 m of a silty loam that was free of concretions. The bulk density was adjusted to approximate the natural soil profile. Grass was then planted over the surface. After three years of settling, a drainage trench was dug at the intersection of the two slopes and the water-sampling instrumentation was installed.
>
> Five troughs, each 40 m long and constructed of fibreglass, were installed longitudinally in the trench. These troughs were stacked on top of each other to create a set of long zero-tension lysimeters. Each trough has a 20 cm aluminum lip that extends horizontally into the soil layer to prevent leakage between layers. Waters collected in each trough pass through V-notch weirs where discharge is continuously monitored and recorded . . . the uppermost trough collects rain; the next lower trough collects surface runoff. The next three troughs collect subsurface flow from soil layers spanning the depths of, respectively, 0–30 cm, 30–60 cm, and 60–100 cm.

For each catchment pair, that is, excepting Hydrohill, one year of excess rainfall and streamflow values was used to estimate model structure and the associated parameter val-

TABLE 2. Identification Statistics for Two Adjacent Catchments in the Australian Capital Territory Using 1 Year of Daily Rainfall and Streamflow Data Beginning in April 1983

Configuration and Model Order (n, m)	Orroral Valley Catchment D	Orroral Valley Catchment ARPE, %	Licking Hole Catchment D	Licking Hole Catchment ARPE, %
One storage (1, 0)	0.761	0.087	0.817	0.033
Two serial (2, 0)	0.768	3.366	0.824	3.442
Two parallel (2, 1)	0.787	0.108	0.870	0.043
Three serial (3, 0)	0.788	1.251	0.842	2.013
One serial, two parallel (3, 1)	0.787	945.474	0.854	4.425
Three parallel (3, 2)	0.785	365.680	0.872	104.358

ues. For Hydrohill, analysis was performed on a storm covering a period of about 34 hours with data spaced at 6 min. For all catchments analyzed, the corresponding sample sizes were sufficient to identify the appropriate configuration of linear storages and their approximate parameter values.

3. RESULTS

For a broad of range of catchments we have found, as we have for the seven catchments in this paper, that the most commonly identified configuration is the one in Figure 1 of two storages in parallel driven by excess rainfall. This configuration of the linear module identified is the same for any values of the nonlinear module parameters, τ_w and f, which yield reasonable fits to streamflow. Indeed it is the same if no nonlinearities are assumed and rainfall is treated as excess rainfall. However, only one storage may be identified if either base flow is absent or data $\{\mathbf{u}_N, \mathbf{q}_N\}$ are sampled over a coarse enough time interval. From monthly data, only one storage was identified by *Littlewood and Jakeman* [1992] for the Thames Basin at Kingston, and the match between observed and model flow was excellent.

3.1. *The Paired Catchments*

Identification results. A general pattern emerges from the results of the modeling approach applied to a year of daily data from each of the six catchments (Tables 2–4). In terms of D, the single-storage and two–serial storage configurations lead to lower values than the two–parallel storage configuration (with the exception of the Monachyle catchment). The former also have fewer parameters, two and three, respectively, compared to four for the latter. In the case of the Monachyle, the slow flow volume fraction is so small (<0.1 as will be seen in Table 5) that the former configurations suffer no reduction in D value by not fitting the low recessions. The slow flow volume fraction at Kirkton is also small enough (estimated as 0.15) to yield only small differences in D among the different configurations.

With all six catchments, more complex configurations than two parallel storages yield no substantial improvement in D values, less than 1.3%, and they may yield a substantial decrease. The average relative parameter error (ARPE) of the single-storage and two–parallel storage configurations is orders of magnitude lower than for the other configurations. Therefore the results show that a rainfall-runoff model configuration with more than two storages is not warranted by the data. If larger configurations are fit, the uncertainty or ambiguity in parameter estimates is exceedingly high. If configurations involving fewer parameters are estimated, the fit to streamflow for all the catchments is visually inferior, especially during long recessions, and is generally manifest as lower D values.

Qualitative relation of DRC values to physical catchment descriptors. The dynamic response characteristics of the linear module for each of the six catchments, derived directly from the parameter estimates $\mathbf{a}$, show a variability reflecting our wide choice of catchment types (Table 5). A comprehensive interpretation of the estimated DRC values is beyond the scope of this paper, but some comments are warranted. Note that size of the catchment bears little relation to the time constants of the quick and slow components. Orroral Valley, the largest catchment by far, has a much smaller τ_q (faster quick response) than the next three largest catchments. Licking Hole, which is much larger than both Coweeta catchments, has a smaller τ_s (faster slow response) than these. If DRCs are related to physical catchment descriptors, as suggested by *Jakeman et al.* [1992], more than catchment size is involved.

Qualitatively, some of the controlling differences between catchments in each pair can be argued. This is because the climatic forcing variables are basically the same and some of

TABLE 3. Identification Statistics for Two Adjacent Catchments Near Balquhidder, Scotland, Using 1 Year of Daily Rainfall and Streamflow Data Beginning in July 1985

Configuration and Model Order (n, m)	Monachyle Catchment D	Monachyle Catchment ARPE, %	Kirkton Catchment D	Kirkton Catchment ARPE, %
One storage (1, 0)	0.686	0.492	0.728	0.175
Two serial (2, 0)	0.686	450.004	0.728	292.149
Two parallel (2, 1)	0.685	1.555	0.733	0.278
Three serial (3, 0)	0.690	29.589	0.740	4.549
One serial, two parallel (3, 1)	0.694	34.147	0.742	836.647
Three parallel (3, 2)	0.695	89.647	0.746	19,528.081

TABLE 4. Identification Statistics for Two Adjacent Catchments in North Carolina Using 1 Year of Daily Rainfall and Streamflow Data Beginning in October 1981

Configuration and Model Order (n, m)	Coweeta 36 Catchment D	Coweeta 36 Catchment ARPE, %	Coweeta 34 Catchment D	Coweeta 34 Catchment ARPE, %
One storage (1, 0)	0.741	0.074	0.808	0.043
Two serial (2, 0)	0.749	4.339	...	∞
Two parallel (2, 1)	0.891	0.063	0.916	0.078
Three serial (3, 0)	0.788	1.502	...	∞
One serial, two parallel (3, 1)	0.893	5.465	0.908	3.264
Three parallel (3, 2)	0.900	7.427	0.922	49.691

Here, ∞ denotes divergent model.

the physical catchment descriptors are similar as described in section 2.3. For the largest pair (the Australian Capital Territory catchments), terrain is of the same type, with the soils and vegetation being the obvious difference. The deep soils and reduced vegetation at Licking Hole provide relatively larger increases in storage (see separation in Figure 3 versus Figure 2) immediately following rainfall, slow flow contributing more to the hydrograph peaks, whereas an underlying less permeable soil layer at Orroral Valley provides a much flashier quick flow response (τ_q = 1.61 compared to 3.79 days) and a slow flow storage which is less responsive to rainfall and recedes more than twice as slowly (τ_s = 97 compared to 36 days). In the case of the Balquhidder catchments, the topography is similar, the main differences also being the vegetation and soils. The larger areas of upland peat may contribute to the greater proportion of quick flow volume in the Monachyle (v_q = 0.93 compared to 0.85 and see separation in Figure 5 versus Figure 4). For the Coweeta pair the differences in soils and topography are substantial. The steep slopes and thin soils at watershed 36 yield a much smaller proportion of slow flow volume than do the deep soils at watershed 34 (v_s = 0.58 compared to 0.80 and see separation in Figure 6 versus Figure 7).

Correspondence between DRC values and stream hydrograph patterns. There is also a very reasonable correspondence between the patterns in the stream hydrographs in Figures 2–7 and the DRC values of the linear module, especially if one does the comparison within each pair where the climatic forcing variables are basically the same. The low value of τ_q for the Orroral Valley stream response accords well with its qualitative flashiness (Figure 2). Perhaps not so easy to appreciate from the figures are the similar relative volumes of quick (and hence slow) flow throughput for Orroral Valley and Licking Hole (Figures 2 and 3). For Orroral, the long slow flow time constant τ_s of almost 100 days and the flashiness of quick flow evidently ensure a strong contribution of slow flow to total stream volume despite the relatively low magnitude of slow flow during rainfall events. For the Balquhidder catchments, Monachyle shows a flashier quick response and a smaller slow flow volume than Kirkton (Figures 4 and 5). The DRC values (Table 5) confirm this view. But it is more difficult from inspection of the stream hydrographs to appreciate the identified longer response times of quick (and slow) components for Coweeta watershed 34 versus 36 (Table 5). This is partly due to the obviously larger slow flow volume for

TABLE 5. Dynamic Response Characteristics of the Linear Module and Minimum 90% Confidence Intervals for the Seven Catchments

Catchment	Dynamic Response Characteristic: τ_q, days	τ_s, days	v_q, fraction	v_s, fraction
Orroral Valley				
Value	1.61	97	0.62	0.38
90% confidence interval	(1.45, 1.78)	(−308, 458)	(0.34, 0.96)	(0.02, 0.66)
Licking Hole				
Value	3.79	36	0.63	0.37
90% confidence interval	(3.44, 4.18)	(25, 64)	(0.56, 0.69)	(0.30, 0.43)
Monachyle				
Value	0.85	39	0.93	0.07
90% confidence interval	(0.72, 1.00)	(−102, 86)	(0.81, 1.07)	(−0.07, 0.20)
Kirkton				
Value	1.35	46	0.85	0.15
90% confidence interval	(1.18, 1.54)	(−176, 251)	(0.73, 0.99)	(0.01, 0.28)
Coweeta 36				
Value	2.32	50	0.42	0.58
90% confidence interval	(2.07, 2.64)	(42, 61)	(0.38, 0.46)	(0.54, 0.62)
Coweeta 34				
Value	3.45	69	0.20	0.80
90% confidence interval	(2.97, 4.12)	(64, 75)	(0.18, 0.22)	(0.78, 0.83)
Hydrohill	0.0054	0.2417	0.77	0.23

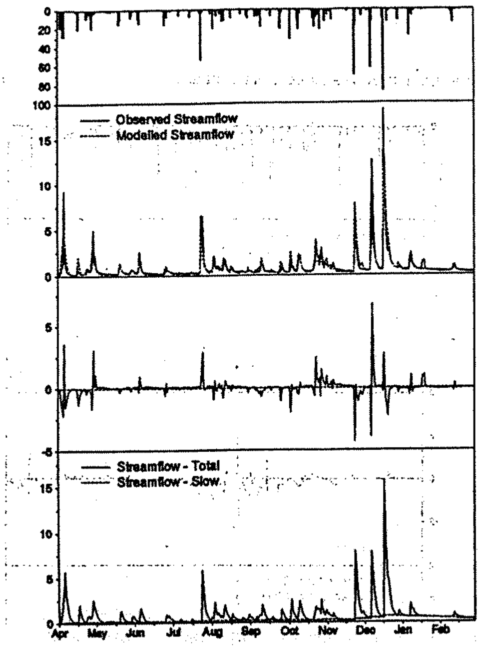

Fig. 2. Rainfall (millimeters), model fit, residuals and total/slow separation for daily streamflow (cubic meters per second) at Orroral Valley in 1983–1984.

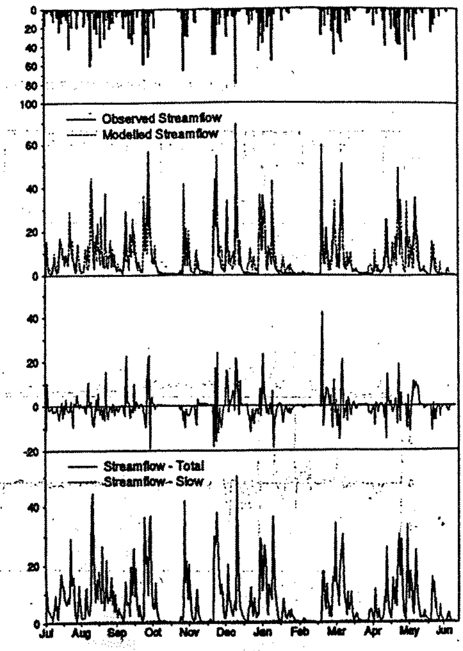

Fig. 4. Rainfall (millimeters), model fit, residuals and total/slow separation for daily streamflow (millimeters) at Monachyle in 1985–1986.

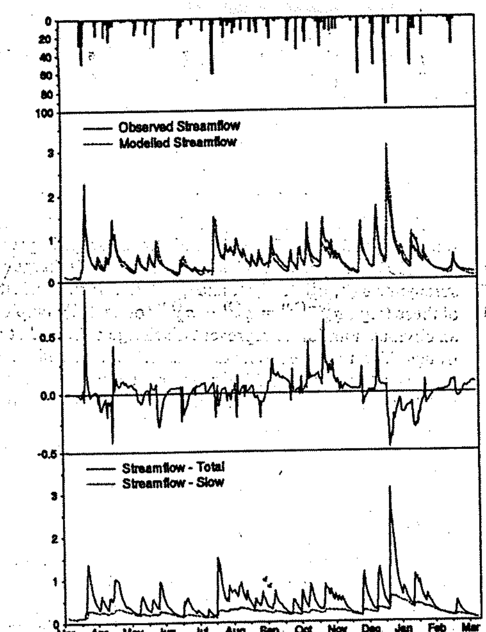

Fig. 3. Rainfall (millimeters), model fit, residuals and total/slow separation for daily streamflow (cubic meters per second) at Licking Hole in 1983–1984.

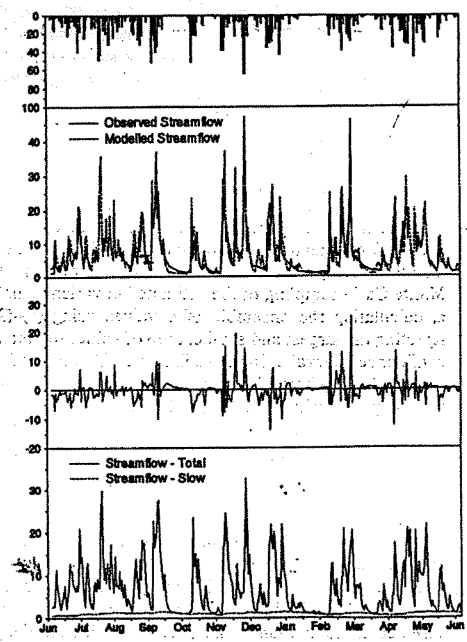

Fig. 5. Rainfall (millimeters), model fit, residuals and total/slow separation for daily streamflow (millimeters) at Kirkton in 1985–1986.

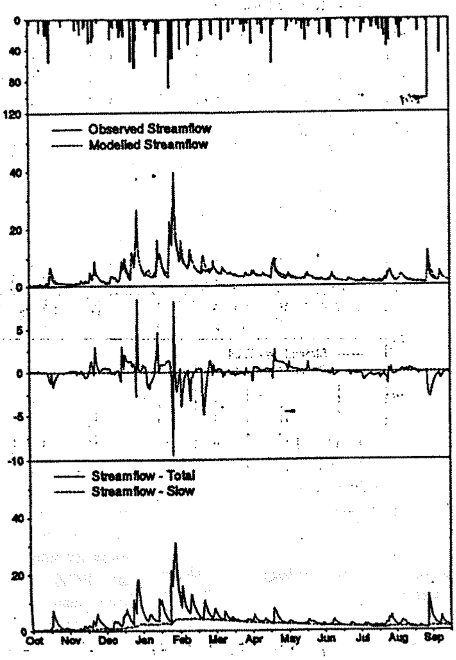

Fig. 6. Rainfall (millimeters), model fit, residuals and total/slow separation for daily streamflow (millimeters) at Coweeta watershed 36 in 1981–1982.

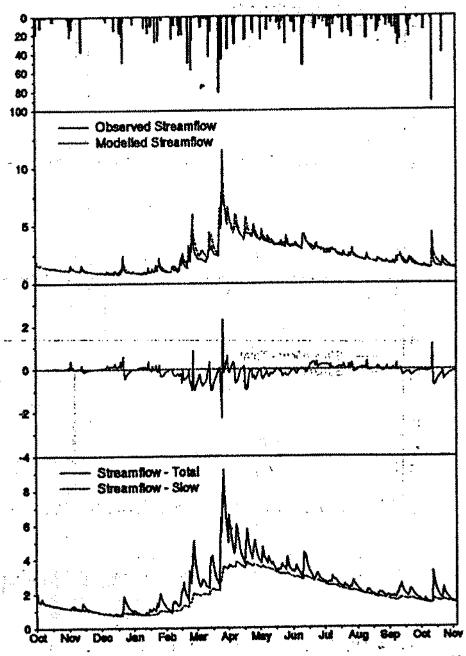

Fig. 7. Rainfall (millimeters), model fit, residuals and total/slow separation for daily streamflow (millimeters) at Coweeta watershed 34 in 1981–1982.

watershed 34 obscuring direct appreciation from the stream hydrograph of the rate of quick response.

Uncertainties in DRC values. A measure of the uncertainties for estimated mean values of the DRCs for the linear module is also provided in Table 5. These were computed by Monte Carlo sampling of the estimated covariance matrix of **a**, calculating the ensemble of τ values using (6)–(9) and rejecting the largest and smallest 5% of values to obtain 90% confidence intervals. The table illustrates that the underlying nature of a catchment's response has a strong effect on the accuracy with which the properties of that response can be calculated. Notice the large uncertainties on τ_s, v_q, and v_s for Kirkton, Monachyle, and Orroral. These large uncertainties are principally due to the low magnitude of slow flow at many time steps. Absolute errors in time series calibration data and model have a larger relative effect on slow flow magnitude, swamping the underlying slow flow signal in the streamflow series to a greater degree.

There is also an effect which compounds these uncertainties. The absolute data errors are probably larger in these three catchments than in the others examined in the paper. For the Balquhidder pair, some precipitation in the winter months occurs as snowfall, and its delayed transfer to excess rainfall as it melted was not attempted (see the model's premature response to snowfall in early January in Figures 4 and 5). In Orroral Valley the rain gauge, being situated in the largest by far of our seven catchments, will not yield as representative a measure of incident rainfall over the catchment.

3.2. *The Hydrohill Catchment, China*

Rainfall time series data $\mathbf{r}_N$ from a storm on July 5, 1989, were analyzed by C. Kendall et al. (manuscript in preparation, 1993) using model (4) separately with discharge measurements $\mathbf{q}_N^{(0)}$, $\mathbf{q}_N^{(30)}$, $\mathbf{q}_N^{(60)}$, and $\mathbf{q}_N^{(100)}$ and various additions of these (e.g., $q_k^{(0+30)} = q_k^{(0)} + q_k^{(30)}$ for each k), employing an obvious notation to represent discharge collected at each trough. The time step selected was 0.1 hours, and analysis was undertaken from 8.7 hours after the storm commenced, still leaving almost 25 hours of record ($N = 250$) before surface discharge ceased. Deletion of the first 8.7 hours of the storm data allows omission from the analysis of some missing discharge measurements in that early period. As will be seen, rainfall after this period, from time steps 88 to 337, can be treated as excess rainfall. That is, $u_k = r_k$ is set for this catchment, allowing omission of the nonlinear module.

Transfer function model identification was applied (C. Kendall et al. (manuscript in preparation, 1993) give detailed results and compare the quick flow–slow flow separations with chemical separations) to the following discharge time series: $\mathbf{q}_N^{(0)}$, $\mathbf{q}_N^{(30)}$, $\mathbf{q}_N^{(0+30)}$, $\mathbf{q}_N^{(60)}$, $\mathbf{q}_N^{(100)}$, $\mathbf{q}_N^{(60+100)}$, $\mathbf{q}_N^{(0+30+60+100)}$, and $\mathbf{q}_N^{(30+60+100)}$, $N = 250$. The resulting model fits were found to be credible in all cases. Figure 8 shows the fit to total discharge, for example, including simulation of rainfall through the model for the first 87 time steps not used for model calibration. The identified configurations of the different models (omitting the subscript N) were a single storage for **u** with $\mathbf{q}^{(0)}$, $\mathbf{q}^{(30)}$, $\mathbf{q}^{(60)}$, and $\mathbf{q}^{(0+30)}$,

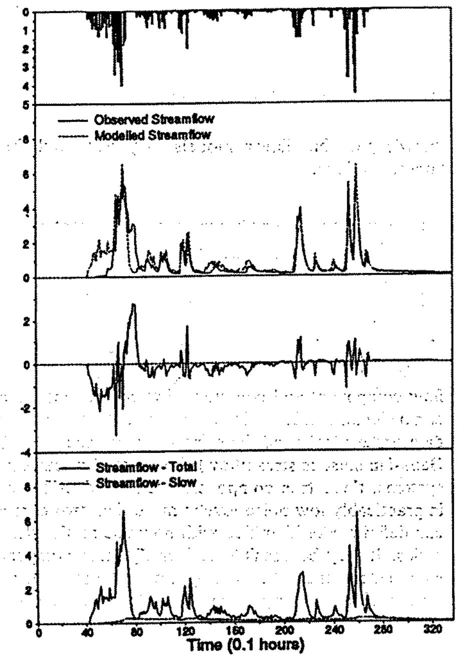

Fig. 8. Rainfall (millimeters), model fit, residuals and total/slow separation for 0.1 hourly streamflow ($\times 10^{-3}$ cubic meters per second) at Hydrohill in July 1989.

and two storages in parallel for $q^{(100)}$, $q^{(60+100)}$, $q^{(30+60+100)}$, and $q^{(0+30+60+100)}$.

A conclusion to be drawn from the Hydrohill results is that higher-order configurations of linear storages can often be well approximated by lower-order linear ones. The sum of individual exponential decay responses (to a pulse of rain) in different parts of the catchment can be well approximated by the linear combination of a smaller number of exponential terms. Using rainfall and total discharge time series data $\{u, q^{(0+30+60+100)}\}$, only two components, compared to five separately identified components from measurements of four individual troughs, are necessary to approximate 89% of the variance of the total discharge at the outlet (Figure 8).

Of course, the response at any level is composed of numerous components. Data and model errors prevent identification of more than a small number of them. Estimation of parameters in higher-order configurations than those identified lead to little improvement (of the order of 1% in D) in accounting for the variance of observed discharge. These configurations also carry high parameter variances. Thus even in cases where several runoff components are observed directly, only a small number (four) of parameters are needed to fit streamflow well and to separate a slow flow component.

4. Discussion and Conclusions

The main issue addressed in this work is that of complexity in rainfall-runoff models parameterized using only data on precipitation, air temperature, and streamflow. In this respect, the actual model used is important only insofar as it allows a statistical evaluation of the number of parameters (taken by us to be an index of model complexity) that can be logically supported by the data. The major result of our study is that, for catchments in temperate climates but over a tremendously wide range of scales, only a handful of parameters can be reliably estimated from rainfall-runoff data. This result confirms suggestions made by others using conceptual or more physically based models [e.g., *Beven*, 1989].

Before proceeding with a discussion of the main inferences drawn from the modeling work, it should be emphasized what our work does not do. First, there are many reasons for building rainfall-runoff models. From a scientific perspective, for example, insights derived from physically based models, whether or not such models can be rigorously parameterized using statistical methods, can be extraordinarily useful. Our work should not be taken as a recommendation to replace in toto physically based models with transfer function models such as used here.

Better definition of the spatial distributions of catchment characteristics, moisture status, precipitation, and so forth is likely to be achieved in the future as a result of promising modern techniques. Such additional information may result in an improved ability to identify some of the complex hydrological mechanisms that drive catchment response. For example, it has been argued that a knowledge of the distribution of precipitation over a catchment would greatly enhance our ability to simulate hydrographs [e.g., *Wilson et al.*, 1979]. This knowledge may become routinely available from data obtained from sophisticated weather radar units that are being installed around the world. The work presented here should not be taken as an implied limitation on possibilities for the future.

Our modeling approach may be utilitarian for such purposes as flow forecasting. However, repeated applications in this style will not, in and of themselves, lead to scientific advances. We believe that there are two main paths to the development of intuition and advances in the sciences that deal with the natural environment. The first, building up from a physically based understanding of local processes to the large scales of natural catchments, is one with which most scientists are comfortable. The second approach, studying relationships at the larger scales with the aim of discovering patterns that may subsequently be explained using conventional scientific wisdom, is also valuable. It is through the second approach that our modeling results ultimately may be applied in a scientific sense. We do not claim that we have arrived at this point. The aim of this paper is to investigate the number of parameters that are supported by rainfall-runoff data sets. Nevertheless, we indulge later in this discussion in some speculation as to how our approach might prove to be useful in empirical scientific exploration.

4.1. *Complexity*

Complexity has been analyzed within a statistical framework using a specific family of models which allows an optional nonlinear rainfall loss model and any parallel and/or series arrangement of linear reservoirs. Strictly, the results are valid only under these conditions. However, we will proffer some implications later for rainfall-runoff modeling in general.

For a broad range of catchments, we have found that the most commonly identified configuration for a rainfall-runoff model is two storages in parallel driven by excess rainfall as illustrated in Figure 1. (If base flow is negligible or the data sampling interval is too coarse, then only one storage may be identified). This four-parameter (two in the case of one identified storage) linear model obviously may require supplementation to allow for antecedent precipitation conditions and fluctuations in evapotranspiration. In humid catchments, this need add only a few more parameters.

It appears to be a robust conclusion that only a small number of conceptual storages is warranted. Orroral Valley with an area of about 90 km^2 is the largest catchment reported here, but similar results can be obtained on much larger catchments especially when one has a good spatial distribution of rain gauges to estimate areal rainfall. For the 894-km^2 Teifi basin in Wales and the 767-km^2 French Broad River basin in North Carolina, for example, *Jakeman et al.* [1993*a*] identified from daily data only a parallel quick flow and slow flow configuration as the linear module. Addition of the loss model (1)–(3) produced a seven-parameter model with good calibration and validation statistics. Use of finer temporal data for small catchments also leads to identification of the parallel configuration. *Jakeman et al.* [1990] used hourly rainfall and streamflow for two small (0.72 and 0.34 km^2) moorland catchments above Llyn Brianne, Wales, and found the two-parallel storage configuration. The Hydrohill results also show how individual discharge responses at different depths become identifiable as either only a quick component or a quick and a slow component when discharge data are aggregated in space at the "stream" outlet.

4.2. *Linearity of Response*

We have observed a predominant linearity in the response of watersheds over a large range of catchment scales, even if only a simple adjustment is made for antecedent rainfall conditions. The linearity assumption of unit hydrograph theory therefore seems applicable in temperate catchments and works just as well for slow flow as for quick flow. The major evidence for this is twofold. First, there is the ability of the exponentiallike response of the transfer function approximation to the convolution integral to fit stream hydrograph recessions generally quite well, indicating that nonlinearities can be described by the transformation of rainfall to excess rainfall. Second, the Hydrohill results reinforce our observations of a predominantly linear response in catchments and add considerable justification for the modeling approach used in the paper to extract information. Despite considerable spatial heterogeneity in the subsurface wetting of the experimental catchment [see *Kendall and Gu*, 1991], the discharge response (without any nonlinear adjustment between rainfall and excess rainfall) is quite linear; and as found by C. Kendall et al. (manuscript in preparation, 1993), it is a linear response at all four troughs. Such linearity is perhaps surprising on so small a scale: The catchment is only 490 m^2 in area. *Chapman* [1992] has also observed linearity from analysis of event data using a nonparametric unit hydrograph method. *Caroni* [1986] used a unit hydrograph approach to consider variation of parameters with flow and, despite findings of nonlinearity, acknowledged that linear models may very well be good approximations.

4.3. *Implications for Current Modeling Practice*

The amount of information that can be gleaned from climatic time series of rainfall and temperature and from streamflow seems a great deal smaller than much of the current modeling practice, largely performed in temperate catchments, would indicate. With or without adjustment of rainfall for antecedent conditions, almost always, one quick flow component and one slow flow component are all that can be identified. Experiments by the authors with synthetic data under ideal conditions (no error in excess rainfall and Gaussian noise in streamflow) show that, for a large range of systems, three true components can be identified only for impracticably low noise levels; more than two components can definitely be identified with no noise on the streamflow series. It may be possible to identify three components in some catchments when the nature of the rainfall, dynamic response, and quality of records allow this. To find more than three components without knowledge of internal states, such as levels of hydraulically connected groundwater, would seem to be a rare achievement.

The ability to identify more than one response component with reasonable accuracy requires data sets of reasonable length and quality, as well as an algorithm that makes few assumptions about the nature of errors and that is numerically stable enough to extract the slow flow component. Even when just two components are identified, high parameter variance (see Table 5) and covariation exist for estimation from daily data of 1 year in length. Such uncertainty is exacerbated when the difference in relative volume between quick and slow flow response is large and absolute errors in rainfall or excess rainfall are high. For a wide variety of catchment types, the calibration of relatively complex models (conceptual, physically based or otherwise), solely from climatic and streamflow data, in the hope of understanding processes or inferring the compartmentalization of storage, is likely to be largely nonprofitable (see also *Beven* [1989] and *Hooper et al.* [1988]).

The inclusion of spatial data on physical catchment descriptors (PCDs) would not appear to resolve the identifiability problem substantially. While the model (1)–(5) does not explicitly allow the incorporation of additional data on PCDs, the framework used permits speculation as to what additional information such data provide for model calibration. With physically based models there tend to be spatial elements (areas or volumes), representative of a level of physical homogeneity, upon which incident precipitation falls and/or to which flow from other elements may travel. Each element requires some minimal level of definition by parameters describing, for example, the rate at which water is transmitted and the potential throughput or storage. One can therefore regard such models as being nonlinear versions of our conceptual model. Indeed, precipitation enters the surface component of elements in parallel, and flow out of these storages may pass to other storages of the same element or storages of neighboring elements. The PCD data permit an attempt to specify the number and configuration of storages. However, the precipitation and streamflow data are still used in physically based models to estimate those key parameters of each storage related to the transport of water. The results obtained in this paper help argue that, basically independent of the scale of representative elements, information on the flow needs to be obtained from time series data on the inputs and outputs of about every second storage that is separately parameterized. If, for three or more connected storages, one has flow data only into the first and out of the last storage, then the uncertainties of estimating the characteristic hydrological properties of all these will be extremely high.

One way around this problem would be to assume that characteristic hydrological properties are related to suitable PCDs in a parametrically efficient way. For example, it may be possible to obtain a simple relation between them wherein each of the parameters associated with an element is designed to have the same calibrated value over all elements or a large subset of element types.

4.4. *Opportunities*

An ability to represent succinctly the response of a catchment to precipitation and other climatic inputs proffers many opportunities for enhancing our knowledge of hydrological phenomena on a local, regional, and global basis. The transfer function–unit hydrograph separation approach allows one to quantify the differences and similarities of catchment behavior in terms of dynamic response characteristics such as our wetness declination time constant τ_w, temperature modulation factor f, change in storage index per unit rainfall c, and the time constants and fractional volumetric throughputs associated with each linear storage. Such a parsimonious, yet effective and physically plausible parameterization at catchment scale, may provide a common basis for accumulating knowledge through collective application of the associated identification procedure to rainfall-streamflow data sets. An obvious research avenue is examination of the relationships between estimated DRC values and physical catchment descriptors. Any such procedure will need to account for systematic and random uncertainties deriving from rain gauge coverage errors and possible drifts and shifts in stream stage-height rating curves. While the amount and quality of time series data required to generate sufficiently reliable DRC values for any catchment require investigation, it is likely that there are a useful number of catchments worldwide with adequate data.

Acknowledgments. This work was undertaken while both authors were visiting scholars, Department of Applied Earth Sciences, Stanford University and visiting hydrologists, Water Resources Division, U.S. Geological Survey, Menlo Park during 1991. The authors would like to thank the following for providing data: Carol Kendall (Hydrohill), ACT Electricity and Water (Orroral Valley and Licking Hole), Paul Whitehead and the Institute of Hydrology (Balquhidder), and Wayne Swank (Coweeta). Tom Chapman, Jud Harvey, Carol Kendall, Ian Littlewood, and Dennis McLaughlin generously provided helpful comments on the manuscript. The three reviewers' comments have also improved the paper.

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(Received June 15, 1992;
revised March 23, 1993;
accepted April 1, 1993.)