Optimal Diameter Growth Equations for Major Tree Species of the Midsouth

Don C. Bragg, *USDA Forest Service, Southern Research Station, P.O. Box 3516 UAM, Room 111, Forest Resources Building, Monticello, Arkansas 71656.*

ABSTRACT: *Optimal diameter growth equations for 60 major tree species were fit using the potential relative increment (PRI) methodology. Almost 175,000 individuals from the Midsouth (Arkansas, Louisiana, Missouri, Oklahoma, and Texas) were selected from the USDA Forest Service's Eastwide Forest Inventory Database (EFIDB). These records were then reduced to the individuals growing at the fastest rate given their species and size, and nonlinear ordinary least squares regression was used to fit equations to a subset of points with maximal increment. Sugarberry is provided as an example of the PRI derivation process. South. J. Appl. For. 27(1):5–10.*

Key Words: Potential relative increment (PRI), Eastwide Forest Inventory Database (EFIDB), nonlinear regression, sugarberry.

Diameter increment equations are cornerstones of many forest models. However, the quality and usefulness of these functions has been inconsistent. Theoretical designs (e.g., Botkin et al. 1972, Pacala et al. 1996) are ecologically appealing but rarely empirically based. In addition, some increment designs are physiologically constrained to fit a particular set of assumptions that do not readily translate to real-world situations (Moore 1989, Bragg 2001). Statistically based models like the Forest Vegetation Simulator (Wykoff et al. 1982) predict growth based on a predefined set of environmental conditions. This growth model style is rigidly limited to the variables included and therefore may suffer from errors of omission. Others have fit curves to realized diameter increment without considering the factors influencing performance (Zeide 1993). This approach usually combines a theoretical design with inventory data, but the factors responsible for deviations from optimal growth are unknown. While workable, all of the aforementioned increment models lack either reasonable ecological assumptions or the ability to reflect environmental uncertainty over time (see Bragg 2001). A procedure that combines the fitting of a model to inventory data with a strong, biologically-founded growth response would seem to represent an ideal compromise.

NOTE: The author can be reached at Phone: (870) 367-3464 ext. 18, Fax: (870) 367-1164, E-mail: dbragg@fs.fed.us. Thanks to the following individuals for their reviews: Jim Guldin (USDA Forest Service, Southern Research Station), Tom Lynch (Oklahoma State University), Boris Zeide (University of Arkansas at Monticello), and Dave Larsen (University of Missouri at Columbia). Several anonymous reviewers also improved the quality of this article. Manuscript received June 19, 2001, accepted October 10, 2001.

The potential relative increment (PRI) approach balances the use of empirical data to predict growth with ecologically robust assumptions (Bragg 2001). The PRI methodology fits a combined power/exponential function to maximal increment data gathered from a large public database. However, the technique is simple enough that any inventory containing information on species, beginning and ending diameters, and remeasurement interval could be utilized. The objective of this paper is to provide optimal diameter growth models for 60 major tree species of the Midsouth region (Arkansas, Louisiana, Missouri, Oklahoma, and Texas) of the United States.

Methods

This section will only briefly review the PRI process since greater detail on the assumptions and behavior of the PRI methodology can be found in Bragg (2001). While the information necessary to derive PRI equations can be taken from virtually any inventory, this effort used the Eastwide Forest Inventory Database (EFIDB) described by Hansen et al. (1992). The EFIDB consists of the computerized records of the periodic Forest Inventory and Analysis (FIA) program of the USDA Forest Service. The original 0.4 ha FIA plots were established on a systematic grid (approximately one plot every 5 km) across most of the United States (note that the new annualized FIA system has redesigned plot size and layout). These plots are rigorously sampled by trained field crews held to high accuracy standards and subjected to periodic quality control checks. The advantages to using this database include the extent, consistency, quality, detail, and availability of the information. However, the FIA emphasis on commercial tree species results in some minor or exotic species not being represented or getting combined with similar taxa.

Inventory records for Arkansas (last surveyed in 1995), Louisiana (1991), Missouri (1989), Oklahoma (1993), and Texas (1992) were downloaded from the EFIDB web site and the necessary data on 60 common species (Table 1) were extracted using a DOS program written by the author. Individual tree records were selected for analysis if the tree was: (1) of the desired species; (2) alive in the most recent inventory; and (3) had a periodic diameter increment greater than 0 cm. While the PRI software makes it possible to further segregate into stands of natural versus planted origin (to distinguish potentially improved stock or very favorable growing conditions), only loblolly pine (*Pinus taeda* L.) was separated for this study.

The raw inventory information extracted from each state database was sorted and written to an ASCII file. Actual relative increment (*ARI*) was then determined for each tree record:

$$ARI = \frac{DBH - DBH_0}{DBH_0}(P / t) \qquad (1)$$

where the relative increment is based on the most recently measured diameter at breast height (dbh), the original dbh (DBH_0), P is the period (in years) over which results are forecast (e.g., for annual increment, P = 1), and t is the inventory remeasurement interval (in years, $t \geq 1.0$). After this, each state's data were combined with the other state inventories into a pooled file, and then refiltered by another program to select the highest ARI values (PRI) by 2 cm dbh class for each species (Figure 1). This subset of maximal ARI values is assumed in the PRI methodology to represent the individual trees growing under the most favorable conditions present in the inventory. Combined with the considerable spatial extent and typically large size of the samples, this assumption produces a conservative estimate of optimal growth performance (Bragg 2001). Nonlinear ordinary least squares PRI models were fit to the following design:

$$PRI = b_1 D_{MAX}^{b_2} b_3^{D_{MAX}} \qquad (2)$$

where D_{MAX} is the dbh for the trees with maximum ARI within each 2 cm diameter class, and b_1, b_2, and b_3 are species-specific regression coefficients. Those individuals that did not realize maximum size class growth (i.e., did not fall along the optimal performance horizon) or were identified as outliers (probably resulting from measurement or transcription errors) were removed, and the model refined until the "best" subset was chosen (see Bragg (2001) for details). While developing a best subset is a partially subjective process, since the goal was to fit an idealized response curve to a biologically intuitive relationship between tree size and growth potential, some bias was unavoidable. However, the elimination of records accruing at a lower level than expected given adjacent diameter class performance or for obviously incorrect growth under known circumstances helped to maintain the integrity of the desired optimal growth model while retaining reasonable predictions (Bragg 2001).

After the model parameters were obtained, predicted optimal increment (I_0) was calculated by substituting tree dbh for D_{MAX} in Equation (2) and multiplying by current tree diameter:

$$I_0 = DBH \times PRI \qquad (3)$$

Equation (3) generates realized diameter increment when modifier function(s) are used to adjust for growth favorability. This approach is commonly used in ecological models and enjoys considerable flexibility since any environmental quality function(s) can be used to constrain growth. However, no specific qualifiers are presented in this article to encourage users to develop their own based on their particular need and perspective of what factors are important.

A key advantage to the PRI methodology is that it does not rely on tree age to determine optimal growth performance. While tree age is highly correlated to diameter (and thus potential increment) for shade intolerant species, the relationship is much weaker for shade tolerant species, especially in mature stands (e.g., Gates and Nichols 1930). Furthermore, age is a time-consuming and often difficult variable to measure and thus is usually not available in large inventories like the EFIDB. However, if age is deemed too valuable to leave out of the increment calculation, it should be possible to include age as an additional variable in the modifier function(s).

Results and Discussion

Almost 175,000 individuals from the 60 chosen species in the five-state inventory pool were identified as usable, with some taxa better represented than others (Table 1). Five species [loblolly pine (n = 37,667), shortleaf pine (n = 17,114), post oak (n = 13,308), white oak (n = 13,085), and sweetgum (n = 12,250)] comprised 53.4% of the total sample. Nine species had fewer than 200 individuals, while shellbark hickory had just 97 representatives. While sample sizes less than 100 are usually discouraged for the PRI methodology (D.C. Bragg, unpublished manuscript), this may arise for some taxa because of their scarcity. Since almost half of the species exceeded 1,000 trees, and 25% had more than 3,000 individuals, the optimal growth equations presented in this study should provide reliable estimates of potential growth.

The extensive EFIDB survey also yielded broad diameter distributions for most taxa. Trees as small as 2.8 cm dbh were available for most species (Table 1). Average dbh typically ranged from 15 to 30 cm, with maximum dbh exceeding 90 cm for more than half of the species in this survey. While no species reached their maximum possible diameter in this study, some very large trees were encountered (the final tallies included a 183.1 cm dbh sycamore and a 250.2 cm dbh baldcypress). Most individuals fell into small to moderate (< 50 cm dbh) diameter classes.

Regression coefficients and final sample size are reported in Table 2. While at least 5 final sample points has been recommended for fitting the PRI equations (D.C. Bragg, unpublished manuscript), this may be unavoidable for some

Table 1. Species names*, Forest Inventory and Analysis (FIA) codes, and sample statistics calculated from original diameter (DBH_O).

Common name	Scientific name	FIA species code	Sample size	Min. dbh	Avg. dbh	Max. dbh	Std. dev.
				(cm)			
Boxelder	*Acer negundo* L.	313	567	2.8	22.7	63.5	11.97
Red maple	*Acer rubrum* L.	316	3,113	2.8	16.1	89.4	12.81
Silver maple	*Acer saccharinum* L.	317	238	2.8	36.4	127.8	23.03
Sugar maple	*Acer saccharum* Marsh.	318	695	2.8	16.1	68.3	12.41
River birch	*Betula nigra* L.	373	312	3.0	30.0	98.8	16.23
American hornbeam	*Carpinus caroliniana* Walt.	391	1,271	2.8	11.8	44.5	7.53
Water hickory	*Carya aquatica* (Michx. f.) Nutt.	401	1,203	2.8	33.9	105.2	18.26
Bitternut hickory	*Carya cordiformis* (Wangenh.) K. Koch	402	489	2.8	20.3	65.8	12.93
Pecan	*Carya illinoensis* (Wangenh.) K. Koch	404	400	2.8	38.3	118.4	20.07
Shellbark hickory	*Carya laciniosa* (Michx. f.) Loud.	405	97	2.8	20.5	68.6	14.28
Shagbark hickory	*Carya ovata* (Mill.) K. Koch	407	979	2.8	19.4	75.7	13.19
Black hickory	*Carya texana* Buckl.	408	3,937	2.8	17.8	70.9	11.01
Mockernut hickory	*Carya tomentosa* Poir. Nutt.	409	2,572	2.8	19.0	85.3	12.84
Sugarberry	*Celtis laevigata* Willd.	461	2,486	2.8	28.5	114.3	14.92
Hackberry	*Celtis occidentalis* L.	462	286	2.8	18.5	76.5	15.12
Common persimmon	*Diospyros virginiana* L.	521	548	2.8	11.7	50.0	8.65
American beech	*Fagus grandifolia* Ehrh.	531	1,009	2.8	44.4	115.1	19.35
White ash	*Fraxinus americana* L.	541	1,268	2.8	21.4	103.9	14.93
Green ash	*Fraxinus pennsylvancia* Marsh.	544	3,058	2.8	28.1	96.5	17.21
Waterlocust	*Gleditsia aquatica* Marsh.	551	126	3.0	28.4	67.8	13.50
Honeylocust	*Gleditsia triacanthos* L.	552	444	2.8	28.8	97.8	16.75
Black walnut	*Juglans nigra* L.	602	502	2.8	25.5	103.4	13.50
Eastern redcedar	*Juniperus virginiana* L.	68	2,435	2.8	15.2	72.9	9.57
Sweetgum	*Liquidambar styraciflua* L.	611	12,250	2.8	24.3	111.3	14.59
Yellow-poplar	*Liriodendron tulipifera* L.	621	171	6.3	42.6	104.9	18.84
Sweetbay	*Magnolia virginiana* L.	653	449	2.8	23.1	72.1	14.04
Red mulberry	*Morus rubra* L.	682	292	2.8	15.1	71.4	10.61
Water tupelo	*Nyssa aquatica* L.	691	2,061	2.8	31.5	95.5	13.09
Blackgum	*Nyssa sylvatica* Marsh.	693	3,942	2.8	22.5	93.0	16.01
Eastern hophornbeam	*Ostrya virginiana* (Mill.) K. Koch	701	980	2.8	8.0	61.0	5.94
Shortleaf pine	*Pinus echinata* Mill.	110	17,114	2.8	25.3	89.7	10.96
Loblolly pine †	*Pinus taeda* L.	131	37,667	2.8	24.8	103.4	14.42
Sycamore	*Platanus occidentalis* L.	731	720	3.3	36.3	183.1	20.90
Black cherry	*Prunus serotina* Ehrh.	762	498	2.8	17.9	65.3	11.93
White oak	*Quercus alba* L.	802	13,085	2.8	25.6	101.9	14.60
Southern red oak	*Quercus falcata* Michx.	812	5,128	2.8	30.5	138.2	16.22
Cherrybark oak	*Quercus falcata* var. *pagodifolia* Ell.	813	2,112	2.8	37.7	117.3	20.22
Shingle oak	*Quercus imbricaria* Michx.	817	133	2.8	23.9	59.2	13.47
Bluejack oak	*Quercus incana* Bartr.	840	101	2.8	18.6	41.4	10.43
Laurel oak	*Quercus laurifolia* Michx.	820	1,023	2.8	33.8	99.8	20.31
Overcup oak	*Quercus lyrata* Walt.	822	2,010	2.8	40.4	129.8	19.92
Bur oak	*Quercus macrocarpa* Michx.	823	104	3.8	37.0	78.2	19.69
Blackjack oak	*Quercus marilandica* Muenchh.	824	2,304	2.8	20.4	72.1	11.35
Swamp chestnut oak	*Quercus michauxii* Nutt.	825	472	2.8	42.3	110.2	20.94
Chinkapin oak	*Quercus muehlenbergii* Engelm.	826	688	2.8	24.3	81.5	14.48
Water oak	*Quercus nigra* L.	827	5,464	2.8	34.8	140.7	20.91
Nuttall oak	*Quercus nuttallii* Palmer	828	855	3.3	44.8	130.8	21.33
Pin oak	*Quercus palustris* Muenchh.	830	146	3.6	37.7	82.5	16.79
Willow oak	*Quercus phellos* L.	831	2,867	2.8	37.6	149.1	20.30
Northern red oak	*Quercus rubra* L.	833	3,319	2.8	29.2	94.5	14.36
Shumard oak	*Quercus shumardii* Buckl.	834	363	2.8	40.1	107.9	22.24
Post oak	*Quercus stellata* Wangenh.	835	13,308	2.8	25.2	104.4	14.02
Black oak	*Quercus velutina* Lam.	837	7,145	2.8	26.3	133.1	15.04
Black locust	*Robinia pseudoacacia* L.	901	122	4.1	22.9	82.0	11.76
Sassafras	*Sassafras albidum* (Nutt.) Nees	931	585	2.8	9.5	53.6	8.94
Baldcypress	*Taxodium distichum* (L.) Rich.	221	3,695	3.3	42.8	250.2	28.44
American basswood	*Tilia americana* L.	951	118	2.8	29.1	77.5	17.59
Winged elm	*Ulmus alata* Michx.	971	3,170	2.8	13.6	67.8	10.32
American elm	*Ulmus americana* L.	972	1,734	2.8	24.3	148.6	18.28
Slippery elm	*Ulmus rubra* Muhl.	975	642	2.8	20.3	102.1	16.02

Total number of trees, all species = 174,872

* Nomenclature from Harlow et al. (1979), Hansen et al. (1992), and Moore (1999).

† Includes 24,344 loblolly pines from natural stands and 13,323 loblollies from plantations.

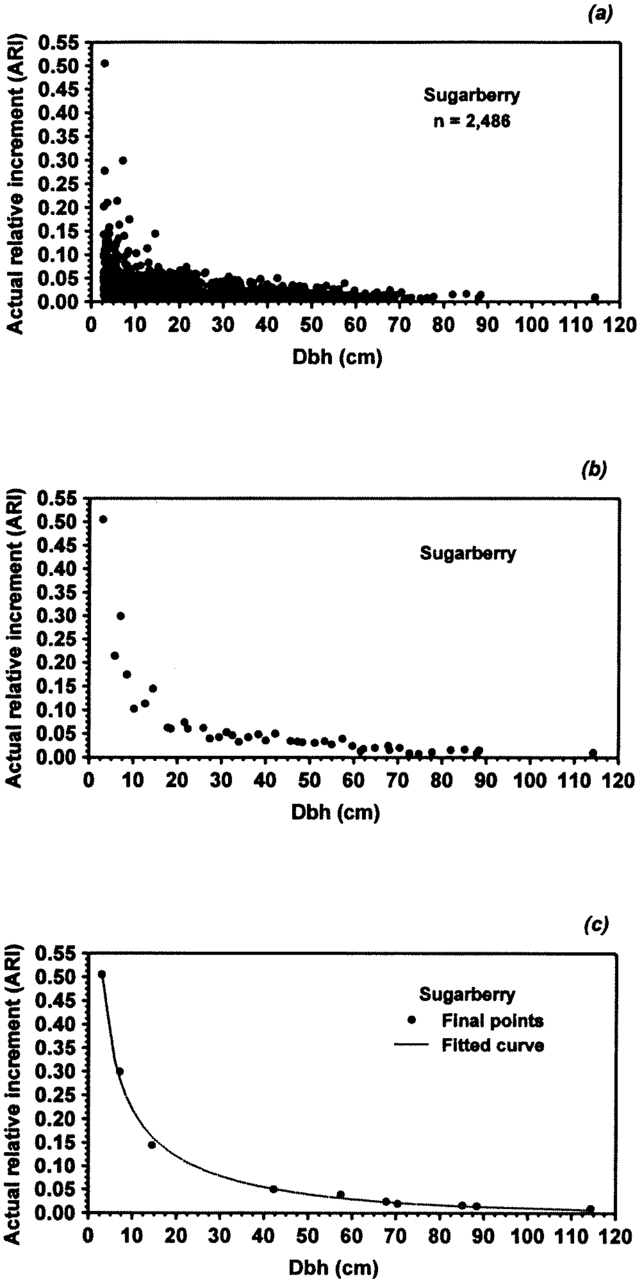

Figure 1. Using sugarberry as an example of the PRI methodology, the 2,486 eligible trees taken from the EFIDB had their actual relative increment (ARI) calculated and plotted as a function of their original dbh (a), from which a set of maximal ARI points was determined (b) and eventually reduced to the best subset (final n = 10) to which the PRI equation was fit (c).

species. Indeed, black locust only had three final points, resulting in a perfect fit (interpolation) given the three parameter PRI design. Since a very small fraction (usually much less than 10%) of initial records were used to fit the optimal growth equations, the proportion of the variance explained by the equations was quite high. In fact, traditional measures of model quality (e.g., fit indices, standard errors of parameter estimates, confidence intervals) are of limited utility with the PRI methodology because the final subset of points used to define optimal growth so closely matched the fitted curve.

Optimal growth performance predicted by the PRI methodology generally produces a skewed modal response, with highest relative increment coming at the smallest diameters and maximum optimal diameter increment appearing at moderate (10 to 30 cm) diameters. Figure 2 presents sugarberry as an example of optimal performance. Note that the highest PRI value occurred at the smallest dbh (dashed line), but that optimal growth did not peak until 20 to 25 cm dbh. The PRI model predicts that large sugarberry trees can add 0.75 cm in diameter per year at 120 cm dbh, which, while substantial, still represents a conservative estimate of potential increment.

Potential Applications

The original application of these optimal increment equations was to support the development of ecologically based growth models. Even though most ecological simulators have been used for research rather than forest management, this does not imply that increment models containing optimal growth equations could not be adapted operationally. If properly designed modifier functions are developed, then systems could be engineered to assess site potential or yield performance. For example, if a landform or soil type was known to have certain characteristics, then treatments intended to alter performance could be modeled from the differences in potential and realized increment.

While this article presented optimal growth models for an existing database, it is possible to adapt the PRI methodology to improve the assessment and management of growth on commercial timberlands with other inventories, even if the range of environmental conditions from which to sample is narrow. The PRI approach lends itself well to landowners with extensive, long-term surveys for which specific trees have been tracked for years. Individuals performing at the highest rates can be used as benchmarks for evaluating different seed sources, genetic improvements, silvicultural treatments, or site conditions. Variation in PRI model response curves can also be applied to the interpretation of what species may perform the best in natural stands, which may be especially important when the differences in production and revenue between taxa are pronounced.

Figure 3 highlights the distinctions between PRI models developed exclusively for natural stands of loblolly pine, loblolly pine plantations, and a pooled model (natural + planted loblollies). PRI models indicate that the magnitude of loblolly pine growth potential for the Midsouth region was virtually indistinguishable between natural stands and plantations (although the average potential would not be as similar). However, the timing of maximum growth appeared to differ, with plantations peaking at smaller diameters than natural stands (~ 10 cm for plantations, ~ 20 cm in natural stands). Natural forests considerably outperformed plantations in older stands, but this inconsistency is at least partially an artifact of limited data on older plantations. Such knowledge can help users evaluate what types of stand origins are optimized for immediate fiber yield versus long-term productivity. For example, the PRI methodology could identify possible trade-offs between early and late growth performance over a species' lifespan. If sufficient controls on local environmental conditions are implemented and the stands are

Table 2. Regression coefficients and final number of trees by species for nonlinear model*.

Common name	b_1	b_2	b_3	Final n[†]
Boxelder	0.483468	−0.294860	0.958936	15
Red maple	0.647564	−0.503425	0.978791	8
Silver maple	0.756076	−0.489618	0.980200	8
Sugar maple	0.394028	−0.582677	0.978206	11
River birch	0.930254	−0.862366	0.994566	7
American hornbeam	0.197567	0.175911	0.935024	6
Water hickory	0.481054	−0.616199	0.992552	13
Bitternut hickory	0.175900	−0.083534	0.953427	9
Pecan	0.629583	−0.691738	0.991713	11
Shellbark hickory	0.385894	−0.564218	0.973063	9
Shagbark hickory	0.435097	−0.813536	0.998634	8
Black hickory	0.275064	−0.024384	0.939325	12
Mockernut hickory	0.510023	−0.741816	0.990071	8
Sugarberry	1.023890	−0.584687	0.980998	10
Hackberry	0.886501	−0.844634	0.997343	8
Common persimmon	0.676177	−0.873767	0.983282	11
American beech	0.374346	−0.470061	0.989114	5
White ash	0.543299	−0.663623	0.990433	7
Green ash	0.252831	0.110854	0.960122	9
Waterlocust	0.624384	−0.838400	0.990496	12
Honeylocust	0.936298	−0.820395	0.987251	14
Black walnut	0.582506	−0.764277	0.988875	17
Eastern redcedar	0.966434	−0.633951	0.977445	6
Sweetgum	0.531074	−0.133884	0.962071	11
Yellow-poplar	0.682570	−0.623197	0.986883	13
Sweetbay	0.648961	−0.611084	0.970136	9
Red mulberry	1.172310	−1.003450	0.993882	6
Water tupelo	0.374860	−0.126429	0.972068	8
Blackgum	0.304936	0.107307	0.950137	14
Eastern hophornbeam	2.248903	−1.158950	0.945845	11
Shortleaf pine	2.658290	−0.897679	0.977186	14
Loblolly pine (both)[††]	2.123323	−0.713648	0.977223	20
Loblolly pine (natural only)	1.517960	−0.608057	0.977180	15
Loblolly pine (planted only)	2.173650	−0.706970	0.968018	17
Sycamore	0.304608	−0.169763	0.971701	6
Black cherry	0.475483	−0.479033	0.972801	8
White oak	1.903511	−0.596252	0.975344	10
Southern red oak	0.797005	−0.538161	0.980522	17
Cherrybark oak	0.417619	−0.460899	0.993015	9
Shingle oak	0.617340	−0.575426	0.974941	10
Bluejack oak	1.160607	−1.198810	0.997653	8
Laurel oak	0.331734	−0.331405	0.986103	12
Overcup oak	0.402881	−0.486610	0.988737	18
Bur oak	0.161181	−0.302171	0.980487	14
Blackjack oak	0.309588	−0.099845	0.962664	9
Swamp chestnut oak	0.122347	−0.097450	0.982234	7
Chinkapin oak	0.268120	−0.389037	0.969627	13
Water oak	0.999875	−0.671970	0.993051	22
Nuttall oak	0.364752	−0.210055	0.982672	8
Pin oak	0.559907	−0.644857	0.988439	12
Willow oak	0.410711	−0.291929	0.980597	9
Northern red oak	0.843941	−0.823358	0.992617	13
Shumard oak	0.772183	−0.712126	0.995513	8
Post oak	2.012296	−1.080410	0.998539	15
Black oak	2.468516	−0.825705	0.977436	13
Black locust	0.107925	−0.096474	0.980874	3
Sassafras	0.549307	−0.719195	0.983459	6
Baldcypress	0.196895	0.138542	0.968257	21
American basswood	0.142845	−0.148052	0.968398	7
Winged elm	1.826372	−1.101460	0.999796	9
American elm	1.284522	−0.900599	0.996408	14
Slippery elm	1.325421	−0.965510	0.994707	12

* $PRI = b_1 D_{MAX}^{b_2} b_3^{D_{MAX}}$

† Final number of points used to fit the optimal PRI curves.

†† Loblolly pine is reported for both (natural + planted), natural stands only, and plantations only, as identified in the EFIDB.

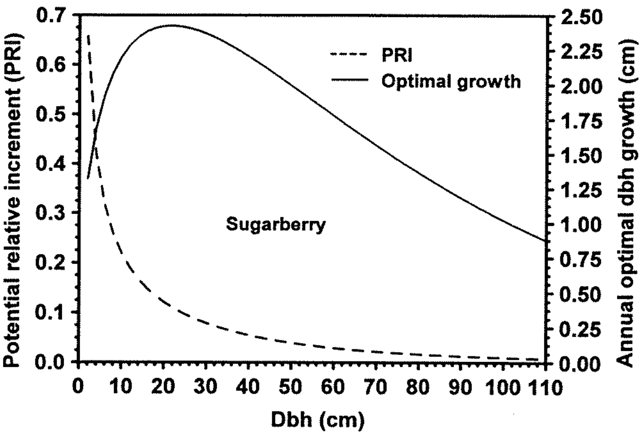

Figure 2. Shape of the PRI and optimal growth curves for sugarberry. For most species, the highest predicted PRI values occur at the smallest diameters, while the greatest annual optimal increment occurs in somewhat larger trees (in this case, somewhere between 20 and 25 cm dbh).

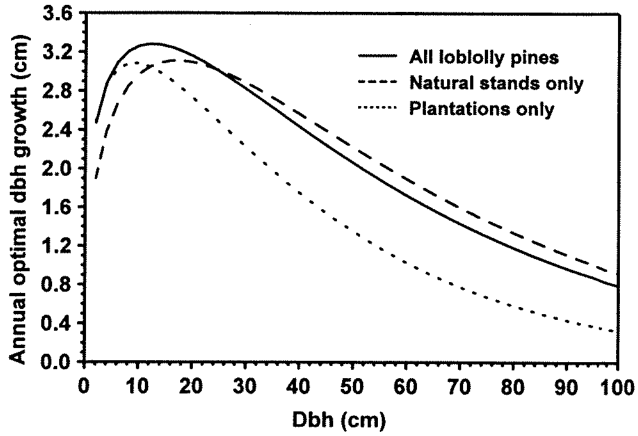

Figure 3. Contrast between PRI models developed for natural (dashed line) stands of loblolly pine, loblolly plantations (dotted line), and natural and plantation loblolly pine pooled for the 5-state Midsouth region. Not surprisingly, the plantation pine had greater potential in the small-end dbh range, but natural stands quickly overtook dominance and retained it throughout the rest of the dbh range. This difference (in part) arose from the scarcity of large-sized loblollies in plantations. The pooled model reached a different peak and decline as a moderated function of the other curves and would appear to do a better job of emulating natural stands.

tracked for an equally long duration, then the variation suggested in Figure 3 may arise from physiological alterations in the timing of tree growth. This, in turn, could have long-term repercussions on market-oriented fiber production or carbon storage.

Conclusions

Potential relative increment has promise as a means to estimate tree optimal diameter growth. While sample size issues are a concern (even when using a large database like EFIDB), supplemental measurements and other inventories can be added to increase the reliability of the information. Even without additional resources, the data provided from EFIDB are often more spatially extensive (especially when multiple states are pooled), cover greater environmental variability, and use more individuals across a wider range of size classes than other efforts. The optimal diameter increment models fit to the sixty species in this study followed growth trends noticed for species in other parts of the country (Bragg 2001). The PRI methodology should be adaptable to other species and geographic regions with the same degree of success if reliable inventory information is available.

Literature Cited

Botkin, D.B., J.F. Janak, and J.R. Wallis. 1972. Some ecological consequences of a computer model of forest growth. J. Ecol. 60:849–872.

Bragg, D.C. 2001. Potential relative increment (PRI): a new method to empirically derive optimal tree diameter growth. Ecol. Model. 137:77–92.

Gates, F.C., and G.E. Nichols. 1930. Relation between age and diameter in trees of the primeval northern hardwood forest. J. For. 28:395–398.

Hansen, M.H., T. Frieswyk, J.F. Glover, and J.F. Kelly. 1992. The Eastwide Forest Inventory Data Base: Users manual. USDA For. Serv. Gen. Tech. Rep. NC-151. 48 p.

Harlow, W.M, E.S. Harrar, and F.M. White. 1979. Textbook of dendrology. Ed. 6. McGraw-Hill, New York. 510 p.

Moore, A.D. 1989. On the maximum growth equation used in forest gap simulation models. Ecol. Model. 45:63–67.

Moore, D.M. 1999. Trees of Arkansas. 5th rev. Arkansas For. Comm., Little Rock, AR. 142 p.

Pacala, S.W., C.D. Canham, J. Saponara, J.A. Silander, R.K. Kobe, and E. Ribbens. 1996. Forest models defined by field measurements: Estimation, error analysis and dynamics. Ecol. Monogr. 66:1–43.

Wykoff, W.R., N.L. Crookston, and A.R. Stage. 1982. User's guide to the stand prognosis model. USDA For. Serv. Gen. Tech. Rep. INT-133. 112 p.

Zeide, B. 1993. Analysis of growth equations. For. Sci. 39:594–616.