NATURAL RESOURCE MODELING
Volume 8, Number 3, Summer 1994

THE ECONOMICS OF TIMBER SUPPLY: AN ANALYTICAL SYNTHESIS OF MODELING APPROACHES

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ABSTRACT. The joint supply of timber and other services from forest environments plays a central role in most forest land debates. This paper defines a general conceptual model of timber supply that provides the context for discussing both individual harvest choice and aggregate supply models. While the structure and breadth of these models has developed considerably over the last twenty years, unresolved issues remain. Supply formulations that account for the quality and vintage distribution of forest capital will be necessary for improving medium- and long-run forecasts. This will be especially important for examining the potential impacts of structural changes in forest production and timber markets. In addition, consistent aggregation of individual owners to total supply will be required to address changing forest land ownership patterns.

KEY WORDS: Timber supply, forest policy, supply models.

Introduction. This paper examines methods for modeling timber supply. The paper begins by presenting an analytical framework, then uses this framework to organize existing literature. The framework is different than that used in earlier surveys of this subject (e.g., Adams and Haynes [1980], Alig, Lewis, and Morris [1984], Binkley [1987]) and presents new insights. As the types of questions examined with timber supply models have changed, it is reasonable to ask whether the existing models can still be used to help answer them. This review of existing

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Revised April, 1994.

models defines research frontiers and identifies opportunities for further research.

Timber supply defines an important contemporary issue for resource analysis. For example, in the Pacific Northwestern United States regulations have recently been implemented under the Endangered Species Act to protect the Northern Spotted Owl (*Strix occidentalis* caurina) from extinction. These regulations have led to dramatic reductions in wood production from National Forests in the Pacific Northwest, a region that has historically produced roughly one third of the total softwood timber in the United States. Contracting wood production has had substantial impact on derivative wood product manufacturing and employment, especially in this region, but also in other wood-producing regions.

Shifts in domestic markets also have implications for the international trade of wood products and the location of wood processing employment, both between Canada and the U.S. and between North America, Europe, and the Pacific Rim. Unraveling complex effects (both domestic and international) has been the subject of considerable research effort; this effort has produced methodological tools that will be invaluable for analyzing the impacts of new policies and programs.

Although trade-offs between wildlife habitat protection and timber production are an important contemporary issue, this is hardly the only issue that could be used to motivate a review of timber supply modeling. Analyzing the nature and extent of forests is required to evaluate topics such as global warming and the role of trees in sequestering carbon, harvesting in tropical forests, the structure of resource concessions, the direct trade of forest products and the derivative trade of environmental hazard. While these specific questions are beyond the scope of this paper, their answers in part must derive from an understanding of the structure of timber supply.

Conceptual framework. Timber supply models summarize the production behavior of forest managers in a market setting. Modeling timber supply requires information on the biological/physical production possibilities of timber growing and inventory adjustment, as well as information on the objectives of forest landowners. When sector-level timber supplies are to be examined, heterogeneous forest land

and owners with heterogeneous objectives must be aggregated.

This section provides an analytical framework for classifying and discussing existing models and defining where gaps between theory and empirical application exist. A timber production function is introduced first to define production possibilities. The section then describes how individual decision models may be aggregated in various ways to define sector-level timber supply.

Timber production function. Underlying any study of production is a production function which translates inputs into outputs. For timber supply, the inputs should include the age of the forest, a, the level of forest management effort, E, and the quality of the land, q. Merchantable timber volume per unit area, V, is given by the yield function:

$$V = v(a, E; q). \tag{1}$$

The marginal physical products of age and management effort are positive and decreasing in the relevant ranges of age and effort. These properties can be summarized (Binkley [1987]) as:

$$\begin{aligned} \frac{\partial v}{\partial a} > 0, \qquad \frac{\partial^2 v}{\partial a^2} < 0, \\ \frac{\partial v}{\partial E} > 0, \qquad \frac{\partial^2 v}{\partial E^2} < 0. \end{aligned} \tag{2}$$

Forest harvesting decisions. Provided that the forest manager's objective function can be specified, then the forest yield function can be used to define if and when a forest stand would be harvested. For example, consider a manager who faces prices p for timber and w for management effort (e.g., effort used to reforest the land after harvest). When the land is maintained indefinitely in forest use, the manager will maximize profit by selecting harvest ages a and levels of effort E to optimize:

$$\begin{aligned} \pi^F &= \max\{a, E\} \sum_{j=0}^{\infty} \{pv(a, E; q)e^{-ra} - wE\}e^{-ra \cdot j} \\ &= \max\{a, E\} \frac{pv(a, E; q)e^{-ra} - wE}{1 - e^{-ra}}, \end{aligned} \tag{3}$$

The optimum profit obtained, π^{*F}, is the present net value for an infinite sequence of identical harvest ages. This formulation provides a valuation for forest land of quality q when there are no trees present at the beginning of the manager's planning horizon. The manager's problem can easily be modified to allow for the benefit provided by existing timber inventories; however, when profit from timber enterprise is the only argument in the objective function (cf. Hartman [1976]) the solution for optimum age (a*) is unaffected by the manager's starting inventory of timber. With this definition of profit, the manager recognizes that there is an opportunity cost to holding old trees rather than faster-growing young trees, and that this opportunity cost influences the harvest timing decision.'

As long as the manager's optimum timber profits are positive and greater than the value of land in alternative uses, then the manager's solution to (3) will identify profit-maximizing harvest dates, harvest volumes, and levels of regeneration effort. The optimum harvest age is obtained where the marginal benefits from delaying the harvest are just equal to the marginal opportunity costs of the delay:

$$pv_a(a^*, E; q) = \frac{r}{1 - e^{-rt}} [pv(a^*, E; q) - wE] \tag{4}$$

Marginal benefits attributable to postponing harvest, $MBD(a, E; q)$, shown on the left-hand side of equation (4), are derived from the value of an additional year's growth. Marginal opportunity costs from postponing harvest, $MOC(a, E; q)$, shown on the right-hand side of equation (4), are the discounted opportunity costs of future harvest revenues.

When forest management decisions are guided by utility rather than profit maximization, the forest management problem may be more complex than that described by equations (3) and (4). For example, non-priced amenity services may be included in the manager's objective function, or forest-level constraints may bind on local decisions. However, even when these questions are addressed in the manager's problem, similar decision rules are obtained (i.e., where marginal benefits and costs of delaying harvest are balanced).[2] For subsequent discussion here, we posit that a decision rule exists which defines the economically optimal harvest age for each forest owner and quality class. If we define the manager's current expectation of future market prices as

p^e, the manager's optimum harvest age under these expectations, $\hat{a}$, is given by

$$(5) \qquad \hat{a}(p,p^e;q) = a \quad : \quad MBD^e(a,E;q) = MOC^e(a,E;q).$$

The manager's optimum harvest age, $\hat{a}$, depends on current market signals (p) and market expectations (p”). This optimum age is not necessarily the same as that given by equation (4), and will likely vary over time as price expectations are revised.[3] Equation (4) is a long-run solution where all elements of p^e are equal to p.

Aggregate supply. Consider the aggregate supply provided by this manager from an ownership of size L when land quality* varies from q^- to q^+, and the age of standing timber varies from a^- to a+. The forest can be described by the density function, $\phi(q,a)$ which gives the relative frequency of land of quality q that is occupied by trees of age class a. Given the manager's harvesting rules, which are derived in equation (5), current timber supply can be defined by integrating across forest area with age greater than $\hat{a}$ for each quality class

$$(6) \quad S_t^{SR} = L\int_{q-}^{q+}\int_{\hat{a}}^{a+} v(a,E;q)\phi(a,q)\,da\,dq = g(p,p^e;\phi(a,q)).$$

The notation S^{SR} indicates short-run (current period) supply, which depends on current and future prices of products, input costs, and the existing age and quality attributes of the forest.[5]

The supply is a short-run quantity because (i) it allows for secular trends in prices (i.e. all elements of p^e are not necessarily equal to *p)* and (ii) harvest quantity is constrained in the short run by $L\phi(a,q)$, the quantity and quality of forest land and the age-distribution of current inventory. Brazee and Mendelsohn [1990] develop a similar supply function that allows for optimal transition to long-run conditions when price is sensitive to aggregate output. The current inventory of timber volume is:

$$(7) \qquad I, = L\int_{q-}^{q+}\int_{a-}^{a+} v(a,E;q)\phi(a,q)\,da\,dq.$$

In the long-run, the manager allocates land among age classes according to equation (4) with $1/a^*(p, q)$ of the land in each age class.[6] The manager allocates land among forest and nonforest uses by identifying the optimum land use margin (see below). The long-run equilibrium, consistent with the forester's definition of a sustained yield forest, is characterized by a uniform age-class distribution between the age of zero and $a^*(p, q)$, the optimal long-run rotation age:

$$(8) \qquad S_t^{LR} = L \int_{q-}^{q+} v(a, E; q)\phi_q(q) \frac{1}{a^*(p, q)} dq = g_L(p, p^{NF}),$$

The function $\phi_q(q)$ is the marginal distribution of land quality that corresponds to the joint distribution $\phi(a, q)$. In the manager's long-run supply equation (equation (8)), the joint distribution of age and land quality, $\phi(a, q)$, has been replaced by the product of the marginal distribution for land quality, $\phi_q(q)$, and the uniform density for a regulated forest on quality class q managed at age $a^*(p, q)$ (i.e., $1/a^*(p, q)$). The manager's optimum age is identified when marginal benefits of delay equal the marginal opportunity costs of delay (i.e., equation (4)). The land quality margin between forest and nonforest uses is given by

$$(9) \qquad q(p, p^{NF}) = q^+ : \pi^{*F}(q^+) = \pi^{*NF}(q^+),$$

where p^{NF} and π^{*NF} are the prices and profits, respectively, associated with the highest-valued nonforest land use. The lowest land quality that can provide nonnegative profits to forestry is given by $\pi^{*F}(q^-) = 0$.

To this point, the manager has supplied only a single representative product. This assumption may hold in areas where, for example, sawtimber is the exclusive product of timber production. It may not hold, however, where both sawtimber and pulpwood or other timber products are produced (e.g., the Southeast). Multiple products may be aggregated into a total product formulation only in the special case of product separability. To account for I multiple products $(i = 1, 2, \ldots, I)$, the manager defines a production function for each product,

$$(10) \qquad V^i = v^i(a, E; q),$$

which possesses the marginal properties given by equations (2). The manager's harvesting decisions yield a complement of forest products that are determined by merchandising rules.[7] Accordingly, the manager's decision is still when to harvest, but the optimum harvest age now depends on the current and expected prices of all of the potential forest products (cf. equation (5)):

$$\text{(11)} \quad \hat{a}(p^1, p^2, \ldots, p^I; p^{e1}, p^{e2}, \ldots, p^{eI}; q) \\ = a \quad : \quad MBD^e(a, E; q) = MOC^e(a, E; q).$$

The manager's profit-maximizing supply of each product now depends on its own price, and the prices of other products:

$$\text{(12)} \qquad S_{it}^{SR} = \int_{q=}^{q+} \int_{\hat{a}}^{a+} v^i(a, E; q)\phi(a, q)\, da\, dq \\ = g^i(p^1, p^2, \ldots, p^I; p^{e1}, p^{e2}, \ldots, p^{eI}; \phi(a, q)).$$

Timber supply models can therefore be organized into two broad categories: (i) those that focus on the individual harvest rules defined by equations (4), (5), or (11) and (ii) those that focus on aggregate timber supply using equations based on (6), (8), or (12). Within each of these categories, this section examines the structure and scope of previous applications, their inherent limitations, questions regarding data, and promising developments in the literature. Existing studies can be further classified as to whether supply is short- or long-run, and whether inventory is treated as heterogeneous (with respect to age of standing timber).

Individual harvest models. From Faustmann in 1849 through several studies in the 1990's the optimal rotation model has proved useful for studying forestry decisions and developing intuitive insights into timber supply (see Newman [1988] and Reed [1986] for recent surveys). Optimal rotation models have been extended to address risk and uncertainty (Routledge [1980]), secular trends in prices and costs (Hardie et al. [1984], Newman et al. [1985]), and nontimber benefits (Hartman [1976], Calish et al. [1976], Swallow et al. [1990]). Optimal rotation models typically employ engineering methods to calculate supplies of resources or services that optimize a specific objective (e.g., discounted net cash flow). However, these normative models are not the exclusive approach to studying individual harvest choices.

Harvesting behavior has also been examined using positive, discrete choice models. These econometric models have proved useful for examining the influence of various site and forest-owner characteristics on harvest decisions in either a utility or a profit-maximizing framework (e.g., Binkley [1981]). Discrete choice models have also been applied to forest investment (regeneration) decisions (e.g., Royer [1987]).

Both normative (engineering) and positive (econometric) harvest decision models may be useful for examining timber harvesting. This section discusses the structure of both types of models. Aggregation of individual owner models is presented in the next section.

Normative harvest models. F'austmann's model [1849] of harvest timing for a perpetual series of rotations on land that is initially bare (equation (3)) has been extensively used to develop more recent insights into timber supply. While the bare land problem is the intellectual antecedent to virtually all normative harvest models, it represents a highly restrictive case. In contrast to this standard model, prices and costs have not been constant through time and are rarely anticipated with certainty. One can, however, generalize the decision model defined in equation (3) for the case where timber prices vary through time; prices are typically allowed to vary during a transition period before they reach steady-state values, p,. This defines a dynamic optimization problem where the decision variables are a sequence of rotations which can be of different lengths (cf. equation (3)):

$$(13)\quad \pi^F = \max\{t\} \sum_{j=1}^{k} \left[p\left(\sum_{l=1}^{j} t_l \right) v(t_j) - wE \right] e^{-r \sum_{m=1}^{j} t_m} - wE$$
$$+ \max\{a_s\} \frac{p_s v(a_s) e^{-r a_s} - wE}{1 - e^{-r a_s}} e^{-r \sum_{j=1}^{k} t_j}.$$

In this formulation, timber price p(t) is a function of calendar time. The manager's problem is divided into two components: one describing a transition period through which prices are variable and another describing an anticipated steady state where price is constant. The second component is equivalent to the bare land model and it possesses a closed form solution similar to equation (4); the solution yields the steady-state rotation, a,, that optimizes the manager's profits when prices are p_s.

In contrast to Faustmann's original problem, solutions to the manager's optimal rotations during the transition period are obtained using dynamic programming rather than calculus. Provided that the manager has nonnegative opportunity costs of capital, the contribution of future rotations to the present value of profits asymptotically approaches zero the more remote the steady state is from the present. Recent studies typically select the steady state conditions so that discounted profits obtained during this period have no effect on the optimal solutions during the transition period. Hardie et al. [1984] develop this specification and simulate optimal rotations using yields from pine in the Southeastern United States. Newman et al. [1985] and McConnell et al. [1983] use this type of model to examine the impact of price trends on financially optimal timber harvesting decisions and therefore define the decision rules comparable to equation (5).

Faustmann models have been used to evaluate the comparative statics of timber supply. Analysts have used bare-land models to examine how changes in various parameters might lead to qualitative changes in forest management strategies. For example, Jackson [1980], Chang [1983], and Williams and Nautiyal [1990] use these types of models to examine the effects of changes in prices, management costs, and taxes on the timing of timber harvesting as well as on long-run (potential) supply response.

Even in the general form defined by equation (13), these models may leave important aspects of the problem unaddressed. One limitation is that the land dedicated to forest production is typically held constant (i.e., q^- and q^+ are held constant). In addition, when applied to aggregate timber supply (see below), these models implicitly assume a regulated forest (age distributed uniformly between zero and the manager's optimum rotation age, a*). In most applications at the scale of individual owners, Faustmann or dynamic Faustmann (i.e., with prices and/or costs that evolve during a transition period) problems address average flows of products from a fixed land base of homogeneous quality. The key decision for the manager is when to harvest (or, in some studies, when to apply intermediate treatments). Finally, while these models are well-suited for the case of a single timber product, failure to meet conditions required for product separability have made them less useful for analyzing multiproduct situations.

Therefore, while normative harvest models have been useful for developing insights into the structure of supply, used alone, they are inherently limited as sector-level supply models. However, when individual harvest models are aggregated in ways which account for interactions between production and price, timber harvest models have provided an important foundation for supply analysis. Aggregate models will be discussed after positive harvest models.

Positive harvest models. Positive models of harvest choice are empirical applications of the decision rules defined by equation (5). Existing applications at the level of the individual forest manager look directly at the implied marginal conditions between harvesting and delaying harvest on a particular stand for a particular owner. Studies constructed at this level of analysis generally assume that the manager maximizes utility, u. Utility is most often a function of wealth (which includes income provided by harvest of merchantable timber products) and other attributes (e.g., amenities from the standing forest, bequest values of standing timber capital) that may enter the manager's utility directly. Utility for forest management choice i for stand j usually takes the form:

$$u_{ij} = f_i(z_j, x_j). \tag{14}$$

Utility depends on a vector of stand attributes, z_j, such as age, management intensity, land quality, as well as owner attributes, x_j, such as education and income.

The manager's optimum harvest decision in this context depends on the marginal utility of delaying harvest and the marginal opportunity cost (in terms of foregone current utility) from delaying harvest. Accordingly, positive harvest choice models are estimated using discrete choice methods (e.g. logit and probit models) fit to cross sectional observations of harvest and delay choices and landowner and forest quality characteristics (e.g. Binkley [1981] and Dennis [1989]). In addition, the simultaneous decision regarding how much to harvest has also been investigated using Tobit formulations (Dennis [1990], Kuuluvainen and Salo [1991]).

By framing these decisions within a household production problem, these models recognize trade-offs between forestry and other household consumption decisions and between timber products (which may be sold to provide income and wealth) and other services (e.g., amenities) from forests which may be consumed (and thereby provide direct utility benefits) by the household. Studies that employ the household production framework have the potential to provide insights into the provision of wood products from a forested landscape with variable forest ownership characteristics and variable forest conditions.

These studies have found that several factors may influence the decision to harvest. In general, income from nonforest sources income holds a significant (negative) influence over the propensity to harvest and age and education are also influential (see Kuuluvainen and Salo for a review of results). Taken as a whole, this set of results suggests that the structure of forest ownership should influence aggregate timber supply. That is, it is unlikely that forests are separable across ownership types, indicating that the actual distribution of ownership characteristics needs to be considered when modeling aggregate timber supply.

While these types of models have not been directly aggregated to model regional supply, the technology is certainly available (see Hardie and Parks [1991] for a study of aggregate regeneration using a discrete-continuous choice approach with an area frame sample). Another plausible extension of these models is to study the joint production of timber and nontimber services. However, the specification of the requisite production relationships, including spatial relationships, remains a substantial challenge (see Max and Lehmann [1988], Swallow and Wear [1993]).

Aggregate supply models. Moving from individual harvest choice models to aggregate supply models shifts the focus from decision rules (equations (4) and (5)) to the supply functions (e.g. equation (6), (8), and (12)). These supply functions have been constructed by mechanically applying estimated decision rules to a variable forest inventory (referred to below as mechanistic or engineering models). More frequently, aggregate supply is estimated as a direct econometric relationship between short-run supply and aggregate measures of price and average forest conditions.

As they have been applied to date, aggregate supply models are generally not pure applications of econometrics or mathematical programming. Rather they are generally hybrid approaches which utilize various combinations of econometrics and optimization on both the demand and supply sides of wood products markets.

Engineering supply models. Individual timber harvest models may be applied across heterogeneous timber inventory and land quality to define regional timber supply. This approach requires some assumptions regarding the overall structure of the market (perfect competition is usually assumed) and information on the existing timber inventory.[8] Another key component is the time-structure of the models: some focus on the eventual long-run supply response of a timber sector; others focus the optimal transition of the sector to a long-run state given anticipated changes in exogenous factors. Both approaches are normative in that they project behavior based, not on historical actions, but on the solution of profit maximization problems.

Long run engineering models have been constructed directly from normative individual harvest models. These engineering models are solved for the optimal harvest date and harvest quantity for each forest quality class. Optimization results can then be translated into annual product flows by assuming that, in the long run, the age distribution of each managed forest class will be uniform between the ages of zero and the optimal rotation age. Accordingly, the total area divided by the rotation age defines the portion of area harvested and the total quantity of product harvested each year in the steady state. The long-run supply function can then be formed by solving this problem over a range of timber prices. The approach can be summarized with the following algorithm:

1. The forest land base is arranged into various categories based on the quality of the forest site and/or its distance from demand centers. $L_j(q)$ is the area of land in quality class q and location class j, $(j = 1, 2, \ldots, J)$.

2. A timber price, p, is specified

3. Equation (3) is solved for the optimal harvest age, a*. The corresponding harvest volume per acre, $v(a^*, E; q)$, and profit, $\pi_j^{*F}(q)$,

are calculated for each quality/location class.

4. If $\pi_j^{*F}(q) > 0$ then this quality of forest is suitable for timber management.

5. By assuming that the age-distribution is approximately uniform between the ages of 0 and a* in the long run, the average annual timber output from each suitable class is defined as

$$S_j(p, q) = v(a^*, E; q)L_j(q)/a^*.$$

6. Total supply for a specific price is then defined as the sum of supplies from quality classes q^- to q^+ and location classes 1 to J:

$$S(p) = \sum_{q=q-}^{q=q+} \sum_{j=1}^{J} Z_j(q)$$

where

$$Z_j(q) = \begin{cases} S_j(p, q) & \text{if } \pi_j^F(q) > 0, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

7. Construct the long-run timber supply for a region by repeating steps 2 through 6 for a range of timber prices.

The advantage of this method is that it defines a production potential consistent with a capital theoretic analysis of timberlands. Lands provide timber only when timber management generates positive profit. The approach is also relatively simple to implement and it is a useful tool for examining the long-run impacts of changes in both market conditions and biophysical production relationships (the $v(a, E; q)$). Because of its relatively simple structure, the approach can accommodate more detail than is usually possible with large scale optimization or econometric models of supply.[9]

The usefulness of the approach is limited by its long-run focus. Supply in the short and medium runs may be highly variable in response to market fluctuations and the most obvious criticism of this approach is that the long run is typically so far off in forestry as to be irrelevant to the supply problems at hand. This is especially problematic when considering cases where the age class distributions of the forest capital is skewed from the long-run uniform condition (e.g. old-growth conditions, where the bulk of the age distribution is concentrated in older age classes). Furthermore, because these models are not based on historical observations of behavior they may be especially susceptible to specification error, and the analyst must be especially careful when specifying the $v(a, E; q)$ functions.

Another class of engineering models directly addresses short-run harvest and inventory adjustments by focusing on intertemporal optimization in timber management (e.g., Berck [1979]). This is accomplished by linking the anticipated effects of production on prices through time recognizing that increasing harvests in one period can increase scarcity in subsequent periods. Accordingly, these models must explicitly account for the age distribution of the existing forest and its influence over production possibilities in the short run, as well as on the evolution of the age distribution through time. Furthermore, the connection between prices and quantities and therefore the market-clearing mechanism must be specified.

Consider a simple linear relationship between the quantity of timber demanded, Q, and its price, p:

$$p = \alpha - \beta Q. \tag{15}$$

An aggregate supply function, $g^{-1}(Q)$, completes the economic sector:

$$P = g^{-1}(Q). \tag{16}$$

For example, the supply function, g-‘(Q), can be derived by inverting the function $g(\cdot)$ in equation (6) or (8) to solve for price. Equilibrium producer and consumer behavior in a competitive market is defined by the simultaneous solution of the demand and supply equations; the solution gives rise to equilibrium prices and quantities.

Obviously if one could specify these aggregate functions, it would be relatively elementary to identify the competitive market solution. However, the situation is much more challenging when heterogeneous production units must be aggregated to define the market supply. The analysis is further complicated by considering trade-offs through time. Optimization has been employed with estimated demand and supply equations to simulate these types of cross sectional and intertemporal trade-offs.

Samuelson [1952] describes a widely-used method that translates the market clearing solution into a geometric problem. Market equilibrium occurs where the area between demand and supply curves is at a maximum. Define the function c(Q) as the total cost of producing at the level Q. The objective is a simple quadratic function of Q :

$$Z = (\alpha \quad \beta Q)Q - c(Q). \tag{17}$$

The present value of a multiperiod objective function such as this is given by:[10]

$$Z^* = \sum_{i=1}^{T} [(\alpha_i + \beta_i Q_i)Q_i - c(Q_i)](1+r)^{-i} + Z^*_{T+1}(1+r)^{-(T+1)}, \tag{18}$$

subject to

$$p_i = \alpha_i - \beta_i Q_i \quad \text{for } i = 1, 2, \ldots, T,$$

where i indexes time and Z_{T+1} is a terminal value assigned to standing inventory at the end of the planning period. This is analogous to the steady state condition in the single stand harvest models (see equation (13)) and defines an approach to an eventual (nonzero) level of forest stocks. If this steady state is programmed far enough into the future, then its impact on short-run behavior will be minimal.

In addition, the parameters of the demand function are indexed by time. This defines a linkage between the timber market and the economy as a whole, allowing demand to shift in response to factors exogenous to timber markets.

We return now to assessing the components of the aggregate quantity (Q) and cost terms (c) in the market model. The Q define the outcomes of the harvesting decision models described earlier. Returning to equation (4), we can modify the problem to account for an existing stock of trees which is clearly important for production in the short run:

$$\begin{aligned} \pi_{\bar{a}}(q) = \max\{t, a_s\}[p(t_1)v(t_1 + \bar{a}, E;\ q) - wE]e^{-rt_1} \\ + \sum_{j=2}^{k} \left[p\left(\sum_{l=1}^{j} t_l\right)v(t_j, E; q) - wE\right] e^{-r\sum_{m=1}^{j} t_m} + S^*(a_s) \end{aligned} \tag{19}$$

where $\bar{a}$ is the existing age of the stand, q is site quality, and S^* is the steady state volume from equation (4). Simulating market behavior requires that quantities produced by solving (19) with given prices, generate these prices when evaluated in model (18).

There are various ways to approach the solution of this problem. One is to simply iterate between the market solution (18) and the individual producer problems (19): define an initial price vector, solve (19) with these prices, solve (18) with the resulting quantities, and then solve (19) again with the resulting prices. This iteration between the market problem and individual subproblems continues until prices and quantities converge. This approach has been applied by Sallnäs and Eriksson [1989] in their analysis of timber markets in Southern Sweden.

The iterative approach provides a straightforward solution technique and an explicit link between harvest decisions and aggregate supply, but may limit the breadth of policy issues which can be addressed. In particular, there are a number of issues that can only be modeled by limiting production in a particular period or constraining decisions across a subset of forest owners or a class of forest land. For these types of questions, an alternative formulation of the linkage between the market and individual behavior is required. This requires building a large scale optimization model which directly incorporates the stand level decisions as decision variables in the market objective function. This kind of model accommodates policies which are simulated with constraint sets. This type of approach is incorporated, for example, in Lyon and Sedjo's [1986] optimal control model of international trade in temperate forest products.

While intertemporal optimization models have the advantage of addressing short- and medium-run market behavior, they still present the same kind of specification problems that face long-run engineering models. That is, they require the model to approximate the essential decision making mechanisms in the optimization problem. The resulting sensitivity to specification error demands careful attention to model calibration (an area which has not received extensive attention).

The strength of the engineering approach is that (given the above qualification) the underlying production models are explicitly defined. Accordingly, these models are especially well-suited for studying the implications of changes in biophysical productivity associated with, for example, global climate change. In general, these models hold advantage for simulating the structure of nonmarginal changes in the timber-producing sector. These would include the effects of productivity shifts related to global climate change.

However, while they hold great advantage for generating hypotheses regarding nonmarginal changes and making detailed forecasts, engineering models have no role in testing hypotheses regarding supply behavior. This is the strength of econometric approaches to modeling timber supply which is taken up next.

Econometric timber supply models. The structure of a statistical model of economic behavior is generally derived from the economic theory of rational behavior and then parameterized using historical observations of actual production and consumption decisions. These models, as a class, offer very powerful tools for testing various economic hypotheses and they have been applied in a wide variety of forms to timber markets. Statistical models of timber supply also define the structure of many forecasting models, including the Timber Assessment Market Model (TAMM, Adams and Haynes [1980]) used by the U.S. government to analyze timber markets and IIASA's Global Trade Model (GTM) international forest trade model (Kallio et al. [1987]).

Nearly all timber market models specify the same form for timber supply. Supply is modeled as a function of price (p) and standing forest inventory (from equation (7)):

$$S = g(p, I, Z) \tag{20}$$

where S is timber supply, and g is a function which is, in effect, an abstract representation of equation (6). This model derives, at least implicitly, from a forest production function, where I represents the accumulated capital inputs to forestry (i.e., the timber inventory, which results from time, effort, land quality, and possibly other capital inputs). Z is a vector of other supply shifters which may or may not be included in the supply model.[11] Timber supply, S, should be positively related to I (i.e. $\partial g/\partial I > 0$) and positively related to price ($\partial g/\partial p > 0$). These supply models are often estimated simultaneously with models of timber demand using a time-series of observations on production, prices, inventory, and other variables.[12]

While these supply formulations have proved empirically powerful, they are not always explicitly tractable to theories of production behavior (Binkley [1987], Wear [1991]). For example, these models cannot distinguish the effects of various structures of forest capital (e.g., different age distributions comprising the same timber volume) which might be represented by the aggregate inventory variable. When the age-distribution and species composition of the forest capital is relatively constant this may not be a problem. However, the misspecification may be serious when these qualities vary substantially. For example, an inventory skewed towards newly planted forests is qualitatively distinct from the same level of inventory represented by old growth forests or a bimodal age distribution. Because of these limitations, which derive from the lack of explicit links to the production technology and the dynamics of forest capital, these models are limited in scope to the study of short-run supply responses.

One reason for the lack of detail in aggregate supply models is the small number of observations available in time series. An alternative approach is to look at production in cross-section (where more detailed data are available) and to impute the temporal dynamics. One approach, taken by Wear and Newman [1991] is to examine output and investment decisions simultaneously using a restricted profit function. This formulation allows sawtimber and pulpwood supply to be assessed in a theoretically consistent framework and allows direct testing of various homogeneity and separability assumptions.

Within this context, pulpwood and sawtimber supply are examined as joint products while land, growing stock, and regeneration input are considered nonseparable inputs to timber production. With land and growing stocks considered quasi-fixed, short-run supply has the following form (cf. equation (12)):

$$S^i = f(p^s, p^p, w, I, L) \tag{21}$$

where S^i is the supply of product i (either sawtimber or pulpwood), p^s is the price of sawtimber, p^p is the price of pulpwood, w is the cost of regeneration, I is the level of growing stocks of timber, and L is the quantity of land dedicated to timber production. This type of model accounts for substitution between timber products by including the different product price variables as well as the impacts of investment costs on production (i.e. the w variable). In addition, the equation distinguishes between land and growing stock components of inventory so that stock:land ratios can impact supply. However, it assumes that growing stocks of various vintages are separable.

Because short-run and long-run responses are associated with the same production function (with and without binding input constraints respectively), long-run response can be imputed from the envelope of short-run behavior (cf. Lau [1976]). Le Chatelier's Rule requires that long-run elasticities be greater than short-run elasticities.[13]

While this model defines the long-run elasticities of supply implied by the envelope of short-run decisions, the actual transition between short and long run remains unaddressed. To date, econometric models have not directly studied this important medium-run (e.g., ten years) phenomenon. This is clearly an important issue, especially given the relatively long life of capital invested in wood product manufacture, to be addressed by forthcoming timber supply work as analysts consider the implications of shifts in forest inventories.

Summary and conclusions. This paper has reviewed various methods for modeling the supply of timber. The conceptual model shows that the question is complicated by substitution both intertemporally and between vintages of capital. It is this essential role of time in production that sets forestry apart from most other applications of resource economics. Accounting for the resulting age structure of forest capital remains as one of the most substantial challenges for timber supply modelers.

Applications are divided into two categories: (i) those that focus on individual timber harvest rules and (ii) those that focus on aggregate timber supply. Research has been extensive in both areas, though perhaps the greatest opportunity for future advances is at the interface between individual harvest rules and their aggregate supply consequences. Aggregate models built through theoretically consistent aggregation of individual choice models hold great promise for improving knowledge and forecasts of timber supply.

Individual harvest models have the benefit of allowing extensive detail in the modeling of forestry decisions. Given the complexity of forest conditions and often the complexity of management objectives, the detail allowed by these models may provide important insights into the structure of timber supply.

Normative harvest choice models, following in the Faustmann tradition, continue to stimulate interest and provide the framework for much of the forest economics literature. Their specifications have become increasingly complex to reflect, among other things, joint production of multiple marketed and nonmarket products. This complexity raises concerns about specification errors. However, the primary purpose of these models is neither to forecast nor to test hypotheses. Rather it is to develop insights and perhaps to define hypotheses for subsequent study.

Empirical forms of these decision models-referred to in this paper as "positive harvest models"-have seen extensive development over the last five years. These models allow the investigator to propose and test for the relationships between various factors and individual timber production decisions. While several interesting results have been obtained, they can generally be summarized as follows: the characteristics of forest owners have a significant bearing on forest management decisions. Accordingly, the structure of ownership matters in determining timber supply and changes in forest ownership can have significant aggregate impacts.

However, accounting for ownership structure is rarely found in aggregate timber supply models. To date, the detail allowed by individual choice models has not been accommodated into large scale applications because of data limitations. This is especially true for econometric supply models that have been the basis for most policy analysis within the U.S. and around the globe. Some recent *efforts* suggest, however, that additional information may be developed from the analysis of cross-sectional datasets.

The lack of detail in the specification of econometric models of timber supply limits the scope of policies that can be analyzed. In particular long-run responses cannot be derived directly from present approaches-though long-term forecasting can be accomplished with ad hoc adjustments to inventory. An important first step in improving timber supply projections is to link long-term inventory adjustment models to economic decision models. The considerable challenge for timber supply modelers is to address the long-run structure of timber supply in theoretically consistent terms.

Extensive production detail and long-run adjustments may be accommodated in mechanistic market models. These approaches have been recently enhanced by improved computer technologies that allow for more efficient solution algorithms and larger data storage capacity. Enormous mathematical programming formulations of timber markets may therefore be accommodated. These models provide expansions of Faustmann-style forest management models to the market as a whole.

These types of models are especially useful for forecasting the effects of structural changes in timber markets (i.e., changes that are nonmarginal or without historical precedent). For example, they may be useful for exploring shifts in timber production technologies from even to uneven aged management systems (Haight [1994]). Other examples include environmental factors such as global climate change or acid rain and policy factors such as large-scale changes in the availability of national forest timber. However, credible models must be based on a foundation of assumptions derived from empirical analysis. So, while these types of models hold considerable promise for improving the precision of forecasts, this promise is inextricably tied to the quality of information provided by econometric models.

ENDNOTES

1. The first correct capital theoretic analysis of forest production is generally attributed to Faustmann [1849]. The problem has been rediscovered or extended by several other researchers. Löfgren [1983] provides a discussion of the early history of this problem and Newman [1988] provides a survey of more than eighty related papers in English-language journals. In addition, see the review of harvesting models by Reed [1986].

2. Although decision rules can become quite complex, especially when spatial juxtaposition of stands is included (e.g., Swallow and Wear [1993]), they generally reduce to this simple structure. When existing stands and nontimber benefits that increase with stand age are considered, corner solutions involving immediate harvest and permanent preservation are also possible (e.g., Hartman [1976]).

3. At an aggregate level expectations may be endogenously formed through an inventory management problem (Lewandrowski, Wohlgenant, and Grennes [1994]). Endogenous prices are considered in more detail below, when the demand for timber is introduced. In addition, price expectations may be sensitive to changes in the regulatory environment surrounding forest management. Anticipation of restrictions and uncertain property rights would necessarily work against long-term investments in timber production.

4. In the long run, both the upper and lower limits of the quality class distribution of land allocated to forest use can change. These changes result from long-run dynamic adjustments of land inputs in response to changes in relative forest and nonforest benefits (see below).

5. The optimum age a is given in equation (5) above. The function $g(\cdot)$ only implicitly depends on prices via $a(p,p^e;\, q)$. Equation (6) presents the function as explicit to facilitate comparison with the abundant applications of this equation in the literature.

6. Equation (4) applies here because p equals p^e in the long run.

7. The use of merchandising rules defines a kind of product separability. It implies that forest managers affect product mix only through harvest timing decisions, but not by substituting between products at a given harvest age. While this may seem restrictive, it appears reasonable, given the very large premium paid for sawtimber over pulpwood-i.e., it is generally unreasonable to assume that sawtimber-size material would be sold as pulpwood.

8. Market structure is an interesting but infrequently addressed topic in forest economics. Early studies of market power include Mead's [1966] study of lumber production in the U.S. Pacific Northwest, and Lowrv and Winfrey's [1974] analysis of the pulp and paper industry. A collection of studies by Johansson and Löfgren [1985] examines various hypotheses regarding market power in the wood products industries in Sweden. Most recently, Murray [1992] tests for market power in both pulpwood and sawtimber markets in the U.S. (Murray also reviews previous work in this area).

9. This type of analysis was initially developed by Vaux [1954]. Hyde [1980] used long-run models to examine various scenarios for the Pacific Northwest region of the United States. Robinson et al. [1978] have also used these models to examine long-run supply in the U.S. South.

10. For a good discussion of the mathematical programming approach to sector modeling in general, see Hazell and Norton [1986].

11. Binkley [1987] points out that the theoretical justification for the inventory variable differs from one application to another. In practice, it may be the only variable available to identify the parameters in the supply and demand equation system.

12. There are several examples of this type of econometric model including Jackson [1983] for the state of Montana, Daniels and Hyde [1986] for the state of North Carolina, Brlnnlund et al. [1985] for Sweden, and Newman [1987] for the U.S. South.

13. We appreciate the efforts of an anonymous reviewer, who pointed this out. Empirical validation of this in a timber supply context is provided in Newman and Wear [1993]. In addition, their results lend support to findings from the qualitative choice models of timber harvesting. That is, they find that ownership characteristics matter to aggregate timber supply and that nonindustrial and industrial owners behave differently in their pursuit of forest management.

REFERENCES

Adams, D.M. and R.W. Haynes [1980], *The 1980 softwood timber* assessment market model: *Structure,* projections and policy simulations, Forest Science Monograph 22 64 pp.

Alig, R.J., B.J. Lewis, and P.A. Morris [1984], *Aggregate timber supply* analyses, United States Department of Agriculture, Forest Service, Rocky Mountain Forest and Range Experiment Station, General Technical Report RM-106, Fort Collins.

Berck, P. [1979], *The* economics ***of*** timber: *A* renewable resource in *the long run,* Bell Journal of Economics 10 (2), 447-462.

Binkley, *C.S.* [1981], *Timber supply* ***from*** nonindustrial ***forests,*** Bulletin No. 92, Yale University, School of Forestry and Environmental Studies, New Haven, 97 pp.

Binkley, C.S. [1987], *Economic models* ***of*** timber *supply,* in *The* global ***forest sector:*** *An* analytical **perspective** (Kallio, Dykstra and Binkley, eds.), John Wiley and Sons, New York, 703 pp.

Brännlund, R., P.-O. Johansson and K.G. Löfgren [1985], An *econometric* analysis ***of*** **aggregate** sawtimber *and pulpwood supply* in *Sweden,* Forest Science 31, 595-606.

Brazee, R. and R. Mendelsohn [1990], *A dynamic* model of timber *markets,* Forest Science 36 (2), 255-264.

Calish, S., R.D. Fight and D.E. Teeguarden [1978], *How* do nontimber *values affect* Douglas-fir rotations?, Journal of Forestry 76, 217-221.

Chang, S.J. [1983], *Rotation* age, management intensity, *and the economic factors* ***of*** *production,* Forest Science 29 (2), 267-277.

Daniels, B. and W.F. Hyde [1986], *Estimation* ***of*** *supply and demand elasticities* **for** North Carolina *timber,* Forest Ecology and Management 14, 59-67.

Dennis, D.F. [1989], *An economic* analysis ***of*** harvest *behavior:* Integrating ***forest*** *and* ownership characteristics, Forest Science 35 (4), 1088-1104.

Dennis, D.G. [1990], *A probit* analysis ***of*** *the* harvest *decision using pooled time* series and cross-sectional data, Journal of Environmental Economics and Management 18, 176-187.

Faustmann, M. [1849], *Calculation* ***of*** *the* value *which* **forest** *land* and immature *stands* possess for forestry, in *Martin* Faustmann and *the* evolution ***of*** *discounted* cash *flow* (M. Gane, ed.) [1968], reprinted in English translation, Oxford Institute, Oxford, 27-55.

Haight, *R.G.* [1994], *Technology* change and *the economics* ***of*** *silvicultural* investment, United States Department of Agriculture, Forest Service, Rocky Mountain Forest and Range Experiment Station, General Technical Report RM-232, Fort Collins.

Hardie, I.W., J.N. Daberkow and K.E. McConnell [1984], *A timber* harvesting model *with* variable rotation lengths, Forest Science 30 (2), 511-523.

Hardie, I.W. and P.J. Parks [1991], Acreage response *to* cost-sharing in *the South,* 1971-1981: *Integrating* individual *choice and* regional acreage *response,* Forest Science 37 (1), 175-190.

Hartman, R. [1976], *The harvest* decision when *the standing forest has value,* Economic Inquiry 14, 52-58.

Hazell, P.B.R. and R.D. Norton [1986], Mathematical programming for economic analysis in agriculture Macmillan, New York, 502 pp.

Hyde, W.F. [1980], *Timber supply, land* allocation, and economic *efficiency*, The Johns Hopkins University Press, Baltimore, 225 pp.

Jackson, D.H. [1980], *The* microeconomics *of the timber* industry, Westview Press, Boulder, 136 pp.

Jackson, D.H. [1983], Sub-regional *timber* demand analysis: Remarks and an approach *for prediction,* Forest Ecology and Management, 5 109-118.

Johansson, P.-O. and K.G. Löfgren [1985], *The* economics *offorestry* and *natural* resources, Basil Blackwell Inc., New York, 292 pp.

Kallio, M., D.P. Dykstra and C.S. Binkley, eds. [1987], *The global* forest *sector: An analytical* perspective, John Wiley and Sons, New York.

Kuuluvainen, J. and J. Salo [1991], *Timber supply* and *life cycle harvest of* nonindustrial private *forest owners: An empirical* analysis of *the* Finnish case, Forest Science 37 (4), 1011-1029.

Lau, L.J. [1976], *A* characterization *of the normalized* restricted *profit function,* Journal of Economic Theory 12, 131-163.

Lewandrowski, J.K., M.K. Wohlgenant, and T.J. Grennes [1994], *Finished products* inventories and *price expectations* in *the softwood* lumber *industry,* American Journal of Agricultural Economics 76 (1), 83-93.

Löfgren, K.G. [1983], *The Faustmann-Ohlin* theorem: *A* historical note, History of Political Economy 15 (2), 261-264.

Löfgren, K.G. [1985], *Effects on the socially* optimal rotation *period in forestry of biotechnological* improvements *of the growth* function, Forest Ecology and Management 10 (2), 233-249.

Löfgren, K.G. [1987], *Effects of* permanent and non-permanent *forest* means on *timber* supply, Silva Fennica 20, 308-311.

Lowry, S.T. and J.C. Winfrey [1974], *The kinked cost* curve and *the* dual resource *base* under *oligopsony* in *the pulp and* paper *industry,* Land Economics 50, 185-192.

Lyon, K.S. and R.A. Sedjo [1986], *Binary* search *SPOC: An* optimal *control* version *of ECHO,* Forest Science 32 (3), 576-584.

Max, W. and D.E. Lehmann [1988], *A* behavioral *model* of timber *supply,* Journal of Environmental Economics and Management 15, 71-86.

McConnell, D.E., J.N. Daberkow and I.W. Hardie [1983], Planning *timber production with evolving prices and costs,* Land Economics 59 (3), 292-299.

Mead, W.J. [1966], Competition and *oligopsony in the Douglas-fir* lumber industry, The University of California Press, Berkeley.

Murray, B.C. [1992], Testing *for* imperfect *competition in spatially* differentiated markets: *The* case *of U.S.* markets *for pulpwood* and *sawlogs*, Ph.D. Dissertation, School of the Environment, Duke University, Durham, NC.

Newman, D.H. [1987], An *econometric* analysis *of the* southern softwood *stumpage market: 1950-1980,* Forest Science 33 932-945.

Newman, D.H. [1988], *The optimal forest* rotation: *A* discussion and annotated bibliography, General Technical Report No. SE-48, U.S.D.A. Southeastern Forest Experiment Station, Asheville.

Newman, D.H., C.B. Gilbert and W.F. Hyde [1985], The optimal *forest rotation with* evolving prices, Land Economics **61** (4), 347-353.

Newman, D.H. and D.N. Wear [1993], *Production economics* ***of*** private ***forestry:*** *A* comparison ***of*** industrial and nonindustrial ***forest*** *owners*, American Journal of Agricultural Economics 75, 674-684.

Reed, W.J. [1986], Optimal harvesting models in ***forest*** management: *A survey,* Natural Resource Modeling 1 (1), 55-79.

Robinson, V.L., A.A. Montgomery and J.S. Strange [1978], GASPLY: *A* computer model for Georgia's future forest, Georgia Forest Research Report 26A, Georgia Forest Research Council, Macon, 21 pp.

Routledge, R.D. [1980], *The effect* ***of*** *potential* catastrophic *mortality* and other unpredictable *events on forest rotation policy,* Forest Science 28 (3), 389-399.

Royer, J.P. [1987], Determination ***of*** *reforestation* behavior among southern landowners, Forest Science 33, 654-667.

Sallnäs, 0. and L.O. Eriksson [1989], Management variation and price *expectations* in an intertemporal ***forest*** sector *model,* Natural Resource Modeling 3, 385-398.

Samuelson, P.A. [1952], *Spatial price* equilibrium and linear programming, American Economic Review 42, 283–303.

Swallow, S.K., P.J. Parks and D.N. Wear [1990], *Policy* relevant *nonconvexities* in *the* production ***of*** *multiple* forest benefits, Journal of Environmental Economics and Management 19, 264-280.

Swallow, S.K. and D.N. Wear [1993], Spatial interactions in *multiple* use forestry *and substitution* and *wealth effects* for *the single* stand, Journal of Environmental Economics and Management 25, 103-120.

Vaux, H.J. [1954], Economics ***of*** young growth sugar pine resources, Bulletin No. 78, Division of Agricultural Sciences, University of California, Berkeley.

Wear, D.N. [1991], *Evaluating* technology and *policy shifts in the forest* sector: Technical issues ***for*** analysis, in *Proceedings of the 1991 Southern Forest* Economics *Workshop* (S.J. Chang, ed.), Washington.

Wear, D.N. and W.F. Hyde [1992], *Distributive issues in* forest *policy,* Journal of Business Administration 21, 297-314.

Wear, D.N. and D.H. Newman [1991], *The* production structure *of forestry: Short*-run and long-run *results,* Forest Science 37 (2), 540–551.

Williams, J.S. and J.C. Nautiyal [1990], *The long-run* timber *supply function,* Forest, Science 36 (1), 77-86.