Fiber Length - Fiber Strength Interrelationship for Slash Pine and Its Effect on Pulp-Sheet Properties

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ABSTRACT. Based on a model developed for hardwood fiber strength-pulp property relationships, multiple-regression equations involving fiber strength, fiber length, and sheet density were determined to predict the properties of kraft pulps of slash pine (*Pinus elliottii*). Regressions for breaking length and burst factor accounted for 88 and 90 percent, respectively, of total variation in these properties resulting from a large number of beater runs. At a given level of sheet density, fiber strength and fiber length had a positive influence on both breaking length and burst, and this effect was more pronounced the higher the level of sheet density. An equation that accounted for 80 percent of total variation in tear factor was also developed. At higher levels of sheet density, shorter fibers gave superior tear resistance. The benefits in tear factor generally ascribed to longer (stronger) fibers may be reconciled with these findings when pulp properties are compared at the densities associated with a common level of freeness rather than at specified levels of sheet density as was done here.

THE ROLE OF FIBER LENGTH and fiber strength in influencing the properties of pulp handsheets has been discussed by many authors. Numerous inconsistencies in experimental findings have been reported. One of the purposes of this study is to explain some of them. In a comprehensive review published in 1965, Dinwoodie (3) summarized the principal fiber factors controlling the strength properties of paper as: fiber density, fiber length, and fiber strength. Fiber density relates to the degree of interfiber bonding in a sheet; fiber length, among other things, is associated with the number of bonding sites available on an individual fiber; and fiber strength assumes its controlling role when interfiber bonding becomes so intense as to transfer stress sufficient to cause fibers to rupture.

Basis for the Model

The senior author in collaboration with others (4, 5, 7) has demonstrated the role of fiber length and fiber strength in populatio of kraft pulps produced from a diverse gro of hardwood species. Handsheets for physi testing were made from pulps before beating well as after Valley-beater processing. In th studies sheet density (a correlate of fiber de sity) was employed in the analysis as a measu of the degree of interfiber bonding. Fi length was the original whole fiber leng based on mildly macerated wood samples, a fiber strength (at least insofar as the pres discussion is concerned) was based on zero-sp

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breaking length of handsheets. Equations summarizing relationships influencing breaking length, burst factor, and tear factor of hardwood handsheets are given in Table 1.

These equations depict for breaking length and burst factor a negligible influence of fiber strength at low sheet densities on the order of 0.30 g/cm^3 and an increasing positive effect of fiber strength at increasing sheet densities at least up to 0.80 g/cm^3. These relationships accounted for 80 and 71 percent of the total variation in breaking length and burst factor, respectively. An attempt to introduce fiber length as a variable in multiple regression contributed less than 1 percent additional accountability to the relationships and was consequently abandoned.

The equation for tear factor shows a similar negligible effect of fiber length and fiber strength at the lowest sheet densities. As sheet density increases, however, increasing fiber length exerts a strong positive effect until maximum tear factor values are attained at sheet densities at or beyond 0.50 g/cm^3. Beyond this point of culmination, the influence of fiber length diminishes and, at the highest levels of sheet density examined, an increase in fiber length actually exerts an adverse effect upon tear resistance. Fiber strength has a positive influence on tear factor at all but the very lowest levels of sheet density, and the effect of this variable continues to increase throughout the entire range of sheet densities studied. In this equation, which accounted for 72 percent of the total variation in tear factor, fiber length alone accounted for 22 percent of the variance. It should be mentioned here that fiber length and fiber strength in these pulps produced from 17 different species were unrelated to each other.

Table 1. — MULTIPLE-REGRESSION EQUATIONS FOR HARDWOOD KRAFT PULPS SHOWING THE INFLUENCE OF FIBER LENGTH AND FIBER STRENGTH ON SHEET PROPERTIES.

Breaking length Y, 100 m

$$Y = 10.00 + 32.34\ SD + 7.50\ (ZBL \times SD)$$

Burst factor Y, $(g/cm^2)/(g/m^2)$

$$Y = 29.16\ SD + 5.19\ (ZBL \times SD)$$

Tear factor Y, $g/(g/m^2)$

$$Y = 4.29\ L - 446.17\ SD + 20.55\ (ZBL \times SD) + 501.80\ (L \times SD) + 912.78\ SD^2 - 1330.50\ (L \times SD^2)$$

in which:

L = fiber length, mm (based on wood samples)

SD = sheet density — 0.30, g/cm^3

ZBL = sheet zero-span breaking length, km

The Present Study

This study of slash pine kraft pulps is an analysis of data available from previous research on relationships between fiber morphology and pulp properties (6). Briefly summarized, these data were based on 66 pulps prepared from top and butt sections of 11 slash pine trees selected to provide a wide range of wood and fiber characteristics within normal limits of variation for slash pine. (The original study involved pulps from 12 trees but one of these was not available for this analysis.) All pulps were cooked at a kraft schedule selected to produce pulps having a uniform permanganate No. 25.

A published report on the original study includes a detailed description of the selection of trees, collection and treatment of bolts, wood and fiber characteristics, pulping, beater processing, and handsheet testing (6). For the purpose of this discussion it may be sufficient to say that our present analysis is based on tests of handsheets made after 0, 30, 50, 70, and 90 minutes of processing in a Valley beater. Of the 330 pulp samples potentially available from this procedure, 312 were actually tested and used in this analysis.

A basic premise of the present study was to employ the parameters shown for mixed hardwoods in Table 1 — zero-span breaking length, fiber length, sheet density, and combinations of these — in multiple regression for the purpose of predicting sheet properties of pulps of slash pine (*Pinus elliottii*). One difference in the significance of fiber length as a predictor of sheet properties was recognized from the start. The equations of Table 1 were based on variation *between* species. The equations to be developed for slash pine involve variation *within* a single species. The variables of fiber length and fiber strength in Table 1 were unrelated to one another. The earlier study of slash pine, however, had established a positive correlation between latewood fiber length and zero-span breaking length within this species (6). Obviously a different interpretation of fiber length effects will be required.

Results and Discussion

A linear regression of zero-span breaking length on latewood fiber length yielded the relationship:

$$ZBL = 11670 + 1456\ L$$

where

ZBL = zero-span breaking length (m)
L = latewood fiber length (mm) based on wood samples

The correlation coefficient was 0.566, indicating a highly significant relationship. This relationship is assumed to result from the more nearly axial fibrillar orientation in the longer latewood fibers. The original study had established the significance of such a relationship between latewood fiber length and cosine of fibril angle (6).

Breaking Length

Multiple-regression equations for breaking length are shown in Table 2. The effect of sheet density alone includes a quadratic term and, as shown in the buildup of the coefficient of determination R^2, accounts for 78.4 percent of the total variance in breaking length. The addition of the $ZBL \times SD$ cross-product increases the accountability to 84.8 percent. Inclusion of the $L \times SD$ cross-product instead of $ZBL \times SD$ has virtually the same effect as might be expected from their interrelationship, and incorporating both cross-products yields a final equation that accounts for 88.0 percent of total variance in this property. All equations are highly significant.

Equation [e] of Table 2 is portrayed graphically in Figure 1. We have chosen to show the independent variables as sheet density and latewood fiber length. However, in this instance an increase in fiber length implies an increase in zero-span breaking length as well. Consequently, instead of holding ZBL constant while varying SD and L in plotting these curves, the probable value of ZBL from the equation

$$ZBL = 11670 + 1456\ L$$

was used in the calculation of breaking length.

The family of curves in Figure 1 shows an increase in breaking length with increasing sheet density and increasing fiber length. The effect

Table 2. — MULTIPLE-REGRESSION EQUATIONS FOR BREAKING LENGTH OF SLASH PINE KRAFT PULPS.*

Eq.	Items**		Coeff.	R^2	Level of signif. (%)	Standard error o estimat
		a	−3,901.8			
a	SD	b_1	22,744.	0.655	1	1,352.0
		a	−26,468.			
b	SD	b_1	110,844.			
	$(SD)^2$	b_2	−83,808.	0.784	1	1,070.8
		a	−22,383.			
	SD	b_1	65,024.			
c	$(SD)^2$	b_2	−60,933.			
	ZBL×SD	b_3	1.4209	0.848		900.5
		a	−22,939.			
	SD	b_1	77,603.			
d	$(SD)^2$	b_2	−64,298.			
	L×SD	b_3	3,636.3	0.858		870.6
		a	−21,175.			
	SD	b_1	56,138.			
e	$(SD)^2$	b_2	−54,351.			
	L×SD	b_3	2,677.8			
	ZBL×SD	b_4	0.9373	0.880	1	719.9

*Equation type $Y = a + b_1X_1 + b_2X_2 + \ldots$ whe Y = breaking length (meters) and $X_1, X_2, \ldots$ = i dependent variables.

**SD = sheet density (g/cm³); L = latewood fiber leng (mm); ZBL = zero-span breaking length (meters).

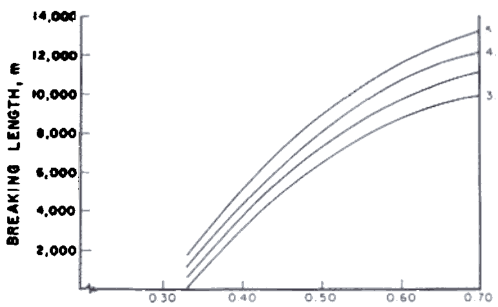

Figure 1. — Effect of latewood fiber length and she density on breaking length of slash pine kraft pulps.

of fiber length increases with increasing sheet density. Because of its interrelationship with *ZBL*, and the lack of a comparable effect in the mixed hardwood pulps (Table 1), the apparent effect of increased fiber length shown in Figure 1 is more likely the result of increased fiber strength than increased fiber length per se. This interpretation recognizes a consistency in the behavior of diverse hardwood and slash pine pulps and helps to explain some of the inconsistencies that have been reported (3).

Burst Factor

Equations for burst factor shown in Table 3 are comparable to those for breaking length. Sheet density alone accounts for 83.2 percent of the total variance in burst factor. The added effect of cross-products for $ZBL \times SD$ and $L \times SD$ increases the total accountability of Equation [e] of Table 3 to 90.4 percent. The family of curves shown in Figure 2 was plotted from Equation [e] of Table 3 as described for breaking length. The discussion accompanying Figure 1 is equally relevant to Figure 2.

Table 3. — MULTIPLE-REGRESSION EQUATIONS FOR BURST FACTOR OF SLASH PINE KRAFT PULPS.*

Eq.	Items**		Coeff.	R^2	Level of signif.	Standard error of estimate
					(%)	
		a	−60.855	0.748	1	11.01
a	SD	b_1	231.79			
		a	−234.04			
b	SD	b_1	907.94			
	$(SD)^2$	b_2	−643.21	0.832		9.01
		a	−206.83			
	SD	b_1	651.58			
	$(SD)^2$	b_2	−492.75			
	L×SD	b_3	28.043	0.880	1	7.62
		a	−197.79			
	SD	b_1	501.25			
d	$(SD)^2$	b_2	−440.17			
	ZBL×SD	b_3	0.01261	0.887	1	7.40
		a	−189.41			
	SD	b_1	439.61			
	$(SD)^2$	b_2	−394.51			
	ZBL×SD	b_3	0.00926			
	L×SD	b_4	18.577	0.904		6.82

*Equation type $Y = a + b_1X_1 + b_2X_2 + \ldots$ where Y = burst factor and $X_1, X_2 \ldots$ = independent variables.

**SD = sheet density (g/cm³); L = latewood fiber length (mm); ZBL = zero-span breaking length (meters).

Tear Factor

Multiple-regression equations for tear factor are shown in Table 4. Sheet density alone, including quadratic and cubic terms, accounts for 75.0 percent of the total variance in tear factor. The inclusion of $ZBL \times (SD)^2$ increases accountability to 78.4 percent, and the final inclusion of $L \times (SD)^2$ and $L^2 \times SD$ brings total accountability up to 80.0 percent. Figure 3 is a graphic representation of Equation [f] of Table 4, plotted in the manner described previously.

The family of curves indicates virtually no effect of fiber length at the lowest sheet densities, a rising trend with increasing sheet density for all fiber lengths culminating at a sheet density of about 0.40 g/cm³, and beyond this a decreasing trend in tear factor with increasing sheet density. Throughout the entire range of declining tear factor the rate of decrease is greater for the longer fiber pulps. This is consistent with the effect of fiber length shown for mixed hardwoods as given by the equation for tear factor in Table 1 and is discussed in an earlier note (7). However, in the present instance the adverse effect of increased fiber length is evident at much lower sheet densities than in the case of the hardwood study. We

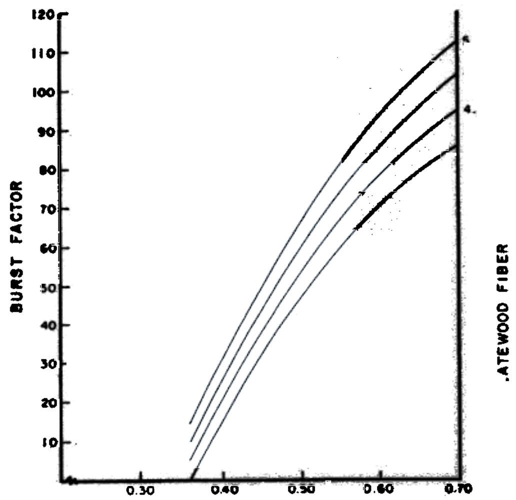

Figure 2. — Effect of latewood fiber length and sheet density on burst factor of slash pine kraft pulps.

Table 4. — MULTIPLE-REGRESSION EQUATIONS FOR TEAR FACTOR OF SLASH PINE KRAFT PULPS.*

Eq.	Items**		Coeff.	R^2	Level of signif.	Standard error of estimate
		a	489.37			
	SD	b_1	−536.33	0.622		34.24
		a	320.79			
b	SD	b_1	121.81			
	$(SD)^2$	b_2	−626.08	0.635	1	33.74
		a	−1,858.4			
	SD	b_1	13,308.			
	$(SD)^2$	b_2	−26,422.			
	$(SD)^3$	b_3	16,383.	0.750		27.94
		a	−1,867.			
	SD	b_1	13,226.			
d	$(SD)^2$	b_2	−25,031.			
	$(SD)^3$	b_3	15,694.			
	$ZBL \times (SD)^2$	b_4	−0.04587	0.784		25.99
		a	−1,843.			
	SD	b_1	13,044.			
	$(SD)^2$	b_2	−24,491.			
	$(SD)^3$	b_3	15,326.			
	$ZBL \times (SD)^2$	b_4	−0.03419			
	$L \times (SD)^2$	b_5	−67.260	0.796	1	25.32
		a	−1,806.6			
	SD	b_1	12,484.			
f	$(SD)^2$	b_2	−22,854.			
	$(SD)^3$	b_3	14,728.			
	$ZBL \times (SD)^2$	b_4	−0.03556			
	$L \times (SD)^2$	b_5	−258.35			
	$(L)^2 \times SD$	b_6	12.843	0.800	5	25.15

*Equation type $Y = a + b_1X_1 + b_2X_2 + \ldots$ where Y = tear factor and $X_1, X_2 \ldots$ = independent variables.

**SD = sheet density (g/cm^3); L = latewood fiber length (mm); ZBL = zero-span breaking length (meters).

are undoubtedly dealing in this instance with a more complex effect of fiber length due to the fiber length-fiber strength interrelationship present here, but which was not involved in the hardwood example.

It must be recalled that fiber length, as used in this study, is original latewood whole fiber length and not the length in the pulp as affected by Valley-beater processing (2). With this qualification it is believed that the shape of the family of curves in Figure 3 is explained by the increased work to tear failure with increasing "frictional drag" in pulling more or less intact fibers out of the web of low density sheets. As interfiber bonding increases with increasing sheet density, a critical level of bonding is reached beyond which point there is a preponderance of fiber rupture—a type o failure that involves less work than does fibe pullout from a relatively compact sheet (7) Consequently, tear factor decreases at highe sheet densities and the dropoff is more pro nounced in the case of longer fibers, inasmucl as they are involved in more fiber-to-fiber bond: Even though fibers are known to be shorten in the Valley beater, available evidence suggest that the *relative* length of original long and sho fibers is retained. It appears that the critica level of bonding in the case of slash pine kra pulps occurs at a sheet density close to 0.4 g/cm^3.

An apparent contradiction between thes results and previous conclusions drawn fro the same basic data has to do with the influenc

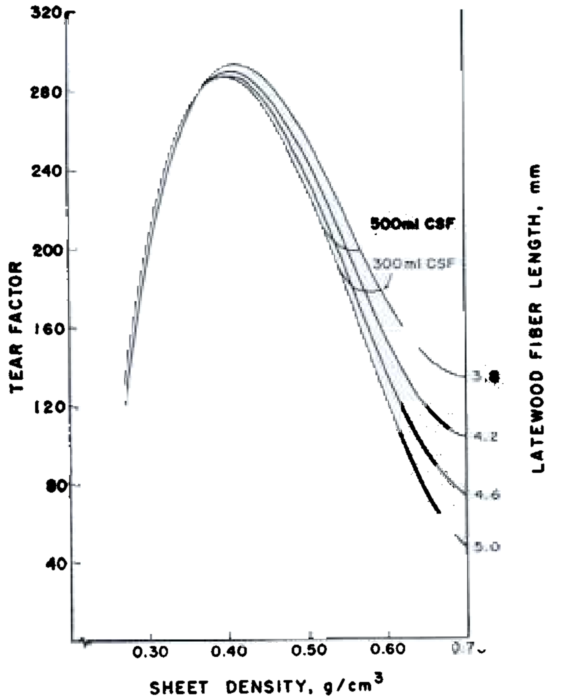

Figure 3. — Effect of latewood fiber length and sheet density on tear factor of slash pine kraft pulps.

of fiber length on tear factor. In the earlier study (6) a positive influence on tear factor at specified levels of freeness was attributed to fiber length. In the present study the effect at a specified level of sheet density is either negative or insignificant. Reconciliation of these opposing statements requires an understanding of the role played by fiber length. Two major studies that have been conducted in the area of fiber morphology-pulp property relationships for southern pine (1, 6) have shown that decreasing sheet density, at specified levels of freeness, is associated with increasing latewood fiber length. From the relationship between sheet density and latewood fiber length previously determined from these same pulps (6), sheet densities attained at 500 and 300 CSF by pulps having different fiber lengths were calculated and superimposed on Figure 3. Compared at a specified level of freeness, as has long been common practice, the conventional relationship between tear factor and fiber length—a higher tear factor associated with longer fibers—is readily seen and the apparent inconsistency is removed.

Summary

"Our evaluation of fibers from various sources having different characteristics must not be limited to a single condition of test such as one level of sheet density or one level of freeness. . .if we are concerned with revealing the full potential of the fiber or determining significant fiber parameters" (7).

The usefulness of the model that we have employed here for this purpose may be better appreciated if it is understood that sheet density is quite readily predictable from a number of morphological parameters (1, 6).

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