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# Projecting Timber Inventory at the Product Level

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**ABSTRACT.** Current timber inventory projections generally lack information on inventory by product classes. Most models available for inventory projection and linked to supply analyses are limited to projecting aggregate softwood and hardwood. The research presented describes a methodology for distributing the volume on each FIA (USDA Forest Service Forest Inventory and Analysis) survey plot to product classes given a type characterization, volume, and average dbh (diameter at breast height, 1.37 m above average ground level) for the plot. A multinomial logit model was developed to estimate sets of product proportion functions to distribute plot volumes by product class for each forest type and size class. A discussion of the performance of the model using Alabama and Mississippi FIA plot level data is provided. *For. Sci.* 45(1):226–231.

**Additional Key Words:** Inventory projection, multinomial logit model, timber product classes, timber supply, FIA data.

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**T**IMBER INVENTORY PROJECTION AND supply analysis are important to industry for strategic activities such as planning for timberlands acquisition, facilities location and expansion, and procurement. In general, models are constructed using the FIA inventory data (e.g., measures of dbh and estimates of volume) as a starting point and then inventories are projected for some period into the future by combining simulations of growth and removals over the period. At the end of the projection period, however, difficulties arise related to determining the product distribution of the future forest since all the trees in a forest do not move uniformly from one product class to another and because the proportions of hardwood and softwood in a stand may change.

This has posed problems for modelers. As a result, most efforts to project timber inventories have been limited to projecting aggregate softwood (pine) and hardwood due to the lack of a method for separating products such as pulpwood and sawtimber from the aggregate data. A procedure to project sawtimber and pulpwood inventories separately is needed to more clearly understand the dynamics of forest inventory and make informed strategic planning decisions.

The overall objective of this research was to develop a methodology to distribute the volume on potential harvest plots to product classes. This will allow modelers to continue to use more aggregate (simpler) growth models to move the

inventories through time and still provide accurate descriptions of the product distribution on those plots in the future. Recent FIA data for Alabama and Mississippi were used to develop the functions to estimate product proportions for each forest type.

## The Data

The data used for this project are Mississippi and Alabama FIA surveys including: MS1994, MS1987, AL1990, and AL1982. These four FIA data sets were pooled for analysis. Only the data representing timberland acres were included, with reserved forest areas and nonstocked timberland acres excluded. We considered all live trees (including all tree classes) rather than focusing on growing stock as many previous modelers have done. The final data set consisted of 13,740 plots in four basic forest types including 1,713 pine plantation plots, 2,739 natural pine plots, 2,000 oak-pine plots and 6,530 hardwood plots. For each forest type and each plot, the percentages of softwood pulpwood, hardwood sawtimber, hardwood pulpwood, and hardwood timber were calculated as new variables and associated with the other plot level data (average plot dbh and volume).

Table 1 shows the live tree volume distribution on timberland by forest type and product class for the most re

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**Table 1. Live tree volume** distribution on timberland by forest type and product size-class in Alabama (FIA90) and Mississippi (FIA94).

	Total	Stwd	Hdwd	SPW	SST	HPW	HST
	.....(%).....						
<b>FIA90 AL</b>							
PP	10.4	9.3	1.1	4.5	4.8	0.6	0.5
NP	23.5	19.6	3.9	4.1	14.9	2.1	1.8
OP	20.1	11.0	9.1	2.3	8.7	4.4	4.7
HD	46.0	5.7	40.3	1.0	4.7	15.6	24.1
Total	100.0	45.6	54.4	12.5	33.1	22.7	31.7
<b>FIA94 MS</b>							
PP	9.5	8.7	0.8	4.3	4.4	0.4	0.4
NP	20.8	17.7	3.1	3.0	14.7	1.6	1.5
OP	17.5	9.6	7.9	1.6	8.0	3.4	4.5
HD	42.3	5.4	36.9	0.8	4.6	14.5	32.4
Total	100.0	41.4	58.6	9.7	31.7	19.9	38.7

NOTES: [Product volume/total volume (all types)], for example, in FIA90\_AL, softwood pulpwood (SPW) in the pine plantation type was estimated to be 104.1 MMCM or 4.5% of total forest inventory, which was estimated to be 2263.6 MMCM (104.1/2263.6 \* 100 = 4.5%).  
 Live tree volume includes all tree classes. Sawtimber volume includes all grades.  
 PP-pine plantation; NP-natural pine; OP-oak-pine; HD-hardwood; SPW-softwood pulpwood; SST-softwood sawtimber; HPW-hardwood pulpwood; HST-hardwood sawtimber.

surveys of Alabama and Mississippi. The tables show a very similar composition among products for both states as a whole. However, the percentage of each product changes as dbh and volume per acre change on individual plots.

The product definitions used in this study are specifically described by Hansen et al. (1992). In essence, the product definitions reflect tree size class. General definitions are: (1) Softwood: pulpwood—dbh greater than or equal to 12.7 cm (5 in.) and less than 22.86 cm (9 in.); sawtimber—dbh greater than or equal to 22.86 cm; (2) Hardwood: pulpwood—dbh greater than or equal to 12.7 cm and less than 27.94 cm (11 in.); sawtimber—dbh greater than or equal to 27.94 cm. The percentages of volume in each product class to total volume on each plot were calculated as separate variables. In addition, we needed an estimate of average dbh for each survey plot. The plot was first classified according to its stand size class, a categorization made by FIA personnel (seedling-sapling, poletimber, and sawtimber). Then an "average" dbh for each plot was calculated based on rules prescribing which trees to include in the average. The rules are listed in Table 2. Volume per acre was estimated based on the reported volume of each tree greater than or equal to 12.7 cm (5.0 in.).

### The Multinomial Model

We separated the data into two groups: those plots with average dbh < 12.7 cm (5.0 in.) and those plots with average dbh ≥ 12.7 cm. The final model estimating product distribution has to satisfy the following properties:

**Table 2. Inclusion rules for calculating average dbh for each plot.**

Size class	Pine plantation	Natural pine	Oak-pine	Hardwood
Seedling-sapling	Softwood only & tree dbh < 22.86 cm	Softwood only & tree dbh < 22.86 cm	Softwood tree dbh < 22.86 cm & hardwood tree dbh < 27.94 cm	Hardwood only & tree dbh < 27.94 cm
Poletimber/sawtimber	Softwood only & tree dbh ≥ 12.7 cm	Softwood only & tree dbh ≥ 12.7 cm	Both softwood & hardwood tree dbh ≥ 12.7 cm	Hardwood only & tree dbh ≥ 12.7 cm

$$0 \leq P_{ij}(dbh, vol) \leq 1.0 \quad \text{for all } i, j$$

$$\sum_i \sum_j P_{ij}(dbh, vol) = 1.0 \quad \text{for each forest type}$$

where  $P_{ij}$  equals the proportion of the live tree volume in each of the four product classes on the plot,  $i = 1$  (softwood),  $2$  (hardwood), and  $j = 1$  (pulpwood), and  $2$  (sawtimber).

We examined the multinomial logit model and used it to solve this problem. The basic multinomial logit model can be expressed as (Maddala 1987):

$$P_j = \frac{\exp(B_j X)}{1 + \sum_{k=1}^{m-1} \exp(B_k X)} \quad (j = 1, 2, \dots, m-1)$$

$$P_m = \frac{1}{1 + \sum_{k=1}^{m-1} \exp(B_k X)}$$

where

$X$  = a vector of explanatory variables (in our problem,  $dbh$  and  $vol$ )

$m$  = categories considered (in our problem, the product classes)

$P$  = proportions associated with the categories

$B$  = a vector of parameters

Table 3. **Multinomial logit** parameter estimates (pine plantation, average dbh  $\geq$  12.7 cm).

Parameters	Products			
	Softwood sawtimber	Hardwood pulpwood	Hardwood sawtimber	
Intercept	-6.1018 (-14.80)	-6.6935 (-9.54)	-7.1908 (-8.92)	
Dbh	0.2595 (11.47)	0.2013 (5.44)	0.2076 (4.98)	
Volume	0.001800 (0.94)	0.000643 (0.18)	-0.000786 (0.19)	

NOTE: Asymptotic t-statistics in parentheses.

The multinomial **logit** model is now being used in a variety of situations in applied econometrics, including occupational choice and transportation choice problems. The only forestry application we are familiar with used the technique to evaluate spruce **budworm** control efforts (Hughes et al. 1991). For our research, the approach is used to simulate the products composition of stands with a given dbh and volume combination. Four product size classes are designated as softwood pulpwood, softwood sawtimber, hardwood pulpwood, and hardwood sawtimber. The proportion of total stand volume associated with each product is a function of average dbh and volume per acre by forest type. The parameters of these proportion functions are estimated by normalizing with respect to the softwood pulpwood proportion for each forest type. Then maximum likelihood estimates were obtained for these defined multinomial **logit** models. As an example, parameter estimates for the pine plantation type stands with an average dbh greater than or equal to 12.7 cm (5 in.) are shown in Table 3.

The corresponding equations are shown in Exhibit A below.

The proportions of these four products at the mean vector (dbh = 21.58 cm or 8.498 in., volume = 96.76 m<sup>3</sup>/ha or 1383

ft<sup>3</sup>/ac.) are:  $P_{11} = 0.531$ ,  $P_{12} = 0.382$ ,  $P_{21} = 0.054$ ,  $P_{22} = 0.033$ . For illustrative purposes, the marginal effects ( $\partial P_{ij} / \partial X_{ij}$ ) are computed at the means of the **Xs** (dbh and volume) and are listed in Table 4. For example, at the data set average dbh of 21.58 cm, if dbh increases one unit (1 cm) while holding volume constant at its mean, the softwood pulpwood proportion will increase by 0.062274, the softwood sawtimber proportion will decrease by 0.055166, the hardwood pulpwood proportion will decrease by 0.004286, and the hardwood sawtimber proportion will decrease by 0.002824.

### Validation of the Multinomial Logit Model for Product Level Projections

As a means of validating the approach and our results, we tested our method by re-estimating the equation parameters using only a portion of the data. The entire pooled data set was stratified by type and size class as described above. Then 60% of the plots in each of these groupings were randomly chosen and used to estimate the model parameters. The fit equations were then used to estimate both the aggregate inventories by product class and the product proportions for the remaining 40% of the plots.

#### Exhibit A

$$P_{11} = \frac{1}{1 + e^{-6.1018 + 0.2595d + 0.001800v} + e^{-6.6935 + 0.2013d + 0.000643v} + e^{-7.1908 + 0.2076d - 0.000786v}}$$

$$P_{12} = \frac{e^{-6.1018 + 0.2595d + 0.001800v}}{1 + e^{-6.1018 + 0.2595d + 0.001800v} + e^{-6.6935 + 0.2013d + 0.000643v} + e^{-7.1908 + 0.2076d - 0.000786v}}$$

$$P_{21} = \frac{e^{-6.6935 + 0.2013d + 0.000643v}}{1 + e^{-6.1018 + 0.2595d + 0.001800v} + e^{-6.6935 + 0.2013d + 0.000643v} + e^{-7.1908 + 0.2076d - 0.000786v}}$$

$$P_{22} = \frac{e^{-7.1908 + 0.2076d - 0.000786v}}{1 + e^{-6.1018 + 0.2595d + 0.001800v} + e^{-6.6935 + 0.2013d + 0.000643v} + e^{-7.1908 + 0.2076d - 0.000786v}}$$

$$\chi^2 = 344.7 \quad N = 882$$

where

$P_{11}$  = proportion of softwood pulpwood volume;

$P_{12}$  = proportion of softwood sawtimber volume;

$P_{21}$  = proportion of hardwood pulpwood volume;

$P_{22}$  = proportion of hardwood sawtimber volume.

Table 4. Marginal effects  $(-P_{ij} / -X_{ij})$  computed at the means of variables ( $d$  = average dbh;  $v$  = volume)

$P_{ij}$	$P_{11}$	$P_{12}$	$P_{21}$	$P_{22}$
	0.062274	-0.055166	-0.004286	-0.002824
$v$	0.000370	-0.000422	0.000003	0.000049

The difference between the estimates of volume provided by FIA for the FIA82 and FIA90 surveys for Alabama and the FIA87 and FIA94 surveys for Mississippi, and the estimates provided using the product functions, are shown in Table 5. All but three of these estimates were within 2% of the value calculated using the FIA data. The remaining three were within 3.5%.

To further test the significance of our results, we compared our estimates of product proportions to those calculated using the FIA data. Estimates of proportions from a logit model are known to be biased after retransformation, and the tests allowed us to understand the extent of this inherent bias. For all models we tested the null hypothesis that the difference between our estimates of the product proportions and the FIA product proportions for each product by type is zero. That is:

$$H_0: \bar{\mu}_{ij} - \mu_{ij} = 0$$

where

$\bar{\mu}_{ij}$  = estimated mean of product proportion for forest type  $i$  and product  $j$ ,  $i = 1, 2; j = 1, 2$ ;

$\mu_{ij}$  = FIA mean of product proportion for forest type  $i$  and product  $j$ ,  $i = 1, 2; j = 1, 2$ .

In the absence of bias, mean differences would be zero for each type and product combination.

As an example, classical paired, t-test results for the pine plantation type in the FIA82 Alabama data set are shown in

Table 6. Entries in the Mean column represent the means of differences between our estimated proportion, and the FIA proportion; the *Sig.* column indicates if the mean of the differences is significantly different from zero. Here, "xx" implies that the mean difference is not significantly different from zero at the  $\alpha = 0.05$  significance level, "x" indicates that the mean difference is not significantly different from zero at  $\alpha = 0.01$  and is significantly different at  $\alpha = 0.05$ . and "#" shows that the mean difference is significantly different from zero at the  $\alpha = 0.01$  significance level.

For the pine plantation type, the product function estimated means and the FIA means for each product class are not significantly different across all four data sets (FIA82 and FIA90 for Alabama, and FIA87 and FIA94 for Mississippi) at the  $\alpha = 0.01$  significance level with the following exceptions: hardwoodsawtimber in the average dbh  $\geq 12.7$  cm group for FIA82 (Alabama) and in the average dbh  $< 12.7$  cm group for FIA87 (Mississippi). Significance of the mean differences for all four forest types by data set is summarized in Table 7. In Table 7, all cells indicate that the means of differences are not significantly different from zero at the  $\alpha = 0.01$  level except ten cells (marked with letters u-j) that show a significant difference between product function estimated product, proportion and FIA estimated product proportion.

## Discussion and Conclusion

Two types of significance are important from the perspective of an analyst: (1) statistical significance (described above) and (2) the importance (significance) of an error in the estimate to the overall analytic objective (in this case, projecting inventory). Table 8 describes the ten significantly different cells (marked a-j in Table 7) in more detail. All of these cells except for "a" are unlikely harvest candidates

Table 5. FIA estimated volume and product function estimated volume by product class (million  $M^3$ ) for a 40% sample of inventory plots.

Alabama	FIA82_AL	FIA82E_AL	Difference (%)	FIA90_AL	FIA90E_AL	Difference (%)
Total	709	709	—	730	730	—
Softwood	356	352	-1.02	341	346	-1.41
Hardwood	353	357	1.02	389	385	-1.24
SPW	103	102	-1.04	90	92	1.92
SST	253	250	-1.01	251	254	1.23
HPW	169	166	-1.67	157	157	0.28
HST	185	191	3.49	233	227	-2.26
Mississippi	FIA87_MS	FIA87E_MS	Difference (%)	FIA94_MS	FIA94E_MS	Difference (%)
Total	850	850	—	821	821	—
Softwood	370	372	0.49	350	352	0.47
Hardwood	480	478	-0.38	471	469	-0.35
SPW	73	75	2.66	85	85	0.04
SST	297	297	-0.04	265	266	0.61
HPW	165	166	0.75	156	157	0.54
HST	315	312	-0.97	315	312	-0.79

NOTES: FIA82\_AL stands for FIA 1982 Inventory for Alabama, FIA82E\_AL stands for estimated 1982 inventory. Inventory in this table does not include public land and nonstocked plots. Percent entries may appear incorrect due to rounding of the volume (million cubic meter) entries.

Table 6. Paired T-tests (FIA82 for Alabama, forest type = pine plantation).

Product <sup>1</sup>	Average dbh < 12.7 cm				Average dbh ≥ 12.7 cm			
	Mean	SD	Prob >  T	sig.	Mean	SD	Prob >  T	sig.
SPW	-0.0955	0.0657	0.1612	xx	0.0318	0.0211	0.1368	xx
SST	-0.0167	0.0553	0.7657	xx	-0.0433	0.0172	0.0147	x
HPW	0.0460	0.0224	0.0533	xx	-0.0107	0.0120	0.3802	xx
HST	0.0662	0.0295	0.0355	x	0.0221	0.0062	0.0007	#

<sup>1</sup> SPW = softwood pulpwood; SST = softwood sawtimber; HPW = hardwood pulpwood; HST = hardwood sawtimber.

since the average dbh for the stand is ≤ 12.7 cm (5 in.). Cell “a” represents hardwood sawtimber proportion in pine plantation stands that could be candidates for harvest from a stand size perspective, but the volume of the hardwood sawtimber in such stands (as measured by FIA) is very small relative to total inventory.

Although statistics show that these cells (a-j) indicate that certain product function estimates are significantly different from their FIA counterparts, the effects of accounting for these differences for product level inventory projection purposes

are not very large. As an example, consider c (hardwood pulpwood in the hardwood type where average dbh < 12.7 cm), which represents the largest percentage total inventory of any of the “problem cells.” The estimate of mean hardwood pulpwood proportion is 0.04658 with a standard error of 0.0214 (the 99% confidence interval is 0.4658 - 0.5942). The product function estimate of proportion for this cell will underestimate on average such that the mean difference is -0.0716 with standard error of 0.0211. The 99% confidence interval is -0.1349 to -0.0083, the

Table 7. Summary of paired T-tests for each product class by forest type.<sup>1</sup>

Data set	Group	Forest type (i) and product classes <sup>2</sup> (j)														
		Pine plantation				Pine natural				Oak-pine				Hardwood		
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3
FIA82 (AL)	Ave. dbh < 12.7 cm	xx	xx	xx	x	xx	xx	xx	xx	xx	xx	x	xx	xx	xx	xx
	Ave. dbh ≥ 12.7 cm	xx	x	xx	a	xx	xx	xx	x	xx	x	xx	xx	xx	xx	x
FIA90 (AL)	Ave. dbh < 12.7 cm	xx	xx	xx	xx	b	xx	xx	xx	xx	xx	xx	x	c	xx	d
	Ave. dbh ≥ 12.7 cm	xx	xx	xx	xx	x	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
FIA87 (MS)	Ave. dbh < 12.7 cm	xx	xx	xx	f	xx	xx	xx	g	xx	xx	xx	xx	h	xx	xx
	Ave. dbh ≥ 12.7 cm	x	xx	xx	xx	xx	xx	xx	xx	xx	xx	x	xx	xx	xx	xx
FIA94 (MS)	Ave. dbh < 12.7 cm	xx	xx	xx	xx	xx	xx	xx	xx	i	xx	x	xx	j	xx	xx
	Ave. dbh ≥ 12.7 cm	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx

<sup>1</sup> xx implies that the mean difference is not significantly different from zero at the α = 0.05 significance level, x indicates that the mean difference is significantly different from zero at α = 0.01 and is significantly different at α = 0.05, and letters a-j indicate that the mean difference is significantly different from zero at the α = 0.01 significance level.

<sup>2</sup> 1 = softwood pulpwood; 2 = softwood sawtimber; 3 = hardwood pulpwood; 4 = hardwood sawtimber.

Table 8. Detailed description of “problem” cells in Table 7.

Cell	Data set	Type <sup>1</sup>	Ave. dbh	Product <sup>2</sup>	FIA estimates <sup>3</sup>	Mean diff. <sup>4</sup>	% of
a	FIA82_AL	PP	≥ 12.7 cm	HST	0.0130 (0.0056)	-0.0662 (0.0295)	0.1
b	FIA90_AL	PN	< 12.7 cm	SPW	0.3465 (0.0466)	0.1393 (0.0463)	0.2
c	FIA90_AL	HD	< 12.7 cm	SPW	0.0936 (0.0131)	0.0458 (0.0146)	0.2
d	FIA90_AL	HD	< 12.7 cm	HPW	0.5300 (0.0214)	-0.0716 (0.0211)	2.3
e	FIA90_AL	HD	< 12.7 cm	HST	0.2422 (0.0180)	0.0442 (0.0164)	1.7
f	FIA87_MS	PP	< 12.7 cm	HST	0.0369 (0.0190)	0.0572 (0.0193)	0.0
g	FIA87_MS	PN	< 12.7 cm	HST	0.0276 (0.0135)	0.0564 (0.0128)	0.0
h	FIA87_MS	H D	< 12.7 cm	SPW	0.0789 (0.0144)	0.0524 (0.0174)	0.2
i	FIA94_MS	OP	< 12.7 cm	SPW	0.2490 (0.0379)	0.1150 (0.0393)	0.4
j	FIA94_MS	H D	< 12.7 cm	SPW	0.0873 (0.0131)	0.0420 (0.0152)	0.2

<sup>1</sup> PP-pine plantation; NP-natural pine; OP-oak-pine; HD-hardwood.

<sup>2</sup> SPW = softwood pulpwood; SST = softwood sawtimber; HPW = hardwood pulpwood; HST = hardwood sawtimber.

<sup>3</sup> Mean product proportion for the forest type-dbh group represented by the “problem” cell. Estimate based on FIA data (standard error in parentheses).

<sup>4</sup> Mean of differences between FIA-based estimate of product proportion and product function estimated product proportion for the forest type-dbh group (standard error in parentheses).

<sup>5</sup> Percent of volume for a certain combination of forest type, dbh group, and product relative to total inventory for a particular survey data set.

ranging from an underestimate of 0.1349 to an underestimate of 0.0083 relative to the FIA point estimate). Therefore, given the unlikely event (1 out of 100) that we underestimated the mean product proportion for cell *d* by 0.1349, our method would yield an estimate for that cell equal to 1.73% of total forest inventory rather than 2.32%, a difference of only 0.59%. (NOTE: FIA estimates are based on sampled data and thus are subject to sampling error. The forest statistics reports produced by FIA outline methods for calculating the sampling errors associated with particular resource items.)

In addition to the relatively small inventory effect associated with possibly underestimating cell proportions, the cell represents stands with an average dbh < 12.7 cm, and it is unlikely that they would be considered harvest candidates in an inventory projection model. The information provided in Table 8 suggests that the estimates using the product functions are more accurate for stands with an average dbh  $\geq$  12.7 cm than for stands with a smaller average dbh. Since the intended uses of these functions are to improve inventory projection, and the merchantable volumes of stands with an average dbh < 12.7 cm are generally small, this bias is one we can live with.

In conclusion, this technique provides a useful tool for inventory projection research. It has been incorporated in the **DPSupply** system (Teeter 1994), replacing a laborious lookup table method which was difficult to work with and problematic when it came to making statements about our confidence in its results. All 16 of the product function models contain the same variables, and the models are used in the **MANAGE**

module of the **DPSupply** system to calculate net present value for each plot based on the plot's mix of products. They are also used in the **HARVEST** module of the program to distribute the products and meet individual product level demands. The product function method appears robust enough to have broad application, including adapting it to other geographic/physiographic regions or to other projection systems, such as **ATLAS** (Mills and Kincaid 1992) or **SERTS** (Abt et al. 1993), which are age based rather than dbh based.

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