Modeling Missing Remeasurement Tree Heights in Forest Inventory Data

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Abstract.—Missing tree heights are often problematic in compiling forest inventory remeasurement data. Heights for cut and mortality trees are usually not available; calculations of removal or mortality volumes must utilize either a modeled height at the time of tree death or the height assigned to the tree at a previous remeasurement. Less often, tree heights are not available for trees that were determined to be missed tally trees in an initial inventory. In these cases, a height is available for the current measurement, but the initial tree height must be modeled or estimated. In this paper, we present a procedure for predicting either a time 1 or time 2 height. The procedure uses actual tree height information for a tree collected at time 1 or time 2 if available. Incorporating the relationship between actual tree height to predicted height provides for an adjustment to height equations that do not incorporate site quality and stand parameters.

Missing tree heights are a common occurrence in broad forest inventories. They are relatively rare in an initial installation of inventory plots. Most missing tree heights are encountered during the remeasurement of a previously installed plot where initial inventory (time 1) tree heights are recorded. During the remeasurement of inventory plots, the need to supply a value for a missing tree height arises primarily from the following situations.

1. A previously measured tree is missing at time 2 due to natural mortality or cutting; a height at the time of tree death is needed to compute the tree's volume, which is assigned to mortality or removals.
2. A previously measured tree grows across a merchantability threshold for volume calculation; a height for the tree at the threshold diameter at breast height (d.b.h.) is needed to accurately calculate ingrowth volume.
3. A time 2 inventory tree is determined to have been missed by the previous field crew or the previous height is known to be an error; a time 1 tree height is needed to compute the tree's time 1 volume.

To illustrate the above examples, Forest Inventory and Analysis (FIA) data from a recent inventory in South Carolina were examined. Out of more than 111,000 trees tallied for current inventory estimates, far less than 1 percent required a modeled height. Data recorder software and editing procedures eliminated almost all missing or invalid heights. However, nearly one out of three trees in the remeasurement sample (trees measured 8 years earlier) required a modeled height. Fifteen percent of the trees were cut, 4 percent died from natural causes, 9 percent grew across the growing-stock or sawtimber volume threshold, and 2 percent were survivor or missed trees that required a modeled height at time 1 for various reasons. These examples illustrate that tree heights must be modeled often in operational inventories. The procedures utilized in dealing with missing tree heights must accurately account for a wide range of site and stand parameters. The values used for missing heights in a remeasurement inventory can have a major impact on growth, removal, and mortality volume calculations.

The FIA program is currently developing nationally consistent procedures for collecting and compiling inventory data. While initial compilation efforts have focused on current inventory procedures, the development of remeasurement procedures is underway. Accurate estimates of change (growth, removals, and mortality) will require viable, consistent procedures for dealing with missing tree heights in remeasurement settings. Large changes in tree heights from time 1 to time 2 in remeasurement settings can have severe adverse effects on components of change estimates. In this paper, we present a procedure for dealing with missing tree heights that is simple, allows the use of any tree height model, and produces a modeled tree height in remeasurement settings that is relatively stable relative to the actual measured height of the tree at time 1 or time 2.

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Methods

The procedures selected for use in FIA inventory computations cannot possibly address all stand, site, and tree parameters because of the diversity encountered across large regions and the Nation. There are many different methodologies utilized by the regional FIA units to model tree heights. We have not assembled a list of those procedures here because no single total height equation or set of equations exists that will perform optimally across all regions. Therefore, the retention of regional FIA methodologies for many computations, including tree height models, is necessary for the immediate future. However, we can implement procedures that improve upon the raw, initial tree heights produced by these equations.

In remeasurement situations, as documented above, we always have some information about each tree that can improve our ability to accurately predict an unknown height. We usually have knowledge of the tree's species, d.b.h. and height, either merchantable length or total height. This information is available for the time 1 inventory or for the time 2 inventory. We can obtain a predicted tree height from a model or equation and use this value unaltered for computations of tree volume and growth. However, unless the model incorporates all site and stand parameters that may impact tree height relationships, abnormal changes in tree height can result. These changes will impact volume and growth computations. One can produce a modeled height that is harmonized with an actual measured height for the tree using the following equation.

$$H_m = \left(\frac{K_a}{K_m}\right) U_m \qquad (1)$$

Where

H_m = final modeled total height
K_a = actual (measured) height
K_m = predicted height for tree with known height
U_m = predicted height for tree with unknown height

This equation may be used to predict a missing tree height at time 2 (cut or mortality tree) or a missing tree height at time 1 (missed tree or erroneous initial tree height). The procedure makes adjustments to raw predicted height values from any equation. The adjustments account for site and stand parameters that have influenced the height development of each individual tree. While stand composition and structural characteristics change due to increasing age and disturbances, the relationship between actual height and modeled height should remain viable for relatively short remeasurement periods of 10 years or less.

Examples

The following examples illustrate the calculation of missing height values for some of the situations described earlier. The examples use data and height equations from the FIA unit at the Southern Research Station (SRS-FIA) to illustrate the effect of the harmonic proportioning equation. D.b.h. and bole length (BL) are the two independent variables in the equations used to derive the initial estimate for a missing height.

$$Y = a + b(d.b.h.) \qquad (2)$$

Trees 1.0 to 4.9 inches d.b.h.

Where

Y = predicted height
$d.b.h.$ = diameter at breast height
a and b are species specific coefficients

$$BL = a + b\left(\sqrt{\log_{10}(d.b.h.)}\right) \qquad (3)$$

Trees 5.0 inches d.b.h. and larger.

Where

BL = predicted length from 1.0-foot stump to 4.0-inch top diameter (outside bark)
$d.b.h.$ = diameter at breast height
a and b are species specific coefficients (table 1)

$$Y = a + b(BL) + c\left(\frac{1}{d.b.h.^2}\right) \qquad (4)$$

Trees 5.0 inches d.b.h. and larger.

Where:

Y = predicted height
BL = bole length
$d.b.h.$ = diameter at breast height
a, b and c species specific total height coefficients (table 2)

Table 1.—*Coefficients for bole length equation by FIA species code*

Species code	a	b	Species code	a	b
131	-156.117206	202.750877	920	-142.109331	185.613178
121	-119.432082	164.99968	621	-148.831031	197.806215
126	-115.074602	152.443899	540	-131.365123	175.827063
128	-124.280736	167.685111	531	-115.243339	151.094033
107	-106.238568	149.364954	370	-85.02753	125.664258
110	-126.330994	169.14767	901	-114.378044	154.378556
111	-152.035297	199.471982	602	-121.652618	161.466442
115	-184.413794	235.884113	491	-26.361267	43.854679
123	-69.543987	109.868908	311	-121.652618	161.466442
132	-106.752125	149.936213	400	-138.431526	179.440462
43	-107.286838	152.696925	591	-89.551605	119.184132
221	-142.478002	186.508392	552	-121.652618	161.466442
10	-130.645957	170.109525	680	-121.652618	161.466442
260	-138.143903	176.104811	521	-121.652618	161.466442
241	-130.645957	170.109525	318	-91.686932	129.421189
222	-134.419057	177.021729	371	-121.652618	161.466442
60	-79.746545	108.851764	837	-121.372298	160.784296
90	-130.645957	170.109525	823	-110.09781	148.608204
129	-135.566314	178.054737	813	-117.535853	162.387287
950	-132.8382	180.305149	832	-94.85546	133.493232
762	-102.309572	139.853199	826	-110.09781	148.608204
694	-150.63215	191.725286	820	-105.69823	143.674925
693	-115.928799	151.979406	838	-107.606289	137.883848
313	-142.109331	185.613178	822	-110.09781	148.608204
330	-142.109331	185.613178	830	-110.09781	148.608204
601	-142.109331	185.613178	835	-95.975318	129.925601
740	-142.109331	185.613178	833	-97.402948	138.559456
651	-96.901447	137.060551	806	-105.862725	146.021142
970	-120.048309	157.758605	817	-110.09781	148.608204
460	-112.391322	149.469203	834	-110.09781	148.608204
555	-108.60678	148.214521	812	-112.741267	149.053737
652	-96.901447	137.060551	825	-116.559834	156.184096
316	-99.705783	137.125344	804	-110.09781	148.608204
580	-142.109331	185.613178	827	-104.639214	143.375171
317	-142.109331	185.613178	802	-109.493036	148.982715
653	-95.979778	136.357591	831	-150.949932	191.23914
611	-161.172361	206.724378	899	-76.443709	103.644195
731	-113.031851	158.006557	999	-87.990247	119.886332
691	-147.46436	193.31287			

The equations use coefficients for 77 species or species-groups in the South. Whereas total stem length is the height value utilized in current volume prediction equations, many of the previous inventories in the South measured merchantable height and modeled total height values from the bole length. Thus, equations are still needed to predict bole length as well as a total height equation utilizing bole length as an independent variable. With total height as the only measured height variable involved in remeasurement plots, a single equation similar to equation (1) can be used to model total heights of trees 5.0 inches and larger.

It should also be noted that d.b.h. is often missing when tree height is missing. For example, both d.b.h. and height are unknown for a tree at the time it was cut. The following examples use modeled d.b.h. values in such instances.

Example 1: Missing Time 1 Tree Height (Missed Tree At Time 1)

Species: chestnut oak (*Quercus prinus*—species code 832)

Time 1 d.b.h.: 19.8 inches (modeled)

Time 2 d.b.h.: 22.0 inches (measured)

Time 2 total height: 72 feet (measured)

Step one: Calculate modeled time 2 bole length (equation 3)

$BL = A + B*SQRT(\log_{10}(d.b.h.))$

$BL = -94.85546 + 133.493232*SQRT(\log_{10}(22.0))$

$BL = 59.813755$

Step two: Calculate modeled time 2 total length (equation 4)

$Y = A + B(BL) + C(1/(d.b.h.)^2)$

$Y = 21.244492 + 0.907202(59.813755) + 141.15111(1/(22.0)^2)$

$Y = 75.799285$

Step three: Calculate modeled time 1 bole length (equation 3)

$BL = A + B*SQRT(\log_{10}(d.b.h.))$

$BL = -94.85546 + 133.493232*SQRT(\log_{10}(19.8))$

$BL = 57.154893$

Step four: Calculate modeled time 1 total length (equation 4)

$Y = A + B(BL) + C(1/(d.b.h.)^2)$

$Y = 21.244492 + 0.907202(57.154893) + 141.15111(1/(19.8)^2)$

$Y = 73.455568$

Step five: Calculate harmonically proportioned total length for time 1 (equation 1)

$H_m = (K_a/K_m)*U_m$

$H_m = (72.0/75.799285) * 73.455568$

$H_m = 69.8$ (final estimate for initial tree height)

In this example, the first estimate of time 1 total height was 73 feet, which was greater than the measured value for the tree at time 2. The proportioning procedure results in a more realistic value that is consistent with d.b.h. change.

Example 2: Missing Time 2 Tree Height (Cut Tree)

Species: longleaf pine (*Pinus palustris*—species code 121)

Time 1 d.b.h.: 6.0 inches (measured)

Time 2 d.b.h.: 8.0 inches (modeled)

Time 1 total height: 54 feet (measured)

Step one: Calculate modeled time 1 bole length (equation 3)

$BL = A + B * SQRT(\log_{10}(d.b.h.))$

$BL = -119.432082 + 164.99968*SQRT(\log_{10}(6.0))$

$BL = 26.118891$

Step two: Calculate modeled time 1 total length (equation 4)

$Y = A + B(BL) + C(1/(d.b.h.)^2)$

$Y = 7.729578 + 1.038906(26.118891) + 460.44451(1/(6.0)^2)$

$Y = 47.65478$

Step three: Calculate modeled time 2 bole length (equation 3)

$BL = A + B*SQRT(\log_{10}(d.b.h.))$

$BL = -119.432082 + 164.99968*SQRT(\log_{10}(8.0))$

$BL = 37.368841$

Step four: Calculate modeled time 2 total length (equation 4)

$Y = A + B(BL) + C(1/(d.b.h.)^2)$

$Y = 7.729578 + 1.038906(37.368841) + 460.44451(1/(8.0)^2)$

$Y = 53.74674$

Table 2.—*Coefficients for total tree height equation by FIA species code*

Species code	a	b	c	Species code	a	b	c
131	11.63601	1.002513	347.272658	920	18.718746	0.973277	237.046549
121	7.729578	1.038906	460.44451	621	20.671028	0.956696	219.505156
126	12.235965	0.949229	110.433871	540	20.026148	0.955226	257.081973
128	9.785433	0.99598	265.638401	531	33.050946	0.781059	-203.793929
107	12.138889	0.976357	255.670995	370	26.749454	0.875348	53.60581
110	8.227609	1.036995	345.204061	901	24.201253	0.890276	45.69745
111	7.627238	1.046602	429.084288	602	21.840821	0.899214	166.601854
115	11.612768	0.998582	340.837362	491	15.84294	0.996554	81.45163
123	3.433555	1.062757	308.1748	311	20.91845	0.967155	143.329004
132	11.719363	0.993829	289.806791	400	20.279042	0.978179	232.507527
43	11.865871	1.000782	235.023676	591	15.702126	0.930761	35.053649
221	20.131836	0.9007	201.316112	552	20.91845	0.967155	143.329004
10	14.527042	0.978106	162.61837	680	22.055232	0.86043	-19.111068
260	20.872663	0.905741	7.644922	521	9.245754	1.051793	407.441276
241	14.527042	0.97816	162.61837	318	16.476717	1.03106	307.64589
222	7.987471	1.02906	497.921189	371	24.394062	0.945372	91.247826
60	14.032483	1.003766	122.450047	837	21.297146	0.952646	226.151486
90	14.527042	0.978106	162.61837	823	17.367545	1.006456	197.818822
129	15.330778	0.95947	219.242547	813	16.436721	1.040216	209.859337
950	24.914346	0.89236	38.929822	832	21.244492	0.907202	141.15111
762	15.738263	0.996829	308.494651	826	21.589084	0.933223	-123.979563
694	15.403016	0.998625	246.855337	820	16.78362	0.985803	238.462787
693	16.746882	0.963007	146.246985	838	17.42442	0.903906	116.917948
313	18.718746	0.973277	237.046549	822	17.367545	1.006456	197.818822
330	9.731165	1.146179	401.643379	830	17.367545	1.006456	197.818822
601	18,718746	0.973277	237.046549	835	20.755906	0.924214	64.282713
740	18.718746	0.973277	237.046549	833	17.354367	1.023787	244.745485
651	16.866377	1.009975	299.861067	806	20.636356	0.964852	128.255717
970	26.041732	0.872665	-6.198928	817	17.367545	1.006456	197.818822
460	31.13191	0.810872	-188.932025	834	17.367545	1.006456	197.818822
555	14.664977	0.990312	242.687846	812	17.40684	0.993855	236.337343
652	13.031815	0.990918	475.180647	825	24.489813	0.891002	10.363523
316	23.972077	0.882021	129.85142	804	17.367545	1.006456	197.818822
580	18.718746	0.973277	237.046549	827	20.649189	0.947504	139.682441
317	18.718746	0.973277	237.046549	802	20.445897	0.961278	148.494802
653	17.455189	0.927531	240.838341	831	19.57347	0.992177	207.241184
611	20.336715	0.950014	235.637303	899	12.147201	1.015202	122.112775
731	25.513236	0.894188	107.211601	999	18.951285	0.920112	122.341224
691	19.550482	0.951712	363.668158				

Step five: Calculate harmonically proportioned total length for time 1 (equation 1)

$H_m = (K_a/K_m)*U_m$

$H_m = (54.0 / 47.65478) * 53.74674$

$H_m = 60.9$ (final estimate for terminal tree height)

This example illustrates the value of using the harmonic proportioning procedure to adjust initial predicted values for site and stand conditions. The first estimate of total height at time of cutting was 54 feet, an average value for the species and d.b.h. If we used this value, there would be no height growth recorded for the tree. However, the relationship between actual height and predicted height at time 1 provides a site- and stand-specific ratio to use in producing a more likely height value of 61 feet.

Example: Calculate Probable Height at Time of Ingrowth
Species: yellow-poplar (*Liriodendron tulipifera*—species code 621)
Time 1 d.b.h.: 2.5 inches (measured)
Time 2 d.b.h.: 6.8 inches (measured)
Time 1 total height: 24 feet (measured)
Time 2 total height: 49 feet (measured)

In this example, we have a known d.b.h. and total height at both time 1 and time 2; we want to obtain a height when the tree crossed the merchantability threshold for volume calculation, which is 5.0 inches d.b.h. We could use the time 1 or time 2 actual values in the calculation of the proportion—we use time 2 values here.

Step one: Calculate modeled time 2 bole length (equation 3)

$BL = A + B * SQRT(\log_{10}(d.b.h.))$

$BL = -148.831031 + 197.806215*SQRT(\log_{10}(6.8))$

$BL = 31.651170$

Step two: Calculate modeled time 2 total length (equation 4)

$Y = A + B (BL) + C (1/(d.b.h.)^2)$

$Y = 20.671028 + 0.956696 (31.651170) + 219.505156 (1/(6.8)^2)$

$Y = 55.69866$

Step three: Calculate modeled bole length at ingrowth d.b.h. (equation 3)

$BL = A + B*SQRT(\log_{10}(d.b.h.))$

$BL = -148.831031 + 197.806215*SQRT(\log_{10}(5.0))$

$BL = 16.543720$

Step four: Calculate modeled total length at ingrowth d.b.h. (equation 4)

$Y = A + B (BL) + C (1/(d.b.h.)^2)$

$Y = 20.671028 + 0.956696 (16.543720) + 219.505156 (1/(5.0)^2)$

$Y = 45.27854$

Step five: Calculate harmonically proportioned total length at ingrowth d.b.h. (equation 1)

$H_m = (K_a/K_m)*U_m$

$H_m = (49.0 / 55.69866) * 45.27854$

$H_m = 39.8$ (final estimate for tree height at ingrowth d.b.h.)

The tree in this example has achieved height growth well less than average based upon the relationship of predicted versus actual height at time 2. As a result, the modeled height values at the time of ingrowth into the merchantable volume category are adjusted downward accordingly.

Discussion

The harmonic proportioning procedure presented above has been operational in remeasurement inventory processing procedures at SRS-FIA for several years. It is effective in preventing unnecessary fluctuations in modeled tree heights and resulting volumes. The procedure is easily incorporated into existing height calculations and works with any equation or model that provides an initial estimate for a missing height. It effectively accounts for much of the influence that site and stand conditions have on tree height if remeasurement cycles are not excessively long.