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RESEARCH ARTICLE

Neutral model analysis of landscape patterns from mathematical morphology

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Abstract Mathematical morphology encompasses methods for characterizing land-cover patterns in ecological research and biodiversity assessments. This paper reports a neutral model analysis of patterns in the absence of a structuring ecological process, to help set standards for comparing and interpreting patterns identified by mathematical morphology on real land-cover maps. We considered six structural classes (core, perforated, edge, connector, branch, and patch) on randomly generated binary (forest, non-forest) maps in which the percent occupancy (P) of forest varied from 1% to 99%. The maps were dominated by the patch class for low P, by the branch and connector classes for intermediate P, and by the edge, perforated, and core classes for high P. Two types of pattern phase changes were signaled by abrupt transitions among the six structural classes, at critical P thresholds that were indicated by increased variance among maps for the same P. A phase change from maps dominated by the patch class to maps dominated by the branch and connector classes was related to the existence of a percolating cluster of forest, and the P threshold varied depending on the co-existence of the core class. A second phase change from the edge class to the perforated class was related to the existence of a percolating cluster of non-core (including non-forest) and represents a change of context from exterior to interior. Our results appear to be the first demonstration of multiple phase changes controlling different aspects of landscape pattern on random neutral maps. Potential applications of the results are illustrated by an analysis of ten real forest maps.

Keywords Pattern analysis · Percolation theory · Phase change · Simulation · Threshold

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Introduction

Mathematical morphology (e.g., Soille 2003) encompasses a wide range of methods that may be useful for characterizing spatial patterns in ecological research and biodiversity assessments. Vogt et al. (2007a) used mathematical morphology to analyze land-cover structural patterns, and demonstrated that the approach is superior to image convolution for identifying 'perforated' and 'edge' conditions (Riitters et al. 2000). Vogt et al. (2007b) showed that the approach also objectively identifies structural corridors (sensu Freemark et al. 2002), which makes it feasible to implement the classic patch-corridor-matrix model (e.g., Forman 1995) in large-area assessments using land-cover maps derived from satellite imagery.

Further application of mathematical morphology in ecological research and assessments would benefit from tests with neutral models (Caswell 1976; see Gardner et al. 1987) to provide standards for comparisons of patterns observed on real maps (Gardner et al. 1987). Indeed, the introduction of new landscape pattern indices without testing by neutral models has been called a "serious omission" (Turner et al. 2001). Pearson and Gardner (1997) defined a neutral model as "a minimum set of rules required to generate pattern in the absence of a particular process." A neutral model analysis does not provide evidence about the operation of any particular process, but it is useful for understanding the individual and correlated behaviors of many indices of landscape pattern (Gardner and O'Neill 1991; Gustafson and Parker 1992; Milne 1992; O'Neill et al. 1992; With and King 1997; Turner et al. 2001).

We conducted a neutral model analysis of landscape structural patterns identified by mathematical morphology on random, binary maps. Our specific objectives were: (1) to investigate neutral models as a basis for setting standards for comparisons of real land-cover maps, and; (2) to explore the behavior of structural patterns in relation to the percent of a map occupied by the focal land-cover class. Standards for comparison are needed so that some degree of significance can be ascribed to observations of different structural patterns in the real world. The relationships between structural patterns and percent occupancy are important because most pattern indices are controlled strongly by percent occupancy, and few indices can be interpreted independently of it (Turner et al. 2001). We considered a neutral model represented by a random binary raster map because that is the simplest neutral model and it has been used extensively in landscape ecology. Analyses of forest patterns on ten real land-cover maps derived from satellite imagery were used to illustrate an application of the results.

Methods

Generation of random maps

We used randomly generated neutral maps to establish empirical frequency distributions for the proportions of a focal class (hereafter, 'forest,' but it could be any focal class of interest) in six types of structural classes (see below), in relation to the percentage of the map that is occupied by forest. Following procedures already well established in the study of landscape pattern indices (Turner et al. 2001), we used Gardner's (1999) RULE software to generate random maps of size $1{,}024 \times 1{,}024$ pixels. Let P be the proportion of the map occupied by forest. RULE assigns the presence or absence of forest to each pixel independently with probability equal to P. The expected proportion of non-forest on the random map is thus given by 1 – P.

Fifty maps were generated for target values of P from 1% to 99% in steps of size 1%, providing 4950 maps for analysis. Analyses of maps with 0% and 100% forest have trivial results and these values of P were not included. The actual P on a generated map was typically within 0.1% of the target P, and the small differences were ignored when later summarizing the results according to the values of target P. A temporary surrounding buffer of non-forest pixels was added to each map to alleviate minor boundary effects when performing the analyses with mathematical morphology. The buffer pixels were excluded from subsequent analyses and thus did not change the map extent or the actual P.

Pattern analysis with mathematical morphology

Mathematical morphology refers to both a theory and a technique for image analysis (Soille 2003) and we used the procedures described by Vogt et al. (2007b) for structural pattern analysis of land-cover maps. Briefly, each forest pixel was labeled according to one of six structural classes (see below). To accomplish that, the forest and/or non-forest pixels were processed by set operations like union, intersection, complementation, and translation. The operations were controlled by structuring elements (SE) which defined the connectivity rule (4- or 8-neighbor) for a given operation, and the sub-region ('window') over which an operation occurred. To explore how structural patterns changed with measurement scale, we tested four SEs of increasing size denoted as SE1, SE2, SE3, and SE4. Although a SE is not always a square shape, its size is analogous to 'window' size in image convolution, and the comparable 'window' sizes are 3×3, 5×5, 7×7, and 9×9 pixels.

We considered six mutually exclusive structural classes called core, perforated, edge, connector, branch, and patch (Fig. 1). For a given SE, a 'core' pixel is a forest pixel that is at the center of a SE that contains only forest pixels. The forest pixels forming the exterior perimeter of a cluster of core pixels are 'edge' pixels. Where there is a hole (i.e., a non-forest inclusion) contained within a cluster of core pixels, the pixels forming the interior perimeter of the core cluster are labeled as 'perforated' unless (1) the hole is so large that it contains at least one SE populated only by non-forest pixels, or (2) the interior perimeter pixels are adjacent to edge pixels. If either condition is met, then the interior perimeter pixels are labeled as 'edge' pixels. The first condition restricts the labeling of perforated pixels to holes that are relatively small or thin in comparison to the size or width of the SE. The second condition is an isolation requirement which guarantees that perforated pixels are never connected to edge pixels; if that were allowed,

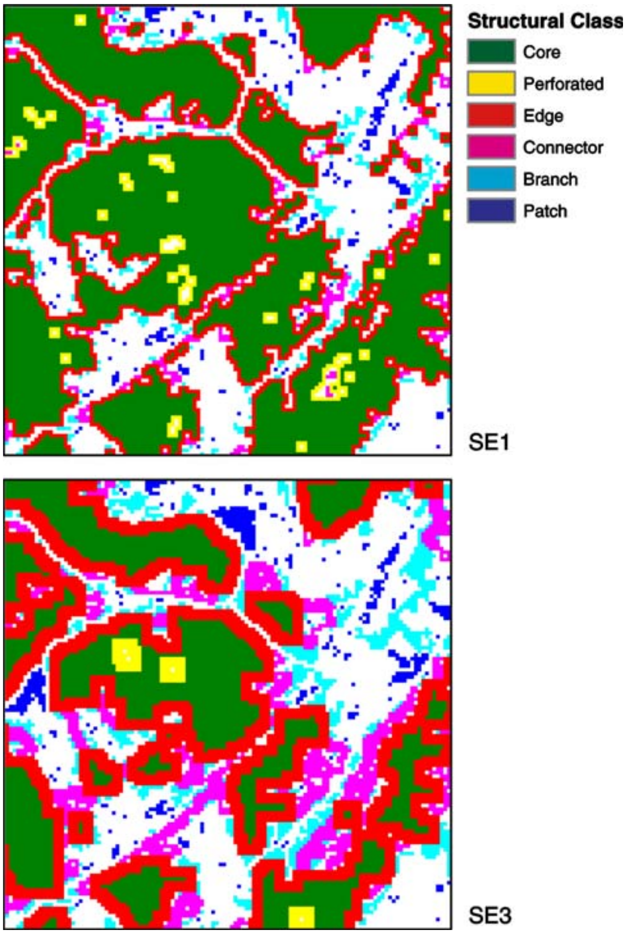

Fig. 1 Structural classes from mathematical morphology for a portion of Alabama, USA. The white pixels are non-forest. *Top*: SE1 (see text). *Bottom*: SE3. Data scale: 1 pixel = 0.09 ha. *Note*: Figures 3, 5, 6 and 7 use the same colors to indicate structural classes

then there would be no core pixels in between to define the difference between 'interior' and 'exterior,' in which case perforation has no meaning.

A 'connector' pixel is part of a cluster of non-core forest pixels connected at two or more locations to edge or perforated pixels. A 'branch' pixel is like a connector pixel except the cluster is connected at only one location to an edge, perforated, or connector pixel. Finally, 'patch' pixels include isolated or disjoint forest clusters that are too small or too thin to contain a core forest pixel. Note that the term 'patch' here defines an object that is not the same as the classical 'patch' in the landscape ecology literature, in which it is defined simply as a set of connected pixels. An important feature of this analysis may be called 'contingency,' referring to the fact that some structural classes depend on the co-existence of core pixels; by definition, a map cannot contain any edge, perforated, connector, or branch pixels unless it also contains at least one core pixel.

Summaries of pattern on random maps

The analyses with four SEs applied to each of 4,950 random forest maps yielded 19,800 maps of structural patterns. For each map, the proportions of all forest pixels that were labeled as each of the six types of structural classes were calculated. The sum of all six proportions therefore equaled one for each map, and that permitted comparisons of maps with different actual P. For each SE, empirical frequency distributions were then prepared for each target P by finding the average proportion (n = 50 maps) of each structural class. The standard deviations (n = 50) of the proportions were also calculated for each structural class, for each target P.

Pattern analysis of real land-cover maps

We selected ten locations in the State of Alabama (Fig. 2) to illustrate a variety of forest patterns. Forest maps with an extent of 1,024 × 1,024 pixels and a 30-m spatial resolution (0.09 ha/pixel) were obtained from the 1992 NLCD land-cover map derived from Thematic Mapper imagery (Vogelmann et al. 2001) with a detailed road map (Geographic Data Technology 2002) superimposed (see Riitters et al. 2004 for details). The three upland forest classes from the NLCD were combined into one 'forest' class, while the remaining NLCD classes, and all pixels containing a road segment, were combined into one 'non-forest' class. Each example map was then processed and summarized using the same procedures that were used with the random maps.

Results and discussion

Neutral models as standards for comparisons

The occurrence and relative abundance of structural classes on random maps were related to P and the size of the SE (Fig. 3). Patch was the dominant class when P was small and the core and perforated classes dominated when P was large. The edge, connector, and branch classes were dominant when P had intermediate values, and within that range, the edge class dominated at larger values of P while the connector class dominated at lower values of P. The branch class was never the most abundant class, and there was less of it with increasing SE size. The range of P over which particular classes occurred depended on SE size. With the exception of SE1, a structural class usually occurred with a relatively high frequency if it occurred at all. In comparison to other SE sizes, for a fixed P there was usually more differentiation among patterns for SE1.

The core class must occupy nearly all of a map when P is large (excepting only the map boundary of edge pixels when P = 100%), and it is logical that the patch class dominates when P is small. To interpret the empirical frequency distributions, it is convenient to consider how the proportions of each class changed with increasing P. Above a certain value of P which depends on SE size, the forest on a random map changed from the patch class to the branch and connector classes, and the branch class usually appeared briefly before disappearing as connectors became more abundant with increasing P (Fig. 3). These transitions were interpreted as shifts from mostly unconnected

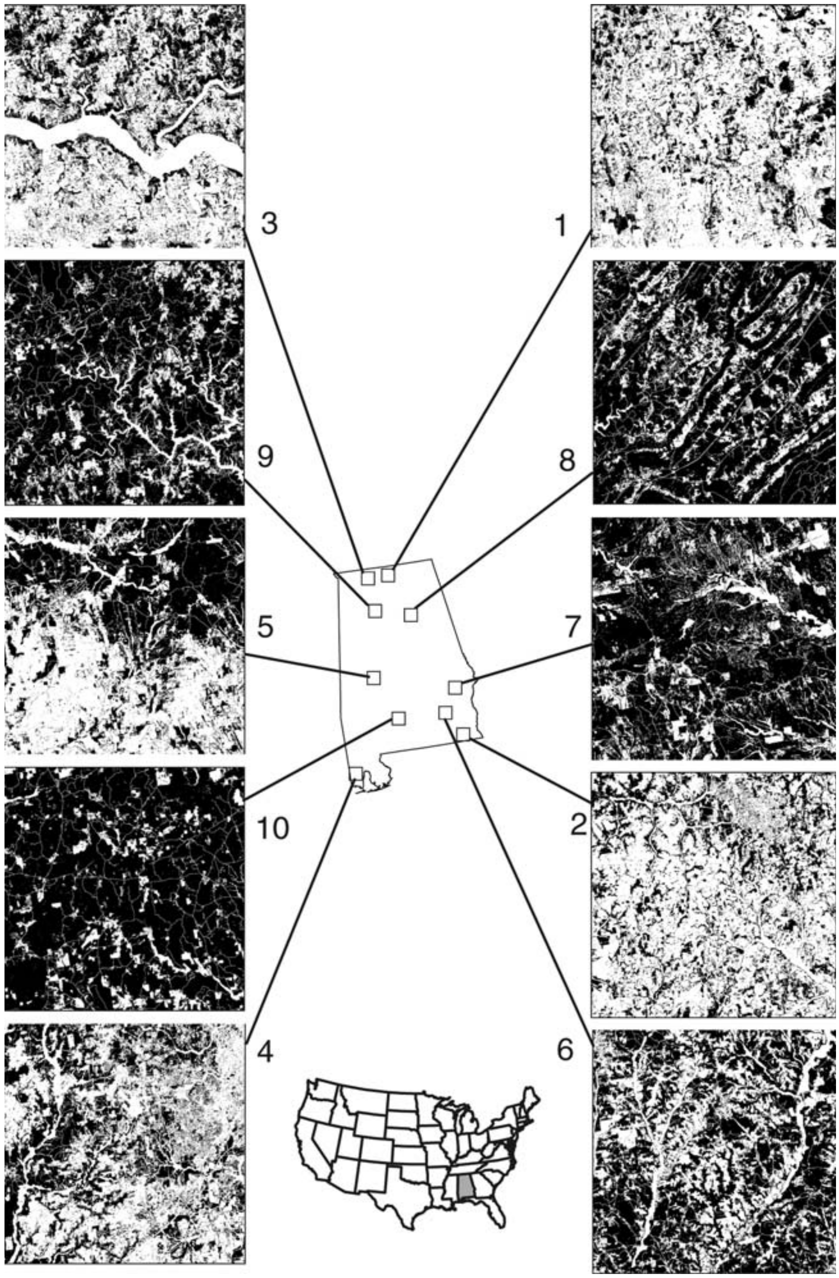

Fig. 2 Location of ten forest maps (Alabama, USA) used to illustrate structural classes on real maps. Each location is 30.72 × 30.72 km. Forest is shown as black pixels. The maps are numbered in order of increasing percent forest

forest (patch), to mostly connected forest (connector), with an intermediate stage of broken connectors (branches). As P increased further, edge forest increased as larger regions of core forest appeared. Perforations appeared when the core forest clusters became sufficiently large to contain recognizable holes.

As the SE size increased, the six curves that describe the proportions of structural classes shifted to the right in Fig. 3 but the sequence of relative abundances in relation to increasing P was the same. The movement to the right was due to the contingency feature of the analysis, whereby a change from patch to other classes depends on the co-existence of the core class. With increasing SE size, a larger P was needed to guarantee at least one core pixel on a random map (Fig. 4).

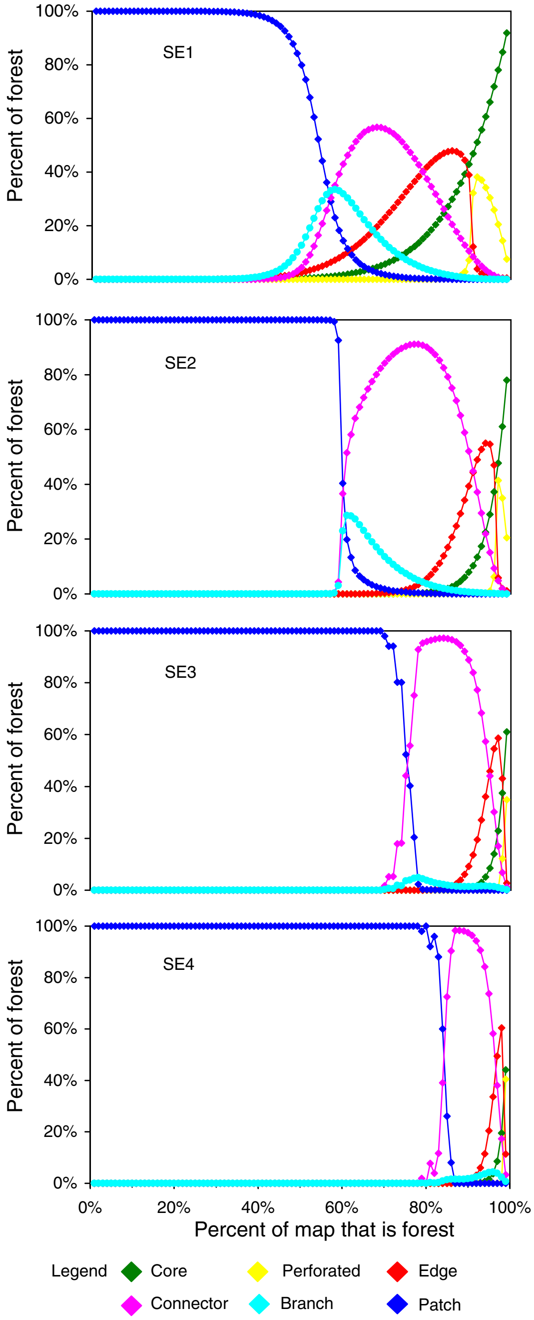

Fig. 3 Trends in structural classes in relation to the percent forest on the map, for four structuring element (SE) sizes. Each data point is the average of 50 random maps. The *x*-axis shows the percent of forest on a map, and the *y*-axis shows the percent of the forest on a map that is in the indicated structural class

With a few important exceptions that will be discussed later, the standard deviations of the proportions shown in Fig. 3 were typically smaller

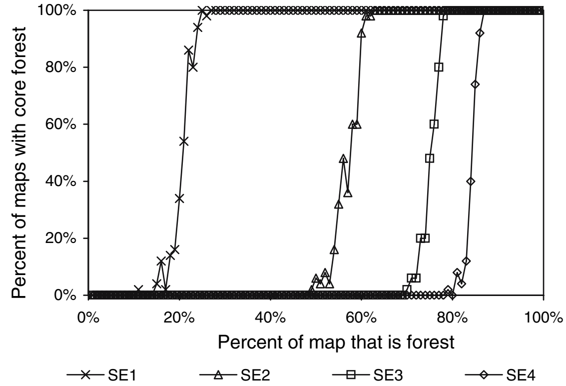

Fig. 4 Trends in percent of maps with core forest in relation to the percent forest on the map, for four structuring element (SE) sizes. Each data point is the average of 50 random maps

than 0.01% (Fig. 5). The relatively small standard deviations indicated that for a given P, the relative abundance of different structural classes was consistent between random maps. In summary, the logical geometric interpretations of the empirical frequency distributions with respect to P (Fig. 3) and the low between-map variance of the distributions (Fig. 5) support the use of random neutral models as a standard for comparisons of structural classes identified by mathematical morphology. Most of the variance in the six structural classes on random neutral maps was accounted for by the size of the SE and the percent of the map that was occupied.

Phase changes among structural classes

For the three largest SE sizes, the visual impression is that the transition from patch-dominated maps to branch- and connector-dominated maps is an abrupt transition with the critical or threshold value of P increasing with SE size (Fig. 3). For the two smallest SE sizes, there is also a visual impression of an abrupt transition from the edge class to the perforated class (Fig. 3). The standard deviations of the proportions (Fig. 5) provide evidence that these abrupt transitions represent 'phase changes' (an abrupt sudden change in one or more physical properties of a system). A common observation in a dynamical system is that phase changes are accompanied by sudden increases followed by sudden decreases in system

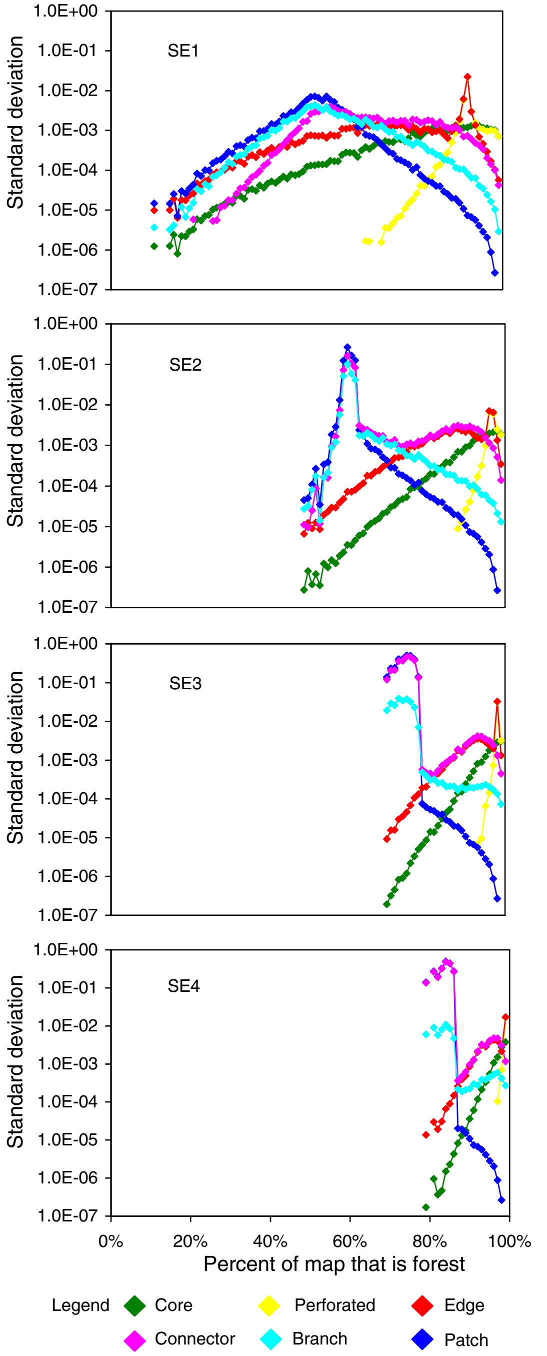

Fig. 5 Variation of the proportions of structural classes among maps in relation to the percent forest on the map, for four structuring element (SE) sizes. Each data point is the standard deviation among 50 random maps

variance, as the system shifts from one stable state to another (e.g., Nicolis and Prigogine 1989). With that rationale, the sudden increases (up to ~2.5 orders of magnitude) followed by comparable decreases in the standard deviations over small (<5%) intervals of P (Fig. 5) indicate the existence of phase changes involving the structural classes exhibiting high variances, and identify the values of P at which they occur. The transitions from patch to branch and connector, and from edge to perforated, are phase changes for some SE sizes.

The phase changes from patch-dominated maps to branch- and connector-dominated maps are explained by percolation theory (e.g., Stauffer and Aharony 1994) together with the contingency feature of the analysis. According to percolation theory, on random binary maps there is a critical P, denoted P*, above which a connected cluster of forest pixels is almost certain to percolate or span the entire map. P* does not apply to individual maps; it is a statistical attribute that is estimated from a sample of random binary maps. Furthermore, P* depends on the connectivity rule, and P* = 0.59275 for 4-neighbor connectivity on a raster map (Plotnick and Gardner 1993). For P < P*, there are many small forest clusters that do not percolate. For P > P*, most of the classical landscape pattern indices are constrained by the existence of the percolating cluster (Gardner et al. 1987; Turner et al. 2001). Since we used the 4-neighbor definition of connectivity in the pattern analysis, we expected that there may be important changes in structural classes for P ~ 59%. However, percolation theory does not guarantee that changes in structural classes from mathematical morphology will be phase changes, nor does it explain why the observed phase changes did not all occur when P ~ 59%.

A full explanation depends on the contingency feature of the pattern analysis. The patch to branch and connector phase change occurred if the appearance of the core class followed the emergence of the percolating cluster of forest. For SE2, 30 of 50 maps had some core when P ~ 59%, and 47 of 50 maps had some core when P ~ 60% (Fig. 4). For this SE, the coincidence of a percolating cluster and the core class for the same P caused abrupt changes in structural classes on most of the random maps. For SE3 and SE4, the core class first emerged when P > P* on all random maps (Fig. 4) and its emergence precipitated immediate changes in structural classes since the percolating cluster was already present. The increase in the threshold P for this phase change can be explained by the fact that the core class is less likely to occur with increasing SE size. For SE1, the core class first occurred when P ~ 11% and all maps had some core when P ~ 25% (Fig. 4). However, a phase change was not observed with SE1 because $P < P^*$, and therefore the transitions among structural classes only occurred over sub-regions of the map, which in turn explains why the changes in the proportions of structural classes were gradual (and not abrupt) with increasing P (Fig. 3).

A different explanation is required for the second phase change from the edge class to the perforated class because the critical values of P are different, and because the core class exists on all maps at values of P that that are less than the critical values (Fig. 4). Let Pc be the percentage of the map (not the percentage of forest) that comprises the core class, and let $\overline{Pc} = 1 - Pc$ be the percentage of the map that comprises all the remaining pixels, including non-forest pixels. The transitions from edge to perforated classes occurred for all SE sizes at the value of P for which Pc first exceeded ~40% (Fig. 6), or equivalently, at the value of P for which $\overline{Pc}$ first dropped below ~60%. This observation was obscured for larger SE sizes in Fig. 6 because Pc changed much more rapidly with respect to P near the critical threshold (to see this, note that the distance between the symbols on the lines in Fig. 6 is proportional to the difference in P).

Assuming a random distribution of core and non-core pixels, percolation theory predicts percolation of the non-core pixels when $\overline{Pc} > P^* = 0.59275$ (equivalently, when $Pc < (1 - P^*) = 0.40725$) which is the case when the value of P (and thus, Pc) is relatively small. By the definition of a percolating cluster, no part of this percolating non-core cluster can be contained within a core forest cluster, and therefore, by the definition of structural classes, none of its parts can be labeled as perforated. As P increases, the proportion of forest that is core increases (Fig. 3),

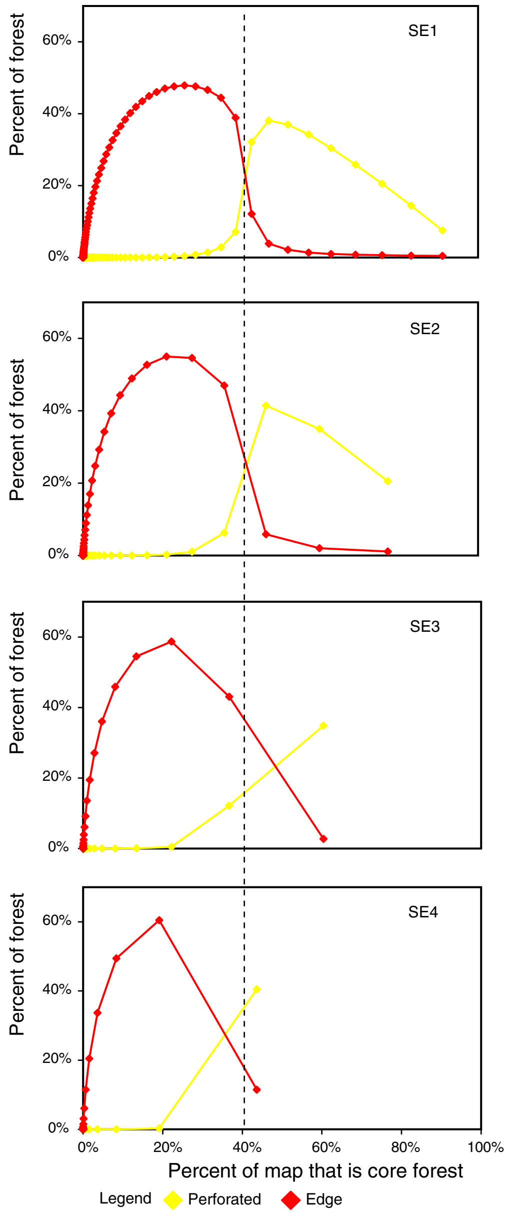

Fig. 6 Trends in the edge and perforated classes in relation to the percent of the map that is core forest, for four structuring element (SE) sizes. The dashed line at 40% core forest is shown for comparisons among SE sizes. *Note*: the spacing of symbols along a trend line reflects increases of percent forest in steps of 1%. The left-most data point represents P = 1% forest and the right-most data point represents P = 99% forest. The data points are concentrated on the left because there are many small values of P for which there is no core forest and hence, no edge or perforated forest. Each data point is the average of 50 random maps

which implies increases of Pc and decreases of $\overline{\text{Pc}}$. At the value of P for which Pc exceeds 0.40725, $\overline{\text{Pc}}$ drops below the critical value of 0.59275, and the percolating cluster of non-core pixels is broken into smaller clusters. Some of those smaller clusters are now contained within clusters of core and the non-forest portions of those non-core clusters become holes in core forest clusters. Because the local context of the non-forest pixels has changed abruptly from 'exterior' to 'interior,' the individual forest pixels that were formerly edge (exterior perimeter adjacent to a non-forest background) now become perforated (interior perimeter adjacent to a non-forest hole). The change of context affects only the edge and perforated classes because they are the only classes that are defined in terms of an exterior or interior context.

Even when the non-core pixels percolate, there still can be internal holes in core clusters; the existence of a percolating cluster does not imply that all non-core pixels are part of the percolating cluster. That is why some perforated forest exists at relatively low values of P for which Pc < 0.40725. However, the frequency of perforated forest is very low because the definition of perforated forest requires that it be isolated from external edge. For small P, most of the core clusters are small in extent, and as a result, any holes that do occur in those clusters are not usually far enough from external edge to meet the definition of perforated forest.

The critical value of P increases with SE size because, for a fixed P, the core class is less likely to occur as SE size increases (Fig. 4), and because the isolation requirement is more constraining for larger SE sizes. Considering the isolation requirement described in the "Methods" section, edges are wider for larger SE sizes (Fig. 1), and therefore forest that is adjacent to holes must be even further from external edge to be labeled as perforated. For a given SE size, the critical values of P for the transition from edge to perforated classes are larger than for the transition from patch to branch and connector classes because Pc increases at a slower rate than P itself, and is therefore always smaller than P.

The transitions from edge to perforated classes are less distinct than the transitions from patch to branch and connector classes because when P is large enough to produce the change of context, it is also large enough to guarantee that most of the forest is in the core class, which in turn limits the proportion of the edge class. These results appear to be the first demonstration of multiple phase changes corresponding to different aspects of landscape pattern on random neutral maps.

Applications

Because structural classes from mathematical morphology depend on SE size, it is natural to ask, "Which SE size is best for analysis of landscape patterns?" Our results suggest that smaller SE sizes provide more differentiation among structural classes over larger ranges of P (Fig. 3), which is equivalent to saying that smaller SE sizes provide more information about pattern. However, a neutral model analysis of random maps should not be used to make decisions about the analysis of real maps; it can only inform those choices and assist in interpreting the results. All pattern measurements are scale-contingent and since SE size is part of the definition of the scale of observation, we recommend using several SE sizes as well as varying other aspects of the scale of observation.

The observed proportions of different structural classes on real maps are shown in Fig. 7 for all SE sizes. Note that each vertical 'stack' of data points in Fig. 7 represents the results for one real map, and that the differences between that map and a random map are expressed as differences in the proportions of structural classes for the value of P on the real map. For a fixed P, the differences between a real map (Fig. 7) and the neutral maps (Fig. 3) represent the degree of non-randomness on the real map. It is easy to conclude based on visual evidence alone (compare Figs. 3 and 7) that none of the real maps exhibited a completely random forest distribution. For example, most of the real maps contained much more of the core, edge, and branch classes than was contained on random maps for the same P. The differences between real and random maps were not consistent across SE sizes, which opens up the possibility that

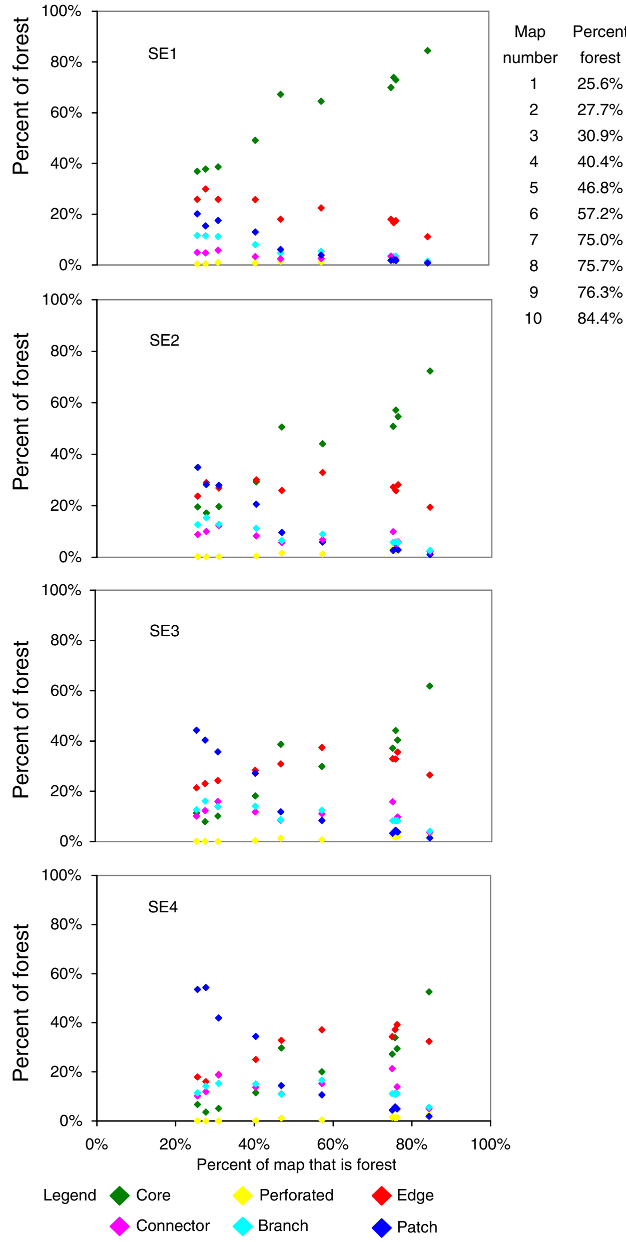

Fig. 7 Summary of structural classes from mathematical morphology on real forest maps. The *x*-axis shows the percent of forest on a real map, and the *y*-axis shows the percent of the forest on that map that is in the indicated structural class. Each real map is represented by a vertical stack of data points showing the percent of forest in each structural class, and each stack is located along the *x*-axis according to the actual percent forest on that real map. See text for additional explanation. The indicated map number can be located in Fig. 2 for comparisons

real-world patterns were ‘caused’ at different scales, in different places. The distribution of forest among structural classes on the real maps suggested that some structural classes are predictable from P and additional research is needed to test the statistical significance of these dependencies and to explore their geographic variance. While the differences between real and neutral maps were visually apparent in these examples, quantitative measures of those differences are needed in practice to assess the departure of real landscapes from either ‘reference’ or ‘desired’ landscape patterns. Candidate measures include the familiar Euclidean and Mahalanobis distances, and the Tran distance (Tran et al. 2006) which could be used to partition total distance into components that are attributable to structural classes.

In summary, a neutral model analysis is part of the foundation needed to move mathematical morphology into wider application for landscape pattern analysis in ecological research and assessment. While random binary maps are not realistic, a neutral model analysis of them was found to be useful for setting standards for comparisons with real land-cover maps. This study also established logical connections between mathematical morphology and percolation theory, which is important because percolation theory is widely appreciated in landscape ecology. On random binary raster maps, phase changes among structural classes from mathematical morphology were explained in part by percolation theory as it pertains to overall map composition (P), and in part as it pertains to pattern composition (Pc). The critical thresholds were not the same as predicted by percolation theory because mathematical morphology considers aspects of pattern that are not included in percolation theory. Our results appear to be the first demonstration of multiple phase changes corresponding to different aspects of landscape pattern on random neutral maps.

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References

Caswell H (1976) Community structure: a neutral model analysis. Ecol Monogr 46:327–354

Forman RTT (1995) Land mosaics: the ecology of landscapes and regions. Cambridge University Press, Cambridge, UK

Freemark K, Bert D, Villard M-A (2002) Patch-, landscape-, and regional-scale effects on biota. In: Gutzwiller KJ (ed) Applying landscape ecology in biological conservation. Springer-Verlag, New York NY, pp 58–83

Gardner RH (1999) RULE: a program for the generation of random maps and the analysis of spatial patterns. In: Klopatek JM, Gardner RH (eds) Landscape ecological analysis: issues and applications. Springer-Verlag, New York, NY, pp 280–303

Gardner RH, Milne BT, Turner MG, O'Neill RV (1987) Neutral models for the analysis of broad-scale landscape pattern. Landsc Ecol 1:19–28

Gardner RH, O'Neill RV (1991) Pattern, process and predictability: the use of neutral models for landscape analysis. In: Turner MG, Gardner RH (eds) Quantitative methods in landscape ecology. Springer-Verlag, New York, NY, pp 289–308

Geographic Data Technology (2002) Dynamap/2000 user manual. Geographic Data Technology, Inc., Lebanon, NH

Gustafson EJ, Parker GR (1992) Relationships between landcover proportion and indices of landscape spatial pattern. Landsc Ecol 7:101–110

Milne BT (1992) Spatial aggregation and neutral models in fractal landscapes. Am Nat 139:32–57

Nicolis G, Prigogine I (1989) Exploring complexity: an introduction. W.H. Freeman and Company, New York, NY

O'Neill RV, Gardner RH, Turner MG (1992) A hierarchical neutral model for landscape analysis. Landsc Ecol 7:55–61

Pearson SM, Gardner RH (1997) Neutral models: useful tools for understanding landscape patterns. In Bissonette JA (ed) Wildlife and landscape ecology: effects of pattern and scale. Springer-Verlag, New York, NY, pp 215–230

Plotnick RE, Gardner RH (1993) Lattices and landscapes. In: Gardner RH (ed) Lectures on mathematics in the life sciences: predicting spatial effects in ecological systems. American Mathematical Society, Providence, RI, pp 129–157

Riitters KH, Wickham JD, O'Neill RV, Jones KB, Smith ER (2000) Global-scale patterns of forest fragmentation. Ecol Soc 4(2):3

Riitters KH, Wickham JD, Coulston JW (2004) Use of road maps in United States national assessments of forest fragmentation. Ecol Soc 9(2):13

Soille P (2003) Morphological image analysis: principles and applications, 2nd edn. Springer-Verlag, Berlin

Stauffer D, Aharony A (1994) Introduction to percolation theory, 2nd edn. Taylor & Francis, London

Tran LT, O'Neill RV, Smith ER (2006) A generalized distance measure for integrating multiple environmental assessment indicators. Landsc Ecol 21:469–476

Turner MG, Gardner RH, O'Neill RV (2001) Landscape ecology in theory and practice: pattern and process. Springer-Verlag, New York, NY

Vogelmann JE, Howard SM Yang L, Larson CR, Wylie BK, Van Driel N (2001) Completion of the 1990s national land cover data set for the conterminous United States from Landsat Thematic Mapper data and ancillary data sources. Photogram Eng Remote Sensing 67:650–662

Vogt P, Riitters KH, Estreguil C, Kozak J, Wade TG, Wickham JD (2007a) Mapping spatial patterns with morphological image processing. Landsc Ecol 22:171–177. doi: 10.1007/s10980–006-9013-2

Vogt P, Riitters KH, Iwanowski M, Estreguil C, Kozak J, Soille P (2007b) Mapping landscape corridors. Ecol Indic 7:481–488. doi:10.1016/j.ecolind.2006.11.001.

With KA, King AW (1997) The use and misuse of neutral landscape models in ecology. Oikos 79:219–229