Factor levels for density comparisons in the split-block spacing design

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The split-block spacing design is a compact test of the effects of within-row and between-row spacings. But the sometimes awkward analysis of density (i.e., trees/ha) effects may deter use of the design. The analysis is simpler if the row spacings are chosen to obtain a balanced set of equally spaced density and rectangularity treatments. A spacing study in poplar (*Populus* spp.) illustrates the approach.

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Un modèle d'espacements en split block est un test compact des effects d'espacements intra-rangée, et inter-rangées. Mais quelquefois, les effets imprévisibles de l'analyse de densité (nombre d'arbres à l'hectare) peuvent détourner l'emploi du modèle. L'analyse est simple si les espacements des rangées sont choisis pour obtenir un ensemble balancé de traitements rectangulaires et de densité également répartie. Une étude d'espacements de peupliers (*Populus* spp.) illustre l'approche.

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Introduction

Many experimental designs may be considered for forestry spacing trials (e.g., Cochran and Cox 1957; Nelder 1962). There is no single best design because there are trade-offs between statistical, biological, and cost considerations (Smith 1959; Curtis 1983). Lin and Morse (1975) introduced the split-block spacing design by randomizing within-row and between-row tree spacings in a split-block design (e.g., Steel and Torrie 1980). The design is attractive because plots are compact and easily maintained in the field. They retain statistical validity and accommodate biological considerations.

One drawback of the split-block spacing design is that although individual plots are characterized by a particular density (trees per hectare) and rectangularity (ratio of within-row to between-row spacings), a convenient set of densities and rectangularities is not necessarily obtained. If interest centers on density, the row spacings could be chosen so that density comparisons are not conditioned upon rectangularity. This report illustrates the approach.

Design and analytical approach

We shall illustrate the general split-block design with two experimental factors, denoted R_i ($i = 1,\ldots,r$ levels) and C_j ($j = 1,\ldots,c$ levels). The $r \times c$ treatments are a factorial combination of the levels of the two experimental factors. Each treatment may be denoted as $\{R_i, C_j\}$. The split-block nomenclature refers to the randomization of treatments to experimental units in the field. Suppose that a matrix with r rows and c columns has cells to be filled with these treatments. In a completely random design, the $\{R_i, C_j\}$ would be assigned at random to the cells in the matrix. In a split-block design, the R_i are assigned at random to the rows, and the C_j to the columns.

When R_i and C_j are, respectively, defined as within-row and between-row spacings of trees, the split-block spacing design is obtained (Lin and Morse 1975). For R_i and C_j in metres, the density of $\{R_i, C_j\}$ in trees per hectare is given by $D_{ij} = 10\,000/(R_i \times C_j)$, and the rectangularity is $L_{ij} = R_i/C_j$. When the spacing is square, L_{ij} equals 1.0. For other rectangular spacings, L_{ij} will be less than or greater than 1.0. L_{ij} measures the deviation from squareness and reflects the orientation of the long axis of each plot.

In the analysis of variance, the main effects of R and C test the significance of within-row and between-row spacings (see Lin and Morse 1975). Comparisons of particular density and rectangularity treatments are then obtained via contrasts of the individual $\{R_i, C_j\}$, accounting for within-row or within-column correlations (if necessary) to compute standard errors. The drawback of this approach is that the density (D) and rectangularity (L) treatments do not necessarily have a factorial structure. Thus, some comparisons of D may not be independent of L. To overcome this, the analyst must compare subsets of density treatments that have the same rectangularity, or ignore rectangularity altogether.

A better alternative is to choose the levels of R and C such that the interesting contrasts of D are independent of L. One possible design has equal numbers of levels for R and C (i.e., $r = c$), the same levels for the two factors (i.e., $R_i = C_j$ for $i = j$), and the levels in geometric progression (i.e., $R_i = f \times R_{i-1}$ for $i = 2,\ldots,r$, where f is a convenient constant). This choice results in geometric progressions for both density and rectangularity treatments. Orthogonal polynomials may be used to find their sums of squares and to partition their regression components

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TABLE 1. Density and rectangularity treatments in a poplar spacing trial (X indicates that the treatment was tested)

	Density[a]								
Rectangularity[b]	748	1057	1495	2114	2990	4228	5980	8456	11960
0.25					X				
0.35				X		X			
0.50			X		X		X		
0.71		X		X		X		X	
1.00	X		X		X		X		X
1.41		X		X		X		X	
2.00			X		X		X		
2.83				X		X			
4.00					X				

[a]Number of trees per hectare.
[b]Ratio of within-row spacing to between-row spacing.

TABLE 2. Plot means of the logarithm of stem diameter used in the numerical example

Within-row spacing (m)	Between-row spacing (m)									
	0.914		1.293		1.829		2.586		3.658	
0.914	0.96	0.95	1.00	0.76	1.14	1.01	1.22	1.01	1.22	1.17
	0.69	0.84	0.80	1.12	0.98	1.20	1.06	1.25	1.01	1.16
1.293	1.08	1.04	1.14	1.02	1.32	1.09	1.32	1.20	1.35	1.22
	0.64	0.76	0.58	0.88	0.85	1.11	1.08	1.11	0.90	1.27
1.829	1.12	0.94	1.31	1.03	1.39	1.27	1.28	1.27	1.40	1.40
	0.89	1.19	0.83	1.20	0.63	1.27	0.79	1.43	0.84	1.29
2.586	0.86	1.15	1.24	1.24	1.37	1.32	1.49	1.31	1.46	1.36
	1.01	1.20	0.96	1.15	0.72	1.01	1.01	1.44	0.99	1.15
3.658	1.29	1.12	1.40	1.28	1.39	1.33	1.44	1.21	1.39	1.28
	0.92	1.13	0.96	0.91	0.96	1.08	1.05	1.16	1.18	1.46

NOTE: The data are scaled to an arbitrary zero. Each treatment was replicated four times.

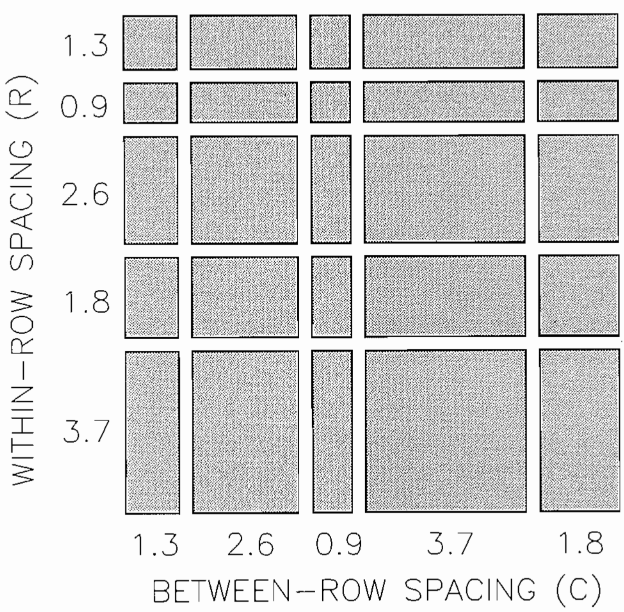

FIG. 1. Field plot layout for one replication of the split-block spacing experiment. The spacings are rounded to the nearest 0.1 m.

TABLE 3. Standard split-block analysis of variance of the example data

Source	df	MS	F-ratio	$P(>F)$
Replications	3	0.6199	—	—
Within-row spacing (R)	4	0.1133	3.74	0.03
Linear	1	0.4287	14.16	<0.01
Quadratic	1	0.0033	0.11	0.75
$R \times$ replications	12	0.0302	2.38	0.02
Between-row spacing (C)	4	0.2097	19.25	<0.01
Linear	1	0.8141	74.73	<0.01
Quadratic	1	0.0057	0.52	0.48
$C \times$ replications	12	0.0109	0.86	0.59
$R \times C$	16	0.0097	0.76	0.72
Error	48	0.0127	—	—
Total	99			

(e.g., Kendall and Stuart 1973). Although not a full factorial, the resulting structure of the density and rectangularity treatments provides some contrasts of density that are independent of rectangularity, as illustrated later in this paper.

The analysis of density and rectangularity proceeds by pooling the sums of squares for R, C, and $R \times C$ into treatments and pooling the interactions with replications into error. This collects all information about density and rectangularity, including that contained in within-row and between-row spacings, into the treatment effect. The pooling of error terms is valid if within-row and within-column correlations are judged unimportant. The validity of the

TABLE 4. Coefficients of orthogonal polynomials used to partition the treatment sums of squares in the examples (lin and quad denote linear and quadratic contrasts, respectively)

Treatment				Coefficient							
				R		*C*		*D*		*L*	
R	*C*	*D*	*L*	Lin	Quad	Lin	Quad	Lin	Quad	Lin	Quad
0.914	0.914	11 960	1.00	−2	2	−2	2	4	12	0	−4
	1.293	8 456	0.71	−2	2	−1	−1	3	5	−1	−3
	1.829	5 980	0.50	−2	2	0	−2	2	0	−2	0
	2.586	4 228	0.35	−2	2	1	−1	1	−3	−3	5
	3.658	2 990	0.25	−2	2	2	2	0	−4	−4	12
1.293	0.914	8 456	1.41	−1	−1	−2	2	3	5	1	−3
	1.293	5 980	1.00	−1	−1	−1	−1	2	0	0	−4
	1.829	4 228	0.71	−1	−1	0	−2	1	−3	−1	−3
	2.586	2 990	0.50	−1	−1	1	−1	0	−4	−2	0
	3.658	2 114	0.35	−1	−1	2	2	−1	−3	−3	5
1.829	0.914	5 980	2.00	0	−2	−2	2	2	0	2	0
	1.293	4 228	1.41	0	−2	−1	−1	1	−3	1	−3
	1.829	2 990	1.00	0	−2	0	−2	0	−4	0	−4
	2.586	2 114	0.71	0	−2	1	−1	−1	−3	−1	−3
	3.658	1 495	0.50	0	−2	2	2	−2	0	−2	0
2.586	0.914	4 228	2.83	1	−1	−2	2	1	−3	3	5
	1.293	2 990	2.00	1	−1	−1	−1	0	−4	2	0
	1.829	2 114	1.41	1	−1	0	−2	−1	−3	1	−3
	2.586	1 495	1.00	1	−1	1	−1	−2	0	0	−4
	3.658	1 057	0.71	1	−1	2	2	−3	5	−1	−3
3.658	0.914	2 990	4.00	2	2	−2	2	0	−4	4	12
	1.293	2 114	2.83	2	2	−1	−1	−1	−3	3	5
	1.829	1 495	2.00	2	2	0	−2	−2	0	2	0
	2.586	1 057	1.41	2	2	1	−1	−3	5	1	−3
	3.658	748	1.00	2	2	2	2	−4	12	0	−4

TABLE 5.
A. Treatment means from the split-block analysis of variance

Within-row spacing (m)	Between-row spacing (m)					
	0.914	1.293	1.829	2.586	3.658	Mean
0.914	0.86	0.92	1.08	1.14	1.14	1.03
1.293	0.88	0.91	1.09	1.18	1.19	1.05
1.829	1.04	1.09	1.14	1.19	1.23	1.14
2.586	1.06	1.15	1.11	1.31	1.24	1.17
3.658	1.12	1.14	1.19	1.22	1.33	1.20
Mean	0.99	1.04	1.12	1.21	1.23	

B. Standard errors from the split-block analysis of variance

	SE	df
Between-row spacing mean	0.023	12
Within-row spacing mean	0.039	12
Mean in body of table	0.056	48

pooling can be gauged by the significance of $R \times$ replications and $C \times$ replications in the analysis of variance, or by other considerations. The total sum of squares for the $r \times c$ treatments is then partitioned into the sums of squares for the $2r - 1$ densities and the $2c - 1$ rectangularities. (The extra $(r \times c) - (2r - 2) - (2c - 2) - 1$ degrees of freedom may be used to repartition the main effects of R and C if that is desired.) In general, the total sums of squares for D, L, R, or C may be obtained in this fashion, but they are not necessarily orthogonal.

Numerical example

In a poplar spacing study, the split-block design was laid out in four replications with $r = c = 5$, and levels of R and C equal to 0.914, 1.293, 1.829, 2.586, and 3.658 m. The geometric constant f equals the square root of 2.0. This design gave nine densities ranging from 748 to 11 960 trees/ha and nine rectangularities ranging from 0.25 to 4.00 (Fig. 1). The balanced structure of density and rectangularity treatments obtained for each replication is evident in Table 1.

Scaled measurements of 3rd-year stem diameter were used for this illustration. Each plot contained 81 trees, of which 25 were measured. The variances of plot means for these unequal-area plots were judged homogeneous in the logarithmic scale. Plot means of transformed data were used in the analysis of variance (Table 2).

Results of the usual split-block analysis of variance are shown in Table 3. In this case, both the within-row and between-row spacing effects are significant. The F-test for within-row spacing by replication interaction ($F_{12,48} = 2.38$) indicates that within-row correlation may be important, but this has been ignored for this example. The linear and quadratic regression components have been partitioned from the main effects (i.e., R and C) using orthogonal polynomials (Table 4). The linear components explain most of the variation in average diameter. In this analysis, the study of density and rectangularity effects would proceed by contrast-

TABLE 6. Alternate analysis of variance of the example data

Source	df	MS	F-ratio	$P\ (>F)$
Replications	3	0.6199	—	—
Treatments	24	0.0603	3.93	<0.01
Density linear	1	1.2122	79.03	<0.01
Density quadratic	1	0.0406	2.64	0.11
Rectangularity				
Linear	1	0.0306	2.00	0.16
Quadratic	1	0.0112	0.73	0.40
Density linear × rectangularity linear	1	0.0002	0.01	0.92
Error	72	0.0153	—	—
Total	99			

ing individual treatment means with appropriate standard errors (Table 5).

The alternate analysis of variance made possible by the special choices of R_i and C_i partitions density and rectangularity effects directly (Table 6). Different sets of orthogonal polynomials (Table 4) were used to obtain the sums of squares for the linear and quadratic components of density and rectangularity, and one interaction. These contrasts are a mutually orthogonal set. Average plot diameter was found to be linearly related to density but unrelated to rectangularity, and there was no interaction between linear components of density and rectangularity. The higher order polynomials will not contribute significantly to the fit of the response surface ($F_{11,72} = 0.90$). Therefore, a simple linear regression of average plot diameter on density will adequately summarize the data. This conclusion is superficially similar to that reached in the previous analysis. The advantage of this formulation is that the conclusion may be stated directly in terms of density rather than within-row and between-row spacings. In addition, regression equations may be more easily developed with density as the independent variable.

Conclusion

There is but one other published application of the split-block spacing design in forestry (Amateis *et al.* 1988). Perhaps the design should be used more often, if only because it efficiently uses experimental area for notoriously large spacing trials. The potential statistical difficulty of within-row and within-column correlations is probably no more a problem in this design than in any other that may be used on nonuniform experimental sites. Although the example showed plots of unequal sizes, the number of planted rows or columns may be modified to obtain equal plot sizes. The increased site homogeneity and cost savings achieved certainly argues for a compact design.

Perhaps the usually awkward analysis of density has been a deterrent to more popular use of the split-block design. In many cases, the comparison of density treatments has to account for different rectangularity, and it is not always easy to test the importance of the latter. The example shows that this can be overcome through judicious choices of within-row and between-row spacings.

The objective of most forest spacing trials is to find a density that is in some way optimum, and the planned regression of yield on density should be considered when selecting treatments. The example tests the widest range of rectangularity at medium density and the widest range of density at square spacing. This is more efficient for finding optimum spacings when the response is nonlinear than for estimating linear response functions. For nonlinear responses, a good selection of treatments would place the expected optimum near the center of the design.

Rectangularity may be practically unimportant, but this is another reason to use the split-block spacing design. The split-block spacing design takes advantage of the practical independence of yield from rectangularity to create a compact field layout. If the effect of rectangularity is unknown, this design provides a direct test of its importance. It also estimates components of the density effect that are independent of rectangularity. More general yield–density–rectangularity models may always be explored by ignoring the split-block design altogether.

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