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Implementing Optimal Thinning Strategies

Kurt Riitters and J. Douglas Brodie

ABSTRACT. Optimal thinning regimes for achieving several management objectives were derived from two stand-growth simulators by dynamic programming. Residual mean tree volumes were then plotted against stand density on density management diagrams. The results supported the use of density management diagrams for comparing, checking, and implementing the results of optimization analyses. FOREST SCI. 30:82–85.

ADDITIONAL KEY WORDS. *Pseudotsuga menziesii, Pinus ponderosa,* growth and yield, relative density, silviculture.

MATHEMATICAL PROGRAMMING TECHNIQUES have been recently applied to stand-growth simulators to derive optimal thinning regimes for achieving specific management objectives (e.g., Adams and Ek 1974, Martin 1978). Criticisms of this approach are that: (a) results are specific to the stands included in the analysis; (b) it is often difficult to check the results of optimization analyses against empirically derived results; and (c) results obtained from different simulators are not easily compared.

When comparing the results of optimization analyses of thinning in Douglas-fir [*Pseudotsuga menziesii* (Mirb.) Franco] and ponderosa pine (*Pinus ponderosa* Laws.), we used a more general descriptor of stand density to facilitiate the comparison. Such a descriptor may also prove to be useful for checking and implementing the results of optimization analyses for other species.

OPTIMAL THINNING REGIMES

Dynamic programming algorithms for deriving optimal thinning regimes for several management objectives have been developed for even-aged stands of Douglas-fir (Brodie and

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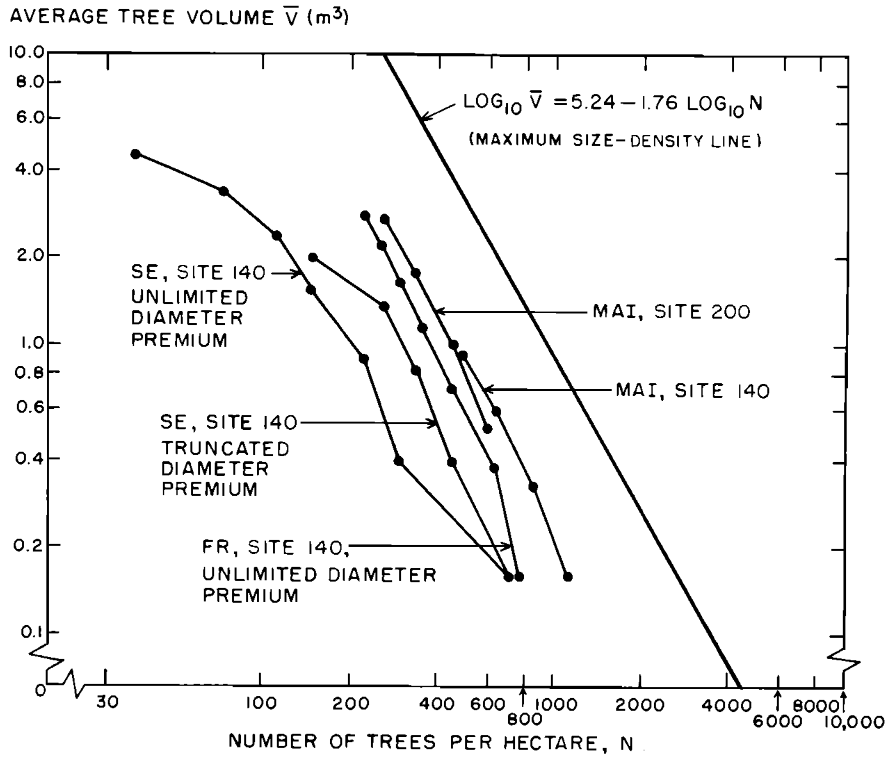

FIGURE 1. Density management diagram for a Douglas-fir stand-growth simulator and time trajectories of residual stand conditions for several examples of optimizations. (Economic assumptions are those described by Riitters and others 1982b. SE is soil expectation criterion. FR is forest rent criterion. MAI is mean annual increment criterion. Each node is separated by 10 years' growth and subsequent thinning. A diameter premium is used to assign higher prices to larger trees; in one example the price increase was truncated at 50 cm dbh.)

Kao 1979) and ponderosa pine (Riitters and others 1982a). The Douglas-fir algorithm uses relationships abstracted from the DFIT stand-growth simulator (Bruce and others 1977) to estimate growth and yield, while the PPINE simulator (Hann 1980) is employed in the ponderosa pine algorithm.

We used these models to derive, for each species, thinning regimes that maximize either soil expectation, forest rent, or mean annual increment. Results of these analyses are known to depend on the particular economic assumptions made (c.f. Brodie and others 1978) and are probably dependent on the particular stand-growth simulator. We wanted to know if the choice of management objective consistently altered the optimal thinning strategy. To answer this question, we needed a suitable descriptor of stand density.

A Density Descriptor

A graph of average tree volume versus number of trees, plotted on a log-log scale and with a maximum size-density line indicated, is known in the North American literature as a density management diagram (Drew and Flewelling 1979). For an even-aged stand, relative density can be computed from such a diagram as the ratio of actual to maximum number of trees for the given average tree volume. Relative density is theoretically independent of stand age, initial density, and site quality, and differences between species are incorporated in the position of the maximum size-density line (Yoda and others 1963). It should be noted that these relationships do not necessarily apply to uneven-aged stands.

It has been demonstrated that empirically or theoretically derived optimal thinning regimes for a particular management objective in an even-aged stand can be characterized

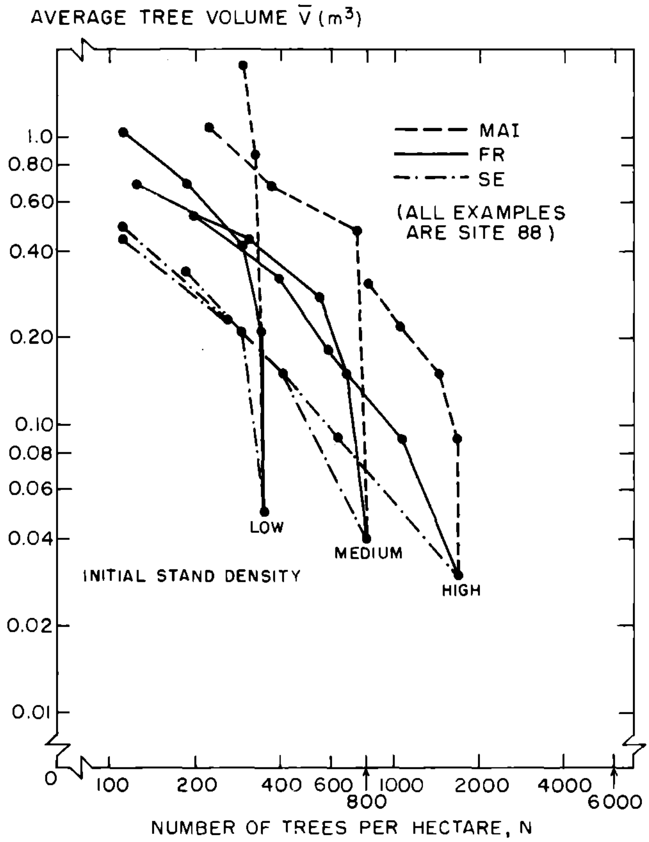

FIGURE 2. Time trajectories of residual stand conditions for three initial densities of ponderosa pine for several examples of optimizations. (Economic assumptions are those described by Riitters and others 1982a. Each node is separated by 20 years' growth and subsequent thinning.)

by a specific relative density (Tadaki 1964, White and Harper 1970, Drew and Flewelling 1979). That is, when graphed on a density management diagram, the time trajectory of residual densities for an optimal thinning regime will approximately parallel the maximum size-density line. That observation forms the basis for our choice of density management diagrams for comparing analytically derived optimal thinning regimes.

COMPARISONS OF OPTIMAL THINNING REGIMES

Theoretical arguments or empirical evidence can be used to position a maximum size-density line on a density management diagram, but for our purposes, the relevant density standards are those that are contained in the growth simulators themselves. In the case of Douglas-fir, a maximum size-density line can be derived from the growth relationships in the DFIT simulator.[1] That line is graphed in Figure 1, along with the results obtained in the optimization analyses. Figure 1 shows that each management objective is roughly

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[1] This we accomplished by relating maximum average tree volume and maximum number of trees to age and site index; evaluating these equations for 28 combinations of age and site; and using the resulting data to estimate the regression equation:

$$\log_{10}(\text{max. mean tree volume in cubic meters}) = 5.24 - 1.76 \log_{10} (\text{max. number of trees per hectare})$$
$$r^2 = 0.9991$$
$$S^2_{y \cdot x} = 0.000582.$$

characterized by a separate line lying parallel to the maximum size-density line. From previous analyses (Riitters and others 1982b) we know that the soil expectation examples maximize average tree volume while achieving near-maximum stand volume production and that the mean annual increment examples maximize total stand volume growth. The trajectories and the relative positions of the different regimes in Figure 1 are therefore consistent with earlier empirical recommendations (White and Harper 1970, Drew and Flewelling 1979).

Because of the complexity of the growth relationships in the PPINE stand-growth simulator, we could not derive a maximum size-density relationship for a ponderosa pine diagram. However, the results of optimization analyses for three initial densities, graphed in Figure 2, bear striking resemblance to those obtained for Douglas-fir. We tentatively concluded that the underlying growth-density relationships are similar for both stand-growth simulators and that the choice of management objective alters optimal thinning regimes in a manner consistent with earlier recommendations.

Discussion

Density management diagrams proved useful for rationalizing the results of optimization analyses of thinning in Douglas-fir and ponderosa pine. Optimal thinning strategies were consistently altered by the choice of management objective, and the results supported earlier empirical recommendations that were based on density management diagrams.

A mathematical programming approach with a stand-growth simulator, logging cost functions, diameter price premiums, and silvicultural inputs such as fertilizer provides much more explicit information for management decisions than do density diagrams alone. However, few such fully optimized models are available—and only for a limited number of species. Furthermore, many useful stand-growth simulators are not amenable to current optimization procedures. Although optimal regimes cannot always be deduced from a maximum size-density relationship alone, a knowledge of the density limits and growth-density relationships for a species, as well as relative density ranges for physical and economic objectives, should provide useful interim optimization guidelines.

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