Individual-Tree Basal Area Growth, Survival, and Total Height Models for Upland Hardwoods in the Boston Mountains of Arkansas

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ABSTRACT. *Models were developed for individual-tree basal area growth, survival, and total heights for different species of upland hardwoods in the Boston Mountains of north Arkansas. Data used were from 87 permanentplots located in an array of different sites and stand ages; the plots were thinned to different stocking levels and included unthinned controls. To test these three tree models, stand developmentfor 5 and 10 yr were simulated in terms of stand basal area/ac, numbers of trees/ac, and quadratic mean diameter. Percent mean differences for the three variables indicated no serious biases. A long-term projection of 100 yr to test model reasonableness showed development that would be consistent with these stands. These equations provideforest managers the first upland hardwood individual-tree growth models specifically for this region. South. J. Appl. For. 22(3):184–192.*

Forest stand growth and development are primary concerns of forest managers, and the information sought today goes far beyond the aggregate stand values they were content to use in the past. Today's managers desire not only aggregate information, but also data by species, diameters, heights, etc., so that they can apply their own merchantability standards to generate stand summaries. Such detailed data are also useful for nontimber applications, such as estimating potential mast production for wildlife. This appetite for detailed information can often be satisfied best with individual-tree growth models, where data for each stem are maintained for later summarization in a number of formats.

Although there are models for upland hardwoods in other regions (Harrison et al. 1986, Hilt 1985, Shifley 1987), we do not have one for oaks and other upland hardwood species for the Boston Mountains. Therefore, the purpose of this analysis is to develop individual-tree equations for basal area growth, survival, and total tree heights for upland hardwood species in this region.

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Study Area and Description

The Boston Mountains are the highest and southernmost member of the Ozark Plateau physiographic province (Figure 1). They form a band 30 to 40 miles wide and 200 miles long from north central Arkansas westward into eastern Oklahoma. Elevations range from about 900 ft in the valley bottoms to 2,500 ft at the highest point. The

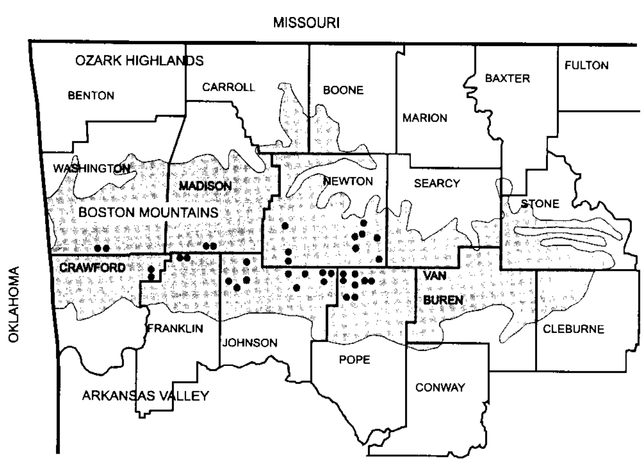


Figure 1. Boston Mountains region of Arkansas and location of stands sampled for upland hardwood individual-tree models (each dot represents 1 to 3 plots per stand).

plateau is sharply dissected, and most ridges and spurs are flat to gently rolling and generally less than one-half mile wide. Mountain slopes consist of an alternating series of steep, simple slopes and gently sloping benches.

Annual precipitation averages 46 to 48 in., with March, April, and May being the wettest months. Extended summer dry periods are common, and autumn is usually dry. The frost-free period is normally 180 to 200 days.

Mixed-oak types occupy most of the commercial forestland in the Boston Mountains, and oaks-white oak (*Quercus alba* L.), northern red oak (Q. *rubra* L.), black oak (*Q. velutina* Lam.)-are the dominant timber species throughout upland hardwood stands in this region. Desirable, fast-growing species such as yellow-poplar (*Liriodendron tulipifera* L.) do not occur in the Ozark region, and valuable species such as black walnut (*Juglans nigru* L.), black cherry (*Prunus serotina* Ehrh.), and white ash (*Fraxinus americana* L.) occur only sporadically, primarily on the better sites. Most associated species in these upland stands are either slower growing or lower in value than the oaks. Oak sites in the Ozarks range from poor on the ridges and south- and west-facing slopes to excellent on some north- and east-facing slopes and mountain benches. However, stand growth and yield on all oak sites are strongly influenced by the hot, dry summers that typify the Ozark region. These hot, dry summers produce severe soil moisture deficits that retard tree growth.

Data were collected from 87 permanent 0.25-0.5 ac plots (Figure I) distributed across the Boston Mountains (Graney 1980). Stand age at the time of initial thinning ranged from 11 to 7.5 yr. Site indexes (Farrar 1985, Schnur 1937) for northern red or black oaks ranged from 46 to 82 ft (base age 50 yr). All plots were established in fully stocked, even-aged upland hardwood stands that showed no evidence of recent fire or cutting. Of the plots studied, 68 plots were installed over a 3 yr period from 1975 to 1977, and 19 were installed later in 1980 and 198 1.

Measurements consisted of a complete inventory of trees larger than 0.5 in. dbh measured to the nearest 0.1 in. Total heights of sample trees, which were selected at random in proportion to the number of stems in each 1 in. dbh class (before and after thinning), were obtained for each plot. Four levels of stocking were created-40, 60, and 80% of full stocking plus an unthinned control. Gingrich's (1967) tree-area equation for upland oak stands was used for stocking calculations. Thinning was mostly from below. Culls and poor-quality stems were removed first, then intermediate and suppressed trees of low quality and vigor. High-quality stems of desirable species were cut only to attain the residual stocking goal and uniform spatial distribution. Data from this study have been used in previous analyses (Graney and Murphy 1991, 1994).

Each of the 87 plots has two 5 yr growth periods. Only live trees 2.6 in. dbh and larger left after the initial thinning were used in the analysis. Ingrowth crossing this 2.6 in. threshold subsequent to thinning was not included. This ingrowth would be composed of more tolerant, slower growing species that remain as understory trees for the remaining life of the stand. Moreover, they will never comprise any significant volume (Graney and Murphy 1994). Therefore, this portion of the stand was excluded from the analysis.

Analysis

Although over 40 tree species were represented on one or more plots, black, northern red, and white oaks accounted for 86% of the total basal area over all plots after thinning (Table 1). Other desirable species-such as black cherry, black walnut, and white ash-were present only as scattered individual trees on most plots. Hickories and blackgum represented 8% of the total basal area after thinning, but these species were more common in the midstory and understory positions on most plots (Table 1).

We segregated the species into groups for each model that had both biological and practical significance. The major groups were: (1) the white oaks (white oak and post oak); (2) the red oaks (black oak, northern red oak, and southern red oak); (3) hickories; (4) ash, cherry, sweetgum; and (5) miscellaneous species. The oaks were the major component (Table 1), and segregating them into the two groups separated the slower growing white oaks from the faster growing red oaks. The hickories were put into a separate group, when possible, because it is a common species and a valuable mast tree. Although ash and cherry are not a large part of the overstory component, they have been reported to be present as large advance reproduction on more mesic sites (Graney 1989, Graney and Murphy 1995, Graney and Rogerson 1985). They are favored species and may make up a larger component of the overstory in the future, although they will probably not be as dominant as the oaks are now. Therefore, they were separated as a group, and sweetgum was added because its growth attributes are closer to ash and black cherry than the other species groups. The remaining species were lumped into a miscellaneous species group.

Basal Area Growth

We used a basal area growth equation recently developed (Murphy and Shelton 1996) for loblolly pine (*Pinus taeda* L.) as the preliminary model. It offers considerable flexibility in how different variables can be introduced, and the function in the numerator can be easily changed. Our full model for individual-tree basal area growth began as:

$$\Delta B = \frac{v_1[1-\exp(-v_2 B)]}{[1+\exp(w_1 BAL + w_2 SBA + w_3 DQ + w_4 A + w_5 SI)]}$$

where

AB = annual tree basal area growth for a 5 yr period (ft^2),

B = average tree basal area (ft^2),

BAL = basal area (ft^2/ac) in trees whose dbh's are equal to or larger than the subject tree (includes the subject tree),

Table 1. Percentage of total number of trees and basal areas by species or species groups for Boston Mountain study after first thinning.

Species or species group	Scientific name	Percentage after initial treatment	
		No. of trees	Basal area
White oaks			
White oak	*Quercus alba* L.	40	47
Post oak	Q. *stellata* Wangenh.	1	1
Subtotal		41	48
Red oaks			
Black oak	*Q. velutina* Lam.	8	25
Northern red oak	Q. *rubra* L.	14	13
Southern red oak	Q. *falcata* Michx.		
Subtotal		22	38
Hickories	*Carya spp.* Nutt.	4	6
Blackgum	*Nyssa sylvatica* Marsh.	6	2
Subtotal		10	8
Ash	*Fraxinus spp.* L.		
Cherry	*Prunus serotina* Ehrh.		
Sweetgum	*Liquidambar styraciflua* L.		—
Subtotal		2	2
Miscellaneous species			
Basswood	*Tilia americana* L.	—	
Beech	*Fagus grandifolia* Ehrh.	—	
Black locust	*Robinia pseudoacacia* L.	—	
Black walnut	*Juglans nigra* L.	—	
Blackhaw	*Viburnum spp.* L.	—	—
Blackjack oak	Q. *marilandica* Muenchh.		
Buckeye	*Aesculus spp.* L.	—	
Chinkapin oak	*Q. muehlenbergii* Engelm.	—	
Cucumber tree	*Magnolia acuminata* L.	—	
Devil's walkingstick	*Aralia spinosa* L.	—	
Dogwood	*Cornusjorida* L.	—	
Elm	*Ulmus spp.* L.	—	
Hackberry	*Celtis occidentalis* L.	—	
Indian-cherry	*Rhamnus caroliniana* Walt.	—	
Hophornbeam	*Ostrya virginiana* (Mill.) K. Koch	—	
Mulberry	*Morus rubra* L.	—	
Ozark chinkapin	*Castanea ozarkensis* Ashe	—	
Papaw	*Asimina triloba* (L.) Dunal.	—	
Persimmon	*Diospyros virginiana* L.	—	
Red cedar	*Juniperus virginiana* L.	—	
Red haw	*Crataegus spp.* L.	—	
Red maple	*Acer rubrum* L.	—	—
Sassafras	*Sassafras albidum* (Nutt.) Nees	—	
Serviceberry	*Amelanchier spp.* Medic.	—	
Shortleaf pine	*Pinus echinata* Mill.	—	
Spicebush	*Lindera benzoin* (L.) Blume	—	
Tree huckleberry	*Vaccinium arboreum* Marsh.	—	
Umbrella magnolia	*Magnolia tripetala* L.	—	
Wild plum	*Prunus spp.* L.		
Witch hazel	*Hamamelis virginiana* L.		
Subtotal		25	4
Grand total		100	100

SBA = stand basal area (ft^2/ac),

DQ = average quadratic mean dbh (in.) of the stand,

A = stand age (yr),

SI = site index (ft), and

v_i's, w_i's = coefficients to be estimated.

The stand variables are the average values for the growth period. All values are for trees 2.6 in. dbh and larger.

The numerator of the equation is the growth portion, and the denominator is a modifier of growth. The full model was fitted for each given species group using nonlinear regression (SAS Institute 1989). Variables were deleted if their approximate t-values had probabilities larger than 0.01. Plots of residuals versus predicted values and independent variables were also examined to determine if the model was correctly specified. The reduced model was refitted, and the residuals were examined for trends and outliers.

Total Tree Heights

Several models for describing tree height were investigated, and the one developed by Harrison et al. (1986) for Appalachian hardwoods worked best for our data. Their model is

$$H = 4.5 + H_a[1 + a, \exp(a_2 H_d)][1 - \exp(-bD / H_d)]$$

where

H = total tree height (ft),

D = tree dbh (in.),

H_d = dominant stand height (ft),

a_i's, b = coefficients to be estimated,

and the other variables are as previously defined. The five species groups initially described were used, but blackgum was added to the hickories. Blackgum was observed to have the same height patterns as the hickories, and it was used to augment the height samples for the hickories to create a big enough data set for the height analysis. The coefficients were fitted using nonlinear regression (SAS Institute 1989). Residual plots were also examined for trends, outliers, and model specification.

Individual-Tree Survival

The logistic function was selected for describing individual-tree survival. The full model is

$$P = \frac{1}{\left[1 + \exp\{-(c_0 + c_1 \frac{D}{DQ} + c_2 BAL + c_3 \frac{1}{N} + c_4 \frac{1}{D} + c_5 SI + c_6 SBA + c_7 \frac{1}{A})\}\right]}$$

where

P = probability that a tree will survive for 5 yrs,

N = trees/ac,

c_i's = coefficients to be estimated,

and the other variables are as previously defined.

For the survival analysis, black and southern red oak were separated from the northern red oak, because their survival patterns differed from northern red oak. The LOGISTIC procedure with stepwise selection in SAS (SAS Institute 1989) was used to fit the model and select variables. This method uses maximum likelihood for fitting. Fitting the logistic model using maximum likelihood is a standard technique to describe tree survival or mortality in forestry. For examples, see Hamilton (1986, 1990), Hamilton and Edwards (1976), and Monserud (1976). Predicted versus actual survivals by 1 in. dbh classes were examined to evaluate model performance.

Results and Discussion

The results for fitting the basal area growth equation are shown in Table 2. The fit indexes were best for the overstory species (the white oaks, red oaks, and ash-cherry-sweetgum) and worst for the understory species, which are grouped in the miscellaneous species category. The positive coefficients for the variables in the denominator of the equation have a dampening effect on growth as these variables are increased. These results are consistent with our understanding of stand dynamics. Conversely, the negative coefficients for quadratic mean dbh and site index indicate that growth increases with an increase in these variables. Although Harrison et al. (1986) used a different equation form that was linear for Appalachian hardwoods, their fit statistics are similar (coefficients of determination ranging from 0.41 to 0.77). No trends were observed in the residual plots.

Table 3 summarizes the coefficient values and associated fit statistics for the total height model for the different species groups. The root mean square errors were from 5.0 to 5.5 ft, and the fit index patterns demonstrate that the model works best for the oaks. Again, the fit statistics are comparable to those of Harrison et al.'s (1986) for Appalachian hardwoods. The residual plots indicated no trends, and the model appeared correctly specified.

The coefficients for the survival equation are depicted in Table 4 with a negative sign for a variable indicating that an increase in value will lower the probability of survival. The variables that describe relative competitive position-(1) stand basal area in trees larger than or equal to the dbh of the subject tree and (2) the ratio of dbh to quadratic mean dbh-appeared most often in the survival equation for the different species groups. A chi-square statistic was used to compare the predicted and actual survival distributions by 1 in. dbh classes for the six species groups. The probability levels for all species groups were close to 1.00, indicating that the actual and

Table 2. Coefficients, number of observations, and fit statistics for tree basal area growth equations of upland hardwood in the Boston Mountains of Arkansas.

	Species group and equation no.				
Coefficient*	White oaks (1)	Red oaks (2)	Hickories (3)	Ash, cherry, sweetgum (4)	Misc. species (5)
“I	0.059594	0.094068	0.45624	0.21928	0.53531
v_2	0.61893	0.74731	1.0682	0.5348	1.0435
w_1	0.025335	0.01723	0.0017067	0.024783	
w_2	0.0011359	0.0028845	0.012793	0.0094439	0.016463
w_3	-0.20362	-0.091378			-0.017572
w_4	0.021362	0.01321	0.03 1293		0.033071
w_5	-0.0089064	-0.0080458			
Observations	7,551	4,148	1,694	374	3,586
Fit index†	0.75	0.74	0.59	0.65	0.58
	....	....	........(ft^2)........	....	....
RMSE††	0.004 1	0.0062	0.0024	0.0073	0.0023

$$* \quad \Delta B = \frac{v_1[1-\exp(-v_2 B)]}{[1+\exp(w_1 BAL + w_2 SBA + w_3 DQ + w_4 A + w_5 SI)]}$$

† Fit index = $1 - \Sigma(y_i - \hat{y}_i)^2 / \Sigma(y_i - \bar{y})^2$

tt RMSE = root mean square error

predicted survival patterns by dbh class are not different. Observed and predicted survivals by dbh classes revealed no model problems.

These results do not demonstrate how the functions will perform when simulating growth at the stand level. Therefore, the basal area growth and survival equations were used to simulate the 5 and 10 yr development of the 87 plot observations used in the analysis. Growth and survival were projected on an annual basis. Observed values of plot and tree variables were used to start the simulation. Because the survival equation is based upon a 5 yr period, the fifth root of the predicted survival was calculated to get the annual survival rate, a procedure recommended by Monserud (1976). The predicted survival was compared to a uniform random number, and the tree was assumed to survive for that year if the predicted survival was less than or equal to the random number. The survival and growth of each tree was calculated for each year, then the tree and plot values were updated from these estimates to get the starting values for the following year. This process was repeated for each year of the projection period. The predicted versus actual values for number of trees/ac, stand basal area, and quadratic mean dbh were then compared. Basal area was used in lieu of volumes, because these variables are very closely related.

Figure 2 shows the dispersion of the percent residuals in relation to the actual values for the three variables. Notice that there is an increase in dispersion for the 10 yr projection, a verification that longer forecasts are inherently more variable than short-term ones. The mean percent differences for the 5 and 10 yr projections, respectively, are the following: basal area, -0.8 and -2.6; quadratic mean dbh, -0.7 an -1.8; and trees/ac, 0.7 and 0.9. There is a tendency for predicted quadratic mean dbh for the young plots (those with a small mean dbh) to be larger than the actual values. Overall, Figure 2 shows acceptable dispersions for 5 and 10 yr projections.

To illustrate an application of these models, suppose that you would like to know the development of an upland oak stand, age 40 yr and site index 62 ft, if it were thinned to 60%

Table 3. Coefficients, number of observations, and fit statistics for total tree height equations of upland hardwoods in the Boston Mountains of Arkansas.

	Species groups and equation no.			
Coefficient*	White oaks (6)	Red oaks (7)	Hickories,blackgum (8)	Misc. species (9)
a_1	0.39300	0.74910	0.8353 1	1.6204
a_2	-0.016420	PO.032945	PO.025586	PO.040793
b	10.659	12.244	9.2513	9.6837
Observations	2,969	2,367	439	211
Fit index†	0.92	0.92	0.88	0.12
	(ft)		....	....
RMSE††	**5.0**	**5.5**	5.1	5.5

• $H = 4.5 + H_d[1 + a_1\exp(a_2 H_d)][1 - \exp(-bD/Hd)]$,

† Fitindex= $1 - \Sigma(y_i - \hat{y}_i)^2 / \Sigma(y_i - \bar{y})^2$

†† RMSE = root mean square error

Table 4. Coefficients and number of observations for survival equations of upland hardwoods in the Boston Mountains of Arkansas.

	Species group and equation no					
Coefficient*	White oaks (10)	Northern red oak (11)	Black and southern red oak (12)	Hickories (13)	Ash, cherry, sweeteum (14)	Misc. species (15)
c_0	-1.4237	4.3254	4.7704	6.4536	1.1689	7.0624
c_1	4.8616	2.2467			2.2258	4.9706
c_2	-0.0228	-0.0418	PO.0352	PO.0446		PO.1366
c_3	351.1			200.8		
c_4		-3.7859	-7.5632	4.5394		-2.7815
c_5			0.038 1			
c_6			-0.0178			0.1 175
c_7	13.6568					
Observations	8,228	2,737	1,714	1,906	391	3,982

$$* \quad P = \frac{1}{\left[1+\exp\{-(c_0 + c_1\frac{D}{DQ} + c_2 BAL + c_3\frac{1}{N} + c_4\frac{1}{D} + c_5 SI + c_6 SBA + c_7\frac{1}{A})\}\right]}$$

stocking. Figure 3 shows the stand tables immediately after thinning and projected 10 and 20 yr later. Notice that the oak species are in the upper diameter classes and that this species stratification not only persists but also becomes more pronounced over time. The oaks grow into the larger classes, while the other species are mostly relegated to the smaller sizes. The number of trees in the smaller dbh classes decrease through mortality as they incur intense competition from the larger trees. This stand progresses from 60% stocking and a basal area of 61 ft^2 immediately after thinning to 76% stocking and 84 ft^2 basal area after 10 yr to 89% stocking and 103 ft^2 of basal area after 20 yr, when the stand is age 60.

Long-term projections are not recommended with these models. However, reasonable behavior for very long-term projections indicates that the models are robust and provide biologically reasonable results. To test these models, the same example was used in which a 40-yr-old stand is thinned to 60% stocking. Stand development was simulated for 100 yr. Figure 4 shows the stand development for this period in terms of basal area and stocking. The stand does not reach any asymptote, but the stocking and basal area levels do not appear unreasonable. What appears to be happening is that tree growth is steadily declining, and tree mortality is keeping density and stocking in check. This simulation is well beyond the data range of the models, but it indicates that the models can be used with confidence for more common stand projections.

We have two specific suggestions for model application. We recommend that formulations of Schnur's site index curves (1937) developed by Farrar (1985) be used to calculate dominant stand height for the tree height equations. No separate height equations for ash, cherry, and sweetgum were developed because of insufficient data. However, for some sites and stand conditions, their height development should be separated from the miscellaneous species. We suggest that Harrison et al.'s (1986) height equation for cherry be used. For your convenience, their equation is included here:

$$H = 4.5 + H_a[1 + 0.0001\exp(0.07590H_d)][1 - \exp(-17.528D / H_d)]$$

As a further aid, Table 5 facilitates the association of what species groups are to be used with each equation.

We recommend these equations for use in the Boston Mountains for projection period lengths of 20 yr or less and within the range of the original data for best results. Application outside these recommended ranges should be reviewed with caution and with knowledge of the growth expectations of the stands under consideration. We also advise users outside the Boston Mountains that they may encounter species not included in this study.

Individual-tree models provide the most comprehensive information about stand behavior. But, heretofore, forest managers of upland hardwood in the Boston Mountains have had to be content with individual-tree models developed for other regions or use local, albeit more limited, stand-level models. These equations developed here should provide a welcome addition to the forester's toolkit for this region.

Computer Software Announcement

The tree models presented here have been incorporated into an Ozark variant of the Forest Vegetation Simulator of FVS (Teck et al. 1996). The FVS software is available directly from the Forest Health Technology Enterprise Team (FHTET) Bulletin Board at 970-498-2187, by anonymous ftp at 162.79.4 1.7 in /pub/products/FVS or from the WEB site at URL http://162.79.41.7/pub/products/FVS. For more information contact Gary Dixon, Forest Management Service Center, U.S. Forest Service, 3825 Mulberry Street, Fort Collins, CO 80524; telephone 970-498- 18 14; fax 970-498-1660; or internet: gdixon/fm@fs.fed.us.

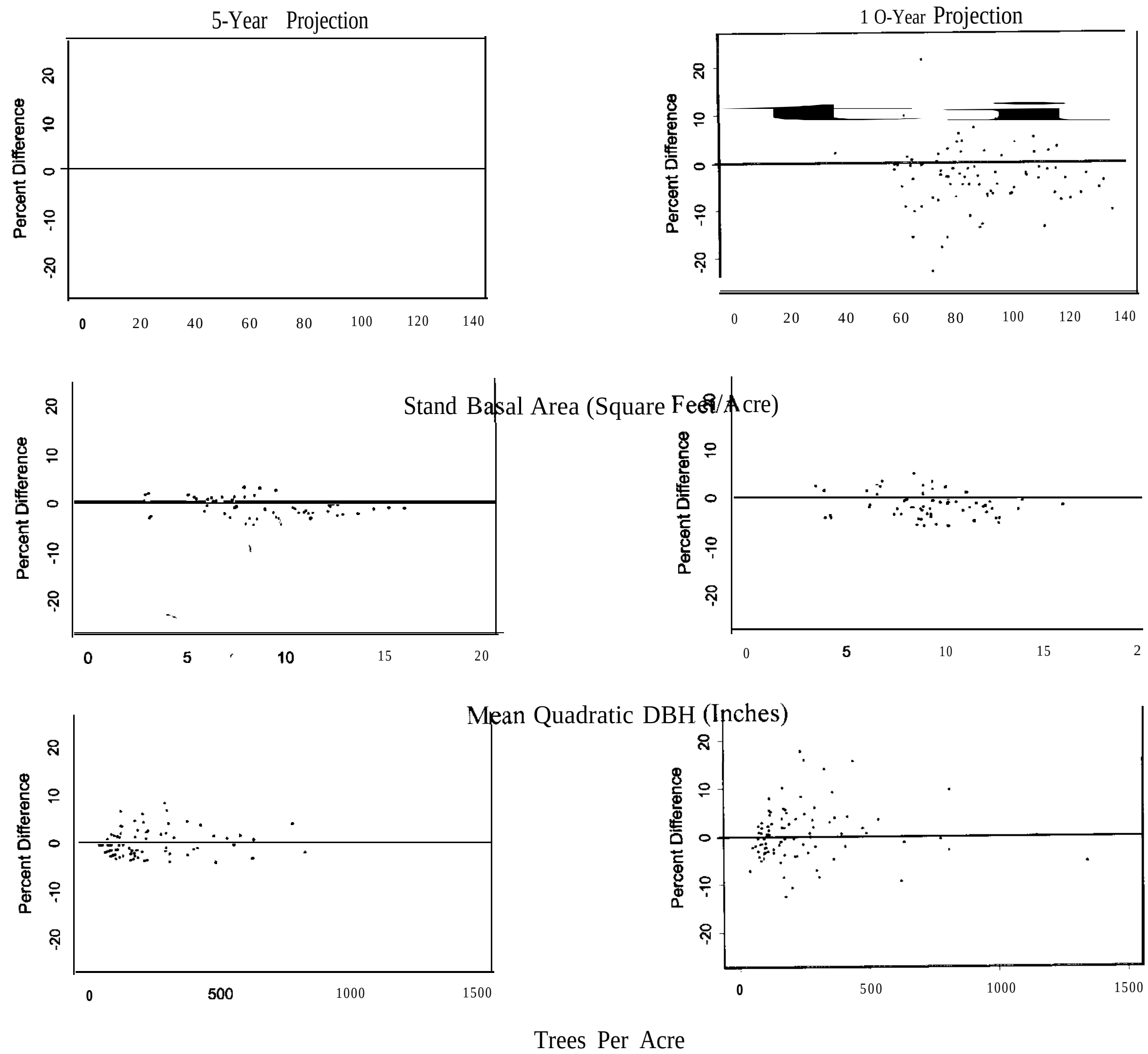


Figure 2. Percent residuals [100 + (Predicted-Observed)/Observedl versus observed values for stand basal area, quadratic mean dbh, and trees per acre for upland hardwood growth plots in Boston Mountams of Arkansas.

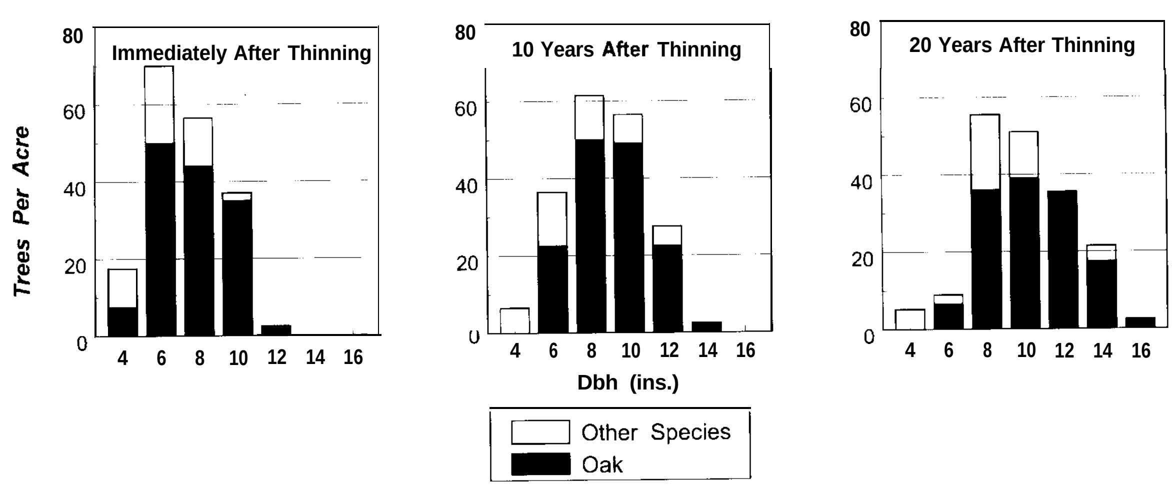
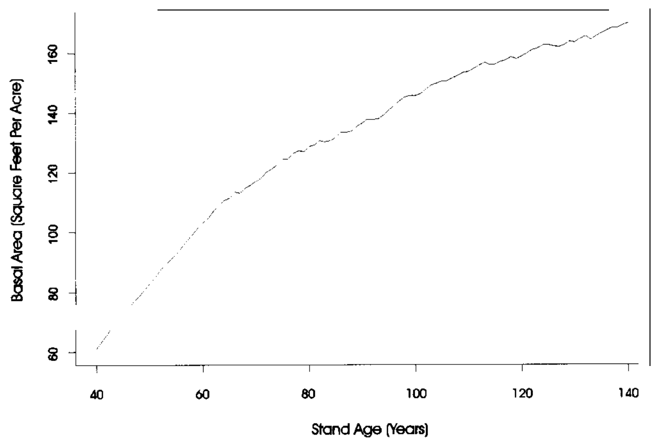


Figure 3. Stand tables for upland-hardwood stand (age 40, site index 62) in the Boston Mountains, Arkansas, after thinning to 60% stocking and simulated stand development 10 and 20 yr after thinning.



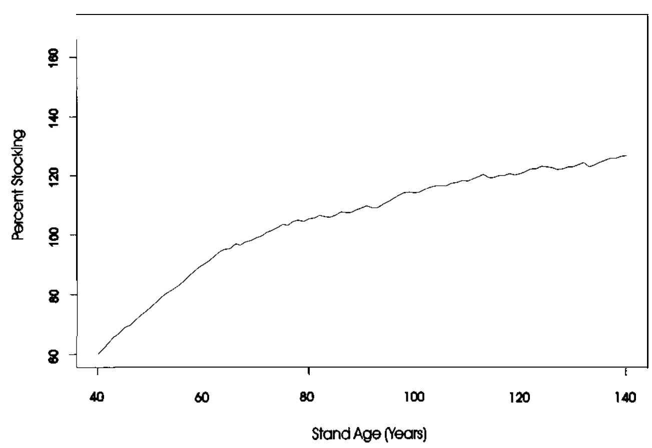


Figure 4. One-hundred-year projection of an upland hardwood stand in Boston Mountains, Arkansas (age 40, site index 62 ft) thinned to 60% stocking.

Table 5. Species sharing common equations for tree basal area growth, total height, and survival models for hardwood stands in Boston Mountains, Arkansas. [Equation numbers (1) through (15) reference the equation to use in Tables 2 through 4. Equation (16) is found in the text].

Tree basal area growth		Total height		Tree survival	
Species	Equation no.	Species	Equation no.	Species	Equation no.
White oak Post oak	(1)	White oak Post oak	(6)	White oak Post oak	(10)
Black oak Northern red oak Southern red oak	(2)	Black oak Northern red oak Southern red oak	(7)	Northern red oak	(11)
				Black oak Southern red oak	(12)
Hickory	(3)	Hickory Blackgum	(8)	Hickory	(13)
Ash Cherry Sweetgum	(4)	Ash Cherry Sweetgum	(16)	Ash Cherry Sweetgum	(14)
All other species	(5)	All other species	(9)	All other species	(15)

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