Active Contours on Statistical Manifolds And Texture Segmentation

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***Abstract*— A new approach to active contours on statistical manifolds is presented. The statistical manifolds are 2-dimensional Riemannian manifolds that are statistically defined by maps that transform a parameter domain onto a set of probability density functions. In this novel framework, color or texture features are measured at each image point and their statistical characteristics are estimated. This is different from statistical representation of bounded regions. A modified Kullback-Leibler divergence, that measures dissimilarity between two density distributions, is added to the statistical manifolds so that a geometric interpretation of the manifolds becomes possible. With this framework, we can formulate a metric tensor on the statistical manifolds. Then, a geodesic active contour is evolved with the aid of the metric tensor. We show that the statistical manifold framework provides more robust and accurate texture segmentation results.**

Keywords–statistical manifolds, active contours, texture segmentation, Kullback-Leibler divergence

I. Introduction

Texture-based segmentation is an essential task in the areas of image analysis and pattern recognition, and thus the task has attracted intensive research for the past three decades [1,2,3,4]. By the nature of difficult of the problem, there exist many segmentation approaches, from transform methods [14,15] and stochastic techniques [16,17] to a combined technique [18].

Curve evolution techniques are becoming increasingly popular [4,5,6,7,8,22]. Most of the reported methods deal with image models that have two or more regions and associated probability density functions. In [5,7,23], statistics of image regions are modeled with parametric methods, while Kim et al. [22] use nonparametric Parzen density estimates for a region descriptor. Also, the authors in [22] utilize an information theoretic approach to image segmentation, in which mutual information between region labels and intensity values is incorporated into a formulation of energy minimizing curve evolution. Meanwhile, a mixture of parametric and nonparametric methods has been proposed in [4], where different techniques are applied to different feature spaces.

The approach proposed in this paper has been inspired by [9,10], and is based on geodesic active contours [11]. In [10], Freedman and Zhang develop a new curve evolution method that utilizes density matching criteria between a model and a target region. They estimate the density function by a nonparametric technique. Meanwhile, Sochen et al. [9] introduce a geometric framework by which images and image feature spaces are considered as 2-dimensional manifolds. We adapt the density matching idea, which is often used as a texture similarity measure, into the geometric framework that provides a metric tensor on manifolds embedded in m-dimensional space. Then, it is straightforward to utilize geodesic active contours because the metric tensor contains geometric information of the manifolds.

Unlike the majority of curve evolution methods, in which feature spaces are based on deterministic measures, our approach considers feature spaces as a set of probability density functions (PDF) of feature values. More precisely, following the terms used in [9], 2-dimensional manifolds embedded in a higher dimensional space consist of a set of maps that transform a 2D parameter space onto probability density functions. We call the 2D manifolds statistical manifolds. It is worth mentioning that our feature PDF is not region-based, but rather it is point-wise. Ordinary distance metric can not be applied to the proposed statistical manifolds, thus we use a modified Kullback-Leibler divergence, known as relative entropy in information theory. With this distance metric, the statistical manifolds become Riemannian which means that their geometric information can be induced. This is an essential part of this framework because geodesic active contours require the geometric information of manifolds.

In the next section we define the statistical manifolds and corresponding distance metric and metric tensor. Section 3 introduces a feature extraction scheme and briefly discusses geodesic active contours. Next, we show some results on texture segmentation and conclude the paper with a brief summary.

II. Statistical Manifolds

A Riemannian manifold $\mathcal{M}^p$ is an abstract surface of arbitrary dimension p with a proper choice of metric. Then, an image $I(\boldsymbol{x})$ parameterized in $\mathbf{R}^2$, that is, $\boldsymbol{x}=(x,y)\in\mathbf{R}^2$, is viewed as a 2-dimensional Riemannian manifold, $\mathcal{M}^2$, embedded in $\mathbf{R}^n$ with $(\boldsymbol{x}, I(\boldsymbol{x}))$ [9], where n=3 for gray scale images and 5 for color images. Similarly, m-dimensional feature spaces of an image can be considered as $\mathcal{M}^2$ embedded in $\mathbf{R}^{m+2}$.

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A. Statistical Embedding

In this paper, we introduce statistically defined manifolds on which each feature at a local coordinate $x \in \mathbf{R}^2$ is represented only by its statistics, for instance, the first and second moments, rather than by deterministic values. Certainly, parametric estimation methods can be used for the feature statistics, but in most cases they are not suitable to model multimodal distributions. Thus, here we will consider only nonparametric methods, such as Gaussian kernel-based estimation. In this framework, therefore, the embedding map for an M-dimensional feature space is $(x, f(\Theta^1;x), \ldots, f(\Theta^M;x))$ that assigns a set of probability density functions to a local coordinate $x \in \mathbf{R}^2$. We call this embedding a statistical embedding. Here, Θ represents a feature in the M-dimensional feature space.

The means and variances of each feature can also be used directly as features, constructing standard (non-statistical) image manifolds. This directly leads to the work of [9]. A performance comparison between the statistical and the non-statistical manifolds is still under investigation.

B. Metric Tensor

Because our manifolds use statistical embeddings as defined in the previous section, it is natural to select a PDF-based dissimilarity measure for feature discrimination. An obvious candidate is Kullback-Leibler (K-L) divergence, defined as

$$\mathbf{kl}(f_A, f_B) = \sum_k f_{k,A} \log\left(\frac{f_{k,A}}{f_{k,B}}\right), \tag{1}$$

where f_A and f_B are two probability distributions, and the subscripts (k, Z) represent the kth element of distribution Z. This is commonly known as relative entropy in information theory, and has values greater than or equal to 0, where 0 indicates a perfect match. However, the K-L divergence is not a metric because it is not symmetric ($\mathbf{kl}(f_A,f_B) \neq \mathbf{kl}(f_B,f_A)$). Thus, we modify the K-L divergence by taking the average of $\mathbf{kl}(f_A,f_B)$ and $\mathbf{kl}(f_B,f_A)$ and use this as a distance measure:

$$KL(f_A, f_B) = \frac{1}{2}\sum_k\left[f_{k,A}\log\left(\frac{f_{k,A}}{f_{k,B}}\right) + f_{k,B}\log\left(\frac{f_{k,B}}{f_{k,A}}\right)\right]. \tag{2}$$

In [12], the modified K-L divergence shows a superior segmentation performance to the non-symmetric version.

Using (2), we are ready to induce a metric tensor defined on a statistical manifold. A metric tensor contains the geometric structure of a manifold and is used to measure distances on manifolds. For a 2D image manifold $(x, I(x))$ introduced above, the metric tensor is expressed as

$$\tau(x) = \begin{pmatrix} 1+I_x^2 & I_xI_y \\ I_xI_y & 1+I_y^2 \end{pmatrix}, \tag{3}$$

and for M-dimensional feature space the metric tensor can be generalized as [9]

$$\tau(x) = \begin{pmatrix} 1+\sum_i \Theta_x^i\Theta_x^i & \sum_i \Theta_x^i\Theta_y^i \\ \sum_i \Theta_x^i\Theta_y^i & 1+\sum_i \Theta_y^i\Theta_y^i \end{pmatrix}, \tag{4}$$

where Θ^i represents i-th feature in the feature space and the subscripts indicate partial derivatives of each feature.

The determinant of the metric tensor is used as an edge detector in various image processing applications. For the proposed statistical manifolds, the metric tensor should be defined somewhat differently since there are no explicit expressions for spatial partial derivatives of probability density functions. Here, we redefine the K-L divergence for a PDF $f(\cdot;x)$ of an arbitrary feature at location x as

$$\begin{aligned} KL_x(f) &= KL(f(x), f(x+\delta x)) \\ &= \frac{1}{2}\sum_k\left[f_k(x)\log\left(\frac{f_k(x)}{f_k(x+\delta x)}\right) + f_k(x+\delta x)\log\left(\frac{f_k(x+\delta x)}{f_k(x)}\right)\right]. \end{aligned} \tag{5}$$

Here, we used $f(x) \equiv f(\Theta^i;x)$ for simplicity. The subscript k in this case represents the kth element of PDF $f(x)$. Then, for a statistical manifold we can rewrite the metric tensor defined in (4) as

$$\tau(x) = \begin{pmatrix} 1+\sum_i KL_x^iKL_x^i & \sum_i KL_x^iKL_y^i \\ \sum_i KL_x^iKL_y^i & 1+\sum_i KL_y^iKL_y^i \end{pmatrix}. \tag{6}$$

This metric tensor provides information concerning the statistical structure of the manifold, and its determinant measures statistical dissimilarity of features on manifolds. The determinant of $\tau(x)$ is much larger than unity when evaluated at locations where the manifold has a high statistical gradient. On the other hand, the value is close to unity when the determinant is evaluated on which the manifold is statistically stationary.

III. Feature Extraction and Active Contours

In this paper so far, a statistical manifold has been introduced by which the embedding creates a set of PDFs of features rather than feature values themselves. We have also formulated a metric tensor, which is defined on the statistical manifold. In this section, we discuss a possible extraction scheme for a feature PDF, and then extend this to a formulation of integrated active contours for texture segmentation. This follows the work in [11] and [20].

A. Feature PDF Extraction

Theoretically, any feature with statistical characteristics that can be estimated from an image neighborhood can be incorporated into this new statistical manifold framework. Possible features include directional information, intensity distributions (histograms), polarity, anisotropy, etc. The directional information can be obtained with Gabor filters, which are extremely useful for extracting texture features over various scales [19]. However, it is known that a Gabor filter tends to have a considerable degree of redundancy from its many feature channels [4]. Also, directional information can be estimated with an average squared gradient known as a structure tensor matrix [4,21].

Here, we used a nonparametric method with a Gaussian kernel to estimate probability density functions of each feature. Also, we used varying window size to incorporate scale-related information.

B. Geodesic Active Contours

Following Caselles et al. [11], a curve evolution through the equation

$$\frac{\partial C(t)}{\partial t}=g(|\nabla I|)\kappa\vec{\mathbf{N}}-(\nabla g\cdot\vec{\mathbf{N}})\vec{\mathbf{N}} \quad (7)$$

minimizes curve length on manifolds, that is, it equivalently finds a geodesic curve in a Riemannian space. Here, κ denotes curvature and $\vec{\mathbf{N}}$ is a unit vector which is normal to the curve. The edge stopping function $g(|\nabla I|)$ is given by

$$g(|\nabla I|)=\frac{1}{1+|\nabla I|^2}=\frac{1}{\det(\tau)}. \quad (8)$$

The function $g(|\nabla I|)$ is defined as an inverse of the determinant of the metric tensor such that it stops the curve evolution at image edges; in our case, these correspond to statistical boundaries on manifolds. A corresponding level set approach, pioneered by [13], is formulated as

$$\frac{\partial u}{\partial t}=div\left(g(|\nabla I|)\frac{\nabla u}{|\nabla u|}\right)|\nabla u|, \quad (9)$$

where the function, $u(x)$, is defined so that the curve C is determined by a level set $\{x \mid u(x) = 0\}$. A typical choice of $u(x)$ is a signed distance function.

Meanwhile, the active contour can be accelerated by adding a region-based term as presented in [20]:

$$\frac{\partial u}{\partial t}=\mu div\left(g(|\nabla I|)\frac{\nabla u}{|\nabla u|}\right)|\nabla u|+\lambda g(|\nabla I|)h\left(KL(f_\Omega,f_C)\right) \quad . (10)$$

The function $h(\cdot)$ penalizes large K-L distances between f_Ω, PDF of the region inside the contour, and f_C, PDF of points on the contour.

IV. Results

This new framework combines statistical manifolds and active contours methods. We have tested the approach on real images of a cheetah and a zebra. In these examples, we used intensity and direction distributions as the manifold statistics. Intensity distributions were approximated by normalized histograms of (gray-scale) pixel values. Direction distributions were estimated by filtering the intensity image with a set of Gabor filters. The scale was fixed at a small value, and 6 directions were selected for the filtering. Then, with different window sizes, the directional distribution was estimated at each parametric point. Fig. 1a to Fig. 1c show independent determinant values $|\tau(x)|$ for each window size. Fig. 1d depicts the determinant of the integrated metric tensor defined in (6). Fig. 2 shows similar outputs to Fig. 1, but for the normalized histogram. Likewise, Fig. 3 shows similar outputs for nonparametric estimation of PDFs of intensity values. In these experiments, the nonparametric PDFs provide the best driving force for the active contours. Fig. 4 shows contour evolutions for two real images. Small initial contours are set inside the objects. Then, the figures show three stages of contour evolution for the cheetah and the zebra images. From the top of each column, contours are captured after 50, 100, and 150 evolutions. Overall, the performance is very good. However, it can be see that the final contours do not include the mouth or the tail of cheetah, nor the front leg of the zebra. This is because image statistics are not stationary at those specific regions. These results demonstrate that the proposed method produces subjectively good segmentation for textured images.

V. Conclusions

This paper presented a novel framework for the use of statistical manifolds. In this framework, an image is considered as a multidimensional manifold which has mappings that transform a 2D parameter domain onto a set of probability density functions. We have defined a metric tensor on the statistical manifolds based on Kullback-Leibler divergence. Given feature data for images, the algorithm estimates feature statistics by applying a kernel-based nonparametric method. Then, the metric tensor is used to generate a stopping function for geodesic active contours. Our experimental results have shown the strength of the proposed technique for texture-based image segmentation. Currently, the method has been tested for single objects only. We are extending this framework to perform segmentation of multiple objects with arbitrary initialization.

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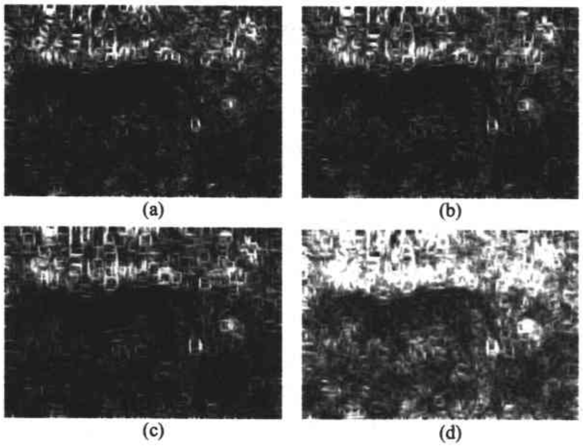

Figure. 1. Determinants of metric tensor, $|\tau(x)|$, for directional distributions with window sizes of (a) W=7, (b) W=9, and (c) W=11, respectively. (d) Integrated determinants of the metric tensor defined in (6).

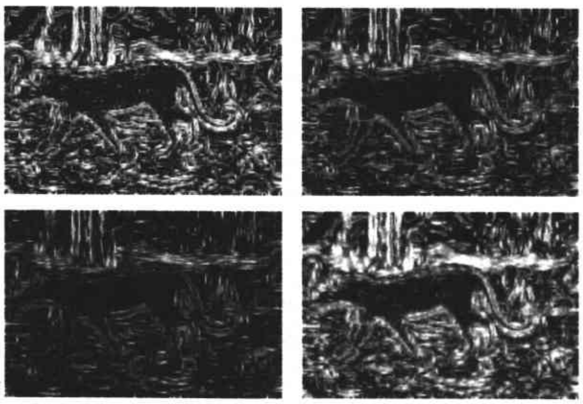

Figure. 2. Determinants of the metric tensor, $|\tau(x)|$, for normalized histograms of intensity values. See Fig. 1 caption.

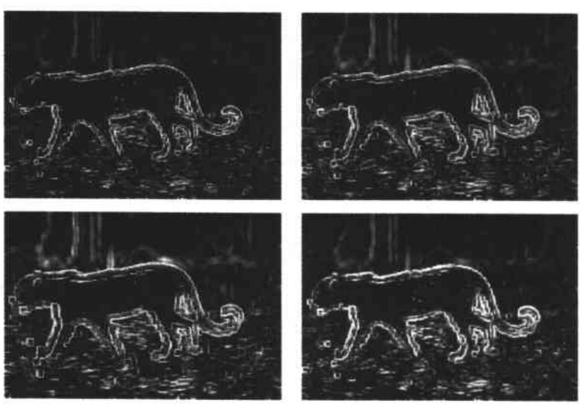

Figure. 3. Determinants of the metric tensor, $|\tau(x)|$, for nonparametric PDFs of intensity values. See Fig. 1 caption.

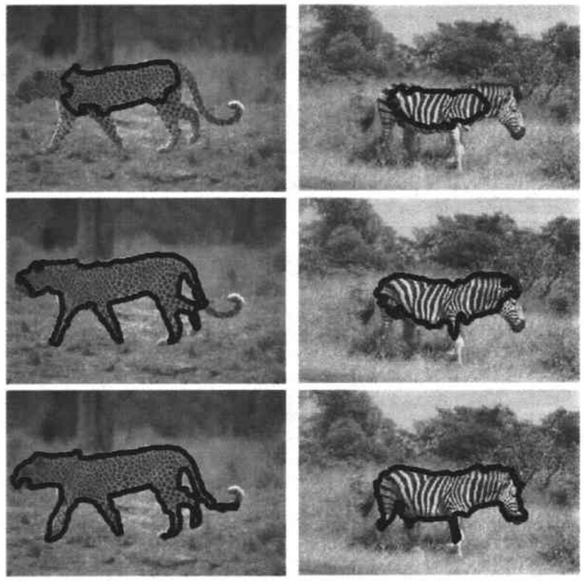

Figure. 4. Examples of active contours on statistical manifolds. Initial contours were manually selected inside each object. The images show various stages of contour evolution. From the top of each column, contours are captured after 50, 100, and 150 evolutions.

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