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An Individual-Tree Growth and Yield Prediction System for Uneven-Aged Shortleaf Pine Stands

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ABSTRACT: *A system of equations modeling the growth and development of uneven-aged shortleaf pine (Pinus* echinata *Mill.) stands is described. The prediction system consists* ***of*** *two main components: (1) a distance-independent, individual-tree simulator containing equations that forecast ingrowth, basal-area growth, probability* ***of*** *survival, total and merchantable heights, and total and merchantable volumes and weights of shortleaf pine trees; and (2) stand-level equations that predict hardwood ingrowth, basal-area growth, and mortality. These equations were combined into a computer simulation program that forecasts future states* ***of*** *uneven-aged shortleaf pine stands. Based on comparisons* ***of*** *observed and predicted stand conditions in shortleaf pine permanent forest inventory plots and examination of the growth patterns of hypothetical stands, the simulator* makes *acceptable forecasts* ***of*** *stand attributes. South. J. Appl. For. 24(2):112–120.*

Forecasting the growth and yield of uneven-aged stands is becoming increasingly important as more public agencies and private forest owners incorporate these techniques into their land-management strategies. This is particularly true of uneven-aged shortleaf pine (*Pinus echinata* Mill.) stands, given the species' wide geographic distribution, economic significance, and general lack of published research. Despite that silvicultural system's lower total merchantable volume output, uneven-aged management of shortleaf and lobloliy (*P. taeda* L.) pines in the West-Gulf region has traditionally been carried out by forest industries producing dimension lumber (Guldin and Baker 1988). Apparently, the combination of low-cost regeneration and comparatively high sawtimber volumes has made intensive uneven-aged management an attractive alternative, especially on lower quality sites (Guldin and Baker 1988, Shelton and Murphy 1994). In today's ever-more-competitive environment, managers need to be able to quantify tradeoffs of various management scenarios. To that end, this article reports the development of a growth and yield simulator for uneven-aged shortleaf pine stands.

Much of the existing growth and yield information for naturally occurring shortleaf pine is based on data collected from even-aged stands. Yield tables have been developed from temporary plot data (USDA Forest Service 1929, Schumacher and Coile 1960, Murphy and Beltz 1981, Murphy 1982). Individual-tree growth and yield equations have been published for even-aged shortleaf pine stands (Miner et al. 1989, Bolton and Meldahl 1990, Huebschmann et al. 1998, Lynch et al. 1999).

Researchers have also proposed a wide variety of quantitative methods for managing uneven-aged stands (Baker et al. 1996). For example, Murphy et al. (1991) applied Marquis' (1978) *BDq* method [controlling residual basal area (BA), maximum tree diameter, and the ratio of the number of trees in a given diameter class to its adjacent class(es)]to managing shortleaf pine.

Moser and Hall (1969) pioneered a methodology for deriving time-dependent BA and volume prediction functions for uneven-aged forest stands by integrating growth-rate equations which do not have time or stand age as an

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independent variable. Lynch and Moser (1986) advanced the technique of integrating a system of first-order ordinary differential equations to predict stand and stock tables for mixed-species stands.

Farrar et al. (1984) presented tables for estimating current and forecasted volumes and volume growth for uneven-aged stands of loblolly-shortleaf pine on average sites in the West Gulf Coastal Plain. Their tables were constructed from stand-level equations that are functions of BA and elapsed time (Murphy and Farrar 1982, 1983).

Hyink and Moser (1983) and Murphy and Farrar (1988a) illustrated the use of parameter prediction and recovery models in uneven-aged applications. Parameter prediction models directly predict the future values of the parameters of a probability density function characterizing a diameter distribution. Stand-average attributes such as volume and BA are then estimated using the diameter distribution. Parameter recovery models, by contrast, directly predict the stand-average attributes, from which the underlying diameter distribution can be estimated.

Farrar et al. (1989) developed equations describing BA and volume growth for uneven-aged loblolly pine stands containing small amounts of hardwoods. They also characterized the effects of different relative amounts of pine and hardwood BA on successful pine regeneration. Murphy and Farrar (1985) used data from continuous forest inventory plots on industrial forestland to create a system of stand-level equations for estimating future BA and current and future volumes in selection-managed stands of shortleaf pine.

Matrix growth models have been used in both even- and uneven-aged forest types (Buongiorno and Michie 1980, Mengel and Roise 1990, Lin et al. 1998), including southern pine stands (Schulte and Buongiorno 1998, Schulte et al. 1998). These models forecast a future stand diameter distribution from the current diameter distribution by using matrices of transition probabilities to predict probability of tree movement to a larger dbh class and probability of mortality during a specified time interval.

Data

Data used in this analysis originate from circular, 0.20 ac continuous forest inventory (CFI) plots established and maintained by an industrial forest landowner in southwest Arkansas. Figure 1 illustrates the locations of the CFI plots used in this analysis.

Measurements were first obtained at the end of the 1965 growing season. Subsequent measurements occurred during dormant seasons at intervals spanning either 5 or 6 yr. Thus, each plot was measured at the ends of up to five growth periods. New plots were established when additional property was purchased, or to replace existing plots lost to harvesting or natural causes. To camouflage the presence of the CFI plots, trees to be monitored were tagged near their bases, and staples were inserted into the bark at breast height; no other marks were allowed.

The dbh of every pine and hardwood 5.1 in. or larger was measured. Merchantable rather than total heights were measured at the ends of the early growth intervals. Consequently,

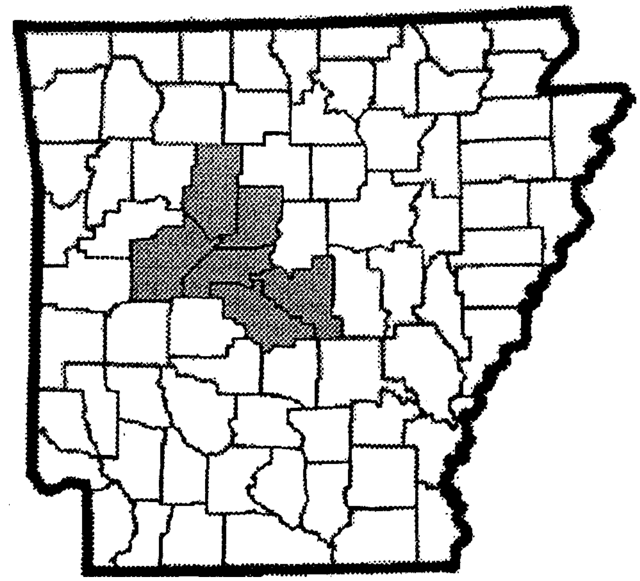

Figure 1. 'Counties in which the 152 uneven-aged shortleaf pine growth and yield plots were located.

height measurements used for this analysis were not obtained until the end of the 1988 growing season. At that time and at the end of the 1993 growing season, total heights of a representative sample of trees on each plot were measured.

Shortleaf pine site index (base age 50 yr) was determined for each plot from Miscellaneous Publication 50 (USDA Forest Service 1929). Although site index is a concept associated more with even-aged management, we followed Murphy and Farrar's (1985) use of it to indicate relative site quality. Baker et al. (1996) state that site index in uneven-aged stands is only an approximation because even the tallest trees were probably overtopped in their seedling and sapling stages. Trees selected for site index determination should be those whose annual rings exhibit no signs of past suppression.

For the datasets used in this analysis, plots containing any loblolly pine were eliminated. If timber stand improvement or thinning/harvest operation occurred on a plot, the growth interval during which the operation occurred was eliminated from the dataset in order to avoid confounding effects. In addition, any plot-interval combination was discarded if: (1) shortleaf pine BA was less than 70% of the total plot BA, (2) mortality during the growth period exceeded 20% of the total plot BA present at the beginning of the interval, or (3) shortleaf pine merchantable BA was less than 30 ft^2/ac or greater than 90 ft^2/ac at the beginning of the interval.

Each plot-interval combination was categorized by shortleaf pine site index, and total merchantable and sawtimber-only BA. If a plot had the same combination of BA classes during more than one growth interval, all but one of the intervals with duplicate characteristics were randomly eliminated. To avoid overrepresenting some stand conditions, the plot-interval combinations in the final dataset were randomly selected such that no more than 25 entries were allowed for each combination of site index and merchantable shortleaf BA classes. Also, to maintain temporal independence, only one interval was allowed in the dataset for each plot selected.

Fifteen plots were selected for the period 1966-19 71, 15 plots for 1972-1977, 40 plots for 1978-1982, 28 plots for 1983-1988, and 54 plots for 1989-1993. Table 1 contains the distribution of plot-interval combinations -classified by site index, and merchantable and sawtimber shortleaf BA—contained in the final dataset. Table 2 summarizes stand-level conditions at the beginning, middle, and end of the 152 plot-interval combinations selected for model development. Mid-period values were obtained by essentially averaging the end-point values. For example, summing the number of trees surviving the entire period with one-half the number of trees that died, and one-half the number of ingrowth trees during the period yields the midperiod trees per acre.

Model Development

The data described above were used to create a distance-independent individual-tree model. The model consists of a combination of individual-tree shortleaf pine equations and stand-level hardwood equations. All coefficients reported in this section are significantly different from zero at the a ≤ 0.05 level unless otherwise noted.

Shortleaf Pine Models

The shortleaf pine component of the model system is comprised of equations for estimating individual-tree BA growth, ingrowth, probability of survival, height and height growth, crown ratio, volume and weight.

Basal-Area Growth

The model reported by Bitoki et al. (1998) was the basis for developing a BA growth model for individual shortleaf pine trees. This model follows Shifley and Brand (1984) in that a modified Chapman-Richards (Richards 1959) function-representing potential tree growth-is multiplied by a logistic modifier. The modifier function (Murphy and Shelton 1996) is constrained to values between 0 and 1, thereby reducing the potential growth on the basis of variables representing stand and tree attributes. Lynch et al. (1999) used this same general form to estimate BA growth of shortleaf pine trees in even-aged natural stands.

Summary statistics from the 3,654 trees used to fit parameters are given in Table 3. The following equation predicts BA growth of individual shortleaf pine trees in uneven-aged stands:

$$G_i = \frac{0.0781 B_i^{0.7311} - \left(0.0781 B_i / B_M^{0.2689}\right)}{1 + \exp(-2.7768 + 0.0225 B_S - 0.0118 SI_P + 1.9302 BL_i)} \quad (1)$$

where G_i is annual growth (ft^2) of tree ***i;*** B_i is BA (ft^2) of tree ***i;*** B_S is stand BA (ft^2/ac) of both pine and hardwoods at the beginning of the growth interval; SI_P is shortleaf pine site index (ft at base age 50 yr); BL_i is proportion of shortleaf BA in the stand on trees as large as, or larger than tree ***i;*** and B_M = 7.07 ft^2, which corresponds to the largest shortleaf pine tree (36 in. dbh) that can be expected in operational stands (Hitch 1994). The dbh of shortleaf pines in the Ouachita Highlands rarely exceeds 30 in. (Smith 1986). Thus, the 36 in. dbh was derived by averaging 30 with 42 in.-the dbh of the champion shot-deaf pine (American Forestry Association 1992).

Nonlinear least squares was used to estimate Equation (1), which had a fit index (I^2) of 0.45 (I^2 = 1 - SSE/SST, where SSE is the uncorrected error sum of squares and SST is the corrected total sum of squares).

Ingrowth

Under uneven-aged management, stand structure is maintained by the ingrowth of submerchantable stems to merchantable size classes. Consequently, an ingrowth model is needed to reflect stand dynamics. Tables 2 and 3 summarize the variables used to fit the following shortleaf ingrowth model:

$$T_{P,I} = 3.1315 + 0.06279 T_P - 0.1283 B_S \quad (2)$$

'where $T_{P,I}$ is number of shortleaf pines per acre growing past the 5.1 in. dbh merchantability limit each year, T_P is number of merchantable shortleaf trees per acre already in the stand, and B_S is as previously defined. Equation (2) was fitted using ordinary least squares and had a R^2 of 0.60.

Probability of Individual-Tree Survival

Forecasting future stand conditions with a distance-independent, individual-tree model requires a method of predicting individual-tree survival. This study followed Lynch et al. (1999) and Murphy and Shelton (1998), who adapted Hamilton's (1974, 1976) and Monserud's (1976) use of a logistic function in order to predict the probability of shortleaf tree survival. Table 4 summarizes the data used to fit the following shortleaf pine logistic annual-survival function:

$$P_i = \frac{1}{1 + \exp\left[-\left(7.3138 - 1.4281 R_i\right)\right]} \quad (3)$$

where P_i is probability of annual survival for shortleaf tree ***i,*** and R_i is the ratio of quadratic mean stand diameter to dbh of tree ***i.***

Trees alive at the beginning of the growth interval were assigned a value of 1 if alive at the end of the interval, or 0 if dead. Nonlinear iteratively reweighted least squares was used to obtain both homogeneity of variance and maximum likelihood estimates of the coefficients (McCullagh and Nelder 1989). The weight was the inverse of the variance $P^t(1 - P^t)$, where P^t is the probability of survival predicted by the logistic model for ***tyears*** in the growth interval. Because the length of the growth interval differed between plots, and an annual survival equation was desired, the logistic model was raised to a power equal to the number of years in the growth interval when fitting parameters.

The propriety of the survival model was evaluated using the chi-square statistic. The chi-square value for live trees was 0.169, and 12.579 for dead trees. The Pvalue for this chi-square test (with 13 degrees of freedom) is 0.47; thus the model is considered acceptable.

Total Height

The total height estimation models developed in this study are patterned after Murphy and Farrar (1988b). They used total height of the tree with maximum diameter as an analog

Table 1. Classification of 152 uneven-aged shortleaf pine growth and yield plots in the Interior Highlands of Arkansas by shortleaf pine site index (base age 50 yr), and merchantable and sawtimber basal area at the beginning of the growth period.

Site index (ft)	Merchantable basal area* (ft^2/ac)	Sawtimber basal area† (ft^2/ac)					
		<11	11–29	30–49	50–69	70-89	Total
		(no. of plots)					
<46	30–49	—††	1	0§			1
	50-69	1	1	2	—		4
	70-89	—	—	—	—	—	—
	Total	1	2	2	—	—	5
46-55	30-49	2	18	3			23
	50-69	0	13	10	2		25
	70-89	1	5	6	2	1	15
	Total	3	36	19	4	1	63
56-65	30–49	4	16	5			25
	50–69	1	8	14	2		25
	70-89	—	3	6	5	—	14
	Total	5	27	25	7	—	64
>65	30–49	0	9	0			9
	50-69	1	4	2	2		9
	70-89	—	0	0	2	—	2
	Total	1	13	2	4	—	20
All sites	30-49	6	44	8			58
	50-69	3	26	28	6		63
	70-89	1	8	12	9	1	31
	Total	10	78	48	15	1	152

* All shot-deaf pine trees with dbh ≥ 5.1 in.
† All shortleaf pine trees with dbh ≥ 9.6 in.
†† No plot with this combination of characteristics was available for selection.
§ Although one or more plots exhibited this combination of characteristics, none was randomly selected.

Table 2. Summary statistics for 152 uneven-aged shortleaf pine growth and yield plots in the Interior Highlands of Arkansas.

Variable	Ave	SD	Min	Max
Treesiac				
Shortleaf pine				
Initial	122.4	48.5	40	265
Midperiod	129.7	52.6	40	280
Final	137.0	58.0	40	300
Hardwood				
Initial	20.3	15.6	0	65
Midperiod	23.2	17.6	0	95
Final	27.1	20.8	0	125
Basal area (ft^2/ac)				
Merchantable shortleaf pine*				
Initial	55.0	15.3	30.1	90.0
Midperiod	60.9	16.2	32.7	95.2
Final	66.8	17.8	32.6	102.0
Shortleaf pine sawtimber†				
Initial	30.4	14.1	2.6	75.2
Midperiod	40.0	14.1	5.6	76.2
Final	42.1	14.9	5.7	77.2
Merchantable Hardwood††				
Initial	6.5	6.4	0	30.1
Midperiod	7.5	6.7	0	30.6
Final	8.6	7.3	0	35.1
Shortleaf site index (ft at base age 50 yr)	56.3	7.3	35	74
Total shortleaf volume (ib, ft^3/ac)				
Initial	1,212	366	545	2,145
Final	1,535	420	716	2,506

* All shortleaf pine trees with dbh ≥ 5.1 in.
† All shortleaf pine trees with dbh ≥ 9.6 in.
†† All hardwood trees with dbh ≥ 5.1 in.

Table 3. Summary statistics of shortleaf pine trees from uneven-aged stands in the Interior Highlands of Arkansas used to fit individual-tree basal-area growth and ingrowth models.

Variable	Ave	SD	Min	Max
Tree basal area (ft^2)				
Initial	0.45	0.27	0.14	2.05
Midperiod	0.49	0.29	0.14	2.14
Final	0.53	0.30	0.14	2.29
Proportion of plot BA in pines as large as, or larger than, the subject tree				
Initial	0.62	0.29	0	0.99
Midperiod	0.59	0.28	0	0.99
Final	0.59	0.28	0	0.99
Mean ingrowth (trees/ac/yr)	3.1	3.5	0	16.0

of dominant stand height common in even-aged models, and then derived heights of smaller dbh trees from a function including theirdbh, maximum dbh, and height of the tree with maximum dbh.

Because total tree heights were measured at the beginning and end of only the last growth interval, a separate dataset was created for this analysis. Data errors excluded seven plots from the height dataset, resulting in 14.5 plots available for model development. Table 5 summarizes the variables used to fit the height models.

The model for predicting the height of the largest-diameter shortleaf pine is:

$$H_{DMax} = \exp\left[3.2593 - 9.0252/DMax + 0.3596\ln(SI_P)\right] \quad (4)$$

where H_{DMax} is total height (ft) of the largest-diameter tree ($DMax$) in the stand, and SI_P is as previously defined. The model, fitted by nonlinear least squares, had an I^2 of 0.39.

The model forecasting the future height of the largest-diameter shortleaf pine is:

$$H_{DMax,2} = \ln\left[\exp\left\{\exp\left(-4.1238H_{DMax,1}\right)\right\} + \exp(-3.8302)\right]/\exp(-4.1238) \quad (5)$$

where $H_{DMax,t}$ is total height (ft) of the largest-diameter tree in the stand at time t. The model was fitted using nonlinear least squares and had an I^2 of 0.89. The double-exponential form was chosen because its residuals did not exhibit bias.

The model for estimating current and predicting future total heights of shortleaf pines other than the largest-diameter tree is:

$$H_i = H_{DMax}\exp\left[23.4162\left(1/D_i - 1/DMax\right)\right]\cdot H_{DMax}^{-6.9053(1/D_i - 1/DMax)} \quad (6)$$

where H_i is total height (ft) of tree i, D_i is dbh (in.) of tree i, and the other variables are as previously defined. This model was fitted using nonlinear least squares and had an I^2 of 0.68.

One might wonder why estimating tree heights requires a system of equations when one equation generally suffices in even-aged situations. In even-aged stands, individual-tree height predictions are often functions of dbh, dominant stand height, and other variables. The uneven-aged situation is more complicated because even-aged crown classes do. not apply, and dominant stand height is not defined (Murphy and Farrar 1988b). If uneven-aged stands are considered as small even-aged clumps, dominant trees in neighboring clumps may have vastly different heights; thus any estimates of dominant height using those disparate "dominant" trees would be imprecise. The method adapted here assumes the tree of maximum diameter is probably also the tallest one. If its height is known, heights of the smaller trees can be modeled as a function of its height.

Volume and Weight Estimation

Total volume and merchantable volumes to desired upper-stem diameter limits were estimated for each tree. Cubic and board foot volumes were obtained by integrating Farrar and Murphy's (1987, or see Lynch et al. 1999) taper function. Because no weight equations are available for natural shortleaf pine in western Arkansas and eastern Oklahoma, approximate green densities were obtained by calculating the ratio of the shortleaf pine weight and volume for a tree of a given dbh and height from Saucier et al.'s (198 1) equations. This density was then multiplied by the appropriate cubic-foot volume supplied by Farrar and Murphy's (1987) taper curves to yield the green weight.

Hardwood Models

Because the hardwood component in many operational uneven-aged shortleaf pine stands either represents only a small proportion of the stand BA or is of submerchantable quality, individual-tree equations were deemed unnecessary.

Table 4. Summary statistics of data from 152 uneven-aged shortleaf pine growth and yield plots and 3,722 shortleaf oine trees in the Interior Hinhlands of Arkansas used to fit a logistic survival model.

Variable	Ave	SD	Min	Max
Mean mortality (treesiaclyr)	0.4	0.7	0	4.0
Quadratic mean diameter (in.)	9.0	1.2	6.6	13.2
Diameter at breast height (in.)	8.7	2.5	5.1	19.4
Ratio of quadratic mean diameter to tree dbh	1.10	0.27	0.43	2.36

Table 5. Summary statistics for the data from 145 uneven-aged shortleaf pine growth and yield plots in the Interior Highlands of Arkansas used to fit total height models.

Variable	Ave	SD	Min	Max
Diameter at breast height (in.)				
All pines				
Initial	8.7	2.7	5.1	20.0
Final	9.1	2.9	5.1	20.7
Pines with largest diameters on their plots				
Initial	14.3	2.1	9.8	20.4
Final	15.2	2.2	5.6	20.7
Total height (ft)				
All pines				
Initial	46.1	11.2	14	88
Final	47.0	12.0	14	94
Pines with largest diameters on their plots				
Initial	58.3	9.6	32	88
Final	60.6	10.1	26	94

Consequently, the following hardwood models represent stand-level changes.

Basal-Area Growth

The following equation estimates annual hardwood BA growth in uneven-aged shortleaf pine stands:

$$G_H = 0.0461 B_H^{0.8074} - 0.000104 B_H B_P \qquad (7)$$

where G_H is survivor growth (ft^2/ac/yr); and B_H and B_P are, respectively, hardwood and pine BA (ft^2/ac) in the stand at the beginning of the growth interval. The model had an R^2 of 0.46. The $B_H B_P$ coefficient was significantly different from zero at only the $\alpha \approx 0.15$ level.

Ingrowth

The equation below predicts the ingrowth BA of hardwoods 5.1 in. dbh in uneven-aged shortleaf pine stands:

$$B_{H,I} = 0.3792 + 0.0211 B_H - 0.0043 B_P \qquad (8)$$

where $B_{H,I}$ is hardwood ingrowth BA (ft^2/ac/yr); B_H and B_P are as previously defined. The model had an R^2 of 0.20.

Mortality

The following equation predicts hardwood mortality BA in uneven-aged shortleaf pine stands:

$$B_{H,M} = 0.0123 B_H \qquad (9)$$

where $B_{H,M}$ is hardwood BA (ft^2/ac/yr) lost to mortality, and B_H is as previously defined. Despite a R^2 of 0.10, we believe Equation (9) is superior to an overall average since it recognizes that mortality differs with changing hardwood BA.

An Individual-Tree Simulator

Equations (1) through (9) were incorporated into the Shortleaf Pine Stand Simulator (SLPSS), described in Lynch et al. (1999) and Huebschmann et al. (1998), so that it simulates growth and yield in both natural even- and uneven-aged shortleaf pine stands. Like its even-aged counterpart, the input to the uneven-aged simulator consists of initial stand condition-either a stand table (number of shortleaf trees by dbh classes) or inventory data from field plots. Also required are the maximum allowed tree age, shortleaf site index ,and initial hardwood BA. If stand table data are supplied, the simulator uniformly distributes the trees in 0.1 in. increments within each dbh class present in the data.

Each shortleaf tree (or group of trees in a dbh-class increment) is grown on an annual basis. Equation (1) estimates each surviving shortleaf tree's yearly BA increment, while Equation (7) calculates the stand-level hardwood BA growth. Equation (2) predicts the number of shortleaf ingrowth trees/ac (defined as submerchantable trees growing past 5.1 in. dbh) that will appear during the year; Equation (8) forecasts the amount of BA contributed by hardwood ingrowth. Equation (3) determines each shortleaf tree's probability of survival. A tree survives until the following year if its probability of survival exceeds the value of a uniformly distributed random number (restricted to the interval 0 to 1) generated for that tree. Hardwood BA per acre lost to mortality is estimated by Equation (9).

If initial tree heights are not supplied to the simulator, the simulator identifies the maximum-dbh tree in the stand. It then uses Equation (4) to estimate that tree's height, and Equation (6) to ascertain the heights of all other pines. During subsequent annual growth iterations, the simulator estimates the height of the largest-dbh tree using equation (5); the other heights are again determined by Equation (6).

If the maximum dbh tree is either harvested or dies, the simulator identifies the next largest tree. If that tree is substantially shorter than the original largest tree, the other trees' estimated heights may be shorter than their previous heights. In that case the simulator retains each tree's previous height until the forecasted height exceeds its previous height during a subsequent iteration. The simulator then estimates each shortleaf pine's volume with Farrar and Murphy's (1987) taper function. Finally, green weights are assigned using Saucier et al.'s (1981) equations.

To determine whether the equations yield reasonable predictions when combined into a system, the uneven-aged shortleaf simulator was evaluated by comparing observed and predicted final conditions in the plots used in the model-fitting phase. Plot conditions at the beginning of the growth interval were supplied to the simulator, which then predicted conditions at the end of the interval. We assumed the following merchantability limits: 0.5 ft stump for pulpwood and 1 ft for sawtimber trees; 4 in. top diameter limit outside bark (dob) for pulpwood and 7 in. top diameter limit inside bark (dib for sawlogs; 5 ft minimum pulpwood stick length and 8.3 ft minimum sawlog length: 15 ft minimum pulpwood tree length and 16.3 ft minimum sawlog tree length. All cubic volumes were reported as inside bark (ib)

Because the system is stochastic, the plot simulations were repeated 10 times, after which the observed final conditions were compared with average predicted conditions. Residual values (observed minus predicted) of final total volume inside bark (ft^3/ac) are plotted in Figure 2. There are no obvious trends in the total volume residual plots. Residual plots were generated for other stand attributes; although not included in this paper, they too show no discernable trends or patterns.

Residual statistics, including stand-level average differences, standard deviations of those differences, and the minimum and maximum differences are summarized in Table 6 for a number of stand attributes. On average, the simulator slightly overestimated most final stand attributes. However, after calculating the t statistic for each attribute, the null hypothesis of a zero mean difference could not be rejected for any attribute at $a = 0.05$.

The simulator was also evaluated by growing hypothetical stands and comparing the Scribner bd ft volumes produced over the projection period. The stands had initial hardwood BA of 6.5 ft^2/ac and initial shortleaf BA of 40, 60, or 80 ft^2/ac. One set of three stands had a site index (base age 50 yr) of 40 ft, while the other set had a site index of 70 ft; in all other respects the two sets of stands were identical. These stands were grown for a period of 10 yr. The linearity of board-foot volume accumulation (Figure 3) closely resembles Moser and Hall (1969) for uneven-aged northern hardwood stands.

Summary

An uneven-aged component of the program SLPSS has been developed from continuous forest. inventory plots located on industrial forest land in the Ouachita Highlands of Arkansas. This distance-independent simulator uses equations that predict ingrowth, BA growth, and survival of individual shortleaf pine trees. It also estimates total heights for trees of given dbh relative to the total height of the stand's maximum-dbh tree. Taper equations estimate total and merchantable volumes of shortleaf pine in cubic and board feet. Green weights are also estimated for user-specified merchantable top limits. The simulator also forecasts changes in stand-level hardwood BA from ingrowth, survivor growth, and mortality.

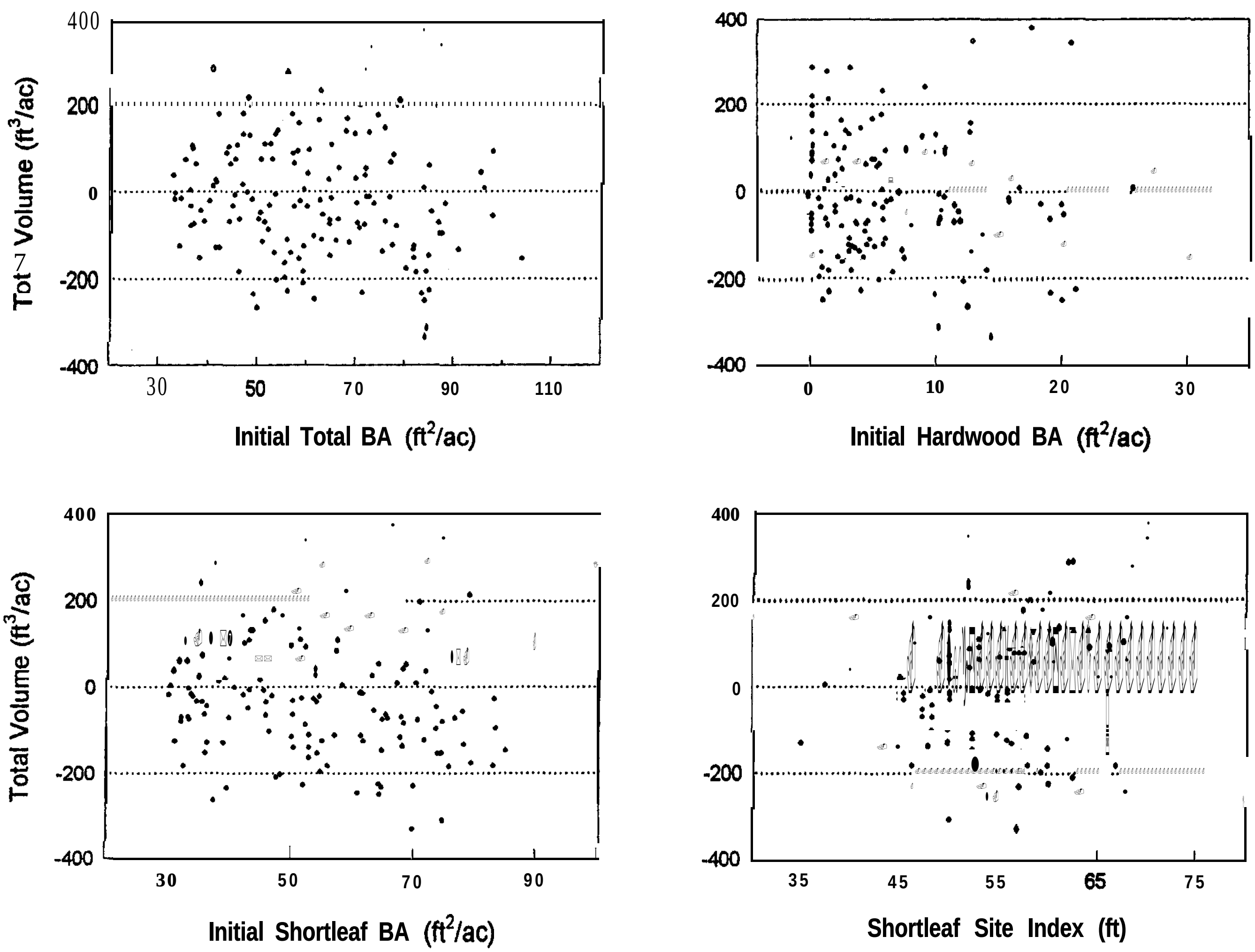

Figure 2. Residual values (observed minus predicted from the Shortleaf Pine Stand Simulator) of final shortleaf pine total cubic volume versus initial total, hardwood and shortleaf basal area; and shottleaf pine site index (ft at base age 50 yr). Based on an average of 10 simulations of 152 uneven-aged shortleaf pine growth and yield plots in the Interior Highlands of Arkansas.

Table 6. Summary of differences between actual final stand conditions and those predicted by the Shot-deaf Pine Stand Simulator based on 152 uneven-aged growth and yield plots in the Interior Highlands of Arkansas.

	Difference (actual-predicted)			
Attribute	Ave	SD	Min	Max
Shortleaf trees/ac	-0.2	15.3	-40.1	47.4
Shortleaf quadratic mean dbh (in.)	0.03	0.4	-1.3	1.1
Merchantable basal area (ft^2/ac)				
Shortleaf	-0.1	5.4	-13.7	13.1
Hardwood	-0.1	2.8	-11.9	13.3
Shortleaf volume/ac				
Cubic (i.b.)				
Total	-11.5	138.1	-331	378
To 4 in. top (dob)	-2.8	129.8	-294	356
To 7 in. top (dib)	-9.5	102.0	-260	218
Board foot				
Doyle	-42	330	-966	844
Scribner	-58	500	-1,372	1,275
International-1/4 in.	-61	589	-1,548	1,324
Shortleaf green weight (tons/ac)				
Total	-0.4	4.8	-11.7	13.2
To 4 in. top (dob)	-0.1	4.5	-10.4	12.4
To 7 in. top (dib)	-0.3	3.5	-9.1	7.6

Stand tables or inventory data of initial conditions are needed in order to conduct growth projections. Predicted stand conditions are reported in units per acre by user-defined dbh classes. Users can thin stands by removing shortleaf trees from desired dbh classes (hardwood reductions can be made on a per-acre basis). The software's flexibility makes SLPSS a useful tool for managers of uneven-aged shortleaf pine stands.

Because the shortleaf survival function is stochastic, and thus causes stand predictions to vary, users may want to average several projections for a particular stand. Information concerning installation and use of SLPSS can be obtained from the correspondence author.

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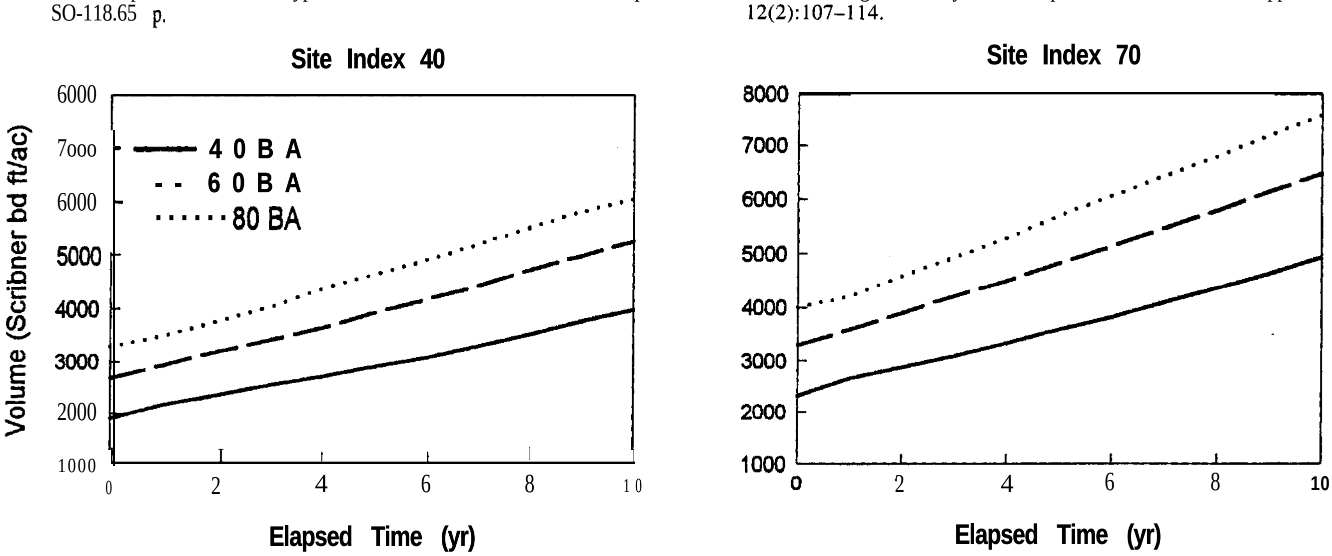

Figure 3. Shortleaf pine volume in hypothetical uneven-aged stands grown for 10 years, with 40, 60 or 60 ft^2/ac of initial shortleaf pine basal area and shortleaf pine site index (ft at base age 50 yr) of 40 and 70 ft.

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