

# A site model for Pyrenean oak (*Quercus pyrenaica*) stands using a dynamic algebraic difference equation

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**Abstract:** This paper presents a growth model for dominant-height and site-quality estimations for Pyrenean oak (*Quercus pyrenaica* Willd.) stands. The Bertalanffy–Richards function is used with the generalized algebraic difference approach to derive a dynamic site equation. This allows dominant-height and site-index estimations in a compatible way, using any desirable base age and allowing estimations to be time independent, which are important properties for site models. The database contains all possible height-growth intervals. The model is fitted considering residual autocorrelation, giving more efficient parameter estimates. Finally, the model behavior is analyzed by calculating error statistics from dominant-height and site-index estimations at different ages.

**Résumé :** Cet article présente un modèle de croissance pour estimer la hauteur dominante et la qualité de station de peuplement de chêne tauzin (*Quercus pyrenaica* Willd.). On utilise la fonction de Bertalanffy–Richards avec la méthode des différences algébriques généralisée pour obtenir une équation dynamique du site. On obtient ainsi des estimations de la hauteur dominante et de l'indice de qualité de station de façon compatible, pour un âge de référence choisi et indépendamment de la période d'estimation qui sont d'importantes propriétés des modèles de croissance. La base de données est structurée, considérant tous les possibles intervalles de croissance. Le modèle est ajusté en tenant compte de l'autocorrélation des résidus, ce qui fournit des estimations plus efficaces des paramètres. Finalement, la précision du modèle est évaluée en calculant les erreurs statistiques provenant des estimations de la hauteur dominante et de l'indice de qualité de station à différents âges.

## Introduction

Site quality directly affects stand growth as well as stand yield, hence determination of site quality is critical for forest management (Hägglund 1981). Dominant height growth modeling is central in the development of forest stand growth and yield models, and dominant tree height is commonly used as a measure of site quality. In forestry, we consider that site quality expresses the total wood yield potential (Avery and Burkhart 1994; Hägglund 1981; Ortega and Montero 1988). According to several authors such as Jones (1969), Kreutzer (1978), and Hägglund (1981), site-quality evaluation can be made using three different approaches or methods, that is, based on stand biometric characteristics, on environmental factors, or on the species composition of the forest. The first method estimates site quality based on parameters related to stand yield and growth. In this approach, factors such as the dominant height at a base age or the maximal stand volume growth are considered intrinsic because

they are attributes of the stand. In contrast, ecological and floristic characteristics are considered extrinsic.

In the present article, site-quality assessment is based on biometric criteria, using stand dominant height and age. Those variables are extensively used and easy to measure (Avery and Burkhart 1994; Curtis 1964; Hägglund 1981). Site index corresponds to the stand dominant height at a particular age considered the reference age and is the most widely used indicator of site quality. In such a way, productivity estimation is specific to a site index. Then, it is possible to classify the yield potential of a stand. The site-index estimation involves, therefore, the projection of actual height to the reference or base age. The use of height-growth equations as a function of tree age allows one to estimate the dominant height ( $h_d$ ) development through time ( $t$ ) and thus determine site index (SI) based on the general expression  $SI = f(h_d, t)$ .

Growth results from the interaction of biological processes that preside over the development of the organism. Growth functions describe changes in the individual or population size through time. Empirical models are defined, according to Burkhart (1997), as flexible models that are well adjusted to a data set and intend to describe real observed situations, independent of their formulation. Sometimes the distinction is not clear, because it is usual for mechanistic models to use empirical functions to describe particular physiological, biochemical, or physical processes (Burkhart 1997). On the other hand, as mentioned by Zeide (1997), process-based models can have such complexity that their practical use is made difficult. The use of empirical functions or models does not mean that they are less creative or scientific than other ap-

Received 29 July 2004. Accepted 13 September 2004.  
Published on the NRC Research Press Web site at  
<http://cjfr.nrc.ca> on 5 February 2005.

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proaches but, as referred by Burkhardt (1997) and Dixon et al. (1990), it depends on the study objectives, the available data, and the level of the process to be described.

Empirical models can express in their structure basic growth processes, with biological interpretation, such as happens with the von Bertalanffy model (Zeide 1993). Von Bertalanffy considered, as a theoretical assumption, that growth results from two opposite processes, one represented by anabolism and the other by catabolism. The anabolic rate is proportional to the organism surface (or its mass to the power of 2/3), while the catabolic rate is proportional to its mass. Then, growth is obtained from the following expression:

$$[1] \quad \frac{dY}{dt} = \eta Y^m - \gamma Y$$

where  $Y$  is the organism mass (or volume),  $\eta$  and  $\gamma$  are the anabolic and catabolic constants, respectively, and  $m$  is an allometric constant of the anabolic rate. This approach is the origin of one of the most widely used growth functions, the Bertalanffy–Richards function. It has a biological interpretation, as shown by Pienaar and Turnbull (1973).

A similar approach was used by Zeide (1993) to examine different growth functions, where plant growth is considered the result of two opposite factors. One expresses the biological potential for unlimited growth and the other represents environmental and increased age impositions. The expansion factor prevails in earlier stages of the tree life, while in the final life stages growth decreases as a result of resources and age limitations. The common tendency is that the growth expansion is proportional to tree dimension, while the decrease of the growth rate is more variable, reflecting a great number of factors that reduce growth (decreasing resources, competition, reproduction, diseases, disturbances). Therefore, differential growth functions can be decomposed into two components that represent growth expansion and decline. Using different transformations (differentiation, decomposition, logarithmization), Zeide (1993) showed that different forest-growth functions (Hossfeld, Gompertz, logistic, monomolecular, Bertalanffy–Richards, Levakovic, Korf, Weibull, Yoshida and Sloboda) could be expressed considering these two factors and thus conferring a biological interpretation to the empirical functions.

In dominant height growth modeling, empirical models have been extensively used for the development of decision tools for practical use in forest management. Based on the objectives of the present work, the function of von Bertalanffy, as modified by Richards (1959), was chosen:

$$[2] \quad Y = A(1 - ce^{-kt})^{1-m}$$

Being a four-parameter function, it has great flexibility, assuming different forms depending on the value of the parameter  $m$ ;  $c$  is a position parameter that usually assumes a value of 1, implying that  $Y = 0$  for  $t = 0$ ;  $m$  is related to curve shape;  $A$  is the asymptote; and  $k$  regulates the growth rate for a fixed value of  $m$ . The growth rate is given by

$$[3] \quad \frac{dY}{dt} = \frac{kY}{1-m} \left[ \left( \frac{A}{Y} \right)^{1-m} - 1 \right]$$

The Bertalanffy–Richards function is one of the most used in forest tree and stand-growth studies (Zeide 1993). It has been used in studies with English oak (*Quercus robur* L.) (Trencia 1992) and Pyrenean oak (Bengoa et al. 1991).

In this work, a dominant height growth model is presented for Pyrenean oak stands (*Quercus pyrenaica* Willd.) in Portugal, with the aim to estimate stand dominant-height development and site quality. Based on the data, an evaluation of the model was done for dominant-height and site-index estimations.

## Data

Information was collected in 33 installed study plots of 500 m<sup>2</sup> from the natural extent of Pyrenean oak stands in Portugal. Stands are pure and even aged and represent a wide range of site conditions (regions of occurrence, geology, elevations, aspect, slope). The stem analysis method was used to obtain information concerning growth of the dominant trees. The criteria proposed by Tennent and Burkhardt (1981) was used, selecting two dominant trees whose diameters correspond to the mean diameter of the 100 largest trees per hectare and whose heights are in an interval of 5% of the mean dominant height. Trees were cut and sectioned in 1-m length logs above the DBH level. From each tree stem, disks were collected from the bottom of each log and from the tree top. Tree diameter at breast height (DBH) and total height were measured. Ring reading was done in a laboratory using a tree-ring measuring device, to obtain tree growth trends.

Breast-height age ranged from 38 to 115 years, with an average of 58.8 years old. The most preponderant age-class was the 50- to 60-year-old class. For tree height, the average was 14.4 m, ranging from 8.5 to 27.0 m. The majority of trees fell within the 10–15 and 15–20 m height classes.

## Model derivation

In the Bertalanffy–Richards function, for simplicity we substitute  $q = [1/(1 - m)]$  and obtain

$$[4] \quad Y(t) = A(1 - ce^{-kt})^q$$

The age used is at breast height ( $t_d$ ). This is in conformity with several authors such as Carmean and Lenthall (1989), Edminster et al. (1992), Ker and Bowling (1991), and Huang (1997). In the first stages, growth is affected in several ways by non-site factors, causing an erratic or non-normal behavior. Competition from spontaneous vegetation, superficial soil characteristics, initial stocking, silvicultural manipulations, wildlife, and frost damage are some of the factors that can influence initial height growth, creating fluctuations. In addition, age is simply easier to measure at breast height.

## The GADA approach

Bailey and Clutter (1974) introduced a technique now known as the algebraic difference approach (ADA). Site equations derived from this methodology are mathematically sound and give consistent predictions. Cieszewski and Bailey (2000) extended this method and presented the generalized algebraic difference approach (GADA), a new generic methodology for the derivation of flexible dynamic equations that allows desired properties such as variable asymptotes, poly-

**Table 1.** Average and standard deviation of the  $A$ ,  $k$ , and  $q$  parameters of the Bertalanffy–Richards function for five site-index classes and the associated  $F$ -statistic probability ( $P$ ) from the analysis of variance.

Class	$\bar{A}$	$s_A$	$\bar{k}$	$s_k$	$\bar{q}$	$s_q$
1	26.914	3.018	$2.503 \times 10^{-2}$	$6.181 \times 10^{-3}$	1.132	$2.630 \times 10^{-1}$
2	20.236	2.370	$2.535 \times 10^{-2}$	$3.020 \times 10^{-3}$	0.966	$1.077 \times 10^{-1}$
3	18.786	3.577	$2.352 \times 10^{-2}$	$8.569 \times 10^{-3}$	1.005	$2.623 \times 10^{-2}$
4	15.924	5.005	$2.569 \times 10^{-2}$	$1.509 \times 10^{-2}$	0.919	$1.257 \times 10^{-1}$
5	12.300	3.898	$2.810 \times 10^{-2}$	$1.468 \times 10^{-2}$	1.021	$4.150 \times 10^{-2}$
$P$	<0.0001		0.957		0.172	

morphism, and parsimony. The application of the GADA implies the selection of a base equation (in our case, the Bertalanffy–Richards function) and the identification of any desired number of site-specific parameters. Then, the parameter changes among different sites are defined and replaced by a new variable  $X$ . This variable is a quantification of particular dynamics associated with the site and individual characteristics of growth. It can be either a variable or a function and can describe the relative rates of change.

To apply the GADA, an analysis of variance (ANOVA) was carried out on the equation parameters in relation to the site index of each plot. The analysis was done by aggregating plots into five classes corresponding to a desired established number of site-quality classes. Table 1 shows the average and standard deviation of the Bertalanffy–Richards equation parameters for each site class and the associated  $F$ -statistic probability from the ANOVA. It can be seen that parameters  $k$  and  $q$  do not present significant variations. Only the parameter  $A$  changes significantly among the site classes, being a site-specific parameter, giving different asymptotes for each site curve.

**The dynamic model**

From the base function  $Y$  (eq. 4) as a function of  $t$  and parameters  $A$ ,  $k$ , and  $q$ , the site-specific parameter  $A$  is dependent on  $X$ . The base equation is changed to the explicit three-dimensional site equation (Cieszewski and Bailey 2000):

$$[5] \quad Y(t, X) = X(1 - ce^{-kt})^q$$

Now the solution for  $X$ , using the equation’s initial conditions  $Y_0$  and  $t_0$ , is

$$[6] \quad X = Y(1 - ce^{-kt})^{-q} = Y_0(1 - ce^{-kt_0})^{-q}$$

which can be substituted into eq. 5, resulting in

$$[7] \quad Y(t, t_0, Y_0) = Y_0 \left( \frac{1 - ce^{-kt}}{1 - ce^{-kt_0}} \right)^q$$

Equation 7 is the dynamic site equation with an implicit initial condition. Based on this methodology, the same equation can be used either for SI or dominant-height estimations. Also, height-growth estimations are time-interval independent.

**Model fitting with all possible growth intervals and results**

In site-index modeling, the trend has been to use algebraic difference equations (Bailey and Clutter 1974; Clutter et al. 1983; Payandeh and Wang 1994; Huang 1997; Goelz and Burk 1998; Bailey and Cieszewski 2000; Cieszewski and Bailey 2000). Such equations allow one to obtain compatible dominant height growth and site-index models. A crucial point mentioned by Goelz and Burk (1992) is that in dominant-height modeling, two processes are involved: (i) site-index estimation for a base age giving height at any age; and (ii) height estimation for a desirable age from SI at the base age. In such estimations, height is assumed to be measured with error when it appears in the left-hand side of the equation but is fixed when it appears on the right-hand side. To simultaneously optimize both processes, that is, the regression of  $Y$  on  $X$  and  $X$  on  $Y$ , and to avoid parameter bias, Goelz and Burk (1992, 1996) suggest the use of an algebraic difference equation that considers all possible growth intervals in both directions in the data structure. Huang (1997) also found that this data structure provides the most stable and consistent results.

The dynamic eq. 7 obtained by using the GADA was fitted to a database of all possible growth intervals by nonlinear regression using SAS/ETS (SAS Institute Inc. 1995). As expected, residuals displayed heteroscedasticity, as can be seen in Fig. 1. Goelz and Burk (1996) proposed with a similar equation the use of the following weight factor,  $p$ :

$$[8] \quad p = \left( \frac{1 - e^{-k \cdot t}}{1 - e^{-k \cdot t_0}} \right)^q$$

where  $t_0 > t$ , and whose positions must be inverted when  $t_0 < t$ . However, we found it more appropriate to use the logarithmic transformation, as was done by Bailey and Clutter (1974) and Amateis and Burkhart (1985). Taking the logarithm of eq. 7 and renaming the variables as  $Y = h_d$  and  $t = t_d$ , we obtain (with  $c = 1$ )

$$[9] \quad \ln h_d = \ln h_{d_0} + q \ln \left( \frac{1 - e^{-kt_d}}{1 - e^{-kt_{d_0}}} \right)$$

The parameter estimates and fit statistics are presented in Table 2. With this equation, residuals displayed a homogeneous distribution (Fig. 2).

**Table 2.** Regression results for eqs. 9 and 12 listing values of parameters  $k$  and  $q$  and their respective standard errors ( $s_k, s_q$ ) in parentheses, parameters  $\rho$  and  $\gamma$ , and statistics  $R^2$  and MSE.

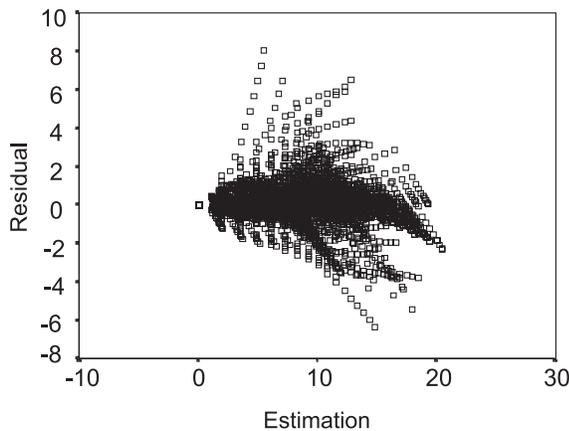
Equation	$k$ $s_k$	$q$ $s_q$	$\rho$	$\gamma$	$R^2$	MSE
Logarithmic	$0.200 \times 10^{-1}$ ( $0.498 \times 10^{-3}$ )	0.915 ( $0.126 \times 10^{-2}$ )			0.998	$0.492 \times 10^{-1}$
Logarithmic + autocorrelation	$0.210 \times 10^{-1}$ ( $0.929 \times 10^{-4}$ )	0.915 ( $0.243 \times 10^{-3}$ )	0.114	0.607	0.999	$0.166 \times 10^{-2}$

**Table 3.** Statistics of the bias from estimations of  $h_d$  and  $SI_{40}$ , for 5-year periods.

Age, $t_d$ (years)	$h_d$				$SI_{40}$			
	ME	SE	MAE	RMSE	ME	SE	MAE	RMSE
10	-0.205	0.532	0.448	0.580	0.560	2.053	1.225	2.528
15	-0.182	0.568	0.470	0.607	0.360	1.222	0.929	1.761
20	-0.147	0.535	0.436	0.564	0.234	0.849	0.692	0.642
25	-0.109	0.449	0.361	0.470	0.148	0.608	0.489	0.164
30	-0.071	0.325	0.260	0.339	0.085	0.389	0.311	0.027
35	-0.035	0.173	0.139	0.179	0.038	0.187	0.151	0.001
40	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
45	0.055	0.193	0.159	0.205	-0.052	0.180	0.149	0.001
50	0.134	0.380	0.314	0.412	-0.119	0.336	0.278	0.018
55	0.235	0.573	0.479	0.635	-0.188	0.486	0.400	0.082
60	0.484	0.711	0.666	0.886	-0.392	0.575	0.539	0.265
65	0.650	0.772	0.842	1.059	-0.508	0.603	0.657	0.467
70	0.672	0.982	0.991	1.190	-0.509	0.744	0.751	0.659
75	0.787	1.143	1.161	1.388	-0.580	0.843	0.856	0.996
80	1.004	1.494	1.500	1.401	-0.723	0.976	1.080	0.822
85	1.052	1.012	1.052	1.056	-0.742	0.663	0.742	0.507
90	1.207	1.010	1.207	1.209	-0.836	0.555	0.836	0.492
95	1.401	1.385	1.401	1.410	-0.955	0.853	0.955	0.830
100	1.511	1.426	1.511	1.516	-1.016	0.965	1.016	1.064

**Note:** mean error,  $ME = \sum_{i=1}^n (y_i - \hat{y}_i) / n$ ; standard deviation,  $SE = \sqrt{\sum_{i=1}^n (e_i - \bar{e})^2 / (n - 1)}$ ; mean absolute error,  $MAE = \sum_{i=1}^n (y_i - \hat{y}_i) / n$ ; and root mean squared error,  $RMSE = \sqrt{\sum_{i=1}^n (y_i - \hat{y}_i)^2 / n}$ . All errors are in metres.

**Fig. 1.** Scatterplot of residuals from height estimations by using eq. 7.



**Autoregressive error structure**

The occurrence of residual serial correlation is usual in repeated measurements made on the same tree, as happens with stem analysis. A nonlinear model can be represented in the following way:

$$[10] \quad h_i = f(t_i, \mathbf{\beta}) + e_i$$

where  $h_i$  is height,  $t_i$  is age,  $\mathbf{\beta}$  is the parameter vector, and the residuals are assumed to be independent and identically distributed with homogeneous variance ( $e_i \sim N(0, \sigma^2)$ ). When residuals are not independent, the standard procedure is to expand the residual term to consider first-order autocorrelation (Monserud 1984; Judge et al. 1988; Johnston 1991; Wang and Payandeh 1994; Huang 1997):

$$[11] \quad e_i = \rho e_{i-1} + \varepsilon_i$$

where the residuals  $\varepsilon_i$  are now independent and identically distributed and have homogeneous variance.

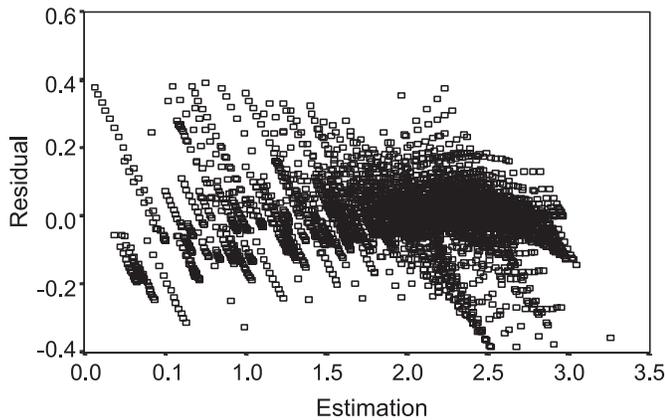
Goelz and Burk (1992) indicated that when using all possible growth intervals, model [10] is more complex and becomes

$$[12] \quad h_{i,j} = f(t_i, h_j, t_j, \mathbf{\beta}) + e_{i,j}$$

where  $h_{i,j}$  is the estimated height at age  $i$  using  $t_i, h_j$ , and  $t_j$ , with age  $i \neq j$  as predictor variables. Consequently, the residual term must be expanded to

$$[13] \quad e_{ij} = \rho e_{i-1,j} + \gamma e_{i,j-1} + \varepsilon_{ij}$$

**Fig. 2.** Scatterplot of residuals from height estimations by using eq. 9.



Equation 13 represents the autocorrelation structure of eq. 12. The  $\rho$  parameter considers the autocorrelation between the current residual and the residual by estimating  $h_{i-1}$  using  $h_j$  as predictor. The  $\gamma$  parameter considers the autocorrelation between the current residual and the residual from estimating  $h_i$  by using  $h_{j-1}$  as predictor.

Residual autocorrelation was evaluated by using Durbin's  $t$  test (Johnston 1991). This test consists in evaluating the significance of the correlation parameters of the residuals structure (eq. 13), by a  $t$  test. The test showed that the residuals are significantly correlated (for  $\hat{\rho}$ ,  $t = 24.88$ ,  $P < 0.0001$ ; and for  $\hat{\gamma}$ ,  $t = 26.47$ ,  $P < 0.0001$ ).

**Final model**

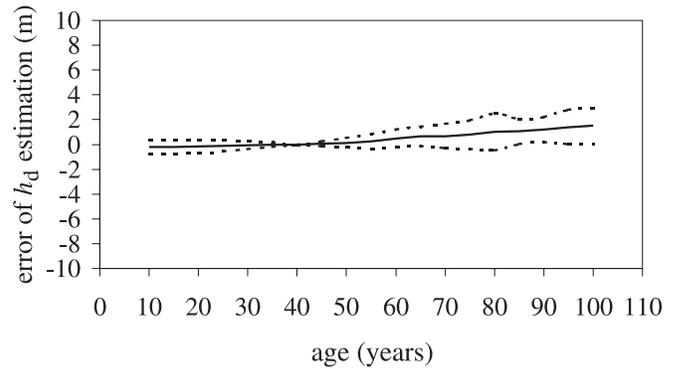
As suggested by Goelz and Burk (1992), it is possible to efficiently estimate parameters by considering the residuals expansion from eq. 13. This was realized with SAS/ETS (SAS Institute Inc. 1995), which allows dynamical updating of residuals. Parameter estimates, parameter standard errors,  $R^2$ , and MSE are presented in Table 2. The values of  $\rho$  and  $\gamma$  are also presented. Goelz and Burk (1996) proposed a correction for the standard errors, because the number of observations are artificially increased, by using the expression  $\sqrt{N_{tpd}/N_{pd}}$ , where  $N_{tpd}$  is the number of observations using all possible differences and  $N_{pd}$  is the number of observations using only the first growth difference.

As we can see, there are no large changes on parameter estimates by using the equation with logarithmic transformation or considering the structure of the residual term. However, the incorporation of the autocorrelation error structure gives lower parameter standard errors and thus results in more efficient parameter estimates. The  $R^2$  value has increased slightly and MSE has been reduced. Thus, the compatible equation for Pyrenean oak dominant height growth and site-index classification is the following

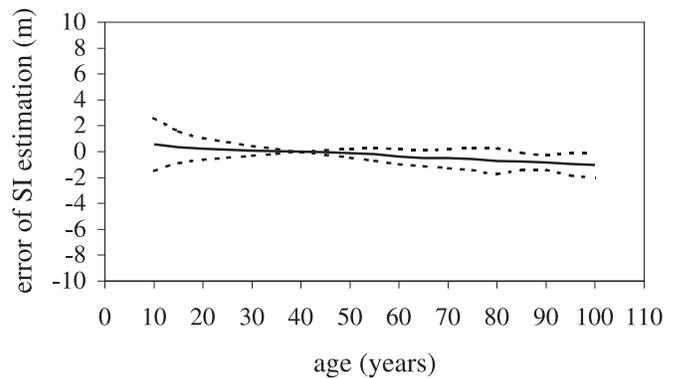
$$[14] \quad \ln h_d = \ln h_{d_0} + 0.915 \ln \left( \frac{1 - e^{-0.0210t_d}}{1 - e^{-0.0210t_{d_0}}} \right)$$

The correction factor to apply for antilogarithmic conversion is  $\exp(\hat{\sigma}^2 / 2) = 0.00083$ , corresponding to an average percentage correction of 0.5%, which is negligible.

**Fig. 3.** Comparative error of dominant height ( $h_d$ ) as estimated by eq. 14 across ages (solid line: mean error; broken lines: mean error  $\pm$  SE).



**Fig. 4.** Comparative error of site index ( $SI_{40}$ ) as estimated by eq. 14 across ages (solid line: mean error; broken lines: mean error  $\pm$  SE).



**Model evaluation**

We intend to give an idea of the accuracy and precision of the presented Pyrenean oak site model. Different statistics were calculated to evaluate the accuracy and precision of both dominant-height and site-index estimations. The statistics are the mean error (ME), standard deviation (SE), mean absolute error (MAE), and root mean squared error (RMSE). Formulas are presented in Table 3. These statistics refer to the bias from observed values in each plot and the estimations from eq. 14, obtained for quinquennial periods from 10 to 100 years old (Table 3). For this purpose an arbitrary base age of 40 years was chosen.

Figures 3 and 4 show the amount of the mean error (solid line) and standard deviation, SE (broken lines: mean error  $\pm$  SE), for total dominant height and site-index estimations, respectively. As expected, the statistics have a value of zero at the reference age (40 years old). For ages further from the reference age, there is an increment on the error of the predictions, proportionally bigger for ages above 40 years for  $h_d$  estimation, and inversely for SI estimations. For dominant-height estimations, error has a negative sign until base age, and error changes inversely after this age. The opposite happens with SI estimations. Globally, and in absolute terms, there is a maximal mean error of 0.5 m (10–15 years old) to 1.5 m (80–100 years old) for  $h_d$  estimations, while the maxi-

mal mean error for SI estimations is 1.2 m (10 years old) to 1.0 m (80–100 years old).

## Discussion

The moment of maximal current increment is given by  $\ln(q/k)$ , which in the present study occurs on average at 3.8 years old, which is very early. This means that the inflection point is imperceptible or not represented in the growth curve, and the relative growth rate decreases with the size  $Y$ . A similar behavior was determined by Carvalho (1995) in stands of the same species located in the Nogueira mountain of northeast Portugal, using the Hossfeld function, where the maximal increment happened at around 5 years of age. The initial fast growth pattern of this species is also evidenced by the number of years to achieve breast height, which is normally about 4 years. Zeide (1993) also observed such a pattern with several forest species.

The use of a dynamic equation derived from the generalized algebraic difference approach (Cieszewski and Bailey 2000) allows two different estimation processes. First, the estimation of a SI using height at some age; second, the estimation of dominant height at a desired age given height at base age. The residual-term expansion, to consider the existence of serial correlation that characterizes data from repeated measurements in the same individual, has led to a decrease of the parameter standard errors (asymptotically more efficient), a slight increase of the coefficient of determination, and a reduction of the mean square error. The site model also has the path of invariance property, that is, iterative computations give the same values at a given final age.

Concerning the precision analysis of dominant-height and site-index estimations, we obtained an average maximal error between  $-0.2$  m at age ( $t_d$ ) 10 years and 1.5 m at age 100 years. The root mean squared error (RMSE) ranged between 0.6 and 1.5 m for the same ages. For site-index estimations, the average maximal error was between 0.6 and  $-1.0$  m, and the RMSE was between 2.5 and 1.1 m, for the same ages. This larger value of 2.5 m refers to a particularly low age, becoming 1.8 m at age 15 years and 0.6 m at age 20 years. The RMSE, expressed as a percentage of the mean, was 6.0% for height estimations and 5.3% for site-index estimations. It was also observed that the size of the errors in these two estimation processes happens in an opposite way with age. That is, site-index estimations have larger bias below the reference age, while for height estimations bias happens in older ages. Errors in site-index estimations are higher in initial growth stages. This is in agreement with some authors such as Carmean and Lenthall (1989) and Huang (1997) when they mention that it is in these ages where major fluctuations in tree dominance status occurs.

## Acknowledgements

The authors thank the support given by the project Mediterranean Coppice – Agriculture and Agro-Industry Including Fisheries 0904 and Praxis Flor 2128/95. We also wish to thank the anonymous reviewers and the associate editor for helpful comments and suggestions.

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