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Additivity in tree biomass components of Pyrenean oak (*Quercus pyrenaica* Willd.)

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Abstract

In tree biomass estimations, it is important to consider the property of additivity, i.e., the total tree biomass should equal the sum of the components. This work presents functions that allow estimation of the stem and crown dry weight components of Pyrenean oak (*Quercus pyrenaica* Willd.) trees. A procedure that considers additivity of tree biomass components is presented, and applied to a particular case. The application of a simultaneous equations system estimation procedure that used parameter restrictions and considered residual contemporaneous correlations allowed more efficient estimates and consistent prediction intervals.

Keywords: *Quercus pyrenaica* Willd.; Biomass; Additivity; Seemingly unrelated regressions

1. Introduction

Pyrenean oak is an important natural species of Portugal, often used as firewood in coppice systems. In high forest it is also important to have a means of biomass estimation. There are few studies concerned with the biomass of this species, and for crown biomass there are no evaluations. A quantitative stand assessment is important for adequate forest management.

When we consider more than one tree component, it is desirable to have the property of additivity in estimations of the components. That is, the predictions for the components sum to the prediction from the total tree regression. Although this property is proposed by several authors (e.g. Kozak, 1970; Chiyenda and Kozak, 1984; Cunia and Briggs, 1984), it is frequently ignored. Some approaches were suggested in Cunia and Briggs (1984), Reed and Green (1985) and Parresol (1999).

The aims of the present work are to: (1) present stem and crown biomass functions for Pyrenean oak trees; (2) guarantee the property of additivity by using restricted joint-generalized least squares; (3) present a reliability analysis with an example case.

2. Data collection

The study area includes the natural extent of Pyrenean oak stands in Portugal. Biomass evaluations were done with 166 trees, cut from 83 plots. Sample

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stands included different ages and stocking levels to consider a wide range of tree dimensions. Field and laboratory procedures followed methodology used by several authors (Johnstone, 1966; Crow, 1973; Hepp and Brister, 1982; Marklund, 1983; Chojnacky, 1992; Maingi and Ffolliott, 1992; Clair, 1993; Bartelink, 1996). Before tree felling, measurements included two perpendicular crown diameters (d_c) and live crown height (h_{lc}). Trees were pruned and cut into logs of 1 m length above breast height up to a 2.5 cm top diameter, similar to Baz et al. (1987) and Bengoa et al. (1991). A wood disk was collected from each log. In the laboratory, specific gravity was obtained for each disk using the methodology of Haygreen and Bowyer (1982). The determination of the saturated wood volume was done in a hydrostatic balance by the water displacement method (Bisch, 1986; Brasil et al., 1994). Dry weight of the stem was obtained from the volume and specific gravity of each log (Haygreen and Bowyer, 1982). Crown dry weight was obtained from the 'fresh weight/oven-dry weight' ratio of crown sub-samples. As done by Baz et al. (1987), Bengoa et al. (1991) and Chojnacky (1992), branches with a diameter less than 2.5 cm were not considered because these branches are not used as firewood, and are usually left in the forest. Leaving the smaller branches is beneficial because their high nutrient content is reclaimed by the site (Bouchon et al., 1985; Lemoine et al., 1990). The tree components considered in this work are stem and crown dry weight, with bark, up to a 2.5 cm top diameter.

3. Methodology and results

A detection of anomalous and influential data was done as recommended by Belsley et al. (1980) and Barnett and Lewis (1995). Graphical distributions of biomass values and diagnostic statistics (leverage values, SDFBETA, Cook's distance, SDFFIT) were analysed. Supplementary fit statistics used were the mean absolute error (MAE), maximum error (EMax), minimum error (EMin), and mean error (ME). Significance of regression was determined at a 5% significance level. Table 1 presents descriptive statistics for mean tree diameter (d), height (h), mean crown diameter (d_c), live crown length ($\mathrm{lcl} = h - h_{lc}$), stem biomass ($w_{\mathrm{stem}\ 2.5}$) and crown biomass ($w_{\mathrm{crown}\ 2.5}$).

Table 1
Tree biomass descriptive statistics for data set (average, S.D., minimum, maximum)

	Average	S.D.	Minimum	Maximum
d (cm)	16.9	7.70	2.5	46.0
h (m)	11.6	3.78	3.3	27.0
d_c (m)	3.6	1.75	1.3	9.6
lcl (m)	5.4	2.12	1.0	14.0
$w_{\mathrm{stem}\ 2.5}$ (kg)	119.0	154.65	0.7	1408.0
$w_{\mathrm{crown}\ 2.5}$ (kg)	25.4	35.37	0.0	212.5

3.1. Stem biomass ($w_{\mathrm{stem}\ 2.5}$)

For stem biomass a nonlinear allometric function was selected. To meet the assumption of homogeneous variance, the logarithmic transformation was used, as suggested by Carroll and Ruppert (1988):

$$\ln w_{\mathrm{stem}\,2.5} = \ln \beta_0 + \beta_1 \ln d^2 h + \ln \varepsilon \qquad (1)$$

where $w_{\mathrm{stem}\ 2.5}$ is the stem biomass (kg) up to a 2.5 cm top diameter, d the diameter at breast height (cm), h the total tree height (m), ln the natural logarithm, the β's are model parameters, and ε the residual error. The variable d^2h is usually used in biomass equations and at the present study it gives the best estimations. Parameter values, standard error of parameters, r^2_{ad} and mean-squared error (MSE), are presented in Table 2. Others fit statistics are shown in Table 3. The residuals of Eq. (1) appear to have a normal

Table 2
Coefficients, standard errors, r^2_{ad} and MSE for tree component biomass equations

Model	Coefficient (standard error)				
	b_0	b_1	b_2	r^2_{ad}	MSE
$w_{\mathrm{stem}\ 2.5}$	ln b_0: -3.323 (5.743×10^{-2})	0.950 (7.257×10^{-3})		0.990	1.568×10^{-2}
$w_{\mathrm{crown}\ 2.5}$	-14.246 (0.611)	2.248 (9.810×10^{-2})	-1.972×10^{-2} (3.131×10^{-3})	0.861	0.473

Table 3
Supplementary fit statistics for stem ($w_{\text{stem 2.5}}$) and crown ($w_{\text{crown 2.5}}$) biomass equations (MAE, EMax, EMin, ME)

	MAE (kg)[a]	EMax (kg)	EMin (kg)	ME (kg)[b]
$w_{\text{stem 2.5}}$	6.485	40.294	−53.671	−0.606
$w_{\text{crown 2.5}}$	6.870	48.176	−41.856	−0.987

[a] $\sum_{i=0}^{n}|yi-\hat{y}i|/n$.
[b] $\sum_{i=0}^{n}(yi-\hat{y})/n$.

distribution and homocedasticity (Fig. 1). The MAE is 6.485 kg and the ME is −0.606 kg which means that, in general, the model slightly underestimates stem biomass. Maximum and minimum errors obtained are 40.294 and −53.671 kg, respectively.

Before converting logarithmic units to arithmetic units the following correction factor is added, $\hat{\sigma}^2/2 = 0.0078$, as suggested by Baskerville (1972) and Wiant and Harner (1979). This permits the correction of biased estimations in the process of anti-logarithmic conversions.

3.2. Crown biomass ($w_{\text{crown 2.5}}$)

For tree crown biomass estimation several models and tree variables were collected from the forestry literature and tested (Johnstone, 1966; Crow, 1973; Ek, 1979; Baldwin, 1986; Rondeux et al., 1987; Hepp and Brister, 1982; Chojnacky, 1992; Clair, 1993). The best estimations are obtained with an equation that includes the variables d^2h and lcl h. In fact, tree biomass is highly correlated with d^2h as shown by others studies, and lcl h gives a measure of relative crown dimensions. The crown biomass equation used is

$$\ln w_{\text{crown 2.5}} = \beta_0 + \beta_1 \ln d^2h + \beta_2 \,\text{lcl}\, h + \ln \varepsilon \qquad (2)$$

with $w_{\text{crown 2.5}}$ the crown biomass (kg) up to a 2.5 cm top diameter, and the others variables defined as before. The equation coefficients and standard errors, r^2_{ad} and MSE are presented in Table 2. It has an acceptable level of multicolinearity (maximum 'variance inflactor factor' = 3.032; 'condition index of matrix $(X'X)' = 21.939$). The residuals from Eq. (2) appear to have a normal distribution and homocedasticity (Fig. 2). Table 3 shows

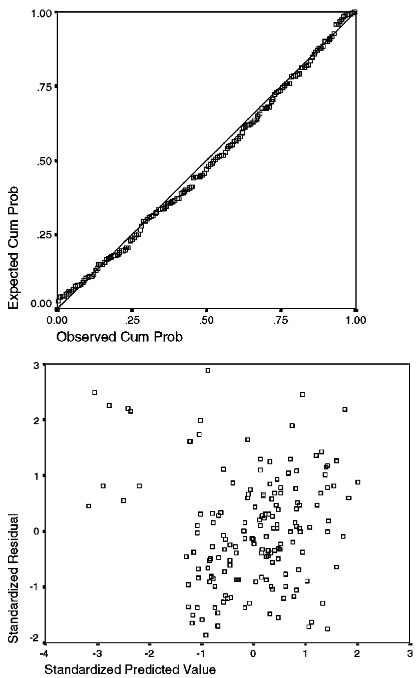

Fig. 1. Plots of residuals for the stem biomass equation (expected and observed cumulative probability; standardized residuals and predicted values).

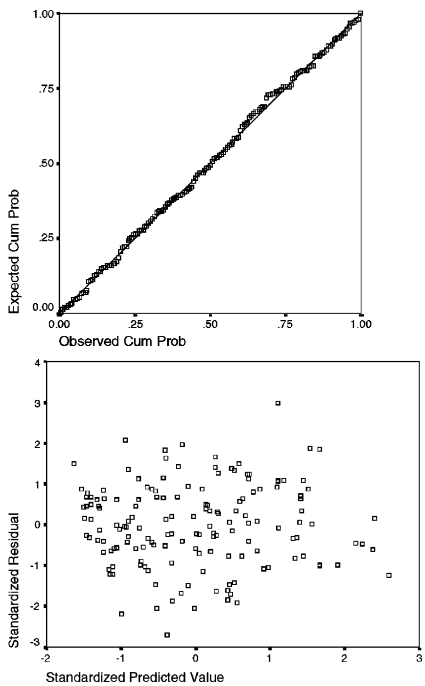

Fig. 2. Plots of residuals for the crown biomass equation (expected and observed cumulative probability; standardized residuals and predicted values).

others fit statistics. The anti-log correction factor used with Eq. (2) is $\hat{\sigma}^2/2 = 0.2365$. As expected, the crown biomass equation is less accurate than the stem model, because of crown weight variations among trees with the same diameter and height. Different stand densities, crown shapes and branch wood variations could originate those variations (Pardé, 1980).

3.3. Tree biomass ($w_{tree\ 2.5}$)

The tree biomass up to a 2.5 cm top diameter ($w_{\text{tree }2.5}$) should correspond to the addition of stem ($w_{\text{stem }2.5}$) and crown ($w_{\text{crown }2.5}$) biomass. For $w_{\text{tree }2.5}$, it is desirable that the tree biomass estimation process ensure the additivity of the tree biomass components. This property has been recommended by authors such as Kozak (1970), Chiyenda and Kozak (1984), and Cunia and Briggs (1984). In this study, the additivity of components to exactly equal tree biomass (i.e., $w_{\text{stem}\,2.5} + w_{\text{crown}\,2.5} \equiv w_{\text{tree}\,2.5}$) is guaranteed using nonlinear joint-generalized regression (more commonly called nonlinear seemingly unrelated regression or NSUR) applied to a system of equations, that includes an equation for tree biomass, with cross-equation constraints. In the present case, a nonlinear system is obtained and NSUR is required.

The additivity can be guaranteed using a tree biomass equation with the same independent variables as the biomass component equations. A new parameter estimation is done, with parameter restrictions in the tree biomass equation (Reed and Green, 1985; Reed, 1986; Parresol, 2001). The tree biomass equation is then a function of the independent variables from each ith component equation,

$$\mathbf{y}_i = \mathbf{f}_i(\mathbf{X}_i, \boldsymbol{\beta}_i) + \boldsymbol{\varepsilon}_i, \quad i = 1, \ldots, k,$$
$$\mathbf{y}_{\text{tree}} = \mathbf{f}_{\text{tree}}(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_k, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_k) + \boldsymbol{\varepsilon}_k \qquad (3)$$

In this analysis we used the method of simultaneous estimation known as SURs (‘seemingly unrelated regressions’). References about the SUR estimation method can be found in Reed (1986), Gallant (1987) and Srivastava and Giles (1987). Through SUR, it is possible to estimate a system of equations that are statistically correlated and to impose parameter restrictions. This technique considers the existence of contemporaneous correlations among the residuals of the equations, which results in lower variance. As Chiyenda and Kozak (1984) said, it is not realistic to consider the components to be independent or the residuals ε_i $(i = 1, \ldots, k)$ to be uncorrelated, because the same tree gives values for more than one biomass component. In this way, specifying the stochastic properties of the residual vectors, more efficient parameter estimates and more reliable prediction intervals are obtained.

In the present case, the system includes the following structural models for stem, crown and tree biomass:

$$\ln w_{\text{stem}\,2.5} = \ln\left[\beta_{10}(d^2h)^{\beta_{11}}\right] + \ln \varepsilon_{\text{stem}\,2.5},$$
$$\ln w_{\text{crown}\,2.5} = \beta_{20} + \beta_{21}\ln d^2h + \beta_{22}\,\text{lcl}\,h + \ln \varepsilon_{\text{crown}\,2.5},$$
$$\ln w_{\text{tree}\,2.5} = \ln\Big[\beta_{10}(d^2h)^{\beta_{11}} + \exp(\beta_{20} + \beta_{21}\ln d^2h + \beta_{22}\,\text{lcl}\,h)\Big] + \ln \varepsilon_{\text{tree}\,2.5} \qquad (4)$$

The inherent model for $w_{\text{tree }2.5}$ cannot be linearized. Thus, a nonlinear system is obtained and parameters can be efficiently estimated using NSUR. The specific system of three models in (4) can be combined into one model written in matrix notation as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{f}_1(\mathbf{X}_1, \mathbf{0}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \\ \mathbf{f}_2(\mathbf{0}, \mathbf{X}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \\ \mathbf{f}_3(\mathbf{X}_1, \mathbf{X}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \boldsymbol{\varepsilon}_3 \end{bmatrix} \qquad (5)$$

or alternatively, in a compact form,

$$\mathbf{y} = \mathbf{f}(\mathbf{X}, \boldsymbol{\beta}) + \boldsymbol{\varepsilon}$$

where subscript 1 refers to the model for stem biomass, subscript 2 refers to the model for crown biomass, subscript 3 refers to the model for tree biomass, and the vectors $\mathbf{y}$, $\boldsymbol{\beta}$, and $\boldsymbol{\varepsilon}$ are stacked column vectors. In particular, $\mathbf{y} = [\mathbf{y}_1' \;\; \mathbf{y}_2' \;\; \mathbf{y}_3']'$ where $\mathbf{y}_1 = \{\ln w_{\text{stem}\,2.5,t}\}$, $\mathbf{y}_2 = \{\ln w_{\text{crown}\,2.5,t}\}$, $\mathbf{y}_3 = \{\ln w_{\text{tree}\,2.5,t}\}$ for $t = 1, \ldots, 161$, $\beta = [\beta_1' \;\; \beta_2']' = [\beta_{10} \;\; \beta_{11} \;\; \beta_{20} \;\; \beta_{21} \;\; \beta_{22}]'$, and the definitions of the remaining vectors and matrices follow logically.

We assume that the elements of the disturbance vector ε follow a distribution with a zero common mean $(E(\boldsymbol{\varepsilon}_i) = 0)$. Further we assume $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \boldsymbol{\Sigma} \otimes \mathbf{I}$, where $\otimes$ is the Kronecker product, Σ is a (3×3) covariance matrix whose (i,j)th element is given by σ_{ij} (the covariance of the errors from equations i and j) and $E(\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_j') = \sigma_{ij}\mathbf{I}$. The unknown covariances, σ_{ij}, are consistently estimated from the following expression:

$$\hat{\sigma}_{ij} = \frac{1}{(T - K_i)^{1/2}(T - K_j)^{1/2}}\mathbf{e}_i'\mathbf{e}_j \qquad (6)$$

Table 4
Coefficients, standard errors, r^2 and MSE for the $w_{\text{stem 2.5}}$ and $w_{\text{crown 2.5}}$ equations, using the NSUR estimation method

Model	Coefficient (standard error)						
	b_{10}	b_{11}	b_{20}	b_{21}	b_{22}	r^2_{ad}	MSE
$w_{\text{stem 2.5}}$	3.688×10^{-2} (0.205×10^{-2})	0.949 (0.698×10^{-2})				0.991	1.580×10^{-2}
$w_{\text{crown 2.5}}$			−13.909 (0.406)	2.131 (0.643×10^{-1})	-0.145×10^{-1} (0.229×10^{-2})	0.852	0.505

where $\mathbf{e}_i = \mathbf{y}_i - \mathbf{f}_i(\mathbf{X}_i, \mathbf{b}_i)$ are the residuals obtained from nonlinear least squares, K_i and K_j are the number of parameters of the ith and jth equations, and $\mathbf{T}$ is the number of observations. If $\hat{\Sigma}$ is the matrix with the $\hat{\sigma}_{ij}$ estimates from Eq. (6), then the NSUR estimate of the vector β is the value $\mathbf{b}$ which minimizes the residual sum of squares (Gallant, 1987; Greene, 1999):

$$\mathbf{R}(\mathbf{b}) = \mathbf{e}'(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I})\mathbf{e} = [\mathbf{y} - \mathbf{f}(\mathbf{X}, \mathbf{b})]'(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I})[\mathbf{y} - \mathbf{f}(\mathbf{X}, \mathbf{b})] \tag{7}$$

The parameters in Eq. (4), were estimated by minimizing Eq. (7) (using SAS/ETS (SAS, 1995)). Table 4 presents the parameter estimates for the $w_{\text{stem 2.5}}$ and $w_{\text{crown 2.5}}$ equations, and the respective standard errors and fit statistics. We can see that the parameter standard errors were decreased, giving more precision in reliability analysis. Table 5 shows the fit statistics from the utilization of the NSUR estimation method for the $w_{\text{stem 2.5}}$ and $w_{\text{crown 2.5}}$ equations. The anti-log correction factors, $\hat{\sigma}^2/2$, are 0.0079 and 0.2525 for $w_{\text{stem 2.5}}$ and $w_{\text{crown 2.5}}$, respectively. As expected, SUR produce little changes in the fit statistics compared to the ordinary least squares method.

To compute variances we need the matrix of partial derivatives of the residual with respect to the parameters. To be clear, there are $T = 161$ observations per equation, $M = 3$ equations, and $K = 5$ parameters ($\boldsymbol{\beta}$ has dimension (5×1)). For our NSUR system the partial derivatives matrix $\mathbf{P}(\boldsymbol{\beta})'$ is a $(K \times MT)$ matrix given by

$$\mathbf{P}(\boldsymbol{\beta})' = \frac{\partial \mathbf{e}'}{\partial \boldsymbol{\beta}} = \left[\frac{\partial \mathbf{f}'_1}{\partial \boldsymbol{\beta}}, \frac{\partial \mathbf{f}'_2}{\partial \boldsymbol{\beta}}, \ldots, \frac{\partial \mathbf{f}'_M}{\partial \boldsymbol{\beta}}\right] \tag{8}$$

The estimated variance–covariance matrix of the parameter estimates is given by

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{b}} = [\mathbf{P}(\mathbf{b})'(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I})\mathbf{P}(\mathbf{b})]^{-1} \tag{9}$$

the NSUR system variance is based on Eq. (7) and is obtained from,

$$\hat{\sigma}^2_{\text{NSUR}} = \frac{\mathbf{R}(\mathbf{b})}{MT - K} = \frac{\mathbf{e}'(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I})\mathbf{e}}{MT - K} \tag{10}$$

and the estimated variance from the ith system equation on the tth observation $\hat{y}_{it}$ (where for simplicity we drop the t subscript) is given by,

$$S^2_{\hat{y}_i} = \mathbf{p}_i(\mathbf{b})'\hat{\boldsymbol{\Sigma}}_{\mathbf{b}}\mathbf{p}_i(\mathbf{b}) \tag{11}$$

where $\mathbf{p}_i(\mathbf{b})'$ is a row vector for the ith equation from the partial derivatives matrix $\mathbf{P}(\mathbf{b})$ (Judge et al., 1988; Greene, 1999).

4. Application and reliability

Now, it is possible to calculate a confidence interval for an estimated value, $\hat{y}_i$, and a prediction interval for a new value, $\hat{y}_{i(\text{new})}$, with $100(1-\alpha)\%$ probability, from the following expressions (Judge et al., 1988; Greene, 1999):

Estimation from ith equation : $\hat{y}_i = f_i(\mathbf{x}, \mathbf{b})$

Confidence interval : $\hat{y}_i \pm t_{(\alpha/2)}\sqrt{S^2_{\hat{y}_i}}$

Prediction interval : $\hat{y}_{i(\text{new})} \pm t_{(\alpha/2)}\sqrt{S^2_{\hat{y}_i} + \hat{\sigma}^2_{\text{NSUR}}\hat{\sigma}_{ii}}$

where $\hat{\sigma}_{ii}$ is the i,i-element of the $\hat{\boldsymbol{\Sigma}}$ matrix.

Table 5
Supplementary fit statistics for $w_{\text{stem 2.5}}$ and $w_{\text{crown 2.5}}$ equations, from the simultaneous estimation method (MAE, EMax, EMin, ME)

	MAE (kg)	EMax (kg)	EMin (kg)	ME (kg)
$w_{\text{stem 2.5}}$	7.282	38.498	−54.091	−1.148
$w_{\text{crown 2.5}}$	7.086	51.500	−38.299	−0.353

Let us consider the case of a tree with $d = 16.4$ cm, $h = 11.4$ m, dc $= 3.7$ m and lcl $= 5.3$ m. In this case, the estimated variance–covariance matrix of the parameters is,

$$\hat{\Sigma}_{\mathrm{b}} = \begin{bmatrix} 4.1928\times10^{-6} & -0.000014 & -0.00006 & 4.1243\times10^{-6} & 2.2038\times10^{-7} \\ -0.000014 & 0.0000488 & 0.000155 & -8.694\times10^{-6} & -8.218\times10^{-7} \\ -0.00006 & 0.000155 & 0.1649 & -0.0250 & 0.000593 \\ 4.1243\times10^{-6} & -8.694\times10^{-6} & -0.0250 & 0.004129 & -0.000120 \\ 2.2038\times10^{-7} & -8.218\times10^{-7} & 0.000593 & -0.000120 & 5.2321\times10^{-6} \end{bmatrix}$$

and the NSUR system variance obtained is, $\hat{\sigma}^2_{\mathrm{NSUR}} = 0.9335$.

The vector $\mathbf{p}_3(\mathbf{b})$, which contains the partial derivatives of the $w_{\mathrm{tree}\ 2.5}$ equation, is specified as follows. Let $u = b_{10}(d^2h)^{b_{11}} + \mathrm{e}^{b_{20}+b_{21}\ln(d^2h)+b_{22}\,\mathrm{lcl}\,h}$, then we have

$$\mathbf{p}_3(\mathbf{b}) = \begin{bmatrix} \dfrac{(d^2h)^{b_{11}}}{u} \\ \dfrac{b_{10}(d^2h)^{b_{11}}\ln(d^2h)}{u} \\ \dfrac{\mathrm{e}^{b_{20}+b_{21}\ln(d^2h)+b_{22}\,\mathrm{lcl}\,h}}{u} \\ \dfrac{\ln(d^2h)\,\mathrm{e}^{b_{20}+b_{21}\ln(d^2h)+b_{22}\,\mathrm{lcl}\,h}}{u} \\ \dfrac{\mathrm{lcl}h\,\mathrm{e}^{b_{20}+b_{21}\ln(d^2h)+b_{22}\,\mathrm{lcl}\,h}}{u} \end{bmatrix}$$

The variance, $S^2_{\hat{y}_3}$, of the tree biomass equation is then, $S^2_{\hat{y}_3} = \mathbf{p}_3(\mathbf{b})'\hat{\Sigma}_{\mathbf{b}}\mathbf{p}_3(\mathbf{b}) = 0.00014192$

Using Eqs. (1) and (2) for $w_{\mathrm{stem}\ 2.5}$ and $w_{\mathrm{crown}\ 2.5}$, with their respective correction factors, $\hat{\sigma}^2/2$, gives in arithmetic units,

$$w_{\mathrm{stem}\,2.5} = 75.68, \qquad w_{\mathrm{crown}\,2.5} = 13.10$$

and

$$\ln w_{\mathrm{tree}\,2.5} = \ln(w_{\mathrm{stem}\,2.5} + w_{\mathrm{crown}\,2.5}) = 4.4861$$

The 95% confidence interval for $w_{\mathrm{tree}\ 2.5}$ becomes ($t_{0.025,161} = 1.975$), in logarithmic units,

$$4.4861 \pm 0.02353 = [4.4626; 4.5096]$$

and in arithmetic units, [86.7; 90.9] kg.

On the other hand, the prediction interval is, with $\hat{\sigma}_{33} = 0.0222$,

$$4.4861 \pm 0.2853 = [4.2008; 4.7714]$$

and in arithmetic units, [66.7; 118.1] kg.

5. Discussion and conclusions

Kozak (1970) discussed the additivity property of biomass components for linear equations. Reed and Green (1985) and Reed (1986) extended the methods for nonlinear equations using generalized least squares estimation. Parresol (2001) showed there were two procedures to force additivity of nonlinear biomass equations and demonstrated the superiority of using nonlinear SURs. The consideration of the additivity in a system of equations insures the consistency among the components. In the present work, the process of additivity was realized with linear logarithmic equations to achieve a normal distribution and homocedasticity of the residuals (Carroll and Ruppert, 1988).

It was verified that the NSUR estimation method gave appreciable improvement in estimation precision. Though the values of r^2 and MSE change little for w_{stem} w_{crown}, there was a decrease in the parameter standard errors, which is a direct result of exploiting the contemporaneous correlation between the related stem and crown biomass equations in the SUR method. Small changes were also obtained on the supplementary fit statistics with the SUR method. For the stem biomass equation, some fit statistics are worse (MAE, ME, EMin), while others are better (EMax). For the crown biomass equation an improvement was obtained for ME and EMin.

The biggest effect using the process of simultaneous estimation is reduce the confidence and prediction intervals of the biomass estimations (Parresol, 2001). These reductions result from the smaller variance obtained by the application of the SUR estimation method, which considers the contemporaneous correlation among the components, resulting in more efficient parameter estimates. This has an important implication in forest inventory estimates and management plans because less variations are obtained in reliability analysis.

For the stem biomass equation, tree diameter and height are used as independent variables. This equation gives good stem weight estimations. The crown biomass equation uses tree diameter, height and live crown length as independent variables. The additional variable LCL was shown to be significant for crown biomass estimation, as happens with other authors (e.g. Johnstone, 1966; Parresol, 1999). The variable lcl h expresses the relative crown dimensions according to tree size and stand density. This equation gives satisfactory estimations of crown biomass in feasibility applications. However, users must consider that the variance in crown biomass estimations is, in relative terms, higher than the variance obtained with stem biomass estimations. This is because of the variability of internal crown structure, number of branches and variations in wood density along the branches, as reported by Pardé (1980). In this analysis, crown diameter has been shown to be of little significance compared with live crown length. The inclusion of these two variables in the crown model will increase multicollinearity because they are significantly correlated ($\rho_{\mathrm{lcl-dc}} = 0.78$). It has been observed that crowns with small diameters also have small lengths.

The estimation procedure presented here has provided considerable gains in reliability and efficiency by improving inferences. We have described the estimation method that guarantees the property of biomass components additivity.

The present paper also provides to forest managers and applied biologists a means for assessing Pyrenean oak forest stand productivity in terms of biomass. The equations allow the users to estimate tree biomass up to a 2.5 cm top diameter, including bark.

Because in forest management it is important to make inferences about the estimations, the example case shows how to obtain the confidence and prediction intervals for reliability analysis.

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References

Baldwin, V.C., 1986. A summary of equations for predicting biomass of planted southern pines. In: Estimating Tree Biomass Regressions and Their Error, Proceedings of the Workshop on Tree Biomass Regressions Functions and Their Contribution to the Error of Forest Inventory Estimates, May 26–30, Syracuse, NY, For. Serv. Exp. St., pp. 157–171.

Barnett, V., Lewis, T., 1995. Outliers in Statistical Data, 3rd ed. Wiley, New York, 584 pp.

Bartelink, H.H., 1996. Allometric relationships on biomass and needle area of douglas-fir. Forest Ecol. Manage. 86, 193–203.

Baskerville, G.L., 1972. Use of logarithmic regression in the estimation of plant biomass. Can. J. For. 2, 49–53.

Baz, S., Prieto, A., González, I., Torre, M., 1987. Evaluación Energética de las Massas de *Quercus pyrenaica* para la Producción de Energia de Origem Termoeléctrico. UNESA-ASINEL, 148 pp.

Belsley, D.A., Kuh, E., Welsch, R.E., 1980. Regression Diagnostics. Identifying Influential Data and Sources of Collinearity. Wiley, New York, 292 pp.

Bengoa, J., San Miguel, A., Allue, M., 1991. Estimacion de Biomassa y Determinacion de Calidad en Tallares de Rebollo (*Quercus pyrenaica* Willd.) de La Rioja. III. Reúnion de la Ass. Esp. Ecologia Terrestre, León, Set., 15 pp.

Bisch, J.L., 1986. Le taux de conversion de volume de bois frais en biomasse: amélioration de ses méthodes d'estimation. Ann. Sci. For. 43 (3), 383–396.

Bouchon, J., Nys, C., Ranger, J., 1985. Cubage, biomasse et minéralomasse: comparaison de trois taillis simples des ardennes primaires. Acta Oecol. 6 (20), 53–72.

Brasil, M.A.M., Veiga, R.A.A., Timoni, J.L., 1994. Erros na determinação da densidade básica da madeira. CERNE 1 (1), 55–57.

Carroll, R.J., Ruppert, D., 1988. Transformation and Weighting in Regression. Chapman & Hall, New York, 249 pp.

Chiyenda, S.S., Kozak, A., 1984. Additivity of component biomass regression equations when the underlying model is linear. Can. J. For. Res. 14, 441–446.

Chojnacky, D.C., 1992. Estimating volume and biomass for dryland oak species. In: Ecology and Management of Oaks and Associated Woodlands: Perspectives in the Southwestern United States and Northern Mexico, Sierra Vista, Workshop, April, pp. 155–161.

Clair, J.B., 1993. Family differences in equations for predicting biomass and leaf area in douglas-fir (*Pseudotsuga menziesii* var. *menziesii*). Forest Sci. 39 (4), 743–755.

Crow, T.R., 1973. The estimation of biomass in a natural stand of jack pine (*Pinus banksiana* Lamb.). Ph.D. Thesis. University of Minnesota, 143 pp.

Cunia, T., Briggs, R.D., 1984. Forcing additivity of biomass tables: some empirical results. Can. J. For. Res. 14, 376–384.

Ek, A.R., 1979. A model for estimating branch weight and branch leaf weight in biomass studies. For. Sci. 25 (2), 303–306.

Gallant, A.R., 1987. Nonlinear Statistical Models. Wiley, New York.

Greene, W.H., 1999. Econometric Analysis. Prentice-Hall, Upper Saddle River, NJ, 1004 pp.

Haygreen, J.G., Bowyer, J.L., 1982. Forest Products and Wood Science. Iowa State University Press, 495 pp.

Hepp, T.E., Brister, G.H., 1982. Estimating crown biomass in loblolly pine plantations in the carolina flat woods. For. Sci. 28 (1), 115–127.

Johnstone, W.D., 1966. Dry-matter production and complete-tree utilization of lodgepole pine in Alberta. Ph.D. Thesis. B.S.F., University of British Columbia, 137 pp.

Judge, G.G., Hill, R.C., Griffiths, W.E., Lütkepohl, H., Lee, T.-C., 1988. Introduction to the Theory and Practice of Econometrics. Wiley, New York, 1024 pp.

Kozak, A., 1970. Methods for ensuring additivity of biomass components regression analysis. For. Chron. 46 (5), 402–404.

Lemoine, B., Ranger, J., Gelpe, J., 1990. Interprétation et modélisation de la distribution des éléments nutritifs dans les différents compartiments de la biomasse dún jeune peuplement de pin maritime. Ann. Sci. For. 47, 101–115.

Maingi, J.K., Ffolliott, P.F., 1992. Specific gravity and estimated physical properties of emory oak in southeastern Arizona. In: Ecology and Management of Oaks and Associated Woodlands: Perspectives in the Southwestern United States and Northern Mexico, Sierra Vista, Workshop, April, pp. 147–149.

Marklund, L.G., 1983. Collection data for biomass equation development: some methodological aspects. In: Mesures des Biomasses et des Accroissements Forestiers, Orléans, Colloque, October, INRA, pp. 37–43.

Pardé, J.D., 1980. Forest Biomass. For. Abstr. 41 (8), 343–362.

Parresol, B.R., 1999. Assessing tree and stand biomass: a review with examples and critical comparisons. For. Sci. 45 (4), 573–593.

Parresol, B.R., 2001. Additivity of nonlinear biomass equations. Can. J. For. Res. 31, 865–878.

Reed, D.D., 1986. Estimation procedures for analytically related growth and yield models. In: Proceedings of the 1986 Annual Meeting of the Society of American Foresters, pp. 89–92.

Reed, D.D., Green, E.J., 1985. A method of forcing additivity of biomass tables when using nonlinear models. Can. J. For. Res. 15, 1184–1187.

Rondeux, J., Laurent, C., Toussant, A., 1987. Méthode d'estimation du volume des branches et des houppiers en peuplements de hêtres et de chênes. RFF 20 (2), 125–129.

SAS, 1995. SAS/ETS User's Guide, version 6, 2nd ed. SAS Institute Inc., Cary, 1022 pp.

Srivastava, V.K., Giles, D.E.A., 1987. Seemingly Unrelated Regression Equations Models. Estimation and Inference. Marcel Decker, New York, 374 pp.

Wiant Jr., H.V., Harner, E.J., 1979. Percent bias and standard error in logarithmic regression. For. Sci. 25 (1), 167–168.