PROJECTING A STAND TABLE THROUGH TIME[1]

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Abstract—Stand tables provide number of trees per acre for each diameter class. This paper presents a general technique to predict a future stand table, based on the current stand table and future stand summary statistics such as trees and basal area per acre, and average diameter. The stand projection technique involves (a) predicting surviving trees for each class, and (b) growing tree diameters such that the stand table at the end of the growth period produces future stand summaries identical to those mentioned above. Linear and nonlinear diameter growth models produced similar results. The stand table projection approach was as good as the parameter-recovery Weibull model for unimodal distributions, and out-performed the Weibull when diameter distributions were bimodal or irregular.

INTRODUCTION

Stand tables give number of trees per acre for each diameter class. They provide information to compute product volumes and are thus a desirable feature in growth and yield modeling. In 1980, Clutter and Jones developed a stand table projection model which was later revised by Pienaar and Harrison (1988). Nepal and Somers (1992) developed an algorithm to project a stand table so that the future stand table would result in trees and basal area per acre identical to observed values. Recently, Tang and others (1997) introduced a distribution-independent approach to project a diameter distribution through time.

The objective of this project was to develop a model to project a current stand table to a future age, such that the resulting future stand table produces stand-level variables such as trees per acre, basal area per acre, and average diameter. These variables are compatible with either actual values or those predicted from a whole stand model.

DATA

Two data sets were used in this research. The first data set consisted of planted loblolly pines (*Pinus taeda* L.) at the Hill Farm Research Station in north Louisiana. These trees were subjected to thinning and pruning treatments at ages 6 and 11 plus a control. There was a total of 35 plots. Diameter at breast height (dbh) was recorded for each tree every 3 to 8 years (table 1).

The second data set was from 63 direct-seeded **longleaf** pine (*Pinus palustris* Mill.) plots established in central Louisiana. Tree diameters were remeasured every 3 or 5 years (table 1). Precommercial thinning was applied to 39 plots at age 7.

Table 2 shows the distribution of observations from stands of planted loblolly pines and direct-seeded **longleaf** pines by stand age and basal area.

METHODS

The stand table projection system consisted of a survival equation and a diameter growth equation.

Survival Equation

For simplicity's sake, we assumed that all mortality occurred at the beginning of the growth period and surviving trees would remain at the end of the period. Number of trees for the ith diameter class after mortality can be computed from

$$n_{2i} = n_{1i} \{1 - \exp[a_1 (D_i - D_{min} + 1)]\} \qquad (1)$$

where

n_{1i} and n_{2i} = trees per acre in the ith diameter class before and after mortality, respectively,
D_i = midpoint of the ith diameter class,
D_{min} = midpoint of the minimum diameter class, and
a_1 = coefficient.

Note that the quantity $(D_i - D_{min})$ represents the distance between the ith diameter class and the minimum diameter. Equation (1) indicates that trees in a small diameter class will suffer more heavily from mortality than those from a larger diameter class. The survival function has only one coefficient, a,, which was computed such that the sum of trees over all diameter classes was equal to total trees per acre after mortality. A numerical method such as the secant method (Press and others 1996) can be used to solve for a,.

Diameter Growth Equations

In this study, we looked at a linear growth equation,

$$D_{2i} = b_1 + b_2 D_{1i}, \qquad (2)$$

and a nonlinear growth equation,

$$D_{2i} = b_1 (D_{1i})^{b_2}, \qquad (3)$$

where

D_{1i} and D_{2i} = midpoints of the ith diameter class at times 1 and 2, respectively, and
b, and b_2 = coefficients.

Bailey (1980) showed that diameter growth was either linear or nonlinear, depending on the diameter distribution. His nonlinear equation form was slightly different from equation (3).

After diameter growth, number of trees per acre in each diameter class was recalculated using Nepal and Somers' (1992) approach which assumed that trees in each class followed a doubly-truncated Weibull distribution.

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[1] Paper presented at the Tenth Biennial Southern Silvicultural Research Conference, Shreveport, LA, February 16-16, 1999.

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Table l--Distribution of 35 planted loblolly pine plots and 63 direct-seeded longleaf pine plots by measurement age

Plots	Measurement ages
No.	
Planted loblolly pines	
26	11, 16, 21, 29
9	13, 18, 21, 25, 29
Direct-seeded longleaf pines	
7	14, 17
7	14, 17, 22
1	17, 22
18	17, 20, 23, 26
8	18, 23
22	22, 27

Table 2-Distribution of observations from stands of planted loblolly and direct-seeded longleaf pines by stand age and basal area

	Stand basal area						
Age	40	80	120	160	200	240	Total
Years	*---------- Square feet per acre ----------*						
Planted loblolly pine							
11-13	23	3	4	5			35
14-16	6	16	4				26
17-19				4	5		9
20-22		14	9	3	8	1	35
23-25					9		9
29		4	13	9	8	1	35
All	29	37	30	21	30	2	149,
Direct-seeded longleaf pine							
14-16	14	2					14
17-19	8	19	11	3			41
20-22		7	19	21	1		48
23-25	2	5	4	14	1		26
26-28			4	34	2		40
All	22	33	38	72	4		169

The two parameters, b, and b_2, of the linear or nonlinear diameter growth equation were found such that summing up the resulting stand table produced the specified average diameter (Davg) and basal area per acre (B). Therefore b, and b_2 are solutions for the following system of 2 equations and 2 unknowns:

$$f_1 = (\Sigma n_i D_i)/N - Davg = 0 \quad (4)$$
$$f_2 = K\Sigma n_i D_i^2 - B = 0 \quad (5)$$

where
n_i = number of trees per acre in the ith diameter class after diameter growth,
N = total trees per acre after mortality, and
K = 0.005454 = a constant to convert diameters in square inches to basal area in square feet.
The summation sign denotes the sum over all diameter classes.

The algorithm used to solve for b, and b_2 is as follows:

1. Guess starting values for b, and b_2.
2. Compute diameter increment for each diameter class.
3. Compute number of trees for each diameter class (n,) after diameter growth.
4. Compute f, and f_2.
5. Compute new values for b, and b_2, then go back to step 2. The loop continues until f, and f_2 are sufficiently close to zero (or less than a predetermined value).

NUMERICAL EXAMPLE

Table 3 shows an example demonstrating the application of the new methods to projecting a stand table from age 20 to age 23 for plot 310109 from the direct-seeded longleaf pine data set. In this example, it was assumed that the observed stand table at age 20 was available, but only stand summaries at age 23 were known (N = 613.45 trees/acre, Davg = 6.33 in., and B = 149.22 sq.ft./acre). Number of trees in each diameter class was reduced after mortality (column 4). Final stand tables were then computed after trees grew according to the linear diameter growth function (column 5) or the nonlinear function (column 6). Note that stand tables produced by these two methods were very similar.

The observed and predicted trees per acre in each diameter class are shown in Figure 1. Also shown is a stand table from a Weibull distribution, obtained using the parameter recovery method. It is obvious from the graph that the Weibull did not perform very well when the actual distribution was bimodal as in this example.

EVALUATION

The stand table projection methods based on linear and nonlinear diameter growth were evaluated against the parameter-recovery Weibull distribution approach, using growth data from planted loblolly pines and direct-seeded longleaf pines. Three criteria were employed in the evaluation:

1 .The K-S statistic (Steel and Torrie 1980).
2. The Chi-square statistic (Steel and Torrie 1980).
3. A simple form of Reynolds and others' (1988) error index, which is the sum over all diameter classes of the difference between observed and predicted number of trees in each diameter class.

Table 3-A numerical example demonstrating the application of the new methods to projecting stand table from age 20 to age 23 for plot 310109 from the direct-seeded iongieaf pine data set

	Observed trees/acre		Predicted trees/acre at age 23		
Diameter (1)	Age 20 (2)	Age 23 (3)	After mortality (4)	Linear D growth (5)	Nonlinear D growth (6)
Inches					
2	33.61		13.76	7.46	7.43
3	134.45	67.23	87.57	57.51	57.50
4	126.05	100.84	100.10	89.81	89.87
5	75.63	58.82	66.44	77.79	77.81
6	100.84	75.63	93.60	72.41	72.43
7	142.86	109.24	136.80	99.80	99.80
8	75.63	100.84	73.74	110.13	110.08
9	33.61	58.82	33.11	61.02	61 .OO
10	8.40	33.61	8.33	29.00	29.00
11		8.40		7.95	7.96
12				.56	.57
N[a]	731.09	613.45	613.45	613.45	613.45
Davg	5.40	6.33	5.73	6.33	6.33
B	132.46	149.22	122.24	149.22	149.22

[a] N is total trees per acre, Davg is average diameter in inches, and B is stand basal area in $ft.^2$ per acre.

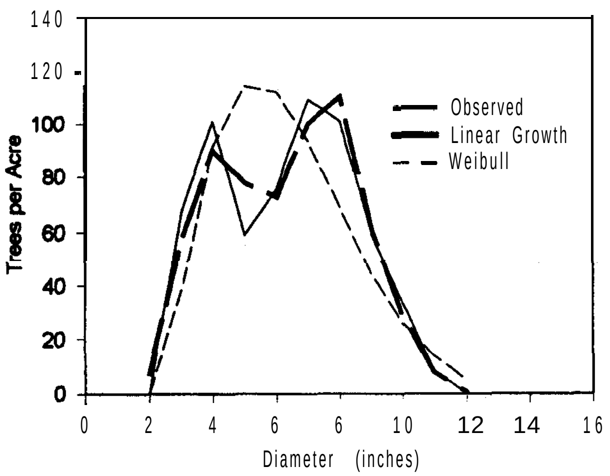

Figure I-Observed and predicted diameter distributions at age 23 for plot 310109 from the direct-seeded **longleaf** pine data set by the stand table projection and Weibull methods.

Results of the evaluation for both data sets are shown in Table 4.

Planted Lobioiiy Pines

The K-S statistics and the error indices for ail three models were not different at the 5 percent level, even though values from the Weibuil distribution model were always higher. Only the Chi-square statistics from the Weibuli distribution were significantly different from those of the other two models. The values for the linear and nonlinear diameter growth models were very close, confirming the fact that these two models produced very similar stand tables. The relatively good performance of the Weibuii model might be due to mostly unimodai diameter distributions from planted iobioiiy pines. These distributions could be adequately characterized by the Weibuii function.

Direct-Seeded Longieaf Pines

As with planted iobioiiy pines, the linear and nonlinear diameter growth models produced essentially identical values for direct-seeded iongieaf pines. The Weibuii values, on the other hand, were significantly higher for ail three criteria. The reason is likely that direct-seeded iongieaf pine data contained some plots that involved bimodai or irregular diameter distributions. As shown in the previous example, the Weibuii model did not perform as well as the stand table projection models in those cases.

Table 4-Means (and standard deviations) of the evaluation statistics by method and species

Method	Evaluation statistic K-S	χ^2	Error index
Planted iobioiiy pine (n = 114)			
Linear growth	**0.051[a]** (0.031)	**3.822[a]** (3.970)	**0.259[a]** (0.152)
Nonlinear growth	**.052[a]** (0.031)	**3.746[a]** (3.479)	**.260[a]** (0.154)
Weibuil	**.058[a]** (0.033)	**5.753[b]** (6.130)	**.285[a]** (0.156)
Direct-seeded iongieaf pine (n = 106)			
Linear growth	**.032[a]** (0.015)	**3.891[a]** (3.133)	**.166[a]** (0.072)
Nonlinear growth	**.033[a]** (0.015)	3.963" (3.003)	**.167[a]** (0.072)
Weibuli	**.046[b]** (0.022)	**7.479[b]** (5.597)	**.221[b]** (0.095)

Note: For each evaluation statistic and for each data set, means with the same letter are not significantly different at the 5 percent level (from the Duncan's multiple range test).

CONCLUSIONS

The two stand table projection models based on either linear or nonlinear diameter growth produced similar results. Therefore the simple linear growth model is recommended. Furthermore, the stand table projection models were as good as the parameter-recovery Weibuli model for unimodai distributions, and may out-perform the Weibull when diameter distributions are bimodal or multimodal.

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