Available online at www.sciencedirect.com

ScienceDirect

Forest Ecology and Management 255 (2008) 3440–3446

Forest Ecology and Management

www.elsevier.com/locate/foreco

Nonlinear mixed modeling of basal area growth for shortleaf pine

Chakra B. Budhathoki [a,*], Thomas B. Lynch [a], James M. Guldin [b]

[a] Department of Natural Resource Ecology and Management, 008C AG Hall, Oklahoma State University, Stillwater, OK 74078, USA

[b] Southern Research Station, USDA Forest Service, P.O. Box 1270, Hot Springs, AR 71902, USA

Received 11 June 2007; received in revised form 10 February 2008; accepted 13 February 2008

Abstract

Mixed model estimation methods were used to fit individual-tree basal area growth models to tree and stand-level measurements available from permanent plots established in naturally regenerated shortleaf pine (*Pinus echinata* Mill.) even-aged stands in western Arkansas and eastern Oklahoma in the USA. As a part of the development of a comprehensive distance-independent individual-tree shortleaf pine growth and yield model, several individual-tree annual basal area growth models were fitted to the data with the objective of selecting the model that has superior fit to the data as well as attributes suitable for practical application in shortleaf pine stand simulator useful as an aid in forest management decision-making. The distance-independent individual-tree model of Lynch et al. [Lynch, T.B., Hitch, K.L., Huebschmann, M.M., Murphy, P.A., 1999. An individual-tree growth and yield prediction system for even-aged natural shortleaf pine forests. South. J. Appl. For. 23, 203–211] for annual basal area growth was improved to incorporate random-effects for plots in a potential-modifier framework with stand-level and tree-level explanatory variables. The fitted mixed-effects models were found to fit the data and to predict annual basal area growth better than the previous model forms fitted using ordinary least-squares. There was also some evidence of heterogeneous errors, the effects of which could be corrected by using a variance function in the estimation process. The revised parameter estimates from the selected mixed model could be utilized in a growth and yield simulator that also takes appropriate dbh–height and mortality functions into account.

Keywords: Mixed-effects; Random-effects; Maximum likelihood estimation; *Pinus echinata* Mill.

1. Introduction

Shortleaf pine (*Pinus echinata* Mill.) is second only to loblolly pine (*Pinus taeda* L.) among the southern pines of the United States in standing volume. It grows in 22 states over more than 1,139,600 km^2, ranging from southeastern New York to eastern Texas (Willet, 1986). Previous shortleaf pine forest growth studies include Murphy (1982, 1986), Murphy et al. (1992), Lynch et al. (1991, 1999), and Lynch and Murphy (1995). However, there is still relatively little published work on shortleaf pine growth modeling compared to other southern pines. An important aspect of shortleaf pine growth research is development of basal area growth models for predicting individual-tree growth rates. Lynch et al. (1999) have developed a complete suite of growth equations to simulate shortleaf pine annual growth based on different management scenarios. Their model parameter estimation was based on ordinary least-squares (OLS) methods. Since typical sample tree measurements are repeated in time and sample trees grow together within plots representing stands an assumption of independent observations for individual trees under OLS appears unrealistic. The problem of spatial and temporal correlation among forestry measurements was well recognized some time ago, for example by Ferguson and Leech (1978), and West et al. (1984). However, methods of OLS assuming a completely random sample have dominated the forest growth and yield modeling literature until recently.

Previous growth models for shortleaf pine have generally been fitted using ordinary or weighted least-squares or seemingly unrelated regression methods. Shortleaf pine individual-tree models fitted in the past using OLS have not accounted for plot-level grouping of tree observations. Mixed-effects models can be used to account for spatial and temporal correlation, providing improved parameter estimates. Lappi and Bailey (1988) presented mixed modeling as an alternative to the then conventional methods of estimation for site index. Gregoire et al. (1995) primarily discussed linear mixed models, but they also considered issues relating to the importance of nonlinear

* Corresponding author at: University of Colorado Health Sciences Center, 4200 E Ninth Avenue B-139, Denver, CO 80262, USA. Tel.: +1 303 315 0994; fax: +1 303 315 3261.

E-mail address: Chakra.Budhathoki@uchsc.edu (C.B. Budhathoki).

0378-1127/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.foreco.2008.02.035

mixed models in forest research. Gregoire and Schabenberger (1996a) used a nonlinear mixed-effects approach for modeling individual-tree cumulative bole volume of sweetgum from east Texas. They later modeled cumulative bole volume by taking spatial correlation between sections of a bole into account (Gregoire and Schabenberger, 1996b). Trincado and Burkhart (2006) used mixed-effects to model stem profiles and developed a framework for calibration to localize the model using additional data from the locality of interest. Budhathoki et al. (2008) developed a mixed model for the shortleaf pine dbh–height relationship using a dataset in which plot specific random-effects were included.

A mixed model typically consists of both fixed-effects parameters and random-effects parameters. When fitting these parameters, data analysts are usually more interested in the variance components associated with random coefficients than in predictions of the random parameters themselves. Fixed-effects coefficients are then assessed through selection of appropriate explanatory variables. Inclusion of random-effects basically helps to account for various sources of variation effectively, thereby increasing the accuracy of testing and estimation for the fixed-effects (Tao, 2002). Mixed models have been increasingly used in forest growth and yield modeling. However, the majority of the mixed model applications to date have used "stand-level" rather than individual-tree level growth models.

2. Methods

2.1. Data

Data collection from over 200 plots permanently established in shortleaf pine natural stands located in western Arkansas and eastern Oklahoma provided individual-tree measurements including total height, crown height, diameter at breast height and survival. These plots were established as part of a collaborative study by the Department of Natural Resource Ecology and Management at Oklahoma State University, and the USDA Forest Service Southern Experiment Station, and Ozark and Ouachita National Forests during the period 1985–1987. Parameters and data ranges for the study design are given in Table 1 (reported in English units by Lynch et al., 1999). This design stipulated the establishment of circular fixed-radius plots 810 m^2 in size for each combination of site index, age and stand basal area classes. An existing shortleaf pine thinning study provided additional plots, which were treated to conform to study design criteria (Lynch et al., 1999). Measurements of diameter at breast height were available for over 8000 trees at three measurement times. A representative sub-sample of trees on each plot provided total height and crown height measurements at each measurement time. Plot ages were determined from ring counts of representative dominant and codominant sample trees on each plot (Avery and Burkhart, 2002). Site index curves for naturally occurring shortleaf pine developed by Graney and Burkhart (1973) were used to determine each plot site index for base age 50 years. The three repeated measurements were used to obtain annual basal area growth for the corresponding two growth periods. Table 2 contains summary statistics for variables used in the development of individual-tree shortleaf pine growth models.

Table 1
Midpoints and ranges for design variables for natural, even-aged shortleaf pine study plots in western Arkansas and eastern Oklahoma (adapted from Lynch et al., 1999)

Design variable	Class midpoint	Class range
Basal area (m^2/ha)	7	≤10.5
	14	10.6–17.5
	21	17.6–24.5
	28	≥24.6
Site index (m at age 50 years)	17	≤17
	18	17.1–19.9
	21	20.0–22.9
	23	≥22.9
Age (years)	20	11–30
	40	31–50
	60	51–70
	80	71–90

2.2. Statistical analysis

The basic objective of this work is to utilize mixed modeling techniques with growth data from three measurement times to develop a basal area growth model with improved parameter estimates relative to a model fitted by OLS to data from two measurement times by Lynch et al. (1999). This basal area growth model is based on a potential-modifier framework (e.g., Murphy and Shelton, 1996).

A basal area growth prediction model having the same form as that given by Lynch et al. (1999) is presented below in Model 1.

$$y_{ij} = \frac{\beta_1 B_{ij}^{\beta_2} - (\beta_1 B_{ij}/B_{\max}^{1-\beta_2})}{1 + \exp(\beta_3 + \beta_4 B_{si} + \beta_5 A_i + \beta_6 R_{ij} + \beta_7 B_{ij})} + \varepsilon_{ij} \quad (1)$$

where y_{ij} = average annual basal area growth (m^2/year) of tree j in plot i, B_{ij} = basal area (m^2) of tree j in plot i, A_i = stand age (year) for plot i, R_{ij} = ratio of quadratic mean stand diameter to the dbh of tree j in plot i, B_{si} = stand basal area (m^2/ha) for plot i, $B_{\max}$ = 0.6566528736 m^2 (the maximum expected basal area for a shortleaf pine tree in managed stands from Hitch (1994) corresponding to dbh = 91 cm), $\beta_1, \ldots, \beta_7$ = model parameters, ε_{ij} = within-plot error, i.e. residual for tree j in plot i, $\varepsilon_{ij} \sim N(0, \sigma^2)$

Lynch et al. (1999) fitted Model 1 using OLS with first two measurements of the data summarized in Table 2. For comparison purposes, the Model 1 was refitted using generalized least-squares (GLS) with the additional third measurement data so that statistics such as Akaike information criterion (AIC) and Bayesian information criterion (BIC) or Schwarz's Bayesian criterion (SBC) could be obtained with S-Plus `gnls` function (Pinheiro and Bates, 2000). This makes it possible to compare a GLS model to a mixed-effects model using the same dataset.

Table 2
Summary of stand-level and tree variables recorded/observed in the study

Variable	No. of observations	Mean	Standard deviation	Minimum	Maximum
Basal area[a] (m^2/ha)	208	21.33	6.68	6.27	29.62
Stand age[a] (year)	208	41.8	19.7	18.0	93.0
Site index (m at age 50 years)	208	17.5	2.9	12.2	26.6
Total height (m)	8,971	18.7	6.4	3.1	36.3
dbh (cm)					
First measurement	8,284	18.8	9.9	2.8	61.9
Second measurement	8,092	20.8	9.9	3.0	64.5
Third measurement	7,591	23.1	10.2	3.8	67.6
Tree basal area (m^2)					
First measurement	8,284	0.0341	0.03385	0.00061	0.26339
Second measurement	8,092	0.04	0.03667	0.00114	0.27982
Third measurement	7,591	0.047	0.04036	0.00183	0.32177
Ratio of qmd[b] to dbh (R)	23,967	1.145	0.4549	0.438	7.362
AABAG[c] (m^2/tree/year)					
Overall	15,669	0.0013	0.00106	0.00059	0.00925
First period	8,083	0.0012	0.00097	−0.00033	0.00643
Second period	7,586	0.0013	0.00114	−0.00059	0.00925

[a] At establishment.
[b] qmd, quadratic mean diameter.
[c] AABAG, average annual basal area growth.

Model 1 was modified to include random-effects associated with plots. The resulting Model 2, a nonlinear mixed model for annual basal area growth with random-effect (b_7) associated with the fixed-effect (β_7) can be written as:

$$y_{ij} = \frac{\beta_1 B_{ij}^{\beta_2} - (\beta_1 B_{ij}/B_{\max}^{1-\beta_2})}{1 + \exp(\beta_3 + \beta_4 B_{si} + \beta_5 A_i + \beta_6 R_{ij} + (\beta_7 + b_{7i}) B_{ij})} + \varepsilon_{ij} \tag{2}$$

where b_{7i} is a random parameter specific to ith plot associated with mid-tree basal area fixed-effect coefficient β_7 that appears in the modifier (denominator), and rest of the terms are described above in Model 1. It is assumed that $b_{7i} \sim N(0, \sigma_b^2)$, $\varepsilon_{ij} \sim N(0, \sigma^2)$, and $\text{cov}(b_{7i}, \varepsilon_{ij}) = 0$. We would usually be interested in an estimate of $\text{var}(b_{7i})$, i.e. $\hat{\sigma}_b^2$, a variance component describing the spread of the random coefficients. Maximum likelihood methods were used to fit Models 1 and 2 with S-Plus `nlme` library. AIC, BIC and residual mean squares can be used to compare Models 1 and 2. The variance component for random-effects can also be used to test the statistical significance of a mixed model, Model 2, versus a model without random-effects, Model 1. Modeling of random-effects is expected to improve estimation and testing of fixed-effect parameters that would help in selecting suitable explanatory variables in a growth model (Pinheiro and Bates, 2000).

More complicated models were examined by fitting the shortleaf pine data with plot-specific random-effects for other associated fixed-effects coefficients (β_1 and β_6). Furthermore, Model 2 was also modified to model heterogeneous errors in order to evaluate whether this modification would improve fit to the shortleaf pine basal area growth data. This modification resulted in the following Model 3:

$$\text{Model 2} + \text{power variance function} \tag{3}$$

where error variance was modeled as $\text{var}(\varepsilon_{ij}) = \sigma^2 |\upsilon_{ij}|^{2\delta}$ with one covariate using variance function $g(\upsilon_{ij}, \delta) = |\upsilon_{ij}|^{\delta}$ (υ_{ij} is covariate and δ is power parameter). This model takes heterogeneous errors into account, so that a constant variance assumption is not necessary. Tree basal area was selected as a covariate in variance function for modeling errors in the basal area growth model.

Model 3 was fitted with the S-Plus `nlme` library using the `varPower` option (Pinheiro and Bates, 2000). Another modification of Model 2, a two-level extension, can be written as Model 4 below:

$$y_{ijk} = \frac{\beta_1 B_{ijk}^{\beta_2} - (\beta_1 B_{ijk}/B^{1-\beta_2})}{1 + \exp(\beta_3 + \beta_4 B_{sik} + \beta_5 A_{ik} + \beta_6 R_{ijk} + (\beta_7 + b_{7ik}) B_{ijk})} + \varepsilon_{ijk} \tag{4}$$

where $k = 1,2$; the index k representing growth period. A variance function to model possible heterogeneous errors was added to Model 4 to obtain Model 5, in which tree basal area was used as a covariate in the variance function similarly to Model 3.

$$\text{Model 4} + \text{power variance function} \tag{5}$$

3. Results and discussion

Summaries of fit statistics and estimates of variance components obtained from fitting the basal area growth models are presented in Table 3.

Table 3 indicates that Model 2 is better than Model 1 due to much smaller AIC and BIC values in Model 2. Furthermore, Model 2 has larger log-likelihood and smaller residual standard deviation (S.D.) than Model 1. These statistics show that addition of plot random-effects improves the model fit. A 95% confidence interval for the b_7 variance component S.D. is [6.505425, 8.432243], with a point estimate of 7.40644, indicating that the component is significantly different from zero. When variance of within-plot errors is modeled as a function of mid-tree basal area instead of assuming constant variance, Model 3 is found to be an improvement over Model 2. The estimated variance component S.D. for Model 3 is larger (14.51838) than that of Model 2, although the residual S.D. is slightly increased.

An attempt was made to fit a two-level hierarchical mixed model (Model 4) with random parameters representing growth period and plots within growth period. However, a convergence problem was experienced when attempting to fit this two-level model. Omission of stand age (with parameter β_5) from the model permitted successful parameter estimation. In the ensuing discussion, we will continue to call this latter model "Model 4." Model 4 appears to be slightly better than Model 2 despite an increase in the number of random parameters to be predicted. However, Model 3 is much better than Model 4 as indicated by fit statistics and other information given in Table 3. Model 5 is a substantial improvement over Model 4 due to inclusion of variance modeling function. Models 3 and 5 show that variance modeling to account for heterogeneous within-plot errors improves the fit irrespective of the number of hierarchical levels selected for mixed modeling (whether single- or two-level).

Table 4
Parameter estimates and other associated statistics for Model 3 (total observations = 15,669, number of plots = 208, and residual d.f. = 15,455)

Parameter	Estimate	Standard error	t-Value	P-value
β_1	0.035478	0.003601	9.85	<0.0001
β_2	0.595154	0.022727	26.19	<0.0001
β_3	−2.885529	0.128518	−22.45	<0.0001
β_4	0.069907	0.001832	38.16	<0.0001
β_5	0.006188	0.001175	5.26	<0.0001
β_6	1.684492	0.053100	31.72	<0.0001
β_7	−9.399331	1.334786	−7.04	<0.0001

Table 3
Summary statistics for fitted basal area growth models (total observations = 15,669, and number of plots = 208)

Model	AIC	BIC	Log-likelihood	$\hat{\sigma}_{T(p)b}$	$\hat{\sigma}_b$	Residual d.f.	Residual S.D. ($\hat{\sigma}$)
1	−186574.8	−186513.6	93295.4	–	–	15,661	0.00062807
2	−191021.0	−190952.1	95519.5	–	7.40644	15,455	0.00052973
3	−198635.6	−198559.0	99327.8	–	14.51838	15,455	0.00327183
4[a]	−191757.0	−191688.1	95887.5	1.70427	6.19544	15,248	0.00050672
5[a]	−198975.0	−198898.4	99497.5	2.67147	10.72944	15,248	0.00303856

$\hat{\sigma}_{T(p)b}$, Estimated standard deviation (S.D.) for variance component for plot random-effects within growth period. $\hat{\sigma}_b$, estimated standard deviation for variance component for plot random-effects.
[a] Stand age dropped from the model due to convergence problem.

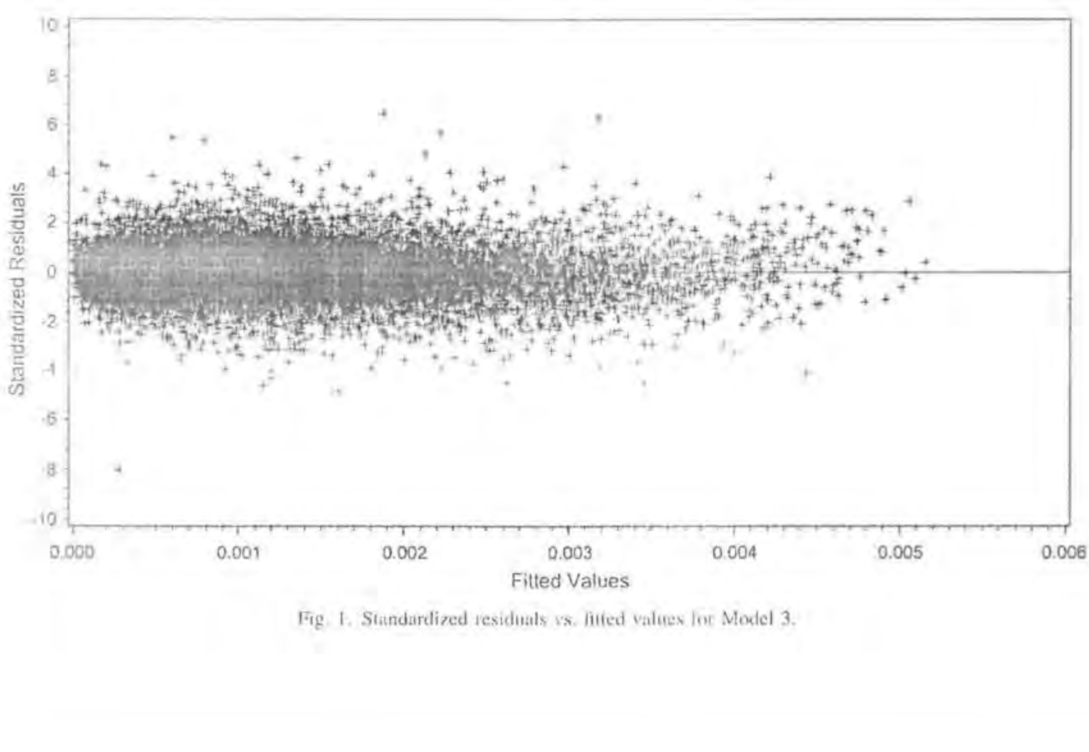

Fig. 1. Standardized residuals vs. fitted values for Model 3.

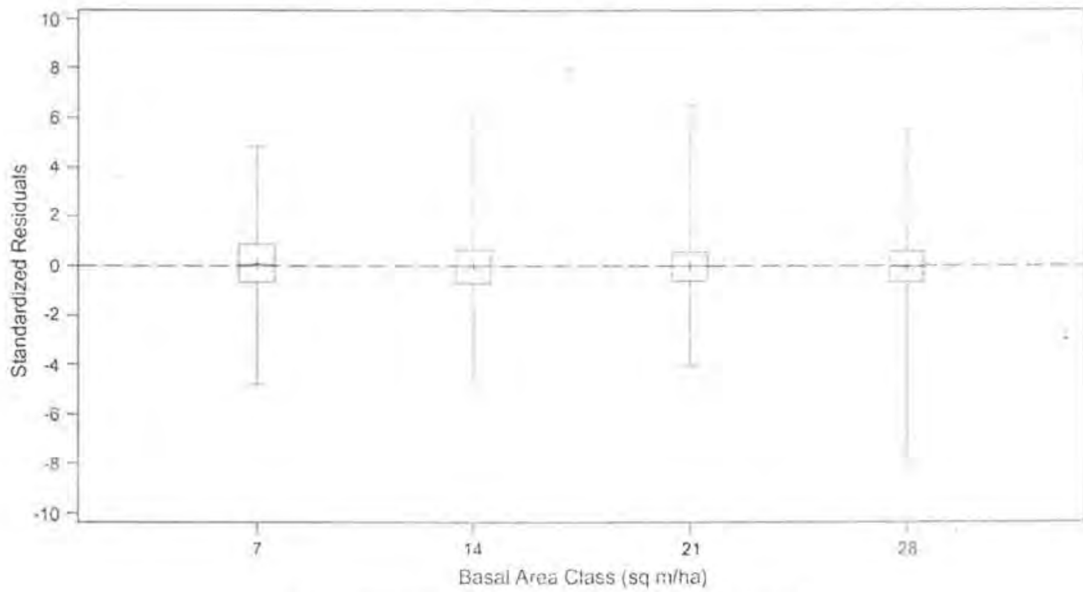

Fig. 2. Standardized residuals vs. stand basal area for Model 3.

The effects of spatially dependent errors were also explored since coordinate data for each tree were available. Three additional models using linear, exponential and Gaussian correlation functions were studied. However, they did not improve fit compared to Model 5. This suggests that the random plot effect essentially explains most of the autocorrelation among trees within plots. Trincado and Burkhart (2006) also found evidence to suggest that the assumption of correlated errors could be relaxed when appropriate tree-level random-effects were included in a stem profile model. Additional details concerning results from the spatial models can be found in Budhathoki (2006).

Because Model 5 does not include stand age as an independent variable, it may be less desirable for practical application than Model 3 despite some modest improvement in fit statistics. Therefore, we prefer Model 3, a model which uses single level random-effects and variance modeling.

3.1. Parameter estimates for Model 3

Parameter estimates and standard errors for Model 3 are provided in Table 4. The estimated variance component S.D. for b_7 ($\hat{\sigma}_{b_7}$) is 14.51838, and that for the residual ($\hat{\sigma}_e$) is 0.00327183. A 95% confidence interval for b_7 variance component S.D. is

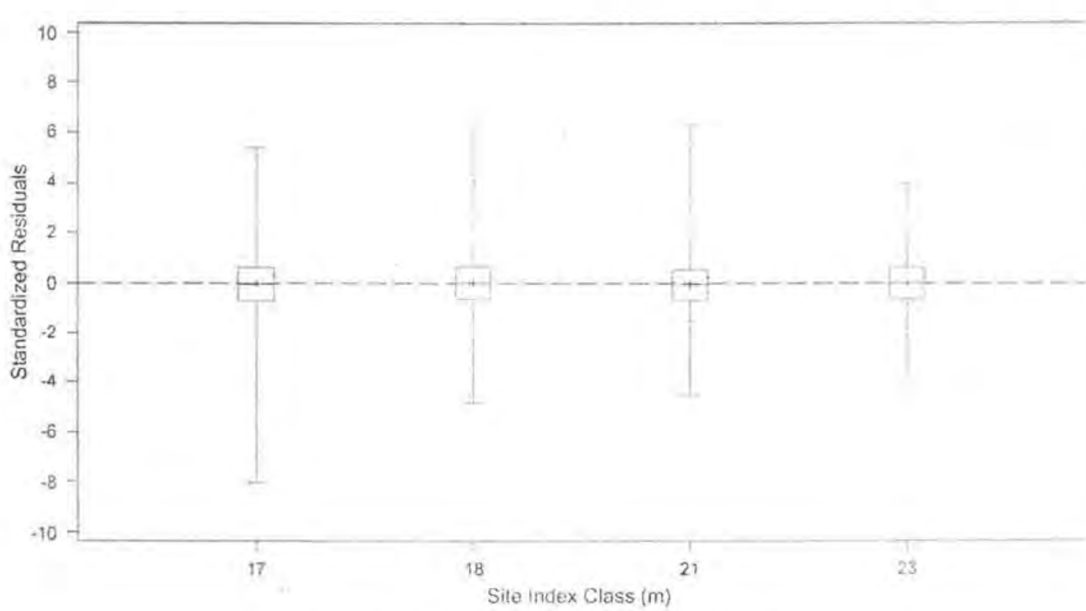

Fig. 3. Standardized residuals vs. site index for Model 3.

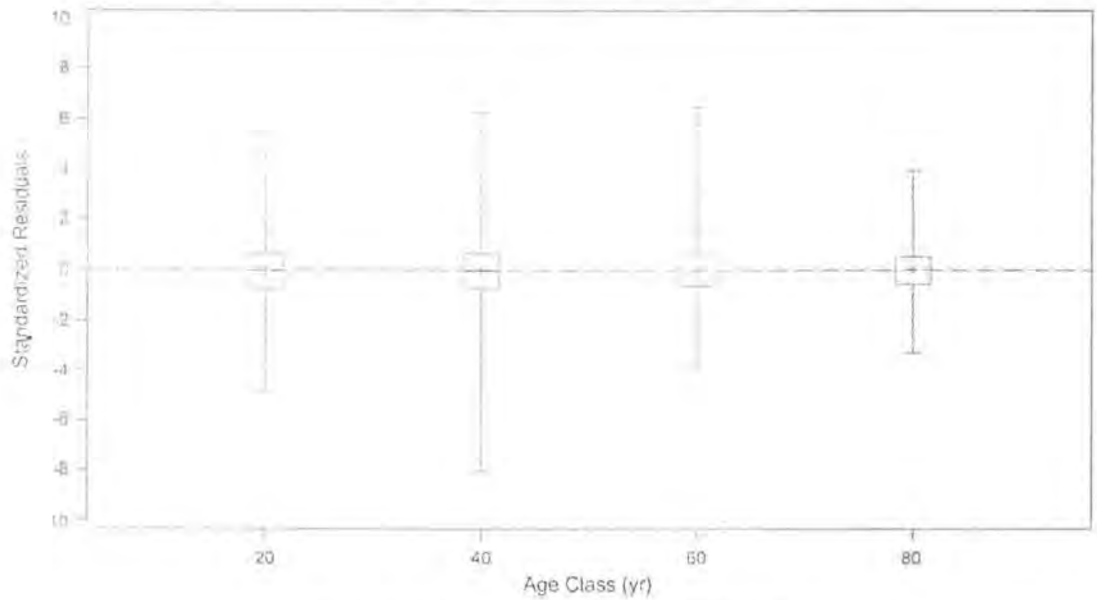

Fig. 4. Standardized residuals vs. stand age for Model 3.

[12.69197, 16.60762], indicating that the variance component is significantly different from zero. The power estimate ($\hat{\delta}$) in the variance function for Model 3 is 0.57274 with 95% confidence interval [0.56048, 0.58499]. Since the interval does not include zero, we conclude that the use of a variance function is beneficial.

3.2. Residual analysis for Model 3

Standardized residuals from the selected model are plotted against the predicted values in Fig. 1. The plot shows random scatter of residuals indicating no violation of model assumptions. Comparison of residual plots from other models (e.g., those not using variance functions to achieve homogeneity of variance) indicates that Model 3 is superior (residual plots of other models not shown) and adequately explains variation in annual basal area growth. Similarly, the residuals are plotted against each of design criteria (stand age, site index and stand basal area). These plots are presented in Figs. 2–4. Figs. 2–4 generally indicate mean residuals for all classes centered near zero, indicating lack of bias for Model 3.

Fig. 2 shows a fairly similar distribution of residuals over four initial stand basal area classes, although there is a slight indication that there is high variability in the highest class (28 m^2/ha). Fig. 3 indicates fairly similar prediction of annual growth values over site index classes except in the highest class (23 m at base age 50 years).

It can be inferred from Fig. 4 that there is no clear prediction bias over initial stand age classes. The graph further reveals that variation in model residuals is higher in slightly younger stands. However, the variability in residuals is decreased in older stands.

4. Conclusions

No shortleaf pine growth models involving random-effects for plots have previously been published in a peer reviewed journal for the Oklahoma and Arkansas region, except a model for diameter–height relationship by Budhathoki et al. (2008). There has also been relatively little work in mixed modeling of basal area increment for other tree species in the region. This study provides evidence that individual-tree growth models with plot random-effects are superior to those fitted with OLS methods (Lynch et al., 1999), due to statistical properties associated with the use of a variance component for plot random-effects. The mixed models are also more attractive for the reasons of interpretation and applicability of the parameter estimates. Model 1 in which parameters were fitted by GLS ignores grouping of trees by plots, while the mixed models conform more closely to actual data structure in which individual-tree measurements are grouped by plots.

The main objective of the fitted models is prediction of the response variables rather than interpretation of individual fixed-effect coefficients. Fixed-effects parameter estimates are given for Model 3, and these could be used in prediction of annual basal area growth in a distance-independent individual-tree growth simulator (e.g., Huebschmann et al., 1998). Residual analysis showed that Model 3 made reasonable predictions over the range of design criteria, without evidence of systematic bias. Overall, there was some evidence of residual variance increase with tree size classes, which was corrected with variance modeling of heterogeneous within-plot errors. Despite somewhat improved statistical properties, Model 5 is not as appealing for practical applications since it does not include age as an independent variable.

Due to limited observations over time (repeated measurements), serial correlations among measurements could not be addressed through modeling of within-subject covariance matrix over time. However, part of temporal correlation was taken into account by using first differences to compute growth between observations for two time points. As coordinates were recorded for each tree, the possibility of spatial correlation for individual-tree errors was also investigated. However, spatial correlation was not statistically significant in presence of plot random-effects. Trincado and Burkhart (2006) suggest that the correlated error assumption could be relaxed for predictive purposes in the presence of tree-level random effects with a stem profile model estimated using multiple measurements on sample trees. Model 3 is preferred for improved predictions of annual basal area growth for individual shortleaf pine trees occurring in natural stands in the Oklahoma and Arkansas region.

Acknowledgments

This article is approved for publication by the Director, Oklahoma Agricultural Experiment Station and supported by Project MS-1887. This publication is based on a part of dissertation research of the first author (Budhathoki, 2006). The assistance received from Department of Forestry, now part of Natural Resource Ecology and Management at Oklahoma State University is highly appreciated. The cooperation of the USDA Forest Service Southern Research Station, and the Ozark and Ouachita National Forests in data collection and financial support for the study is much appreciated. We are also thankful for the comments of the reviewers to improve this article.

References

Avery, T.E., Burkhart, H.E., 2002. Forest Measurements, fifth ed. McGraw-Hill, New York, p. 456.

Budhathoki, C.B., 2006. Mixed-effects modeling of shortleaf pine (*Pinus echinata* Mill.) growth data. Ph.D. Dissertation (unpublished). Oklahoma State University, Stillwater, OK, USA, 168 pp.

Budhathoki, C.B., Lynch, T.B., Guldin, J.M., 2008. A mixed-effects model for dbh–height relationship of shortleaf pine (*Pinus echinata* Mill.). South. J. Appl. For. 32, 5–11.

Ferguson, I.S., Leech, J.W., 1978. Generalized least squares estimation of yield functions. For. Sci. 24, 27–42.

Graney, D.L., Burkhart, H.E., 1973. Polymorphic site index curves for shortleaf pine in the Ouachita Mountains. USDA For. Ser. Res. Pap. SO-85, 12 pp.

Gregoire, T.G., Schabenberger, O., 1996a. A non-linear mixed-effects model to predict cumulative bole volume of standing trees. J. Appl. Stat. 23, 257–271.

Gregoire, T.G., Schabenberger, O., 1996b. Nonlinear mixed-effects modeling of cumulative bole volume with spatially correlated within-tree data. J. Agric. Biol. Environ. Stat. 1, 107–119.

Gregoire, T.G., Schabenberger, O., Barrett, J., 1995. Linear modelling of irregularly spaced, unbalanced, longitudinal data from permanent-plot measurements. Can. J. For. Res. 25, 137–156.

Hitch, K.L., 1994. A distance-independent individual tree basal area growth model for natural even-aged stands of shortleaf pine in eastern Oklahoma and western Arkansas. M.S. Thesis. Oklahoma State University, Stillwater, OK. 78 pp.

Huebschmann, M.M., Lynch, T.B., Murphy, P.A., 1998. Shortleaf pine stand simulator: an even-aged natural shortleaf pine growth and yield model user's manual. Res. Rep. P-967. Okla. State Univ. Agric. Exp. Sta., 25 pp.

Lappi, J., Bailey, R.L., 1988. A height prediction model with random stand and tree parameters: an alternative to traditional site index methods. For. Sci. 34, 907–927.

Lynch, T.B., Murphy, P.A., 1995. A compatible height prediction and projection system for individual trees in natural even-aged shortleaf pine stands. For. Sci. 41, 194–209.

Lynch, T.B., Murphy, P.A., Lawson, E.R., 1991. Stand volume equations for managed natural even-aged shortleaf pine in eastern Oklahoma and western Arkansas. Res. Rep. P-921. Okla. State Univ. Agric. Exp. Sta., 12 pp.

Lynch, T.B., Hitch, K.L., Huebschmann, M.M., Murphy, P.A., 1999. An individual-tree growth and yield prediction system for even-aged natural shortleaf pine forests. South. J. Appl. For. 23, 203–211.

Murphy, P.A., 1986. Growth and yield of shortleaf pine. In: Murphy, P.A. (Ed.), Proc. Symp. on the Shortleaf Pine Ecosystem. Arkansas Coop. Ext. Ser., University of Arkansas, AR, pp. 159–177.

Murphy, P.A., 1982. Sawtimber growth and yield for natural even-aged stands of shortleaf pine in the west Gulf. USDA For. Ser. Res. Pap. SO-181, 13 pp.

Murphy, P.A., Shelton, M.G., 1996. An individual-tree basal area growth model for loblolly pine stands. Can. J. For. Res. 26, 327–331.

Murphy, P.A., Lawson, E.R., Lynch, T.B., 1992. Basal area and volume development of natural even-aged shortleaf pine stands in the Ouachita Mountains. South. J. Appl. For. 16, 30–34.

Pinheiro, J., Bates, D., 2000. Mixed-Effects Models in S and S-Plus. Springer-Verlag, New York, p. 528.

Tao, J., 2002. Mixed Models Analyses Using the SAS System: Course Notes. SAS Institute, Inc., Cary, NC, p. 493.

Trincado, G., Burkhart, H.E., 2006. A generalized approach for modeling and localizing stem profile curves. For. Sci. 52, 670–682.

West, P.W., Ratkowsky, D.A., Davis, A.W., 1984. Problems of hypothesis testing of regressions with multiple measurements from individual sampling units. For. Ecol. Manage. 7, 207–224.

Willet, R.L., 1986. Foreword. In: Murphy, P.A. (Ed.), Proc. Symp. on the Shortleaf Pine Ecosystem. Arkansas Coop. Ext. Ser., University of Arkansas, AR pp. iii–iv.