Optimal Tree Increment Models for the Northeastern United States

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Abstract.—I used the potential relative increment (PRI) methodology to develop optimal tree diameter growth models for the Northeastern United States. Thirty species from the Eastwide Forest Inventory Database yielded 69,676 individuals, which were then reduced to fast-growing subsets for PRI analysis. For instance, only 14 individuals from the greater than 6,300-tree eastern white pine sample were used to fit its PRI model. The Northeastern northern red oak model predicted faster small tree growth than those derived for the Lake States or Midsouth, but it soon fell behind the other regional models and never again matched their performance. Predicted maximum increment differences between regions rarely exceeded 0.25 cm, however. The PRI methodology also can help identify possibly erroneous individual tree records.

Introduction

For tree growth modelers, increment "optimality" is often defined as an idealized or maximal rate of increase in a specified dimension (usually diameter or height). This concept assumes that all environmental conditions are at their most favorable, and, thus, anything suboptimal decreases growth accordingly. The primary advantage to an optimized approach is that the modifiers influencing increment can be separated from the model used to predict growth, allowing for many different constructs to be applied (Bragg 2003a). Not surprisingly, potential increment models have become the cornerstone of many ecological simulators (e.g., Botkin *et al.* 1972).

Potential growth formulations have their critics. Purely theoretical designs, while often intellectually appealing, are problematic because they rarely incorporate real-world measurements and sometimes contain biological flaws. For example, the gap model's potential increment design includes a number of unsupportable assumptions about diameter accumulation and maximum tree dimensions (Bragg 2001). Lessard *et al.* (2001) dismissed potential growth constructs because they cannot be directly observed and may be difficult to estimate. Finally, some have argued that empirical models predicting average (realized) growth are more precise, even if they lack mechanism (Fleming 1996).

Biologically meaningful optimal growth curves can be empirically derived, however. The potential relative increment (PRI) methodology (Bragg 2001) uses the Eastwide Forest Inventory Database (EFIDB) (Hansen *et al.* 1992) to estimate optimal growth based on actual inventories. A set of simple post-processors (Bragg 2002a), when properly applied to data on rapidly growing individuals fit to a nonlinear model, produce response patterns identified as crucial by Shvets and Zeide (1996) and Zeide (1993). PRI models have been developed for the Lake States (Michigan, Minnesota, and Wisconsin) and Midsouth (Arkansas, Louisiana, Missouri, Oklahoma, and Texas) (Bragg 2001, Bragg 2002b, Bragg 2003b). This article presents PRI models for the common tree species of the Northeastern States of Connecticut, Maine, Massachusetts, New Hampshire, New York, Rhode Island, and Vermont.

Methods

A detailed description of the PRI methodology is beyond the scope of this article (rather, see Bragg [2001] and Bragg [2002a]). The PRI approach is a type of boundary line analysis (Webb 1972). Boundary line analysis has shown promise for identifying the role of maximal growth in ecological and mensurational applications (for example, Black and Abrams 2003). Briefly, PRI calculates actual relative increment (ARI) from:

$$ARI = \frac{d.b.h._C - d.b.h._O}{d.b.h._O} \qquad (1)$$

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where the initial (d.b.h.$_O$) and final (d.b.h.$_C$) inventory diameters are in centimeters. ARI values were then annualized by dividing by the remeasurement period. A record was considered eligible if the tree was alive during both inventories, was of the species of interest, and showed a positive increment (d.b.h.$_C$ > d.b.h.$_O$).

The PRI methodology does not consider every ARI value. Because virtually all trees are negatively affected by local environmental conditions, e.g., competition or poor site quality, their diameter growth decreases markedly. Easily identified by their slower growth rates, these individuals were eliminated from further consideration, leaving only a handful of the fastest growing trees in a specified diameter class. Maximally performing individuals that fail to reach the levels of adjacent diameter classes are also removed from further consideration. The final subset represents only a fraction (usually 6 to 12 trees) of the original data, for which the following model was fit:

$$E(PRI) = \hat{b}_1 d.b.h._{MAX}^{\hat{b}_2} \hat{b}_3^{d.b.h._{MAX}} \quad (2)$$

where d.b.h.$_{MAX}$ is the d.b.h. of an individual tree growing at the highest rate in its respective diameter class, and $\hat{b}_1, \hat{b}_2$, and $\hat{b}_3$ are nonlinear ordinary least squares regression parameter estimates.

As an example, ARI values were calculated for 6,348 eastern white pines (*Pinus strobus*) from the Northeastern EFIDB (fig. 1a). Selecting only the pines (by 2-cm d.b.h. classes) with maximal ARI reduced this number to 51 individuals (fig. 1b). Because most of this subset of d.b.h. class maximal ARI points fell appreciably below the "optimal" frontier, they were removed before the final curve fitting. Hence, a PRI model for eastern white pine in the Northeastern United States was generated with only 14 trees (fig. 1c). Eastern white pine displayed a characteristic curve (fig. 1d), with the highest predicted PRI in the smallest pines. Multiplying the result of equation (2) by the tree's current diameter yielded an increment curve (fig. 2), with the greatest optimal annual growth of approximately 2 cm occurring at 20- to 40-cm d.b.h.

Figure 1.—*Step-by-step PRI methodology for eastern white pine taken from the Northeastern United States EFIDB. After the original 6,348 eligible pines were identified (a), the 51 individuals growing at the highest rate per 2-cm diameter class (b) were retained and further reduced to the final subset (c) of 14 data points, to which the actual PRI equation was fit (d).*

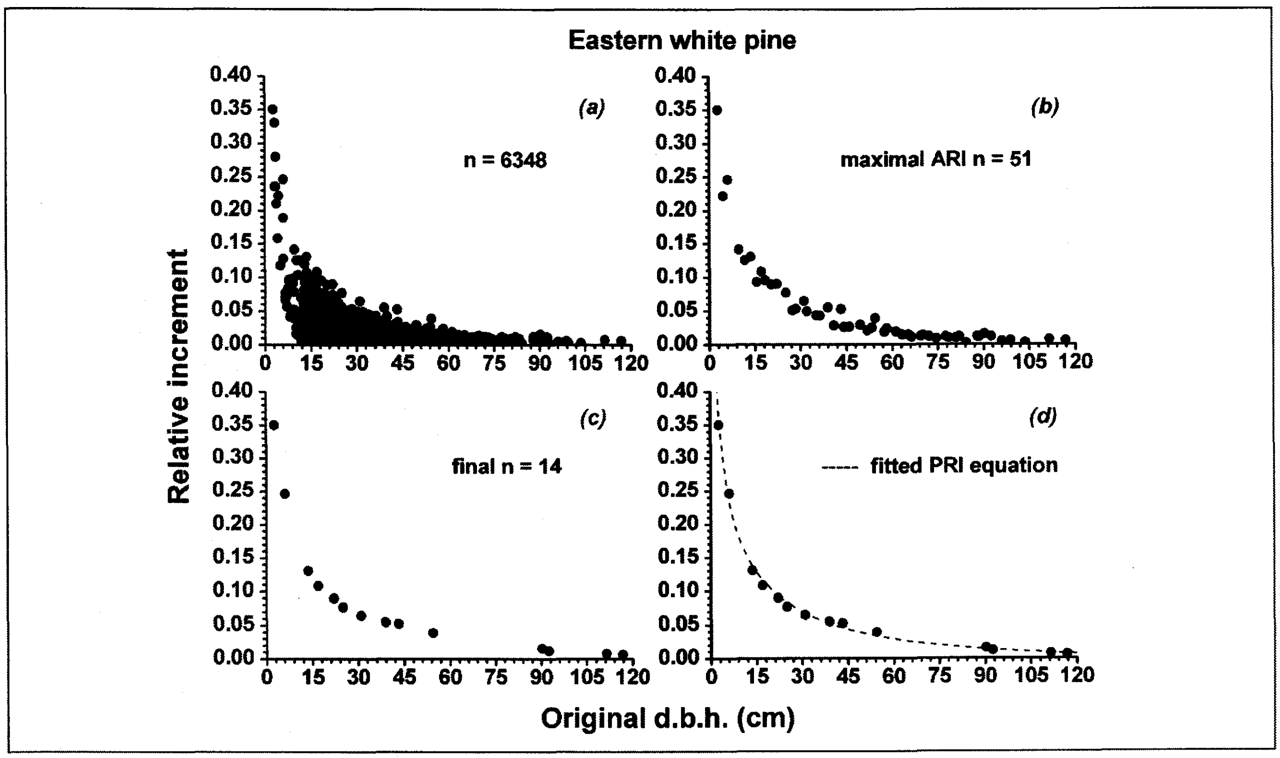
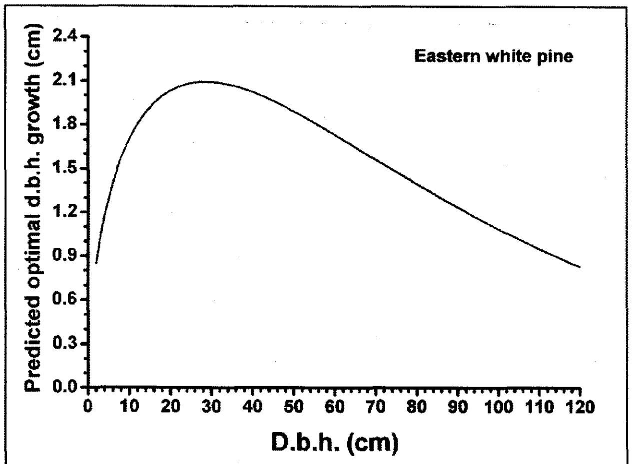


Figure 2.—*Predicted optimal d.b.h. annual increment for eastern white pine in the Northeastern United States.*



Similar steps were used to produce PRI models for the most common species of the Northeastern EFIDB. Additionally, I compared northern red oak PRI curves for three areas (Northeast, Lake States, and Midsouth) to highlight regional differences in predictions of optimal diameter increment. Finally, two examples of extremely fast growing individuals were used to demonstrate the potential of PRI to identify inventory outliers.

Results and Discussion

Northeastern PRI Results

I found 30 species sufficiently abundant ($n \geq 100$) in the Northeastern EFIDB for the PRI methodology (table 1). Most (23 of 30) species produced at least 450 individuals, and 1 in 6 had more than 5,000 trees. Combined, these taxa yielded 69,676 individuals for preliminary analysis. A sample size of this magnitude, even if most are rejected for growing too slowly, is far more comprehensive than typical growth modeling efforts.

Although individuals greater than 10 cm and less than 50 cm in d.b.h. (averaging 20- to 30-cm d.b.h.) predominated, this sample contained very small and very large trees (table 1). For example, 10 species had individuals greater than 100-cm d.b.h., including a 165.9-cm d.b.h. northern red oak (*Quercus rubra*) and a 185.4-cm d.b.h. black willow (*Salix nigra*).

Depending on the species, only 6 to 15 individuals were needed to develop the PRI curves. All parameter estimates were significant at a significance level of $\alpha = 0.05$ (table 2).

Regional Comparison Using Northern Red Oak

Very few obvious differences arose between the regional PRI models (fig. 3a). After converting the PRI values to potential increments, relative growth performance primarily differed by absolute tree diameter. Up to about 10-cm d.b.h., the Northeastern northern red oak model predicted the highest optimal increment. It was then replaced by the Lake States model (to 76-cm d.b.h.), after which the Midsouth version produced the highest predicted optimal northern red oak increment (fig. 3b). Estimated optimal increments peaked at approximately 1.25 cm for the Northeastern and Midsouth models, and at just over 1.4 cm for the Lake States model. These maxima were reached at about 15-cm d.b.h. for the Northeastern model, roughly 25-cm d.b.h. for the Midsouth model, and approximately 30-cm d.b.h. for the Lake States model.

Overall, potential diameter increment differences among the regions were minor, with residual differences rarely exceeding 0.2 cm annually at any given diameter (fig. 3c). This difference is not trivial when accumulated over years of growth, however, especially because PRI-based growth projection systems are nonlinear functions of current tree diameter.

Identifying Potential Inventory Errors With PRI

As a conservative estimate of optimal growth, PRI curves can identify individuals growing dramatically faster than expected. For instance, two individuals from the New York data set were obvious outliers when maximal ARI points were plotted. An 80.3-cm d.b.h. black cherry grew to 107.4-cm d.b.h. in just 12 years (fig. 4a), while an 80.5-cm d.b.h. white oak increased to 106.7-cm, also in 12 years (fig. 4b). Although this growth is possible for vigorous young individuals of either species, this level of productivity was highly suspect in trees of 80-cm d.b.h.

These extremely fast-growing outliers came from plots of low stand density (basal areas of 6.9 m^2/ha for the black cherry and 3.7 m^2/ha for the white oak), and thus could reflect the pronounced release of previously suppressed individuals. More likely, they probably reflect measurement or transcription errors. Given their large girth, these outliers could prove highly influential in any extrapolations based on their size.

Table 1.—*Species, preliminary counts, and diameter at breast height (d.b.h.) ranges of species used in the Northeastern United States PRI analysis.*

Species[a]	EFIDB code[a]	Initial number	Minimum d.b.h. (cm)	Mean d.b.h. (cm)	Maximum d.b.h. (cm)
Balsam fir (*Abies balsamea*)	12	3,387	3.3	17.9	45.7
Tamarack (*Larix laricina*)	71	357	11.7	20.5	52.1
White spruce (*Picea glauca*)	94	763	4.6	21.9	55.9
Black spruce (*Picea mariana*)	95	499	12.7	18.6	39.1
Red spruce (*Picea rubens*)	97	5,384	7.1	22.2	67.1
Red pine (*Pinus resinosa*)	125	501	3.8	24.7	72.4
Pitch pine (*Pinus rigida*)	126	165	9.9	24.7	53.6
Eastern white pine (*Pinus strobus*)	129	6,348	2.8	29.0	116.8
Northern white-cedar (*Thuja occidentalis*)	241	4,222	6.6	23.1	73.4
Eastern hemlock (*Tsuga canadensis*)	261	6,424	5.8	25.5	105.4
Red maple (*Acer rubrum*)	316	11,283	3.0	23.6	114.0
Silver maple (*Acer saccharinum*)	317	157	4.6	29.7	74.2
Sugar maple (*Acer saccharum*)	318	7,540	3.0	27.3	125.2
Yellow birch (*Betula alleghaniensis*)	371	2,905	4.8	25.9	109.5
Sweet birch (*Betula lenta*)	372	810	3.6	23.4	75.4
Paper birch (*Betula papyrifera*)	375	2,538	9.4	20.9	59.7
American beech (*Fagus grandifolia*)	531	3,430	4.6	24.2	76.5
White ash (*Fraxinus americana*)	541	2,402	3.6	24.1	115.6
Black ash (*Fraxinus nigra*)	543	257	8.9	19.0	43.9
Bigtooth aspen (*Populus grandidentata*)	743	757	9.1	24.3	59.9
Quaking aspen (*Populus tremuloides*)	746	1,572	3.0	21.3	97.3
Black cherry (*Prunus serotina*)	762	1,449	4.1	27.1	85.9
White oak (*Quercus alba*)	802	744	9.9	28.8	93.0
Scarlet oak (*Quercus coccinea*)	806	351	6.9	25.1	65.0
Chestnut oak (*Quercus prinus*)	832	484	8.1	25.5	83.6
Northern red oak (*Quercus rubra*)	833	3,384	5.3	28.6	165.9
Black oak (*Quercus velutina*)	837	651	3.8	28.7	100.1
Black willow (*Salix nigra*)	922	104	6.1	46.9	185.4
American basswood (*Tilia americana*)	951	568	4.1	29.3	119.1
American elm (*Ulmus americana*)	972	240	3.6	18.6	46.2
		TOTAL = 69,676			

[a] Species nomenclature consistent with the EFIDB as reported by Hansen *et al.* (1992).

Table 2.—*Parameter estimates by species for Northeastern United States PRI models.*

Species	Final number	Estimated parameters		
		$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
Balsam fir	14	0.42	− 0.21	0.937051
Tamarack	11	1.63	− 1.13	0.991027
White spruce	15	1.69	− 1.11	0.989269
Black spruce	7	0.87	− 0.59	0.930458
Red spruce	7	1.06	− 0.56	0.959002
Red pine	6	0.45	− 0.31	0.955000
Pitch pine	7	0.67	− 0.56	0.942390
Eastern white pine	14	0.61	− 0.47	0.981844
Northern white-cedar	7	0.82	− 0.36	0.945433
Eastern hemlock	11	1.02	− 0.83	0.984948
Red maple	15	0.96	− 0.66	0.986284
Silver maple	7	0.51	− 0.27	0.968776
Sugar maple	10	0.40	− 0.46	0.988446
Yellow birch	13	0.66	− 0.67	0.976746
Sweet birch	6	0.46	− 0.29	0.946431
Paper birch	8	0.48	− 0.36	0.945841
American beech	14	0.33	− 0.51	0.977587
White ash	12	0.98	− 0.86	0.996465
Black ash	7	0.58	− 0.58	0.949105
Bigtooth aspen	7	1.97	− 1.06	0.995803
Quaking aspen	12	1.14	− 0.64	0.955174
Black cherry	14	1.04	− 0.80	0.988091
White oak	8	0.41	− 0.23	0.957937
Scarlet oak	7	3.05	− 1.42	0.999900
Chestnut oak	11	0.12	− 0.38	0.979383
Northern red oak	12	0.88	− 0.80	0.988439
Black oak	12	0.52	− 0.75	0.993657
Black willow	7	0.95	− 0.66	0.989004
American basswood	9	0.58	− 0.74	0.985406
American elm	9	0.78	− 0.30	0.935231

Figure 3.—*PRI comparison for northern red oak between the Northeastern (NE), Midsouth (MS), and Lake States (LS) regions. PRI curves differed slightly for all three regions (a), which translated into noticeable increment differences (b and c).*

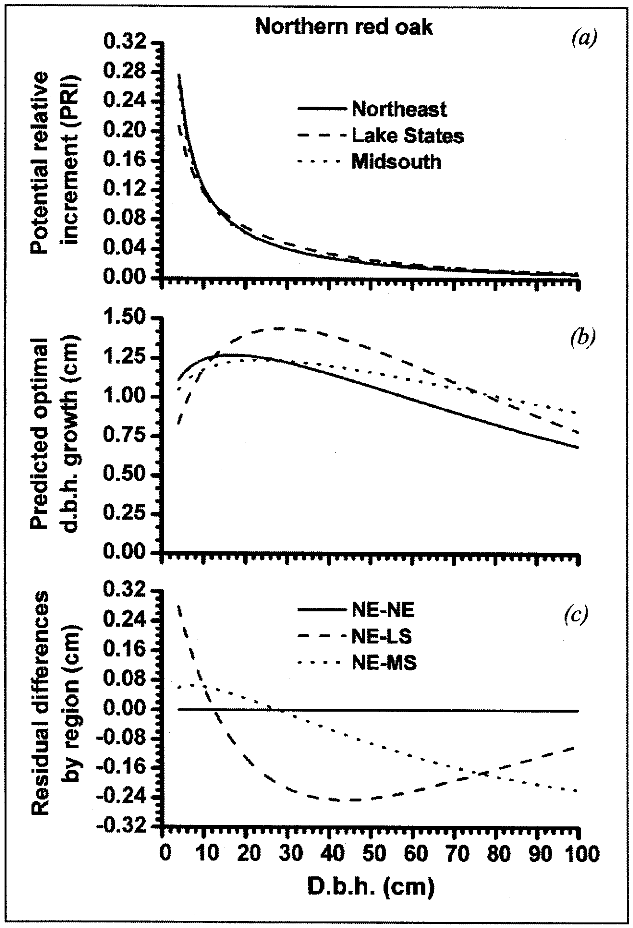


Figure 4.—*Very prominent outliers (large open symbols) identified by the PRI methodology. Both the 80-cm d.b.h. black cherry (a) and white oak trees (b) grew at a very high rate, given their large size, identifying them as individuals of concern.*

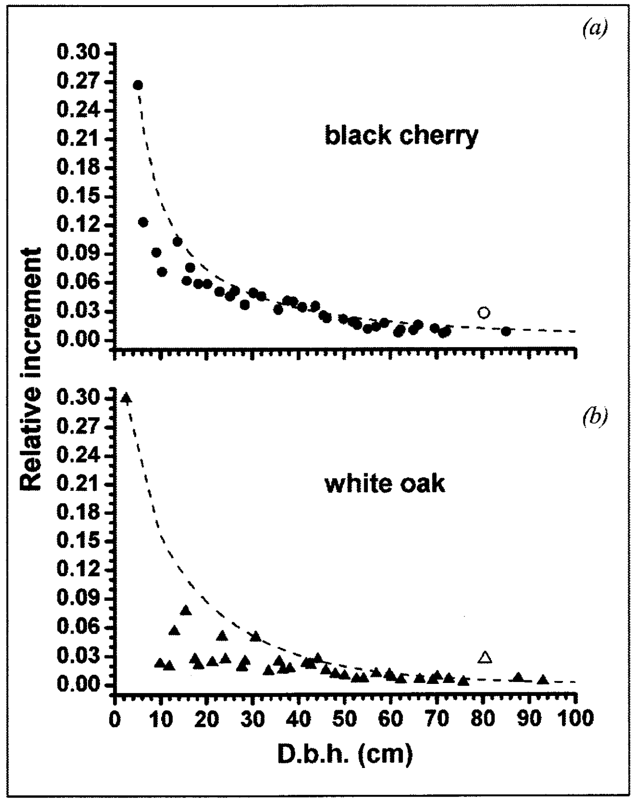


Table 3.—*Annualized parameters for northern red oak models developed from the Northeastern United States (this article), the Lake States (Bragg 2001), and the Midsouth (Bragg 2003b).*

Parameter	Northeast	Lake States	Midsouth
$\hat{b}_1$	0.880309	2.241167	0.843941
$\hat{b}_2$	−0.800826	−0.506656	−0.823358
$\hat{b}_3$	0.988439	0.983046	0.992617

Conclusions

The EFIDB for the Northeastern United States contained enough data to construct PRI growth models for 30 tree species. A comparison of northern red oak models among several regions, including the Northeast, produced noticeable differences in the magnitude and timing of the predicted maximal increment (table 3). Because diameter growth is cumulative, even subtle differences over time would lead to substantial variation in tree size, assuming all other environmental conditions are held constant. Key to any effort, however, is ensuring that the inventory records accurately reflect tree dimensions before they are incorporated into any type of predictive environment.

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