Development of a basal area growth system for maritime pine in northwestern Spain using the generalized algebraic difference approach

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Abstract: A basal area growth system for single-species, even-aged maritime pine (*Pinus pinaster* Ait.) stands in Galicia (northwestern Spain) was developed from data of 212 plots measured between one and four times. Six dynamic equations were considered for analysis, and both numerical and graphical methods were used to compare alternative models. The double cross-validation approach was used to assess the predictive ability of the models. The data were best described by a dynamic equation derived from the Korf growth function using the generalized algebraic difference approach (GADA) by considering two parameters to be site specific. The equation was fitted in one stage using the base-age-invariant dummy variables method. In addition, the system incorporates a function for predicting initial stand basal area, in which the site-related variable was expressed as a power function of site index. This function can be used to establish the starting point for the projection equation when no inventory data are available. The two equations are compatible. The effect of thinning on basal area growth was examined; the results showed that there was no need to use a different equation to reliably predict postthinning basal area development. The nonlinear extra sum of squares method indicated differences in the model parameters for the two ecoregions (coastal and interior) defined for this species in the area of study.

Résumé : Un système d'équations de croissance en surface terrière a été développé pour les peuplements purs et équiennes de pin maritime (*Pinus pinaster* Ait.) à partir des données de 212 placettes mesurées entre une et quatre fois en Galice dans le nord-ouest de l'Espagne. Six équations dynamiques sont considérées pour l'analyse. Tant les méthodes graphiques que numériques ont été utilisées pour évaluer les différentes équations. L'approche de la double validation croisée a été utilisée pour analyser la capacité prédictive des équations. Une équation dynamique dérivée de la fonction de croissance de Korf, dont les deux paramètres qui sont considérés comme spécifiques à la station sont estimés par l'approche de la différence algébrique généralisée (GADA), décrit le mieux les données. L'équation a été ajustée en une seule étape en utilisant la méthode des variables fictives indépendantes de l'âge. En outre, le système incorpore une fonction pour prédire la surface terrière initiale dont la variable spécifique à la station est exprimée comme une fonction de puissance de l'indice de qualité de station. Cette fonction peut être utilisée pour établir le point de départ du système d'équations de prédiction lorsque aucune donnée d'inventaire n'est disponible. Les deux équations constituant le système de prédiction sont compatibles. L'effet de l'éclaircie sur la croissance en surface terrière a été examiné. Les résultats montrent qu'il n'était pas nécessaire d'utiliser une équation différente pour prédire de façon fiable l'évolution de la surface terrière après une éclaircie. La méthode de la somme additionnelle des carrés des résidus révèle l'existence de différences dans les paramètres du modèle pour les deux écorégions (région côtière et région intérieure) définies pour le pin maritime dans la zone d'étude.

[Traduit par la Rédaction]

Introduction

Maritime pine (*Pinus pinaster* Ait.) is the most important coniferous species in Spain, with 1 680 000 ha of single-species or mixed stands occurring in both artificial plantations and natural forests that have regenerated after clearcutting or wildfires. The wide distribution and the variety of sites occupied by maritime pine have made this species

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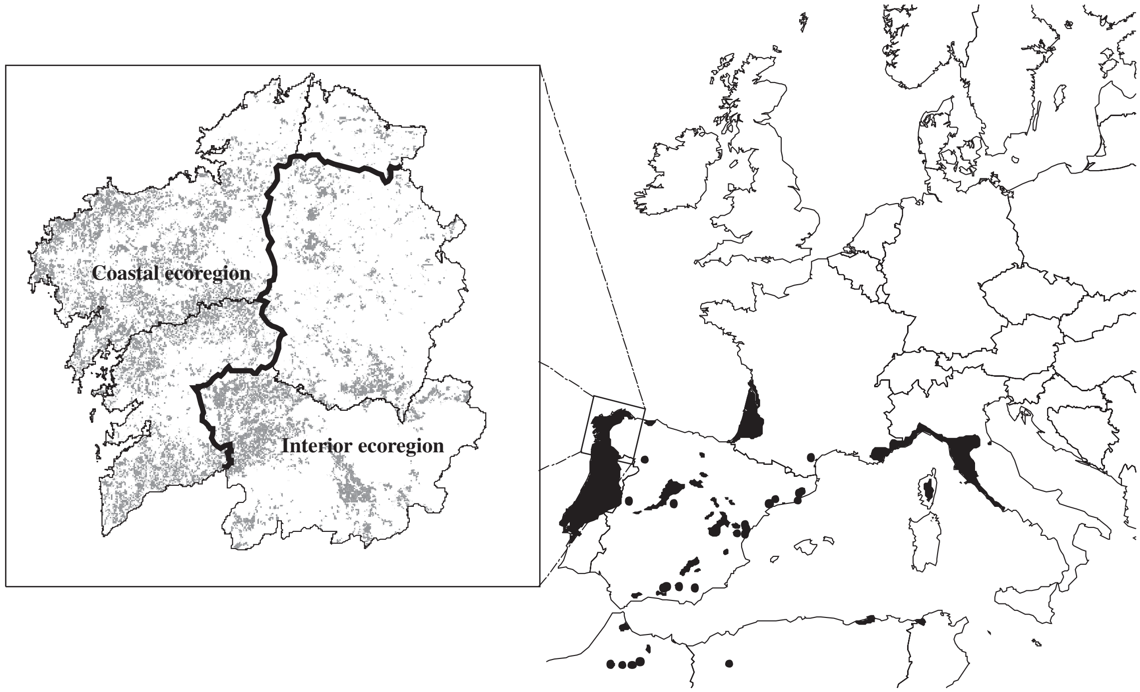
Fig. 1. Natural distribution of maritime pine in the world and limit of the ecoregions (coastal and interior) in Galicia.



highly important in the forestry industry of northern Spain, with more than 50 × 10^6 m^3 of standing timber and an annual harvest volume of 2 380 000 m^3 in the period 1992–2001 (Xunta de Galicia 2001).

Populations of this species show a high level of genetic and phenotypic diversity owing to the planting of seeds of different origins and to genotype–environment interactions (Alía et al. 1995, 1997). These features lead to significant differences in the growth patterns in the two ecoregions (coastal and interior) in Galicia (Fig. 1), as defined by Vega et al. (1993).

Álvarez González et al. (1999) developed an ecoregional stand-level growth and yield model for maritime pine stands in Galicia, which provided rather limited information about the forest stand. With the consideration that forest management decisions require more detailed information about stand structure and volume, a revision of the model has been carried out. Several submodels have been developed to date: (*i*) a diameter-distribution function (Álvarez González et al. 2002), (*ii*) a site-quality system (Álvarez González et al. 2005), (*iii*) a merchantable-volume equation (Rojo et al. 2005), and (*iv*) a generalized height–diameter model (Castedo et al. 2005). The final modification remaining is the development of a basal area growth system. This system can be used for a variety of purposes including inventory updating, harvest scheduling, and predicting wood yields for different stand conditions.

The stand basal area growth system is a key component of stand-level models, since basal area is directly related to other very important economic variables, such as total stand volume and quadratic mean diameter. Furthermore, estimations of basal area growth functions can be used to provide a constrain on size-class or individual-tree models and thus form a link between models of high and low resolution (Gadow et al. 2001).

Basal area growth equations must possess three main properties for obtaining consistent estimates (Clutter et al. 1983; Amaro et al. 1997): (*i*) biological meaning: they must have a coherent inflection point and an asymptotic value when the projected stand age approximates infinity; (*ii*) path invariance: the result of projecting first from t_0 to t_1, and then from t_1 to t_2, must be the same as that of the one-step projection from t_0 to t_2; and (*iii*) simplicity: models that are too complex and include many interactions between independent variables may be affected by correlations among the variables, making them unstable and with a lower predictive capacity.

Fulfilment of these properties depends on both the construction method and the mathematical function used to develop the model. Most of them can be achieved by using the algebraic difference approach (ADA) proposed by Bailey and Clutter (1974) or the generalized algebraic difference approach (GADA) of Cieszewski and Bailey (2000). GADA can be applied in modelling the growth of any site-dependent variable involving the use of unobservable variables substituted by the self-referencing concept (Northway 1985) of model definition (Cieszewski 2004), such as dominant height, basal area, stand volume, trees per unit area, biomass, or carbon sequestration.

The objective of the present study was to develop an ecoregional basal area growth system for maritime pine stands in Galicia (northwestern Spain). To accomplish this, a basal area projection function for thinned and unthinned stands was developed using the GADA approach, and a compatible basal area prediction model was then derived on the basis of the same base equation as the one used for deriving a projection model.

Table 1. Characteristics of the sample plots used for model fitting.

	Coastal ecoregion				Interior ecoregion			
Variable	Mean	Min.	Max.	SD	Mean	Min.	Max.	SD
Unthinned plots								
t (years)	17.7	8	39	7.3	18.3	9	40	5.7
N (trees·ha^{-1})	1746	430	4475	910.8	1855.3	654	3142	519.5
G (m^2·ha^{-1})	31.1	5.9	56.5	12.0	29.1	5.1	72.5	14.4
H_0 (m)	12.1	4.7	24	4.0	10.2	4.6	20.4	2.7
S (m)	14.1	8.5	19	2.9	11.5	7.1	15.2	2.1
Thinned plots								
t (years)	15.7	10	31	4.0	20.2	13	27	4.1
N (trees·ha^{-1})	2235	480	6576	1056.1	1618	701	3267	557.7
G (m^2·ha^{-1})	23.8	3.6	44.1	8.3	28.5	12.3	59.0	10.9
H_0 (m)	11.3	4.8	19.8	3.3	11.1	7.5	14.5	1.6
S (m)	14	9.5	18.5	2.2	11.2	8.0	13.5	1.6
G_{re}/G_b (%)	21.2	11.1	50.6	9.9	21.3	12.0	47.7	10.6

Note: For the unthinned plots, there were 95 inventories for the coastal ecoregion and 124 inventories for the interior ecoregion; for the thinned plots, there were 197 inventories for the coastal ecoregion and 159 inventories for the interior ecoregion. See text for definition of variables. SD, standard deviation.

Material and methods

Data

Three different sources of data corresponding to single-species, even-aged maritime pine stands located throughout the area of distribution of this species in Galicia were used to develop the basal area growth system.

The first data set was gathered by the Instituto Forestal de Investigaciones y Experiencias to develop yield tables for this species (Echeverría and De Pedro 1948) and consisted of 148 inventories of 62 plots measured between one and four times.

The second data set consisted of 249 inventories from 10 thinning trials installed between 1965 and 1972 by the Instituto Forestal de Investigaciones y Experiencias. At each location four treatment plots were installed: an unthinned control, a lightly thinned plot (approximately 15% of the basal area removed), a moderately thinned plot (approximately 30% of the basal area removed), and a heavily thinned plot (approximately 45% of the basal area removed). The stands were thinned once from below, immediately after plot establishment, and were remeasured at different age intervals.

The third data set, a total of 178 inventories of 51 plots, was gathered by the Centro de Investigaciones Forestales de Lourizán with the objective of quantifying the site quality and the effect of fertilization in single-species, even-aged maritime pine stands (Bará and Toval 1983). This data set covered a broad range of stand conditions (stand ages, stand densities, and sites) in coastal and interior ecoregions, and most of the plots were measured between one and four times.

The following stand variables were calculated for each inventory: stand age (t), stand basal area (G), number of trees per hectare (N), dominant diameter (D_0) and dominant height (H_0) (defined as the mean diameter and mean height of the 100 largest trees per hectare, respectively), and site index (S, defined as the dominant height of the stand, in metres, at a reference age of 20 years), which was obtained from the site-quality system developed by Álvarez González et al. (2005) for coastal and interior ecoregions.

Only live trees were included in the calculations of basal area and number of trees per hectare. In addition, data on the number of trees per hectare and basal area removed in thinning operations were available. Thinning weight was computed as the ratio between the basal area removed and the basal area before thinning (G_{re}/G_b). Summary statistics, including the mean, minimum, maximum, and standard deviation of these stand variables, are given in Table 1.

Basal area projection function

Many stand basal area projection functions for thinned and unthinned stands have been reported (e.g., Clutter 1963; Sullivan and Clutter 1972; Pienaar et al. 1985; Tomé et al. 1997). However, most of these are empirical-based equations, not derived directly from growth functions.

The use of dynamic equations derived from the integral form of differential equations is highly recommended for projecting stand basal area over time, since these equations fulfil the three previously outlined desired characteristics. Bailey and Clutter (1974) presented a technique for dynamic equation derivation that is known in forestry as the algebraic difference approach (ADA), which essentially involves replacing a base-model site-specific parameter with its initial-condition solution. The main limitation of this approach is that all models thus derived are either anamorphic or have single asymptotes (Bailey and Clutter 1974; Cieszewski and Bailey 2000). Cieszewski and Bailey (2000) extended this method and presented the generalized algebraic difference approach (GADA), which can be used to derive the same models as those derived by ADA. The main advantage of GADA is that one can expand the base equations according to various theories about growth characteristics (e.g., asymptote, growth rate), thereby allowing more than one parameter to be site specific and permitting the derivation of more flexible dynamic equations (see Cieszewski and Bailey 2000;

Cieszewski 2001, 2002, 2003). GADA includes the ability to simulate concurrent polymorphism and multiple asymptotes. The first step in GADA is to select a base equation and then identify any desired number of site-specific parameters within this equation. It must then be explicitly defined how the site-specific parameters change across different sites by replacing them with explicit functions of X (one unobservable independent variable that describes site productivity as a summary of management regimes, soil conditions, and ecological and climatic factors) and new parameters. In this way, the initially selected two-dimensional base equation ($Y = f(t)$) expands into an explicit three-dimensional site equation ($Y = f(t, X)$) describing both cross-sectional and longitudinal changes with two independent variables t and X. Since X cannot be reliably measured or even functionally defined, the final step of GADA involves the substitution of X by equivalent initial conditions t_0 and Y_0 ($Y = f(t, t_0, Y_0)$) so that the model can be implicitly defined and practically useful (Cieszewski and Bailey 2000; Cieszewski 2002). During this process, redundant parameters are often eliminated, resulting in a final explicit GADA-based model that has the same number of parameters or fewer parameters than the initial explicit site equation.

Basal area prediction function

To project stand basal area using a projection function, it is necessary to have an initial value at a given age for this variable. Usually, the initial condition value is obtained from a common forest inventory where diameter at breast height is measured; however, when this is not available, a basal area prediction equation is required.

The base growth equations from which the dynamic projection functions were derived were used to predict basal area at any specific point in time. Since stand basal area depends on the age of the stand and other stand variables (theoretically the productive capacity of a site and any other measure of stand density), it is generally necessary to relate the site-specific parameters of the growth function to these variables to achieve good estimations. The compatibility between both equations is ensured by relating only the site-specific parameters to stand variables that do not vary over time (e.g., site index), while the remaining parameters are shared by the prediction and projection functions. Thus, for a given stand basal area curve obtained from the prediction function, whatever point on this curve is used as the initial condition value in the projection function, the estimated stand basal area will always be a point on that curve.

There exist two possibilities for estimating the parameters of both equations to maintain compatibility. The first is fitting both equations simultaneously using an appropriate regression fitting technique that accounts for the correlations between the right-hand-side endogenous variables and the error component of the left-hand-side endogenous variables (this is called simultaneous equation bias) (SAS Institute Inc. 2004*a*). The second is estimating independently the parameters of the projection equation, substituting their values into the prediction equation, and then fitting the latter to obtain the estimates of the remaining parameters. This approach gives priority to the projection function and is generally preferred because the dynamic model will be most frequently used to project basal area, given an initial stand condition obtained from a common forest inventory.

Thinning effect on basal area growth

Thinning operations provide an opportunity to obtain intermediate cash flows from the harvested wood, improve the quality of the remaining stand by removing slow-growing and damaged or diseased trees, and shift future growth of the stand to the larger, better-quality remaining trees.

When a forest stand is thinned, its growth characteristics and dynamics change. Studies involving thinning experiments in even-aged stands have shown inconsistent results in terms of the effects of stand density variation on stand basal area growth (Clutter et al. 1983, p. 68). Several studies have shown that there is no difference in the unit-area basal area growth between thinned and unthinned stands of the same age, site index, and stand basal area (e.g., Clutter and Jones 1980; Cao et al. 1982; Matney and Sullivan 1982). In contrast, other studies have shown that basal area growth rates in thinned stands exceed those in unthinned stands with the same stand characteristics (e.g., Pienaar 1979; Hamilton 1981; Pienaar and Shiver 1984; Pienaar et al. 1985; Amateis et al. 1995; Hasenauer et al. 1997; Amateis 2000).

Two approaches have been used to consider the effect of thinning operations on stand basal area growth:

(1) Development of different basal area growth functions for different types of stands (unthinned and after the first, second, and subsequent thinning operations) that have the same mathematical structure but that have been parameterized using different data sets (Pienaar 1979; Woollons and Hayward 1985; Knoebel et al. 1986; Zarnoch et al. 1991).

(2) Inclusion of a thinning response function that expresses the basal area growth of a thinned stand as a product of a reference growth and the thinning response function (Hynynen 1995) — the reference growth accounts for the factors affecting stand growth in unthinned stands, while the thinning response function predicts the relative growth response following thinning. Several attempts have been made to model the thinning response on the remaining basal area growth (Bailey and Ware 1983; Pienaar et al. 1985; Pienaar and Shiver 1986; Falcao 1997; Hasenauer et al. 1997; Chikumbo et al. 1999; Amateis 2000), mostly with stands derived from plantations. Theoretically, the effect of thinning described by these functions must gradually increase from the treatment time to a maximum level and then gradually diminish over time (Amateis et al. 1995; Snowdon 2002). In addition, the effect must respond to the intensity of thinning, the time since thinning, and the age of the stand at thinning (Knoebel et al. 1986; Hynynen 1995; Hasenauer et al. 1997).

In this study, we used the first approach to take into account the effect of thinning on basal area growth, by employing dummy variables.

Models considered

A large number of mathematical equations can be used to describe basal area growth (see, for example, the 74 equations documented by Kiviste et al. 2002). In the present study, three well-known growth functions were selected for analysis: Korf (cited in Lundqvist 1957), Hossfeld (Hossfeld 1822), and Bertalanffy–Richards (Bertalanffy 1949, 1957; Richards 1959).

On the basis of these equations, several dynamic models were formulated using GADA for developing the projection function. Most of the equations considered for modelling basal area growth did not assume anamorphic growth for this variable (e.g., Amaro et al. 1997; Falcao 1997; Tomé et al. 1997, 2001); therefore, only the possible polymorphic solutions of the above-mentioned equations were considered for analysis. Some of these solutions were earlier discarded because the fitting curves did not describe the observed trends in the data well. We thus focused our efforts on six dynamic equations, the formulations of which are shown in Table 2. All of the equations are base-age invariant.

As general notational convention, $a_1, a_2, \ldots, a_n$ were used to denote parameters in base models, while $b_1, b_2, \ldots, b_m$ were used for global parameters in subsequent GADA formulations. All the GADA-based models have the general implicit form of $Y = (t, t_0, Y_0, b_1, b_2, \ldots, b_m)$.

Models E1, E3, and E5 were derived by applying GADA to the Korf, Hossfeld, and Bertalanffy–Richards functions, respectively, by considering only parameter a_2 to be site specific. In this case GADA is equivalent to ADA. Model E1 has commonly been used to describe basal area growth in many studies (e.g., Amaro et al. 1997; Falcao 1997; Tomé et al. 1997, 2001; Lei 1998). Model E3 is the polymorphic equation described by McDill and Amateis (1992) for estimating the site quality, and it has also been used for basal area growth modelling (e.g., Falcao 1997 and Fonseca 2004 for maritime pine stands). Model E5 has also been frequently used in forestry applications, including stand basal area growth modelling (Pienaar and Turnbull 1973; Pienaar and Shiver 1984; Kotze and Vonck 1997; Lei 1998), because of its theoretical flexibility. All of these models are polymorphic with a single asymptote.

Dynamic models E2, E4 and E6 were developed by considering two parameters to be site specific. Model E2 was derived on the basis of the Korf function by considering both parameters a_1 and a_2 to be dependent on X. To facilitate such derivation, the base equation was reparameterized into more suitable form for manipulation of these two parameters, using $\exp(X)$ instead of a_1. Parameter a_2 was expressed as a linear function of the inverse of X. Model E4 was derived by Cieszewski (2002) from the Hossfeld function, by replacing a_1 with a constant plus the unobserved site variable X, and a_2 by b_2/X. Model E6 was developed by Krumland and Eng (2005) by expressing the asymptote as an exponential function of X and the shape parameter as a linear function of the inverse of X.

For the base equations with two site-specific parameters, the solution for X involved finding roots of a quadratic equation and selecting the most appropriate one to substitute into the dynamic equation. Only solutions involving addition rather than subtraction of the square root were used because they are more likely to be real and positive (Cieszewski and Bailey 2000).

In summary, both recently developed dynamic equations with two site-specific parameters and frequently used dynamic equations with only one site-specific parameter were tested. The prediction function was developed on the basis of the base growth function from which the dynamic model that provided the best results on projection was derived.

Model fitting and validation

The basal area growth system was developed in two stages: first, a model for projecting basal area over time was fitted; second, a prediction function was developed ensuring the compatibility between the estimates of both models.

Data measurements generally contain environmental and measurement errors. If basal area is assumed to be error free when it is on the right-hand side of the equation, but possessing error when it is on the left-hand side of the equation, a conflict exists. Therefore, basal areas that appear on the right-hand side of the models should represent points on the global model (estimates) that cannot be evaluated until the global parameters are estimated. However, the estimated basal areas must be known to obtain unbiased estimates of the global model parameters (Krumland and Eng 2005). Several methods have been suggested to overcome this problem (e.g., Bailey and Clutter 1974; García 1983; Cieszewski et al. 2000). These have been generally applied for fitting site-quality equations and involve simultaneous estimation of the global model parameters and of the measurement and environmental errors associated with the site-specific parameters.

We used the dummy variables method proposed by Cieszewski et al. (2000). In this method, the initial conditions are specified as identical for all the measurements belonging to the same unthinned growth period within a single plot, hereafter the individual being investigated. During the fitting process the basal area corresponding to the initial age (which can be arbitrarily selected for each unthinned interval, although age zero is not allowed) is simultaneously estimated for each individual and all of the global model parameters. The dummy variables method recognizes that each measurement is made with error, and therefore, it seems unreasonable to force the model through any given measurement. Instead, the curve is fitted to the observed individual trends in the data.

As an example of this procedure, consider model E1. The Y_0 variable must be substituted by a sum of terms containing a site-specific or local parameter (an initial basal area) and a dummy variable for each individual:

$$[1] \qquad Y = b_1\left[\frac{(Y_{01}I_1 + Y_{02}I_2 + \ldots + Y_{0n}I_n)}{b_1}\right]^{\left(\frac{t_0}{t_1}\right)^{b_3}}$$

where Y_{0i} is the site-specific parameter for each individual i, and I_i is a dummy variable equal to 1 for individual i and 0 otherwise.

The sum of terms of the initial basal area times the dummy variable collapses into a single parameter (an estimated basal area at the specified initial age) that is unique for each individual during the fitting process.

The parameter estimation of regression equations describing the behaviour of individuals over time often has associated problems of serial correlation (i.e., correlation between the residuals within the same individual). To account for this possible autocorrelation, we modelled the error term using a first-order continuous autoregressive error structure (CAR (1)) that allows the models to be applied to irregularly spaced,

Table 2. Base models and GADA formulations considered.

Base equation	Parameter related to site	Solution for X with initial values (t_0, Y_0)	Dynamic equation	
Korf: $Y = a_1 \exp(-a_2 t^{-a_3})$	$a_2 = X$	$X_0 = -\ln\left(\frac{Y_0}{a_1}\right) t_0^{a_3}$	$Y = b_1 \left(\frac{Y_0}{b_1}\right)^{\left(\frac{t_0}{t_1}\right)^{b_3}}$	(E1)
	$a_1 = \exp(X)$ $a_2 = (b_1 + b_2)/X$	$X_0 = \frac{1}{2} t_0^{-b_3} \left\{ b_1 + t_0^{b_3} \ln(Y_0) + \sqrt{4b_2 t_0^{b_3} + [-b_1 - t_0^{b_3} \ln(Y_0)]^2} \right\}$	$Y = \exp(X_0) \exp\left[-\left(\frac{b_1 + b_2}{X_0}\right) t^{-b_3}\right]$	(E2)
Hossfeld: $Y = \frac{a_1}{1 + a_2 t^{-a_3}}$	$a_2 = X$	$X_0 = t_0^{-a_3} \left(\frac{a_1}{Y_0} - 1\right)$	$Y = \frac{b_1}{[1 - (1 - b_1/Y_0)(t_0/t)^{b^3}]}$	(E3)
	$a_1 = b_1 + X$ $a_2 = b_2/X$	$X_0 = \frac{1}{2}\left[Y_0 - b_1 + \sqrt{(Y_0 - b_1)^2 + 4b_2 Y_0 t_0^{-b_3}}\right]$	$Y = \frac{b_1 + X_0}{1 + b_2/X_0 t^{-b_3}}$	(E4)
Bertalanffy–Richards: $Y = a_1[1 - \exp(-a_2 t)]^{a_3}$	$a_2 = X$	$X_0 = \frac{-\ln\left[1 - \left(\frac{Y_0}{b_1}\right)^{1/b_3}\right]}{t_0}$	$Y = b_1 \left\{1 - \left[1 - \left(\frac{Y_0}{b_1}\right)^{1/b_3}\right]^{1/t_0}\right\}^{b_3}$	(E5)
	$a_1 = \exp(X)$ $a_3 = b_2 + b_3/X$	$X_0 = \frac{1}{2}\left[\ln Y_0 - b_2 L_0 + \sqrt{(\ln Y_0 - b_2 L_0)^2 - 4b_3 L_0}\right]$ with $L_0 = \ln[1 - \exp(-b_1 t_0)]$	$Y = Y_0 \left[\frac{1 - \exp(-b_1 t)}{1 - \exp(-b_1 t_0)}\right]^{(b_2 + b_3/X_0)}$	(E6)

unbalanced data (Gregoire et al. 1995; Zimmerman and Núñez-Antón 2001). The CAR (1) expands the error terms in the following way:

$$[2] \qquad e_{ij} = \psi_1 \rho^{t_{ij}-t_{ij-1}} e_{ij-1} + \varepsilon_{ij}$$

where e_{ij} is the jth ordinary residual on the ith individual (i.e., the difference between the observed and the estimated basal area of plot i at age measurement j), ψ_1 is equal to 1 for $j > 1$ and to zero for $j = 1$, ρ is the first-order autoregressive parameter to be estimated, $t_{ij} - t_{ij-1}$ is the time distance separating the jth from the jth – 1 observations within each individual ($t_{ij} > t_{ij-1}$), and ε_{ij} is now the error term under conditions of independence and no heteroscedasticity.

The dummy variables method including the CAR (1) error structure was programmed using the MODEL procedure of SAS/ETS® (SAS Institute Inc. 2004*a*), which allows for dynamic updating of the residuals. The Marquardt algorithm was used for model fitting, since it is most useful when the parameter estimates are highly correlated (Fang and Bailey 1998; Parresol 2001).

Once the projection function was fitted, the common parameters were substituted in the prediction function and the remaining unknown parameters were estimated using ordinary nonlinear least squares (ONLS) with the NLIN procedure of SAS/STAT (SAS Institute Inc. 2004*b*). Only data from inventories corresponding to ages younger than 15 years were used, and it was assumed that if projections are required based on ages older than this threshold, the initial basal area should be obtained directly from inventory data.

The comparison of the estimates for the different models was based on numerical and graphical analyses of the residuals. Two statistics were examined: the root mean square error (RMSE) and the coefficient of determination for nonlinear regression (R^2). The expressions of these statistics are as follows:

$$[3] \qquad \mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p}}$$

$$[4] \qquad R^2 = r^2_{y_i \hat{y}_i}$$

were y_i and $\hat{y}_i$ are the observed and predicted values of the dependent variable, respectively, n is the total number of observations, and p is the number of model parameters; $r_{y_i \hat{y}_i}$ is the correlation coefficient for a linear regression between the observed and predicted values of the dependent variable (see Ryan 1997, pp. 419 and 424).

Another important step in evaluating the models was to perform graphical analyses of the residuals and the appearance of the fitted curves overlaid on the trajectories of the basal area for each individual. Visual or graphical inspection is an essential point in selecting the most appropriate model because curve profiles may differ drastically, even though fit statistics and residuals are similar.

Because the quality of the fit does not necessarily reflect the quality of the prediction, assessment of the validity of the model with an independent data set is desirable (Myers 1990; Huang et al. 2003). One of the most common methods proposed for accomplishing model validation is splitting the data set in two parts, using one for fitting and the other for validation (cross-validation). However, the reported outcomes from such an approach are always identical because the two groups of data are not really independent, which is a basic prerequisite in model validation (Huang et al. 2003). Moreover, according to Myers (1990) and Hirsch (1991) the final estimation of the model parameters should come from the entire data set, because the estimates obtained with this approach will be more precise than those obtained with the model fitted from only one portion of the data. Taking these considerations into account, we carried out a double cross-validation of each model estimating the residual for one plot by fitting the model without that plot. For this, the mean age of the plot and its corresponding basal area, estimated using linear interpolation, were used as initial condition values for t_0 and Y_0 in GADA formulations. The RMSE and the model efficiency (MEF, equivalent to the R^2 of the fitting phase) were calculated from the residuals from the double cross-validation. Although this approach is not a real method of model validation (Vanclay and Skovsgaard 1997; Pretzsch et al. 2002; Huang et al. 2003), it has been used as an additional criterion for selecting the best model (Myers 1990) while waiting for a new independent data set for assessing the true quality of the predictions.

Analysis of the effect of ecoregions and thinning operations on basal area growth

The variability in basal area growth was analyzed by including the regional and thinning effects as dummy categorical variables. To compare the differences between ecoregions and type of stands (thinned and unthinned), the nonlinear extra sum of squares method for detecting simultaneous homogeneity among parameters was used (Bates and Watts 1988; Judge et al. 1988). This test has frequently been applied to analyze differences among geographic regions (e.g., Pillsbury et al. 1995; Huang et al. 2000; Peng et al. 2001; Zhang et al. 2002; Calama et al. 2003; Álvarez González et al. 2005) and silvicultural treatments (e.g., Castedo et al. 2005; Zhao et al. 2005).

This method requires the fitting of the reduced and full models. The reduced model corresponds to the same set of global parameters for both ecoregions and treatments. The full model corresponds to different sets of global parameters for each ecoregion and treatment and is obtained by expanding each global parameter by including an associated parameter and a dummy variable to differentiate the ecoregions and treatments:

$$[5] \qquad b_i + \phi_i I_\mathrm{e} + \varphi_i I_\mathrm{t} \quad i = 0{,}1$$

where b_i is a global parameter of the model, ϕ_i and φ_i are the associated parameters of the full model, and I_e and I_t are dummy categorical variables for considering the ecoregions and the silvicultural treatments, respectively. Variable I_e is equal to 0 if the observation belongs to the interior ecoregion and to 1 if it belongs to the coastal ecoregion. Variable I_t is equal to 0 for unthinned stands and to 1 for thinned stands.

The appropriate test statistic uses the following expression:

Table 3. Parameter estimates and goodness-of-fit statistics for the models analyzed.

					Fitting phase		Cross-validation phase	
Model	Parameter	Estimate	Approx. SE	Approx. $p >\|t\|$	RMSE	R^2	RMSE	MEF
E1	b_1	212.084 4	29.2505	<0.0001	1.1309	0.9944	1.4638	0.9896
	ϕ_1	–102.783	31.02	0.001				
	b_3	21.099 33	0.1008	<0.0001				
	ϕ_3	10.180 48	1.3569	0.0481				
	ρ	0.324 646	5.5617	0.0014				
E2	b_1	–167.466	60.6759	0.0061	1.1256	0.9944	1.1582	0.994
	b_2	999.084 7	316.3	0.0017				
	ϕ_2	–50.342 6	14.868	0.0008				
	b_3	0.893 627	0.0551	<0.0001				
	ρ	0.306 914	0.1047	0.0036				
E3	b_1	116.568 8	7.5823	<0.0001	1.1616	0.9941	1.5109	0.989
	ϕ_1	–37.241 8	8.5854	<0.0001				
	b_3	2.199 107	0.0751	<0.0001				
	ϕ_3	0.293 445	0.1206	0.0155				
	ρ	0.340 389	0.0965	0.0005				
E4	b_1	95.567 93	6.5405	<0.0001	1.1609	0.9941	1.1855	0.9931
	ϕ_1	–22.017 2	4.8741	<0.0001				
	b_2	–15 344.1	8025.1	0.0568				
	b_3	2.335 579	0.0596	<0.0001				
	ρ	0.323 573	0.1023	0.0017				
E5	b_1	106.928 6	6.6563	<0.0001	1.1549	0.9941	1.5002	0.9891
	ϕ_1	–33.466 8	7.4637	<0.0001				
	b_3	2.807 079	0.1683	<0.0001				
	ϕ_3	0.804 045	0.3148	0.0111				
	ρ	0.337 663	0.0968	0.0006				
E6	b_1	0.073 26	0.004	<0.0001	1.1264	0.9943	1.1586	0.9935
	b_2	–2.483 08	1.5275	0.105				
	ϕ_2	–12.903 6	4.6142	0.0055				
	b_3	26.299 6	7.0183	0.0002				
	ϕ_3	55.569 5	20.4848	0.007				
	ρ	0.321 75	0.0983	0.0012				

Note: RMSE, root mean square error; MEF, model efficiency; SE, standard error. See text for definition of parameters.

$$[6] \qquad F = \frac{SSE_R - SSE_F}{df_R - df_F} \bigg/ \frac{SSE_F}{df_F}$$

where SSE_R is the error sum of squares of the reduced model, SSE_F is the error sum of squares of the full model, and df_R and df_F are the degrees of freedom of the reduced and full models, respectively. The nonlinear extra sum of squares follows an F distribution.

If the homogeneity of parameters test reveals significant differences between ecoregions and (or) silvicultural treatments, separate basal area growth models are necessary for each ecoregion and (or) treatment.

Results and discussion

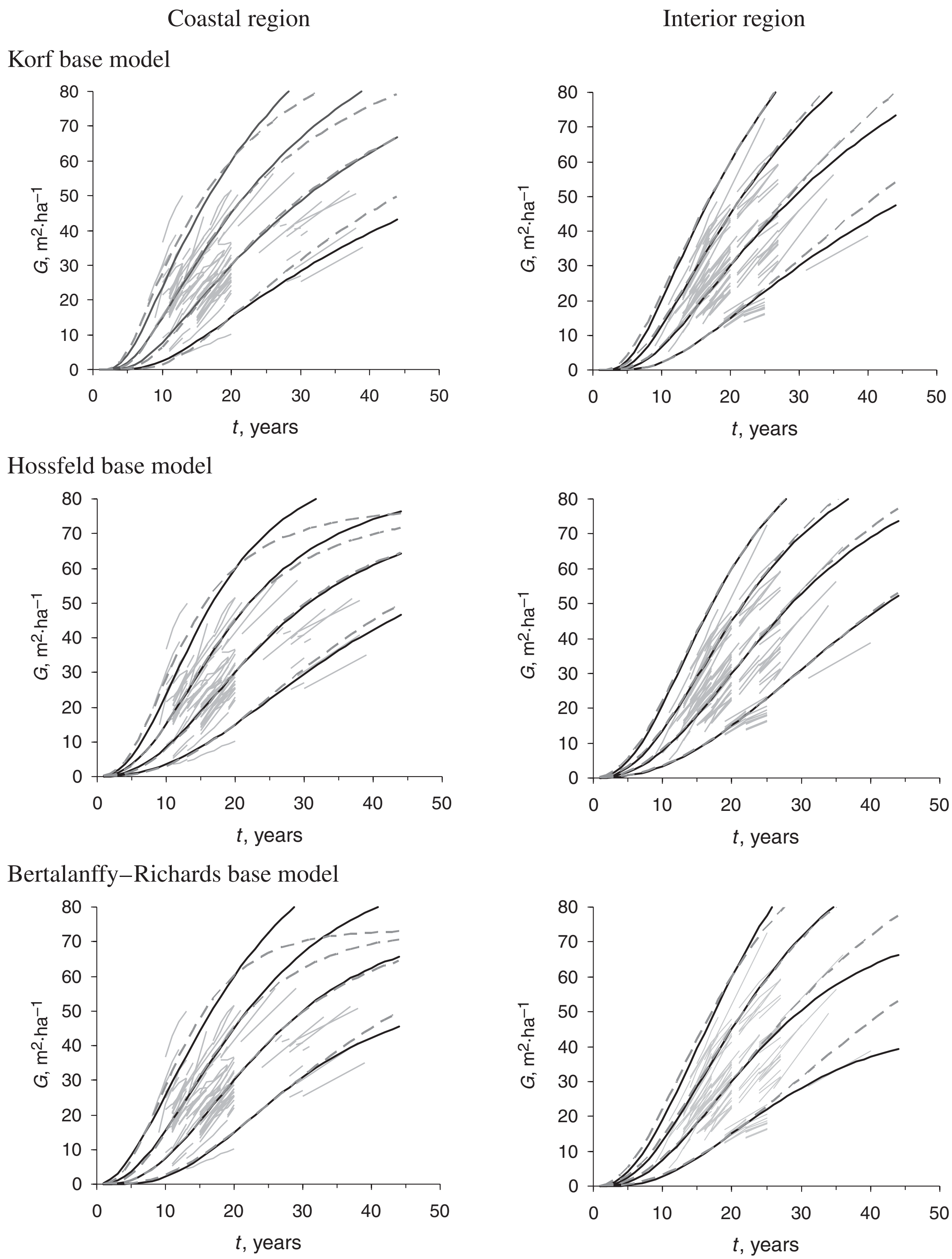
Basal area projection function

Initially, the models were fitted using nonlinear least squares without expanding the error terms to account for autocorrelation. However, a slight trend in residuals as a function of age lag1 residuals within the same individual was apparent in all the models. To account for this autocorrelation, we refitted all the models using a first-order continuous-time autoregressive error structure (eq. 2). After applying this correction, all the parameters associated with the error term were significant at the 5% level, and the error trends in residuals disappeared. The sole purpose of autocorrelation correction was to improve the interpretation of the statistical properties of the model, and it has no use in practical applications unless one is working repeatedly on the same individual.

The parameter estimates for each model, including its standard errors and p values, and the corresponding goodness-of-fit statistics are shown in Table 3. A dummy variable (I_e) was used to determine whether there were any differences between the two ecoregions. The results of a t test indicated that the estimates of some associated parameters of the full models E2, E4, and E6 were not significant at the 5% level. Thus, the models were refitted without these parameters.

Among all the equations analyzed, the models with only

Fig. 2. Basal area (G) growth curves for basal areas of 15, 30, 45, and 60 $m^2 \cdot ha^{-1}$ at 20 years for two-site-specific models (solid line) and one-site-specific models (broken line) overlaid on the trajectories of observed values over time.



one site-specific parameter (models E1, E3, and E5) provided poorer results for the goodness-of-fit statistics than the corresponding models with two site-specific parameters (models E2, E4, and E6, respectively) derived from the same base equation. All of the models accounted for approximately 99.4% of the total variation in the fitting phase and more than 98.9% in the double cross-validation phase and provided a random pattern of residuals around zero with homogeneous variance and no discernable trends.

As previously commented, visual or graphical inspection of the models was considered an essential point in selecting the most accurate representation. Therefore, plots showing the curves for basal areas of 15, 30, 45, and 60 $m^2 \cdot ha^{-1}$ at 20 years overlaid on the trajectories of observed values over time, were examined. A graphical comparison of the fitted curves overlaid on the trajectories of the observed basal area for both ecoregions is shown in Fig. 2. The equations derived from the same base model (with one and two site-specific parameters) were overlaid on the same graph. This comparison allowed us to discard some models that did not provide a good description of the trends in the data. This occurred with the models derived from the Hossfeld and Bertalanffy–Richards base functions, which also provided asymptotic values that appeared to be too small, if we consider that they were lower than the observed basal area values obtained in other nearby regions (e.g., Portugal (Fonseca 2004)). Model E2 provided the best graphical behaviour. Although the asymptotic value for the highest basal area growth curves appeared to be too large, this did not have any apparent harmful consequences for the quality of the predictions within the rotation ages usually applied for maritime pine in Galicia. The predictive ability of this model was also very high, as inferred from the statistics obtained in the double cross-validation phase (Table 3). Taking into account all these considerations, we propose the use of the E2 dynamic model derived from the Korf equation for projecting the basal area of maritime pine stands in Galicia.

Ecoregion and thinning effects on basal area growth

The nonlinear extra sum of squares statistic used to determine the homogeneity between the two ecoregions in terms of significant parameters provided an F value of 26.98 ($p > F$ was less than 0.001). This result reveals, as expected, growth differences between coastal and interior ecoregions and suggests that it is necessary to develop a model with a different set of parameters for each region. Thus, considering the significant ecoregion-associated parameter ϕ_2, the final basal area projection equation is written as follows:

[7]
$$Y = \exp(X_0)\exp\{-[b_1 + (b_2 + \phi_2 I_e)/X_0]\,t^{-b_3}\}$$
with
$$X_0 = \frac{1}{2}t_0^{b_3}\left\{b_1 + t_0^{b_3}\ln(Y_0) + \sqrt{4(b_2 + \phi_2 I_e)\,t_0^{b_3} + [-b_1 - t_0^{b_3}\ln(Y_0)]^2}\right\}$$

where Y is the value of the function (i.e., predicted stand basal area using the projection function) at t, and Y_0 is the reference variable defined as the value of the function at t_0, the parameter estimates b_i and ϕ_i are shown in Table 3, and I_e is a dummy variable that is equal to 1 for the coastal region and to 0 for interior region.

The effect of thinning on basal area growth was analyzed including a new dummy variable to test whether there were any differences between thinned and unthinned plots. Thus, eq. 7 was modified as follows:

[8]
$$Y = \exp(X_0)\exp\{-[(b_1 + \varphi_1 I_t) + (b_2 + \phi_2 I_e + \varphi_2 I_t/X_0]\,t^{(-b_3+\varphi_3 I_t)}\}$$
with
$$X_0 = \frac{1}{2}t_0^{-(b_3+\varphi_3 I_t)}\left\{(b_1 + \varphi_1 I_t) + t_0^{(b_3+\varphi_3 I_t)}\ln(Y_0) + \sqrt{4(b_2 + \phi_2 I_e + \varphi_2 I_t)\,t_0^{b_3} + [-(b_1 + \varphi_1 I_t) - t_0^{(b_3+\varphi_3 I_t)}\ln(Y_0)]^2}\right\}$$

where φ_i's are the parameters associated with the new dummy variable I_t, which is equal to 1 when thinning has been carried out and to 0 otherwise.

The results showed that all the parameters associated with the dummy variable had very large asymptotic confidence intervals, which even included zero, and therefore were not included in the model. These results suggest that for our data set, the basal area growth pattern after thinning is close to the basal area growth pattern of a stand with similar stand conditions but that has not been recently treated. Since the data used to develop the model were obtained from both thinned and unthinned stands, it seems reasonable to assume that the thinning effect is built into the model. Therefore, it is not necessary to incorporate any explicit thinning term into the dynamic model.

Similar results were also reported for maritime pine stands in the Mediterranean area. Fonseca (2004) modelled the basal area growth of this species in northern Portugal (which borders Galicia) and did not find any differences in growth patterns between thinned and unthinned stands. In fact, no stand growth models have considered a thinning response factor as necessary for maritime pine in other European regions (Lemoine 1991; Luis and Guerra 1999; Bravo-Oviedo et al. 2004), even if the data were derived from thinning trials where either low or heavy thinnings were considered.

Examination of the residuals — by applying eq. 7 to thinned and unthinned plots (Fig. 3) — indicated no trends of underestimation of the basal area of unthinned plots or overestimation of the basal area of thinned plots. Therefore, eq. 7 was proposed for projecting the stand basal area over time for both thinned and unthinned stands. The precision analysis of the basal area estimations obtained with that equation showed a root mean square error of 1.12 $m^2 \cdot ha^{-1}$, which represents only 4.1% of the observed mean value.

Basal area prediction function

Since the accurate estimation of the basal area projection function was considered to be the main objective of the system developed, the basal area prediction function was derived from the same base model as the one in eq. 7. Parameters b_1, b_2, and b_3 were substituted with those obtained

Fig. 3. Plot of residual versus predicted values for the basal area projection function E2 for thinned plots (plus signs) and unthinned plots (circles).

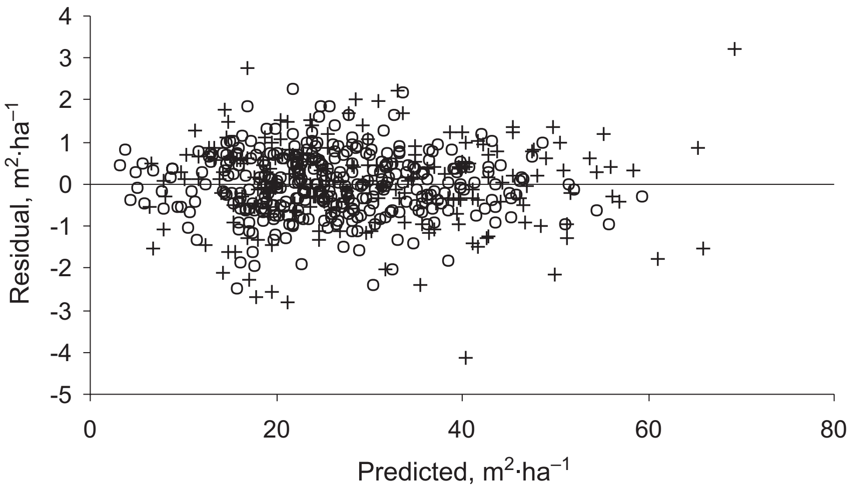


Table 4. Parameter estimates and goodness-of-fit statistics for the prediction function selected (eq. 9).

			Approx. 95% confidence limits		Fitting phase		Cross-validation phase	
Parameter	Estimate	Approx. SE	Lower	Upper	RMSE	R^2	RMSE	MEF
b_4	4.3631	0.061 7	4.2411	4.4851	5.002	0.735	5.3078	0.721
ϕ_0	–0.1489	0.007 15	–0.1630	–0.1347				
b_5	0.0738	0.005 48	0.063	0.0847				

Note: RMSE, root mean square error; MEF, model efficiency; SE, standard error. See text for definition of parameters.

for the projection function, and the unknown site-dependent function X was related to site variables. The best result was obtained by substituting X with a power function of site index. No other stand variables that may affect the amount of basal area at any specific moment, such as other variables related to stand density, were significant. The inclusion of site index in this relationship is consistent with the philosophy of GADA and directly warrants the compatibility between the projection and prediction functions because site index is considered to be a stable stand attribute over time. Moreover, this formulation implies that basal area development will proceed at a faster rate to a higher asymptotic maximum on better sites than poorer sites.

The nonlinear extra sum of squares statistic provided an F value of 6940.70 ($p > F$ was less than 0.001) and again revealed the existence of differences between the two ecoregions when predicting the basal area at any point in time. A different set of parameters for each ecoregion was therefore considered, and the final expression for basal area prediction is written as follows:

$$Y = \exp(X_0)\exp\{-[-167.466 + (999.0847 - 50.3426 I_e)/X_0]t^{-0.893627}\}$$

[9] with

$$X_0 = (b_4 + \phi_0 I_e)S^{b_5}$$

The asymptotic 95% confidence intervals obtained for the parameter estimates from eq. 9 showed reasonable values, with all the parameters being significant (Table 4). The precision analysis of basal area estimations produced a root mean square error of 5.1 $m^2 \cdot ha^{-1}$, which represents 18.3% of the observed mean basal area. As with the projection function, the double cross-validation approach showed a good predictive ability for the prediction function, with the RMSE 6% higher and the R^2 2% lower than the values obtained in the fitting phase.

A new associated dummy variable was not considered in this prediction function for taking into account the thinning effect because the data used for model fitting came from young unthinned stands.

Conclusions

Three well-known growth functions were considered for developing an ecoregional basal area growth system for even-aged maritime pine stands in northwestern Spain. Among the six dynamic equations finally evaluated for basal area projection, the GADA formulation from the Korf equation that considered parameters a_1 and a_2 to be site specific was selected. Graphical representation of the fitted curves overlaid on the trajectories of the observed basal area over time was essential in the final decision making about the dynamic model selected.

The equation with two site-specific parameters that was selected allowed simulation of concurrent polymorphism and multiple asymptotes, two desirable characteristics of growth equations. Furthermore, the dummy variables method used for model fitting is a base-age invariant method that accounts for site-specific and global effects, addresses the error structure of the data, and fits the curves to observed individual trends in the data.

The compatibility of the basal area system was ensured in the sense that the projection equation is a different form of the prediction equation, with identical parameters. The unknown site-dependent function X of the projection function was related to a power function of site index. The compatibility ensures that the basal area projected to age t_2 from the basal area at age t_1 with eq. 7 will be the same as the predicted basal area at age t_2 given by eq. 9 for a predetermined site index.

As expected, the different biogeoclimatic conditions in the two maritime pine ecoregions in Galicia lead to different basal area growth patterns, making it necessary to develop an ecoregion-based basal area growth model.

Although it is usually accepted that the relative rate of basal area production for thinned stands is greater than that for unthinned stands (at least for some period of time following thinning) for the data set analyzed, initial basal area and age provided sufficient information about the future trajectory of the basal area of the stand. It was therefore not necessary to consider the thinning effect in the dynamic model for projecting basal area in thinned stands. These results were consistent with those obtained in similar studies for maritime pine in the Mediterranean area.

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