Modeling Bilateral Forest Products Trade

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Abstract
The focus in this chapter is on the development of mathematical programming models used to model bilateral forest products trade. Theoretical outlines are provided of a multi-region, single product trade model and of an integrated, multi-region, multi-product trade model. The objective function and constraints are described mathematically, while the analysis takes into account horizontal and vertical chains and the need to calibrate the model using observed trade flows. Data sources are discussed, and the GAMS code is provided for the uncalibrated and calibrated versions of the model.

The Canada–U.S. softwood lumber dispute is the raison d'être for much applied work in modeling forest products trade, especially on Canada’s side. In this chapter, we examine several spatial price equilibrium (SPE) trade models that are currently used to investigate the implications of trade barriers imposed on Canadian exports of softwood lumber to the United States. The reason we consider bilateral trade is so that we can determine the impacts of trade restrictions on various regions in North America. We begin in the next section by specifying a general but vertically integrated SPE trade model.

4.1 Integrated Vertical and Horizontal Chain Model Specification

In this section, we develop a general spatial price equilibrium trade model that assumes forestry policy aims to prevent exports of logs from one or more jurisdictions. The SPE model tracks the use of logs in the production of lumber and other wood products (see Fig. 3.6 in Chapter 3, this volume), and aims to determine the price, quantity, and welfare policy implications across regions and products. The policy instrument is assumed to be a simple per unit tax on exports.
4.1.1 Objective function

Consider first the wood processing sector. Each region is assumed to have a set of linear (inverse) demand and supply curves for each downstream product $k$:

$$
P_d^k = \alpha_d^k - \beta_d^k q_d^k, \quad \alpha_d^k, \beta_d^k \geq 0, \quad \forall d = 1, \ldots, M, \forall k, \quad (4.1)$$

and

$$
P_s^k = a_s^k + b_s^k q_s^k, \quad a_s^k, b_s^k \geq 0, \quad \forall s = 1, \ldots, N, \forall k, \quad (4.2)$$

where $k \in \{\text{lumber, plywood, particleboard, fiberboard, pulp, wood pellets}\}$, $q_d^k$ refers to the quantity of commodity $k$ consumed in demand region $d$, and $q_s^k$ refers to the quantity of wood product $k$ produced by supply region $s$. For convenience, we use $d$ to denote a net demand region and $s$ a net supply region, although a region may be simultaneously a supplier and demander of the commodity in question. There are $M$ demand (import) regions and $N$ supply (export) regions.

One objective of the forest trade model is to maximize the sum of the consumer and producer surpluses across all relevant wood-processing sectors. The consumer and producer surpluses are found by maximizing the sum of the areas under the $M$ demand schedules (4.1) and subtracting the sum of the areas under the $N$ supply schedules (4.2). These respective areas are given by:

$$
B_d^k = \int_0^\infty (\alpha_d^k - \beta_d^k x) dx = \alpha_d^k q_d^k - \frac{1}{2} \beta_d^k q_d^k^2, \quad \text{and} \quad (4.3)
$$

$$
C_s^k = \int_0^\infty (a_s^k + b_s^k x) dx = a_s^k q_s^k + \frac{1}{2} b_s^k q_s^k^2, \quad (4.4)
$$

where $x$ is an integration variable, $B_d^k$ is the total benefit (area under the demand function) in demand region $d$ for product $k$, and $C_s^k$ is the total cost (area under the marginal cost/supply function) in supply region $s$ for product $k$ (see Vercammen, 2011, p. 22). Given lack of good data, a supply elasticity of one is often assumed, which implies that the supply schedules pass through the origin. Vercammen also provides the welfare equation if supply schedules intersect the horizontal axis so that they have negative intercepts.

Now consider the markets for industrial roundwood (pulp logs and coniferous logs). As noted earlier, the demand for logs is a derived demand that depends on the production of downstream lumber, plywood, pellets, pulp, etc. For each wood product $k$, its derived demand is given by its output price multiplied by the marginal physical product of the input (logs) in the production of the $k^{th}$ commodity: $P_k \times MP_{\text{logs}} \times k$. The total derived demand for logs is given by the horizontal sum of the individual $k$ derived demands for logs. However, as shown in Chapter 3, the change in consumer surplus in the log market caused by a policy shock affecting logs can be evaluated in the downstream markets as the sum of the changes in the producer surpluses (quasi-rents) in the downstream wood processing markets – changes in the consumer surplus in the log market are measured by the changes in producer surpluses in the downstream markets. Thus, it is necessary to include in the objective function only the producer
surplus in the log market. Assume that the supply (marginal cost) of logs in region $u$ is linear: $r_u = m_u + n_u Q_u$, $m_u, n_u \geq 0$, where $Q_u$ is the quantity of logs in country $u$. The producer surplus from supplying logs from any region $u$ is:

$$QR_u = r_u Q_u - \int_0^{Q_u} (m_u + n_u x) dx = \frac{1}{2} n_u Q_u^2$$

(4.5)

where $U$ regions supply logs.

The overall objective in the forest trade model is to maximize the sum of the necessary producer and consumer surpluses provided above, while subtracting the shipping and handling costs and associated taxes. The objective function to be maximized can thus be written as:

$$W = \sum_{k=1}^{K} \left[ \sum_{d=1}^{D} B_d^k - \sum_{i=1}^{N} C_i^k - \sum_{d=1}^{D} \sum_{i=1}^{N} (t_{s,d}^k + \tau_{s,d}^k) q_{s,d}^k \right] + \sum_{u=1}^{U} \left[ QR_u - \sum_{i=1}^{N} (\delta u,s + \tau u,s) Q_{u,s} \right]$$

(4.6)

where $W$ refers to the overall global wellbeing from trade in forest products, $\tau_{s,d}^k$ is the cost ($/m^3$) of transporting processed forest product $k$ from supply region $s$ to demand region $d$, and $\delta u,s$ is the cost of transporting industrial roundwood (logs) from region $u$ to region $s$, where $\delta$ is a parameter that takes into account the extra cost of transporting logs because they occupy more space per cubic meter than lumber (whose cost of transport from region $u$ to region $s$ is given by $t_{u,s}$). Finally, $\tau u,s$ is the tax on logs ($/m^3$) originating in log supply region $u$ and sold to wood product producing region $s$, while $\tau_{s,d}^k$ is the tax on wood product $k$ originating in supply region $s$ and exported to region $d$.

### 4.1.2 Constraints

Objective (4.6) is maximized subject to a series of biophysical and economic constraints relating to the availability of timber harvests, log supply and wood product manufacturing limits. The essential constraints are material flows and productivity constraints that ensure total supply equals total demand for each region/country and each product. The model constraints are summarized as follows. First, the quantity of industrial roundwood of each type $L \in \{ \text{saw logs, veneer logs, pulpwood logs} \} = \{ SL, VL, PL \}$ produced by any log producing region $u$ must be no greater than its harvest of logs ($h_u$), and the region’s ability to convert harvested timber into various industrial roundwood components:

$$Q_u = \sum_{L \in \{SL,VL,PL\}} Q_u^L \leq \phi_u^L \times h_u, \forall u.$$  

(4.7)

The parameter $\phi_u^L$ indicates how much coniferous industrial roundwood of each type is recovered from the timber harvest in region $u$, which depends on size and species of trees, as well as the region's technical skills, capital, and other factors. The aggregate of the various log types in region $u$ is denoted $Q_u$. 
The sale of logs by region $u$ to log consuming regions $s$, including domestic sales, must not exceed the total supply of logs in region $u$:

$$\sum_{s=1}^{N} Q_{u,s}^{i} \leq Q_{u}^{i}, \forall \; u, \; L. \tag{4.8}$$

The quantity of logs supplied to region $s$ must be greater than or equal to the amount required for the production of downstream wood products:

$$\sum_{u=1}^{U} Q_{u,s}^{i} \geq Q_{s}^{i}, \forall \; s, \; L. \tag{4.9}$$

Logs are used as inputs into the production of the $K$ downstream wood products. It follows that the sale of downstream wood products from supplying region $s$ to all consuming regions must be no larger than what is produced in region $s$:

$$\sum_{d=1}^{M} q_{s,d}^{k} \leq q_{s}^{k}, \forall \; s, \; k. \tag{4.10}$$

Similarly, the supply of downstream products from all supply regions to region $d$, and including domestic supply, must be greater than or equal to the demand of region $d$:

$$\sum_{s=1}^{N} q_{s,d}^{k} \geq q_{d}^{k}, \forall \; d, \; k. \tag{4.11}$$

We do distinguish between primary and secondary products processed from logs in our model - our defined primary products are generally more valuable than our secondary products (but not always). Secondary product may be more valuable depending on the relative quality of logs and location of plant facilities. Consider, as an example, fast-growing pine plantations located next to a power plant; the pine is grown primarily to be used as a biomass fuel. Alternatively, such trees might be best used to produce pulp if no sawmill is in the vicinity.

Lumber and plywood must necessarily be considered primary products for sawlogs ($SL$) and veneer logs ($VL$), while wood pulp is the primary product from pulp logs ($PL$). In addition, secondary products (particleboard, fiberboard, and wood pellets) can employ wood fiber from logs in direct competition with the primary products, with wood pulp also considered a secondary product when it comes to non-pulp logs. Therefore, we denote $f \in \{\text{particleboard, fiberboard, pulp, pellets}\} \subset K$ and $nf \in \{\text{lumber, plywood}\} \subset K \supset f \cup nf = K$.

Secondary products rely on chips and residuals from sawmilling and plywood manufacture for the most part. For simplicity, however, we assume that industrial roundwood gets allocated to each of our six (primary plus secondary) products so that all of the roundwood is utilized. This can be described using the following relation:

$$q_{s}^{k,l} \leq \rho_{s}^{k,l} \times \eta_{s}^{k,l} \times Q_{s}^{i}, \forall \; k, s, L. \tag{4.12}$$

In equation (4.12), the total available output in processing region $s$ of wood product $k$ from logs of type $L$, $q_{s}^{k,l}$, is determined by the proportion of the logs of
type $L$ used to produce $k$, denoted $\rho_{s,k,L}$, multiplied by the recovery factor, $\eta_{s,k,L}$, that converts logs into product and the amount of logs of type $L$ available in region $s$. To ensure that all of the wood fiber is fully used we require

$$\sum_{k=1}^{K} \rho_{s,k,L} = 1. \quad (4.13)$$

The manufacture of lumber and plywood results in chips and other residuals (sawdust, planer shavings, residues) that are joint products that can be used to produce particleboard, fiberboard, wood pulp, and wood pellets. The total amount of wood chips and residuals produced in region $s$ depends on the production of lumber and plywood, and can be determined from the following relation:

$$R^z_s = \sum_{nf} \left( q_{s,nf}^f \times V_{s,nf}^{nf,z} \right), \forall s, z \in \{\text{wood chips, other residuals}\}, \quad (4.14)$$

where $R^z_s$ is the amount of $z$ (wood chips, sawdust, planer shavings or other residuals) produced in region $s$ and $V_{s,nf}^{nf,z}$ is the region’s ability to recover residual $z$ from each of the sawmilling and plywood manufacturing sectors.

The production of products $f$ directly from logs in region $s$ is denoted $q_{s,k,L}^f$ and is determined from equation (4.12). In addition, we find the amount of product $f$ produced from chips and residual fiber using the following relationship:

$$q_{s,rf}^f = \sum_z (w_{s,j}^z \times \theta_{s,f}^z \times R^z_s), \forall s, \quad (4.15)$$

where $q_{s,rf}^f$ denotes the quantity of wood product $f$ produced from residual fiber (and not directly from logs). In addition, $w_{s,j}^z$ refers to the proportion of residual fiber of type $z$ in region $s$ that is used to produce product $f$, while $\theta_{s,f}^z$ is a parameter that converts residual fiber of type $z$ into product $f$. The condition requiring that all residual fiber is exhausted is given by:

$$\sum_z w_{s,j}^z = 1, \forall s, z. \quad (4.16)$$

Pulp mills use chips from sawmilling and manufacture of plywood to the extent that such chips are not used for other products. OSB, particleboard, fiberboard and pellets can employ wood chips and other residuals from sawmilling and plywood production.

Finally, the total amount of product $k$ produced by region $s$ can now be determined as:

$$q_{s,k}^k = q_{s,rf}^k + \sum_{f \in \{S, VL, PL\}} q_{s,rf}^k, \forall s, k, f \in k. \quad (4.17)$$

The constrained optimization program maximizes objective (4.6) subject to constraints (4.7) through (4.17), plus non-negativity conditions on the decision variables. For each of the relevant regions, the decision variables are the supply of industrial roundwood (sawlogs and veneer logs) and pulpwood ($Q_{L,u}$); bilateral flows of logs from supplying to wood processing regions ($Q_{L,u,s}$); production and consumption of product $k$ in each region ($q_{s,k}^k$ and $q_{d,s}^k$, respectively); and
the bilateral trade flows of product $k$ ($q_{k,s,d}^*$). The proportions of the logs of type $L$ used to produce $k$ ($\rho_{s,k,L}$), and the proportions of residual fiber of type $z$ used in $f$, namely $w_{s,f}^z$, can also be determined endogenously in the model, although we have tended to let these be exogenously provided.

4.2 Calibrating the Integrated Vertical and Horizontal Chain Model

It is important that the forest trade model is calibrated so that the user can be confident that model projections are realistic. The calibration must be based on observed values and must be rooted in economic theory. Although trade models rely on observed data, it is often the case that computational deficiencies require an aggregation of firm and market characteristics. As a result, mathematical programming models of trade often experience extreme specialization in supply responses. As well, discrepancies between modeled and observed optimal values may arise due to mis-specified parameters, often originating from transaction costs per unit of product traded between two countries (e.g. non-tariff trade barriers). To deal with such problems, several calibration techniques have evolved.

One method is referred to as the historical mixes approach, which attempts to address the problem of extreme solutions (McCarl, 1982; Önal and McCarl, 1991). This approach is based on the fact that optimal solutions are often found at corners (extreme points), particularly when working with aggregate representative producers. (The simplex method that is used in solving linear and quadratic programming problems finds only corner solutions (Paris, 1991)). Since aggregation may bias the data, and hence the original problem, a region may be assigned a subset of possible production ‘mixes’ based on observed levels. This last point is justified as observed mixes must be optimal, or why would they have occurred in the first place? This calibration method takes historical choices (mixes) into account by constraining the current optimal values to be a weighted average of those observed choices. The weights may be determined endogenously within the mathematical programming framework, with the sum of the weights equaling 1. Chen and Önal (2012) extended this method by including decisions that are not historically observable. Simulated mixes of the ‘new’ decision variables are added to the historical mixes, allowing the optimization procedure to choose the weights, and again constraining the sum of the historical and synthetic weights to equal 1.

A second calibration method based on an approach originally proposed by Howitt (1995), referred to as positive mathematical programming (PMP), is increasingly applied to problems in agriculture and forestry (see de Frahan et al., 2007; Paris, 2011, pp. 340–411; Heckelei et al., 2012). Positive mathematical programming uses the notion that any calibration constraint can be represented in the objective function (e.g. a linear calibration constraint might be represented as a nonlinear cost function in the objective). Rather than adding arbitrary calibration constraints to ensure that the optimal solution to a
mathematical program replicates what is observed (as in the historical mixes approach), the PMP method uses the shadow prices associated with such constraints to re-specify the objective function. The calibrated model is then solved to replicate the observed values exactly.

In trade models, elasticities of supply and demand along with observed production and consumption levels enable the construction of linear supply and demand functions. The greatest uncertainty relates to the transaction (including S&H) costs of moving goods from one region to another. This does not include the export taxes or import tariffs as these are usually known, considered separate from the transaction costs, and, in any event, constitute policy variables. Rather, it is the actual or realizable transaction costs that are unknown, but which can be determined using PMP as a calibration method. Thus, the PMP-calibrated transaction costs represent the ‘effective’ transaction costs between export and import regions. They are derived from the shadow prices on the calibration constraints relating to the observed bilateral flows of forest products (logs, lumber, wood pellets, etc.). Again the calibration is motivated by the fact that there is a discrepancy between the true transaction costs and the observed transaction costs, as determined by shipping, loading and unloading, insurance, and administrative costs, plus unaccounted-for tariffs and non-tariff barriers, such as phytosanitary standards. The main reason for this discrepancy occurs because (observed) transaction costs are measured with a significant degree of uncertainty (Paris et al., 2011). To deal with and measure the hidden or unknown transaction costs (bribes, non-tariff barriers, etc.), one can utilize a two-stage positive mathematical programming model (Paris et al., 2011).

The PMP specification in stage I maximizes objective (4.6) subject to constraints (4.7) through (4.17), with the addition of the following constraints:

**Dual Variable**

\[
Q_{u,s}^i = \overline{Q}_{u,s}^i \\
q_{s,d}^k = \overline{q}_{s,d}^k
\]

In this specification, it is assumed that we observe trade flows for industrial roundwood and \( k \) downstream wood products, \( \overline{Q}_{u,s}^i \) and \( \overline{q}_{s,d}^k \), respectively, along with their respective transaction costs \( \delta t_{u,s}^i \) and taxes or tariffs \( \tau_{s,d}^k \).

Upon obtaining the shadow prices \( \lambda_{u,s}^i \) and \( \lambda_{s,d}^k \) associated with the primal model, the objective function for stage II can be re-specified as follows:

Maximize \( W = \sum_{k=1}^{K} \left[ \sum_{d=1}^{D} B_g^k - \sum_{s=1}^{S} C_s^k - \sum_{d=1}^{D} \sum_{s=1}^{S} \delta_{s,d}^k q_{s,d}^k \right] \)

\[
+ \sum_{L \in \{U, V, P\}} \sum_{u=1}^{U} \left[ Q R_u - \sum_{s=1}^{S} \lambda_{u,s}^i Q_{u,s}^i \right]
\]

where \( T_{s,d}^k \) now equals \( t_{s,d}^k + \tau_{s,d}^k + \lambda_{s,d}^k \), and \( T_{u,s} \) equals \( t_{u,s} + \tau_{u,s} + \lambda_{u,s}^i \). In the second stage, the modified objective function (4.20) is maximized subject to the original constraints (4.7) through (4.17). With this modification, the model precisely duplicates the inter-regional fiber trade flows.
The fact that the shadow prices $\lambda_{w,s}^l$ and $\lambda_{x,d}^k$ can be negative indicates that the original transaction cost data fail to include missing information, perhaps even policy instruments such as export subsidies. Indeed, Paris et al. (2011) indicated that, in some instances, the overall effective transaction costs between two countries might even be negative, as when export subsidies are larger than the sum of other transaction costs. In some circumstances, this may provide additional insight into the potential restrictiveness of trade measures that are otherwise difficult to quantify, such as non-tariff trade barriers.

4.3 Economic Surplus and Income Redistribution

In Chapter 3, we identified the appropriate welfare measures used in trade models, where trade models consist of various products that are vertical and horizontal to the market impacted by a forestry policy. A forest policy might be an export tax on logs, a quota on lumber imports, or some other policy scenario. In addition to consumer surplus and quasi-rent (producer surplus), one must obtain estimates of resource rents, policy-induced quota rents and/or tax/tariff revenues. (In practice, some part of any quota rent might simply be squandered away through rent-seeking activities or simply wasted due to other inefficiencies, although we make no attempt to measure such waste.) In bilateral SPE trade models, the welfare measures and income transfers are usually calculated after the model has been solved for the optimal bilateral trade flows. The following equations provide the mathematical derivation of these welfare measures.

4.3.1 Wood processing sector

Consider first the $k$ downstream wood processing markets in the vertical supply chain. The consumer surpluses in each of these markets and each commodity are given by:

$$CS_d^k = \int_0^{q_d^k} \left( \alpha_d^k \beta_d^k x - P_d^k q_d^k \right) dx = \left( \alpha_d^k q_d^k - \frac{1}{2} \beta_d^k q_d^k \right) - \left( \alpha_d^k - \beta_d^k q_d^k \right) q_d^k$$  

$$= \frac{1}{2} \beta_d^k \left( q_d^k \right)^2, \forall s, k,$$

where $P_d^k$ is the demand price for product $k$ in the domestic market, and $q_d^k$ is the quantity of product $k$ consumed. Likewise, the producer surpluses or quasi-rents in these $k$ downstream markets are given by:

$$QR_s^k = P_s^k q_s^k - \int_0^{q_s^k} \left( a_s^k + b_s^k x \right) dx = \left( a_s^k + b_s^k q_s^k \right) q_s^k - \left( a_s^k q_s^k + \frac{1}{2} b_s^k (q_s^k)^2 \right)$$  

$$= \frac{1}{2} b_s^k \left( q_s^k \right)^2, \forall s, k,$$  

(4.22)
where \( P^k \) is the supply price for product \( k \) in the domestic market, and \( q^k_s \) is the quantity of product \( k \) that is produced.

It might be worth recalling that, in principle, a region might only be a supply or demand region, but in practice regions are usually both suppliers and demanders of each of the \( k \) products. Further, given that the regions in the model are quite large, each supplies some amount of harvested timber to its domestic market for processing.

In each of the downstream markets, a variety of distortions might exist. These consist of tariff and non-tariff trade barriers, export taxes, illegal fees, quotas, and so on. These distortions to trade are captured in the PMP calibration process so that they are included in the revised S&H and other transaction costs. However, when we examine the impact of various policies (e.g., tariff or export tax, quota), income transfers will occur and these can be measured in two ways. First, with tariffs or taxes, the income accrues to government and is calculated simply as the quantity affected (traded or sold) multiplied by the tariff/tax rate. Second, some policies create distortions that result in a wedge between the demand price and the marginal cost (supply price). This leads to a policy-induced scarcity rent that is calculated as follows:

\[
SR^k_y = \left( P^k_{y, Demand} - P^k_{y, Supply} \right) \bar{q}^k_y \\
= \left( (\alpha^k_y - \beta^k_y \bar{q}^k_y) - (a^k_y + b^k_y \bar{q}^k_y) \right) \bar{q}^k_y, \forall k, y \in \{s, d\}, s \neq d, \tag{4.23}
\]

where \( \bar{q}^k_y \) refers to the quantity of \( k \) consumed in market \( y \). In essence, since the producer surplus calculated in equation (4.22) does not include the policy-induced scarcity rent, it is necessary to include this scarcity rent as a transfer, although it is not clear who captures it; it is simply determined by the size of the wedge between the demand price and the marginal cost (supply price).

### 4.3.2 Upstream Log Markets

Now turn to the market for logs. As noted earlier, because the demand for logs is a derived demand, the consumer surpluses in log markets are measured as quasi-rents in the \( K \) downstream markets (lumber, plywood, OSB, wood pellets, etc.). It is necessary, therefore, only to measure the quasi-rent in the log markets. The best measure of the quasi-rent in any log market is given by equation (4.5) and is similar to equation (4.22); it is given by:

\[
QR^L_u = \frac{1}{2} n^L_u \left( Q^L_u \right)^2, \forall u, L, \tag{4.24}
\]

where \( n^L_u \) is the slope of the type-\( L \) log supply curve in region \( u \).

To this must be added any scarcity rent associated with resource scarcity or some policy that creates a wedge between the demand and supply price of logs, thereby creating a policy-induced rent. Because we do not explicitly include demand functions for logs in the trade model, we rely on the shadow
price of logs. The shadow price of logs gives the addition to global wellbeing, as defined in objective function (4.6), if an additional log were available. Therefore, policy-induced rent plus the rent from resource scarcity in the log market can be calculated by the shadow price of logs times the volume produced:

\[ SR_u^L = \lambda_u^L \tilde{Q}_u^L, \forall u, L, \]  

(4.25)

where \( \lambda_u^L \) is the shadow price and \( \tilde{Q}_u^L \) is the equilibrium production of logs of type \( L \) in region \( u \).

The surpluses in equations (4.21) through (4.25) are summed to obtain the total surplus from trade.

4.4 Forest Trade Model Specifications

In the previous sections, we described a general SPE trade model that takes into account vertical and horizontal chains. Here we discuss several models that have been developed to examine various aspects of the softwood lumber trade dispute between Canada and the U.S., as well as policies implemented outside North America, specifically a European policy to increase generation of electricity from wood biomass. However, we also consider models that are more encompassing in terms of the regions explicitly modeled and the number of commercial forest products. Of course, the simplest model would focus only on softwood lumber trade between Canada and the United States, with a single ‘rest of the world’ region included to make the model tractable. The reason for this narrow focus is twofold. First, the main reason for harvesting timber is to produce lumber, with processing of other wood products dependent on activities related to sawmilling. Harvest levels would be much reduced if lumber was not the main objective; it is too costly to harvest logs only to produce wood products other than lumber. Second, the vast majority of Canada’s softwood lumber exports are to the U.S. (as indicated in Chapter 2), although other countries may be modeled so as to evaluate the potential of markets outside the U.S. (e.g. China, Japan). Depending on its purpose, the rest of the world might be aggregated into a single region, or might be considered with greater disaggregated detail.

In Appendix 4.A, we provide the GAMS code for a straightforward model of softwood lumber trade that disaggregates Canada into five regions and the U.S. into three regions, and aggregates countries outside North America into 13 regions rather than one, thereby resulting in a 20-region bilateral SPE trade model. The data required to run the model and model outputs are also provided in the appendix. Then, in Appendix 4.B, we provide the GAMS code required to calibrate the basic model, but, unlike in Appendix 4.A, no model outputs are provided. Finally, a trade model that has the same 20 regions but now with eight forest products (see Fig. 3.6, this volume) is provided in
Appendix 4.C. The actual GAMS code for these three models as well as some additional GAMS code for other forest trade models, along with the data needed to run them, are provided under the ‘Trade Models’ tab at https://www.vkooten.net/ (accessed September 4, 2020).

The general mathematical programming formulation for these models is given by:

Maximize

\[ W = \sum_{d=1}^{D} (\alpha_d - 0.5\beta_d q_d) q_d - \sum_{s=1}^{S} (a_s + 0.5b_s q_s) q_s - \sum_{s=1}^{S} \sum_{d=1}^{D} (t_{s,d} - \tau_{s,d}) x_{s,d} \]

Subject to:

\[ \sum_{d=1}^{D} x_{s,d} \leq q_s \text{ (sales cannot exceed production)} \]

\[ \sum_{s=1}^{S} x_{s,d} \geq q_d \text{ (production must exceed demand)} \]

\[ x_{s,d} = x^0_{s,d} \text{ (calibration constraint) } [\lambda_{s,d}] \]

\[ x_{s,d}, q_s, q_d \geq 0 \text{ (non-negativity)} \]

In the above specification, \( x_{s,d} \) refers to the trade between supply region \( s \) (of which there are \( S \)) and demand region \( d \) (of which there are \( D \)); \( q_s \) is the total production of region \( s \); and \( q_d \) is the total demand in region \( d \). As before, \( \alpha_d \) and \( \beta_d \) are the parameters of the demand function in region \( d \), while \( a_s \) and \( b_s \) are the marginal cost parameters for supply region \( s \). Finally, \( t_{s,d} \) and \( \tau_{s,d} \) refer to the S&H and other transaction costs and taxes, respectively, that exist in softwood lumber trade between regions \( s \) and \( d \).

The shadow prices associated with the calibration constraints, \( \lambda_{s,d} \), can be positive or negative, as the constraint is an equality rather than an inequality. The shadow prices are then used to modify the transaction costs as follows:

\[ t'_{s,d} = t_{s,d} + \lambda_{s,d} \]

where \( t'_{s,d} \) are the calibrated transaction costs to use in the second stage of the program (see Appendices 4.B and 4.C).

### 4.5 Model Data

Our discussion of model data relies on and updates Johnston and van Kooten (2016), who employed a multiple-product, 20-region model to examine an increase in European demand for wood pellets to produce electricity. Data availability is the greatest challenge that the modeler faces, with the degree of difficulty enhanced where country-level data need to be disaggregated into regions. Disaggregating national consumption data can be done using proportions of regional population, incomes or housing starts, while information on regional shares of national softwood lumber production is usually available.
from domestic statistical agencies. Aggregating national consumption and production data from the Food and Agriculture Organization (FAO) of the United Nations (UN) into regions is somewhat easier. (We rely on UN data from FAO and the UN’s ComTrade Database, available at https://comtrade.un.org/, accessed September 4, 2020, aggregating country-level and bilateral trade data into pre-specified regions using the ‘tidyverse’ package in R.)

The greatest challenges are finding appropriate prices and elasticities of supply and demand, particularly at the sub-national (regional) level. This information is needed along with production and domestic consumption data to construct linear demand and supply functions. Data on bilateral trade flows are also readily available. However, information on S&H costs between regions is difficult to find. This remains a problem despite our ability to adjust S&H costs in the second stage of a numerical SPE trade model using the shadow prices on the calibration constraints in the solution to the first stage problem to adjust the S&H data, as indicated in section 4.4.

The underlying data that we use to construct forest trade models come from a variety of sources, with data from FAO (2020) constituting the primary source for forestry statistics. Supplementary data are available from the Government of Canada (2019), BC Government (2019), Random Lengths (various years), the Global Forest Products Model (GFPM) at the University of Wisconsin (available at https://buongiorno.russell.wisc.edu/gfpm/, accessed September 4, 2020), the U.S. Department of Agriculture’s Forest Service (USDA Forest Service, 2019), the United Nations Economic Commission for Europe (UNECE, 2019), and van Kooten and Johnston (2014). (Data were previously available from the University of Washington’s Center for International Trade in Forest Products (CINTRAFOR). For information on the CINTRAFOR model, see Perez-Garcia, 1993). Where FAO data are either unavailable, or observations are missing, supplementary data are used.

The FAO provides annual production and trade data for a number of forest products dating back to 1961. The data are collected through annual questionnaires conducted by the FAO Forestry Department in partnership with the International Tropical Timber Organization, the Statistical Office of the European Communities (Eurostat), and the UNECE. In cases where countries fail to provide information through the questionnaire, the FAO estimates production and trade of wood products through trade journals, statistical yearbooks and other sources. Where data are unavailable, the FAO repeats historical information from the previous years. Although in some instances the quality of the FAO data may be less than desired, they are nonetheless consistently available at a country level, and provide information on the destinations of various forest product exports and the origins of imports. Information on country-to-country trade flows is critical for implementing the positive mathematical programming calibration method. Since Canada and the U.S. are broken down into seven and five sub-regions, respectively, the FAO data need to be adjusted using local information (see also Chapter 5, this volume). Further, information from Canada and the U.S. is then used to reconcile missing observations in the FAO dataset.
The data analysis begins with the collection and calculation of each region’s technical ability to produce logs and wood products. First, a region’s ability to produce logs is a function of the annual allowable cut (AAC), which is the amount of wood permitted to be sustainably harvested, and a region’s ability to convert coniferous logs into industrial roundwood. Data on AAC are available from FAO (2020), the USDA Forest Service (2019), also Howard (2001), Oswalt and Smith (2014), and the Canadian Forest Service’s National Forestry Database (Government of Canada, 2019). Factors converting harvested coniferous timber into industrial roundwood can be determined by taking ratios of each region’s production of roundwood to harvests. Industrial roundwood is assumed to be broken down into two sub-categories: (i) sawlogs and veneer logs; and (ii) round and split pulpwood. For both categories, the FAO provides regional production and trade flows. However, in the model in Appendix 4.C, we are not concerned to replicate pulpwood trade as there is simply too little trade of pulpwood.

The ability to recover coniferous wood products (lumber, plywood, particleboard, fiberboard, pulp and wood pellets) from their respective log inputs can be calculated as the ratio of production to inputs. The FAO differentiates coniferous from non-coniferous lumber, allowing for a simple calculation of regional coniferous lumber recovery factors. This is not the case for other wood products, however. First, plywood and veneer sheets are reported as an aggregate of coniferous and non-coniferous fiber by the FAO. Thus, to estimate regional coniferous plywood and veneer sheet production, the reported aggregate data would need to be adjusted by taking the proportion of coniferous sawlogs and veneer logs consumed in a region and multiplying by total regional production of plywood and veneer sheet. A similar adjustment would be applied to particleboard. Fiberboard and pulp are also reported as an aggregate of softwood and hardwood by the FAO. Since these products use fiber primarily from pulpwood, they can be adjusted using the reported proportion of regional coniferous pulpwood consumption multiplied by the total aggregated production of the respective product. Wood pellet data can be collected irrespective of whether pellets are produced using coniferous or non-coniferous fiber. The FAO does not currently report wood pellet statistics directly; thus, we rely on other sources (e.g. Lamers et al., 2012; Government of Canada, 2019; EuroStat, 2019) and adjust regional production based on the proportion of coniferous industrial roundwood consumption by each region. Finally, a region’s ability to recover chips and residuals from sawmilling is determined from various sources such as the BC Government (2019), UNECE (2019), and Government of Canada (2019).

Regional consumption of logs and wood products is based on apparent consumption (production plus imports minus exports) since the FAO only reports production and trade. For Canada and the United States, regional consumption of logs can be determined from production, while regional exports of logs can be allocated on the basis of various statistical sources (e.g. BC Government, 2019) and trade publications. Reports from Random Lengths (various years) are employed. Regional production of lumber needs to be
determined first from regional production of coniferous roundwood. The forestry statistics for doing so can be found in Government of Canada (2019) and BC Government (2019) for Canada, and USDA Forest Service (2019). Canadian and U.S. population data, which are used to disaggregate Canada and the U.S. into regions, are available from Statistics Canada and the U.S. Census Bureau, while world population data can be obtained from the FAO (2020).

Regional wood product consumption in Canada and the U.S., on the other hand, can be determined by allocating total consumption across regions by their proportion of population. The same can be done for regional imports – national imports allocated across regions according to population. Exports from any Canadian or U.S. region to any other country/region in the model must be determined by allocating national exports to those countries/regions by regional production, but then making adjustments based on other sources of information.

It is important to note that, in many circumstances, bilateral trade flows of wood products are reported by the FAO as an aggregate of coniferous and non-coniferous products. Thus, the trade matrices for aggregated products need to be adjusted in a similar fashion to production. Specifically, exports from a given country are adjusted based on the proportion of coniferous inputs used in the respective region.

Data on prices can be found in the various issues of Random Lengths and the timber database of the UNECE (2019). The price elasticities of demand for wood products are difficult to find but can be calculated from information provided in various sources (Niquidet and Tang, 2013; Oswalt and Smith, 2014; van Kooten and Johnston, 2014; BC Government, 2019; FAO, 2020). Various elasticity estimates are also provided in the data files discussed in the Appendices to this chapter. In many cases where data are missing, supply elasticities are set to 1.0, because then the slopes of the products in question are simply given by the ratio of the base production (manufacturing) costs divided by output, while the supply function begins at the origin (intercepts equal zero). Manufacturing costs can be found in UNECE (2019) (also see van Kooten and Johnston, 2014).

The shadow prices associated with the calibration constraints in the first phase of the PMP procedure are then used to adjust the observed shipping and handling and other transaction costs; associated shadow prices may be either positive (e.g. an unobserved non-tariff cost) or negative (perhaps a subsidy not accounted for). For log shipments, the transportation costs are multiplied by 1.27 to account for the extra volume required to transport logs compared with lumber.

### 4.6 Grouping Regions

The task of creating bilateral trade flows among regions is exacerbated by the need to identify the nations that are to be grouped into regions. Once countries are grouped into regions, bilateral trade flows between regions (inter-regional)
must be identified, while trade flows within regions (intra-regional) must be excluded. Intra-regional trade is treated the same as movement of commodities within the same jurisdiction. Sources of trade data and the allocation of information into regions are discussed in this section.

As noted in the previous section, much of the data employed in SPE forest-product trade models comes from the FAO (2020). While the FAO provides information on wood products trade, the United Nations Comtrade Database (https://comtrade.un.org/data/, accessed September 4, 2020), a pseudonym for the UN’s International Trade Statistics Database, provides data on bilateral trade. Comtrade delivers data on exports of numerous commodities from one country to each of its trading partners, and, at the same time, imports by any one country from each of its trading partners. In some cases, country A may report a level of exports to country B that differs from the imports that country B reports as coming from country A. As a result, the analyst needs to reconcile these sometimes disparate data; reconciliation often requires the use of data from other sources such as the FAO, U.S. Department of Agriculture, WTO (2019), or an industry source. Despite this problem, the FAO and Comtrade data are likely the best and most consistent data that one can find.

A first step in collecting the data is to determine the harmonized system (HS) code for the product(s) of interest. For example, the international 2-digit HS codes for ‘wood & wood products’ range from HS44 (‘wood and articles of wood’) to HS49 (‘printed books, newspapers, picture …’) as found at https://www.foreign-trade.com/reference/hscode.htm (accessed September 4, 2020). The 4-digit HS code for sawnwood is 4407, while the 6-digit HS codes for sawnwood are 440711 (pine), 440712 (spruce and fir), and 440719 (coniferous sawnwood other than spruce-pine-fir (SPF)). The analyst might want to consider all coniferous (softwood) sawnwood. In that case, it is necessary to go to the Comtrade data and sum these three categories for each country’s exports and imports. Once that is accomplished, the countries can be grouped into larger regions – this is necessary in the REPA trade model and, in some cases, in the Global Forest Products Model (see Chapter 6).

In Appendix 4.D, an R program is provided that enables one to group country-level bilateral trade data for any given commodity into region-level bilateral trade data for that commodity. Also provided in the Appendix is an example of grouping countries into regions for trade in sawnwood – the 4-digit commodity classification HS4407.

**APPENDIX 4.A: GAMS Code for Base Model**

The base model and the data required to run it are provided in the ‘Base Lumber Trade Model’ folder under the ‘Trade Models’ tab at the website https://www. vkooten.net/ (accessed September 4, 2020) Other models discussed in this book are also provided at that site.
GAMS Code

* Spatial equilibrium lumber trade model

$Oneolcom
$eolcom #

SETS s supply regions /aus, bcc, bci, al, ac, roc, chl, chn, fin, jap, nz, rus, swe, usn, uss, usw, rola, ROE, roa, row /

; alias(s,d); #All regions considered to be producers and consumers

TABLE sup(s,*) supply info
    Supply     sprice    selast
    * (000s m3) ($/000m3) elas of supply
    aus  4602.7  226000  0.5
    bcc  4603.0  204300  0.529
    bci  16528.9 195100  0.529
    al   4608.3  196100  0.415
    ac    3227.6 214700  0.57
    roc  9715.4  225400  0.415
    chl  5862.7  211100  0.5
    chn  25145.7 255000  0.5
    fin   9008  227800  0.5
    jap  15508.9 325000  0.5
    nz   3946.6  212000  0.5
    rus  23467.5 211100  0.5
    swe  16173.2 248700  0.5
    usn  4026.3  227200  0.2
    uss  37391.7 193100  0.937
    usw  16934.3 212500  0.866
    rola  14429.2 205000  1
    ROE  62519.5 275000  1
    roa  15497.9 230000  1
    row  23006.3 208900  1.5

;

TABLE dem(d,*) demand info
    demand   dprice   delast
    * (000s m3) ($/000m3) elas of demand
    aus  5171.1  226000  -0.16
    bcc  1837.4  204300  -0.16
    bci  1923.3  196100  -0.16
    al   1923.3  196100  -0.16
    ac   1198.4  214700  -0.16
    roc  12087.3 225400  -0.16
    chl  3777.7  211100  -0.21
TABLE lumflow(s,d)
$Include lumberflow.gms
;
TABLE SH(s,d)   Shipping & handling costs on lumber exported from s to d
$Include LumSH.gms
;
TABLE taxduty(s,d) Duties on lumber exported from s to d
$Include Duty.gms
;
* ------- Lumber Manufacturing Cost and Lumber Demand Parameters -------
PARAMETERS slope(s) Slope parameter of lumber manufacturing cost
   sint(s)   Intercept parameter of manufacturing cost
dslope(d) Slope of lumber demand function
dint(d)  Intercept of lumber demand function
tax(s,d)  lumber tariffs imposed by demand region
);
* Assume elasticity of supply equals 1.0 so supply of lumber goes thru origin
slope(s) = sup(s,‘sprice’)/sup(s,‘supply’);
sint(s) = 0;
dslope(d) = -1*dem(d,’dprice’)/(dem(d,’delast’)*dem(d,’demand’));
dint(d) = dem(d,’dprice’) + dslope(d)*dem(d,’demand’);
tax(s,d) = dem(d,’dprice’)*taxduty(d,s); #Converts ad valorem tax to per unit

* ------------ CONSTRUCTION OF THE MODEL EQUATIONS ------------
POSITIVE VARIABLES
   qd(d) lumber consumption
   qs(s) lumber supply
   x(s,d) lumber trade from supply to demand regions
;
VARIABLE
   Z    total surplus in phase I problem ;
EQUATIONS
   obj  objective for PMP calibration phase
   ys(s) lumber supply constraint
   yd(d) lumber demand constraint
;
obj..  Z =E= sum(d, (dint(d)-.5*dslope(d)*qd(d))*qd(d)) - sum(s, (sint(s)+.5*slope(s)*qs(s))*qs(s))
       - sum((s,d), SH(s,d)*x(s,d)) # multiply by 1000 to get price
       - sum(s, sum(d, x(s,d)*tax(s,d)))) ;
ys(s)..  sum(d, x(s,d)) =L= qs(s);
yd(d)..  qd(d) - sum(s, x(s,d)) =L= 0;

MODEL lumbertrade /obj, ys, yd/;

OPTION qcp = cplex;
SOLVE lumbertrade using QCP maximizing Z;

* ------------------------------------------------------
* ---------- Parameters to store results ---------------
PARAMETERS lumtax(s,d) lumber taxes
Trade(s,d)  Bilateral lumber trade
Produced(s) Total lumber production by region
Consumed(d) Total lumber consumption by region
TotLumTax(s) Total lumber tax paid by region $ mil
pd(d)       Demand price of lumber $ per m3
ps(s)       Supply price of lumber $ per m3
CS(d)       Consumer surplus ($ mil)
QR(s)       Producer surplus or quasi rent ($ mil)
;
   lumtax(s,d) = dem(d,'dprice'*taxduty(d,s));
   Trade(s,d) = round(x.l(s,d),2);
   Produced(s) = sum(d, x.l(s,d));
   Consumed(d) = sum(s, x.l(s,d));
   pd(d) = yd.m(d); # Also calculate as: pd(d) = dint(d) - dslope(d)*Cons(d);
   ps(s) = ys.m(s); # Also calculate as: ps(s) = sint(s) + slope(s)*Total(s);
   TotLumTax(s) = sum(d, x.l(s,d)*pd(d)*taxduty(s,d))/1000000;
   CS(d) = (.5*(dint(d)-pd(d))*qd.l(d))/1000000; # Consumer surplus
   QR(s) = (.5*ps(s)*qs.l(s))/1000000;                # Quasi rent

OPTION Trade:1:1:1; option pd:2:0:10; option pd:2:0:10;
OPTION TotLumTax:1:0:1; option CS:1:0:10; option QR:1:0:10;

DISPLAY Trade, pd, ps, TotLumTax, CS, QR;
execute_unload "TradeResult.gdx" pd, ps
* The following three lines need to appear as a SINGLE line of code in GMAS
execute 'gdxxrw.exe TradeResult.gdx O=ExportData.gdx SQ=N text="Lumber Demand Prices" rng=Price!a1 par=pd rng=Price!a2 rdim=1 text="Lumber Supply Prices" rng=Price!c1 par=ps rng=Price!c2 rdim=1'

execute_unload "TradeResult.gdx" CS, QR
* The following three lines need to appear as a SINGLE line of code in GMAS
execute 'gdxxrw.exe TradeResult.gdx Output=TradeResult.xlsx SQ=N text="Consumer Surplus ($mil)" rng=Welfare!b1 par=CS rng=Welfare!a2 rdim=1 text="Producer Surplus ($mil)" rng=Welfare!d1 par=QR rng=Welfare!c2 rdim=1'

execute_unload "TradeResult.gdx" Trade
* The following two lines need to appear as a SINGLE line of code in GMAS
execute 'gdxxrw.exe TradeResult.gdx Output=TradeResult.xlsx SQ=N text="Bilateral trade (00s m3)" rng=Tradeflow!a1 par=Trade rng=Tradeflow!a2 rdim=1 cdim=1'

The following files are needed to run the above program:
Solution to the base model

Upon solving the GAMS model, we obtain the results in Tables 4.A.1 and 4.A.2. The first table provides the trade flows among the regions, while the second provides the domestic prices, consumption and production, and welfare measures.
Table 4.A.1. Regional trade flows projected by the base SPE trade model ('000s m³).

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The table shows the projected regional trade flows in '000s m³ for various regions, with values ranging from 14,981 to 56,277.
APPENDIX 4.B: GAMS Code for Calibrated Base Model

The GAMS code found in the folder ‘Extended Lumber Trade Model’ under the ‘Trade Models’ tab at the website https://www.vkooten.net/ (accessed September 4, 2020).

GAMS Code

* Spatial equilibrium lumber trade model
$Oneolcom
$eolcom #

SETS s supply regions /aus, bcc, bci, al, ac, roc, chl, chn, fin, jap, nz, rus, swe, usn, uss, usw, rola, ROE, roa, row /

; alias(s,d); #All regions considered to be producers and consumers

*=== IMPORT DATA FROM SPREADSHEET ===*
* Build gdxxrw instructions file
$onecho > import.txt
par=sup rng=Data!B2:E22 Rdim=1 Cdim=1
par=dem rng=Data!G2:J22 Rdim=1 Cdim=1
par=lumflow rng=Trade!A1:U21 Rdim=1 Cdim=1
par=SH rng=Transport!A1:U21 Rdim=1 Cdim=1
par=taxduty rng=Duty!A1:U21 Rdim=1 Cdim=1
$offecho

* Create and import GDX file with import data
$call gdxxrw.exe InputData.xlsx o=ImportData.gdx @import.txt
$gdxin ImportData.gdx

PARAMETERS
  sup(s,*)     Supply data matrix (see below)
  dem(d,*)     Demand data matrix (see below)
  taxduty(s,d) Matrix of lumber import duties
  SH(s,d)      Shipping and handling costs ($ per m3)
  lumflow(s,d) Lumber trade flow matrix for calibrating model (observed)

$load sup dem lumflow SH taxduty
$gdxin

* ------- Lumber Manufacturing Cost and Lumber Demand Parameters -------
PARAMETERS slope(s)  Slope parameter of lumber manufacturing cost
  sint(s)  Intercept parameter of manufacturing cost
  dslope(d) Slope of lumber demand function
  dint(d)   Intercept of lumber demand function
  tax(s,d)  lumber tariffs imposed by demand region

;  * Assume elasticity of supply equals 1.0 so supply of lumber goes thru origin
  slope(s) = sup(s,’sprice’)/sup(s,’supply’);
  sint(s) = 0;
  dslope(d) = -1*dem(d,’dprice’)/(dem(d,’delast’)*dem(d,’demand’));
  dint(d) = dem(d,’dprice’) + dslope(d)*dem(d,’demand’);
  tax(s,d) = dem(d,’dprice’)*taxduty(d,s); #Converts ad valorem tax to per unit

* ---------------------------------------------------------------------
*   Stage I
* ---------------------------------------------------------------------
POSITIVE VARIABLES
  x(s,d)  lumber trade from supply to demand regions
  qd(d)  quantity demanded in each region
  qs(s)  quantity supplied in each region

;  VARIABLE
    Z     total surplus in phase I problem;
EQUATIONS
  obj1   objective for PMP calibration phase
  obj2   objective for stage II
  ys(s)   lumber supply constraint
  yd(d)   lumber demand constraint
  calibrate(s,d) marginal cost exceeds marginal revenue
;
  obj1..   Z = E= sum(d, (dint(d)-.5*dslope(d)*qd(d))*qd(d))
            - sum(s, (sint(s)+.5*slope(s)*qs(s))*qs(s))
            - sum(s, sum(d, x(s,d)*tax(s,d)))
            - sum(s, sum(d, x(s,d)*tax(s,d))) ;
  ys(s)..   sum(d, x(s,d)) =L= qs(s);
  yd(d)..   qd(d) - sum(s, x(s,d)) =L= 0;
  calibrate(s,d)..   x (s,d) =E= lumflow(s,d);

MODEL tradeCal /obj1, ys, yd, calibrate/;

OPTION QCP = CPLEX; # Other solvers are Minos5 en Conopt
SOLVE tradeCal using QCP maximizing Z;

* ---------- Parameters to store results ---------------
PARAMETERS cal(s,d) Shadow prices of lumber flows from PMP phase
  Trade(s,d)   Bilateral lumber trade
  Produced(s) Total lumber production by region
  Consumed(d) Total lumber consumption by region
  ps(s)   supply price
  pd(d)   demand price
  CS(d)   Consumer surplus ($ mil)
  QR(s)   Producer surplus or quasi rent ($ mil)
  SellerDuty(s) Total lumber duties imposed on sellers ($ mil)
  BuyerDuty(s) Total lumber duties paid by buyers ($ mil)
  lumtax(s,d) Duty paid on lumber shipped from s to d
  differ(s,d) Difference in trade flows calibrated vs observed
;
  cal(s,d) = calibrate.m(s,d);
  Trade(s,d) = round(x.l(s,d));
  pd(d) = yd.m(d); #dint(d) - dslope(d)*qd.l(d);
  ps(s) = ys.m(s); #sint(s) + slope(s)*qs.l(s);
  CS(d) = (.5*(dint(d)-pd(d))*qd.l(d))/1000000; # Consumer surplus
  QR(s) = (.5*ps(s)*qs.l(s))/1000000; # Quasi rent
  Produced(s) = sum(d, x.l(s,d));
  Consumed(d) = sum(s, x.l(s,d));

OPTION Trade:1:1:1; option pd:2:0:10; option pd:2:0:10; option BuyerDuty:1:0:1;
option SellerDuty:1:0:1; option CS:1:0:10; option QR:1:0:10;
DISPLAY Trade, pd, Produced, Consumed, CS, QR;

* -------------------------------------------------------------

* Stage II

* -------------------------------------------------------------

\[
\begin{align*}
\text{obj2..} \quad Z &= E= \sum(d, (dint(d) - 0.5 \times dslope(d) \times qd(d)) \times qd(d)) \\
& - \sum(s, (sint(s) + 0.5 \times slope(s) \times qs(s)) \times qs(s)) \\
& - \sum((s,d), (SH(s,d) + cal(s,d)) \times x(s,d)) \\
& - \sum(s, \sum(d, x(s,d) \times tax(s,d)))
\end{align*}
\]

MODEL tradeFin /obj2, ys, yd/;

SOLVE tradeCal using QCP maximizing Z;

\[
\begin{align*}
\text{Trade}(s,d) &= \text{round}(x.l(s,d)); \\
\text{pd}(d) &= \text{dint}(d) - \text{dslope}(d) \times \text{qd}(d); \\
\text{ps}(s) &= \text{sint}(s) + \text{slope}(s) \times \text{qs}(s); \\
\text{lumtax}(s,d) &= \text{pd}(d) \times \text{taxduty}(d,s) \times x.l(s,d); \\
\text{SellerDuty}(s) &= \sum(d, \text{lumtax}(s,d))/1000000; \\
\text{BuyerDuty}(d) &= \sum(s, \text{lumtax}(s,d))/1000000; \\
\text{Trade}(s,d) &= \text{round}(x.l(s,d)); \\
\text{Produced}(s) &= \sum(d, x.l(s,d)); \\
\text{Consumed}(d) &= \sum(s, x.l(s,d)); \\
\text{lumtax}(s,d) &= \text{pd}(d) \times \text{taxduty}(d,s) \times x.l(s,d); \\
\text{CS}(d) &= \frac{0.5 \times (\text{dint}(d) - \text{pd}(d)) \times \text{qd}(d)}{1000000}; \quad \# \text{Consumer surplus} \\
\text{QR}(s) &= \frac{0.5 \times \text{ps}(s) \times \text{qs}(s)}{1000000}; \quad \# \text{Quasi rent} \\
\text{differ}(s,d) &= \text{Trade}(s,d) - \text{lumflow}(s,d);
\end{align*}
\]

OPTION Trade:1:1:1; option pd:2:0:10; option BuyerDuty:1:0:1; option SellerDuty:1:0:1; option CS:1:0:1; option QR:1:0:1; option dispwidth=4;

DISPLAY Trade, pd, ps, SellerDuty, BuyerDuty, CS, QR, differ;

$call gdxxrw.exe TradeResult.xls o=ExportData.gdx SQ=N @export.txt

* NOTE: Indented lines need to be programmed as a single ‘execute’ line in GAMS.

execute_unload "TradeResult.gdx" pd, ps
execute 'gdxxrw.exe TradeResult.gdx O=TradeResult.xls SQ=N text="Lumber Demand Prices" rng=Price!a1 par=pd rng=Price!a2 rdim=1 text="Lumber Supply Prices" rng=Price!d1 par=ps rng=Price!d2 rdim=1'

execute_unload "TradeResult.gdx" Produced, Consumed
execute 'gdxxrw.exe TradeResult.gdx O=TradeResult.xls SQ=N text="Lumber Production" rng=Production!a1 par=Produced rng=Production!a2 rdim=1 text="Lumber Consumption" rng=Production!a23 par=Consumed rng=Production!a24 rdim=1'
execute_unload "TradeResult.gdx" CS, QR
execute 'gdxxrw.exe TradeResult.gdx Output=TradeResult.xls SQ=N text="-
Consumer Surplus ($mil)" rng=Welfare!b1 par=CS rng=Welfare!a2
rdim=1 text="Producer Surplus ($mil)" rng=Welfare!d1 par=QR
rng=Welfare!c2 rdim=1’

execute_unload "TradeResult.gdx" Trade
execute 'gdxxrw.exe TradeResult.gdx Output=TradeResult.xls SQ=N text=
"Bilateral trade (00s m3)" rng=Tradeflow!a1 par=Trade rng=Tradeflow!a2
rdim=1 cdim=1’

Discussion

There are two distinct differences between the GAMS code provided in this
Appendix compared with that in Appendix 4.A. The current model includes a
snippet of code that reads data from an Excel file called ‘InputData.xlsx’, which
is available in the same folder as the GAMS code. Further, the model requires
a first stage that solves for the shadow prices for the calibration constraints as
discussed in section 4.2. The shadow prices are then used to revise the shipping
and handling (S&H) costs matrix, which is then employed in the second stage
of the model. Output will differ from that in Appendix 4.A, but will look similar.
The reader is encouraged to run the above model and compare the results with
those in Appendix 4.A.

APPENDIX 4.C: GAMS Code for Calibrated Model using MCP

The listing in this Appendix is for a 20-region model with eight forest products,
including logs. The model consists of two stages – a first stage is used to cali-
brate the model, followed by a second stage that solves the calibrated model.
Again the GAMS code and Excel file with the underlying data are found in the
folder ‘Multiple regions & products’ under the ‘Trade Models’ tab at the website

GAMS Code

$TITLE Multi-Product Forest Trade Model
* This model consists of 20 regions and seven products

$Oneolcom
$eolcom #
SETS
  r  region names
  p  all products
  s(r) product supply regions
  d(r) product demand regions
  ;
  alias(r,rr);

* --- IMPORT DATA FROM SPREADSHEET ----*
* Build gdxrwx instructions file
$onecho > import.txt
par=produce rng=Production!B4:L24 Rdim=1 Cdim=1
par=consume rng=Consumption!B4:L24 Rdim=1 Cdim=1
par=price rng=Prices!B4:L24 Rdim=1 Cdim=1
par=manucost rng=ManuCost!B4:L24 Rdim=1 Cdim=1
par=elastic rng=Elastic!B4:L24 Rdim=1 Cdim=1
par=recover rng=recover!B4:L24 Rdim=1 Cdim=1
par=data rng=data!B2:D22 Rdim=1 Cdim=1
par=lumtax rng=lumtax!A1:U21 Rdim=1 Cdim=1
par=logtax rng=logtax!A1:U21 Rdim=1 Cdim=1
par=trans rng=trans!A1:U21 Rdim=1 Cdim=1
par=lumcal rng=lumberflow!B2:V22 Rdim=1 Cdim=1
par=logcal rng=logflow!B2:V22 Rdim=1 Cdim=1
par=pulplogcal rng=pulplogflow!B2:V22 Rdim=1 Cdim=1
par=pulpcal rng=pulpflow!B2:V22 Rdim=1 Cdim=1
par=residcal rng=residflow!B2:V22 Rdim=1 Cdim=1
par=plycal rng=plyflow!B2:V22 Rdim=1 Cdim=1
par=pbcal rng=osbflow!B2:V22 Rdim=1 Cdim=1
par=fbcal rng=mdfflow!B2:V22 Rdim=1 Cdim=1
set=r rng=lumtax!A2:A21 Rdim=1 Cdim=0
set=p rng=production!D4:M4 Rdim=0 Cdim=1
set=s rng=lumtax!A2:A21 Rdim=1 Cdim=0
set=d rng=lumtax!A2:A21 Rdim=1 Cdim=0
$offecho

* Create and import GDX file with import data
$call gdxrwx.exe TradeData.xlsx o=ImportData.gdx @import.txt
$gdxin ImportData.gdx
PARAMETERS
produce(r,p) Data matrix (see below)
consume(r,p) Data matrix (see below)
price(r,p) Data matrix (see below)
manucost(r,p) Data matrix (see below)
elastic(r,p) Data matrix (see below)
data(r,*) Data matrix (see below)
recov(r,p) Data matrix (see below)
lumtax(r,rr) Matrix of lumber import duties
logtax(r,rr) Matrix of log export taxes
trans(r,rr) Transportation costs
lumcal(r,rr) Lumber trade flow matrix for calibrating model
logcal(r,rr) Log trade flow matrix for calibrating model
pulplogcal(r,rr) pulp trade flow matrix for calibrating model
pulpcal(r,rr) pulplog trade flow matrix for calibrating model
residcal(r,rr) residual trade flow matrix for calibrating model
plycal(r,rr) plywood & veneer trade flow matrix for calibrating model
pbcal(r,rr) osb trade flow matrix for calibrating model
fbcal(r,rr) mdf trade flow matrix for calibrating model

$load r p s d data produce consume price manucost elastic recov lumtax logtax
trans lumcal logcal pulplogcal pulpcal residcal plycal pbcal fbcal
$gdxin

SETS
  dn(p) downstream products /lum, ply, pulp, pb, fb, pell/
  mc(dn) Products whose MC shifts as Ind RW prices change /lum, ply, pulp/

* ----------- Recovery calculations -----------------------------------

SCALARS
  recov_chips_lumber Pulp chip recovery from lumber manufact /0.35/
  recov_chips_plywood Pulp chip recovery from plywood manufact /0.20/
  recov_residuals Bark planer shavings & sawdust recovery sawmill /0.21/
  recov_pb Particleboard recovery from fibre /0.79/
  recov_fb Particleboard recovery from fibre /0.86/

PARAMETER Impact(dn) Impact of shift variable on MC curves
/lum 0.7
/ply 0.6
/pulp 0.5/

PARAMETERS
  bs(r,p) Slope of MC curve
  as(r,p) Intercept of MC curve
  bd(r,p) Slope of demand curve
  ad(r,p) Intercept of demand curve

* -------------- MC and Demand curve parameters --------------
bs(r,p) = manucost(r,p)/produce(r,p);
as(r,p) = 0;
bd(r,p) = -1*price(r,p)/(elastic(r,p)*consume(r,p));
ad(r,p) = price(r,p) + bd(r,p)*consume(r,p);

* ------- Log and Lumber Tax Parameters and Misc -------
PARAMETERS
ytax(r,rr) lumber taxes
xtax(r,rr) log taxes that later change in the analysis
shadow(r,rr,p) Shadow prices from PMP constraints

ytax(r,rr) = price(r,'lum')*lumtax(r,rr);
xtax(r,rr) = price(r,'saw')*logtax(r,rr);

* ----------- CONSTRUCTION OF THE MODEL EQUATIONS ------------- *
POSITIVE VARIABLES
* Pulp and pulpwood variables
pw_pulp(r) pulpwood going to pulp
pw_other(r) pulpwood going to other (panels)
chips_pulp(r)
* flow variables
chips(r) pulp chips produced from sawmilling and plywood
residuals(r) bark planer shavings sawdust from sawmills & plywood
* Residual variables
fibre_pb(r)
fibre_fb(r)
fibre_pellet(r) fibre going to make pellets
ind_rw(r) proportion to sawlogs & pulpwood cannot exceed 1
resid_flow(r,rr) residual & chip trade from supply to demand regions

* ------------
qs(r,p) supply quantity
qd(r,p) demand quantity
flow(r,rr,p) product trade flow matrix
zs(r,dn) downstream product MC shifter based on log input price

VARIABLES
Z1 total surplus in phase I problem
Z2 total surplus with calibrated objective function

* DEFINE EQUATIONS
*-------------------------------------------------------------
EQUATIONS
obj1 objective for PMP calibration phase
obj2 revised objective after calibration
supply(rr,p)
demand(r,p)
shift(r,mc)

* --------------- Logs
mai(r) MAI constraint for sustainable harvests
log_supply1(r) sawlog supply constraint for total production
log_supply2(r) sawlog supply constraint for recovery from ind rw
log_demand(rr)
pmpsaw(r,rr) calibration constraint for sawlog trade flows

* --------------- Lumber
lum_prod(r) sawnwood production constraint
pmplum(r,rr) calibration constraint for lumber trade flows

* --------------- Chips and residuals
chips_prod(r) Pulp chips produced from sawmilling
residuals_prod(r) residuals bark planer shavings sawdust from sawmills

* --------------- Plywood
ply_prod(r) Plywood production constraint
pmply(r,rr) calibration constraint for plywood trade flows

* --------------- Particleboard
pb_prod(r) Particleboard production constraint
pmpbpb(r,rr) calibration constraint for Particleboard trade flows

* --------------- Fibreboard
fb_prod(r) Fibreboard production constraint
pmpfb(r,rr) calibration constraint for Fibreboard trade flows

* --------------- pulp
pulp_prod(r) pulp production constraint
pmppulp(r,rr) calibration constraint for pulp trade flows

* --------------- pulpwood
pw_prod(r) pulpwood production constraint
pw_supply(r) pulpwood recovery from ind rw

* --------------- pellets
pellet_prod1(r) pulp production constraint
pellet_prod2(r) pulp production constraint

* --------------- Fibre flow
Fibre(r) pulpwood and pulp chips fibre flow constraint
residual_flow(r) residual trade flow
log_sum(r) sum of sawlog and pulpwood must not exceed ind rw

;

* ******************************************************

* MODEL EQUATIONS

* ******************************************************

obj1.. Z1 =E= sum((r,dn), ad(r,dn)*qd(r,dn)-.5*bd(r,dn)*qd(r,dn)*qd(r,dn))
    - sum((r,dn), (as(r,dn)+impact(dn)*zs(r,dn))*qs(r,dn)
    +.5*bs(r,dn)*qs(r,dn)*qs(r,dn))
- sum((r,rr,dn), trans(r,rr)*flow(r,rr,dn)) - sum(r, 0.5*bs(r,'log')*qs(r,'log')*qs(r,'log'))
- sum((r,rr), 1.27*trans(r,rr)*flow(r,rr,'saw')) - sum((r,rr), flow(r,rr,'lum')*ytax(r,rr)))
- sum((r,rr), trans(r,rr)*resid_flow(r,rr))

\[ \text{obj2.. } Z2 = E= \sum((r,dn), ad(r,dn)*qd(r,dn)-0.5*bd(r,dn)*qd(r,dn)*qd(r,dn)) \]
- sum((r,dn), (as(r,dn)+impact(dn)*zs(r,dn))*qs(r,dn)) + 0.5*bs(r,dn)*qs(r,dn)*qs(r,dn))
- sum((r,rr), (shadow(r,rr,dn) + trans(r,rr))*flow(r,rr,dn))
- sum(r, 0.5*bs(r,'log')*qs(r,'log')*qs(r,'log') - sum((r,rr), (shadow(r,rr,'saw') + 1.27*trans(r,rr))*flow(r,rr,'saw')) - sum(r, sum(rr, flow(r,rr,'lum')*ytax(r,rr)))
- sum(rr, sum(r, flow(r,rr,'saw')*xtax(r,rr))) - sum((r,rr), trans(r,rr)*resid_flow(r,rr))

* ------ MODEL CONSTRAINTS --------------------------

* ------- Logs -------------------------------------
mai(r).. q(r,'log') =L= data(r,'aac')*recov(r,'log')*1.01;
log_supply1(r.. sum(rr, flow(r,rr,'saw')) =L= qs(r,'saw');
log_supply2(r.. qs(r,'saw') =L= qs(r,'log')*recov(r,'saw');
log_demand(rr.. sum(r, flow(r,rr,'saw')) =G= qd(rr,'saw');
pmpslaw(r,rr.. flow(r,rr,'saw') =E= logcal(r,rr);
shift(r,mc.. zs(r,mc) =E= as(r,'log') + bs(r,'log')*qs(r,'log');

* ------ Downstream product constraints -------------
supply(r,dn.. sum(rr, flow(r,rr,dn)) =L= qs(r,dn);
demand(rr,dn.. sum(r, flow(r,rr,dn)) =G= qd(rr,dn);

* -------- Lumber constraints ---------------------
lum_prod(rr.. qs(r,'lum') =L= recov(rr,'lum')*qd(rr,'saw');
pmpleum(r,rr.. flow(r,rr,'lum') =E= lumcal(r,rr);

* ---------- Plywood constraints -------------------
ply_prod(rr.. qss(r,'ply') =L= recov(rr,'ply')*qd(rr,'saw');
pmpply(r,rr.. flow(r,rr,'ply') =E= plycal(r,rr);

* -------- Particleboard constraints ---------------
pb_prod(r.. qs(r,'pb') =L= recov(r,'pb')*fibre_pb(r);
pmpbpb(r,rr.. flow(r,rr,'pb') =E= pbcal(r,rr);

* -------- Fibreboard constraints ------------------
fiber_prod(r.. qs(r,'fb') =L= recov(r,'fb')*fibre.fb(r);
pmpfib(r,rr.. flow(r,rr,'fb') =E= fbcal(r,rr);

* -------- Pulp constraints ------------------------
pulp_prod(r.. qs(r,'pulp') =L= 0.4*(pw_pulp(r)+ chips_pulp(r));

$OnText # With this command the following lines are not read by the solver
0.4 converts m3 roundwood to mt of pulp; Other conversions 2.5 to 6.0 mt
pulp per m3
forestry-statistics-2016-introduction/sources/timber/conversion-factors/ (accessed
September 4, 2020)
$OffText

pmppulp(r,rr).. flow(r,rr,'pulp') =E= pulpcal(r,rr);
* ------ Pulpwood constraints --------------------------
pw_prod(r).. pw_pulp(r) + pw_other(r) =L= qs(r,'pw');
pw_supply(r).. qs(r,'pw') =L= qs(r,'log')*recov(r,'pw');
* -------- Pellet constraints -------------------------
pellet_prod1(r).. qs(r,'pell') =L= 0.65*0.667*fibre_pellet(r);
* See above reference: 0.65 converts m3 roundwood to m3 of pellet;
* 0.667 converts mt to m3 of pellet (650 kg per m3)
pellet_prod2(r).. qs(r,'pell') =L= data(r,'PelCap');
* -------- Material flow constraints -------------------
chips_prod(r).. chips(r) =E= qs(r,'lum')*recov_chips_lumber +
qs(r,'ply')*recov_chips_plywood;
* chips produced depend on chips from sawmills & from plywood manufacturing
residuals_prod(r).. residuals(r) =E= qs(r,'lum')*recov_residuals;
* Residuals produced are from sawmilling
fibre(rr).. fibre_pb(rr) + fibre_fb(rr) + fibre_pellet(rr) + chips_pulp(rr) =L= sum(r,
resid_flow(r,rr));
* pulp and residual fibre goes to particleboard, fibreboard and pulp
residual_flow(r,.. sum(rr, resid_flow(r,rr)) =L= chips(r) + residuals(r) +
pw_other(r);
log_sum(r).. qs(r,'saw') + qs(r,'pw') =L= qs(r,'log');
* ------ End of model Constraints ------------------------

MODEL tradePMP /obj1, mai, log_supply1, log_supply2, log_demand, supply,
    demand, lum_prod, ply_prod, pb_prod, fb_prod, chips_prod, residuals,
    prod, fibre, shift, residual_flow, pulp_prod, pw_prod, pw_supply, pellet,
    prod1, pellet_prod2, log_sum, pmppaw, pmplum, pmpply, pmppb,
    pmppfb, pmppulp /;

* ------------ Calibration phase ------------------------
SOLVE tradePMP using QCP maximizing Z1;
* ------------ Store PMP shadow prices ------------------
shadow(r,rr,'log') = 0;
shadow(r,rr,'saw') = pmppaw.m(r,rr);
shadow(r,rr,'lum') = pmplum.m(r,rr);
shadow(r,rr,'ply') = pmpply.m(r,rr);
shadow(r,rr,'pb') = pmppb.m(r,rr);
shadow(r,rr,'fb') = pmppfb.m(r,rr);
shadow(r,rr,'pulp') = pmppulp.m(r,rr);
shadow(r,rr,'pell') = 0;
* -------- CALIBRATED MODEL ----------------------------
MODEL trade /obj2, mai, log_supply1, log_supply2, log_demand, supply,
    demand, lum_prod, ply_prod, pb_prod, fb_prod, chips_prod, residuals,
    prod, fibre, shift, residual_flow, pulp_prod, pw_prod, pw_supply, pellet,
    prod1, pellet_prod2, log_sum/;
SOLVE trade using QCP maximizing Z2;

$OffText
VARIABLES
  log_flow(r,rr)
  lum_flow(r,rr)
  ply_flow(r,rr)
  pb_flow(r,rr)
  fb_flow(r,rr)
  pulp_flow(r,rr)
  pellet_flow(r,rr) ;

VARIABLES PS(*,*), PD(*,*), QR(*,*), CS(*,*), SR(*,*);
  PS.m(r,p) = 0.01;
  PD.m(r,p) = 0.01;
  QR.m(r,p) = 0.01;
  CS.m(r,p) = 0.01;
  SR.m(r,p) = 0.01;

  log_flow.m(r,rr) = 0.01;
  lum_flow.m(r,rr) = 0.01;
  ply_flow.m(r,rr) = 0.01;
  pb_flow.m(r,rr) = 0.01;
  fb_flow.m(r,rr) = 0.01;
  pulp_flow.m(r,rr) = 0.01;
  pellet_flow.m(r,rr) = 0.01;

log_flow.l(r,rr) = flow.l(r,rr,'saw');
  lum_flow.l(r,rr) = flow.l(r,rr,'lum');
  ply_flow.l(r,rr) = flow.l(r,rr,'ply');
  pb_flow.l(r,rr) = flow.l(r,rr,'pb');
  fb_flow.l(r,rr) = flow.l(r,rr,'fb');
  pulp_flow.l(r,rr) = flow.l(r,rr,'pulp');
  pellet_flow.l(r,rr) = flow.l(r,rr,'pell');
  qd.l(r,'log') = qs.l(r,'log');
  qd.l(r,'pw') = qs.l(r,'pw');

loop(p,
  PS.l(r,p) = 0.001*(as(r,p) + bs(r,p)*qs.l(r,p));
  PD.l(r,p) = 0.001*(ad(r,p) - bd(r,p)*qd.l(r,p));
  QR.l(r,p) = 0.000001*0.5*bs(r,p)*qs.l(r,p)**2;
  CS.l(r,p) = 0.000001*0.5*bd(r,p)*qd.l(r,p)**2;
  SR.l(r,p) = 0.000001*supply.m(r,p)*qs.l(r,p);)
  SR.l(r,'log') = 0.000001*mai.m(r)*qs.l(r,'log');

DISPLAY shadow, zs.l, impact, PS.l, PD.l, QR.l, CS.l, r, p, dn, as, bs, ad, bd;
DISPLAY qs.l, qd.l, flow.l, resid_flow.l, pw_pulp.l, pw_other.l, chips.l, residuals.l, fibre_pb.l, fibre_fb.l;
execute_unload "Output.gdx" shadow
execute 'gdxxrw.exe Output.gdx Output=Calibrate.xlsx SQ=N par=shadow rng=shadow!a1'

execute_unload "Results.gdx" qd, qs, PS, PD, QR, CS, SR
execute 'gdxxrw.exe Results.gdx Output=Result01.xlsx SQ=N var=qs.l rng=production!a1 var=PS.l rng=S-price!a1 var=PD.l rng=Dprice!a1 text="Quasi-Rent" var=Welfare!a1 var=QR.l var=Welfare!a2 text="Consumer Surplus" var=Welfare!a24 var=CS.l var=Welfare!a25 text="Scarcity Rent" var=Welfare!a47 var=SR.l var=Welfare!a48'

execute_unload "Result.gdx" log_flow, lum_flow, ply_flow, pb_flow, fb_flow, pulp_flow, pellet_flow, resid_flow
execute 'gdxxrw.exe Result.gdx Output=Result02.xlsx text="Inter-regional Sawlog Trade Flows" SQ=N rng=log_flow!a1 var=log_flow.l rng=log_flow!a2 rdim=1 cdim=1 text="Inter-regional Lumber Trade Flows" SQ=N rng=lum_flow!a1 var=lum_flow.l rng=lum_flow!a2 rdim=1 cdim=1 text="Inter-regional Plywood + Veneer Trade Flows" SQ=N rng=ply_flow!a1 var=ply_flow.l rng=ply_flow!a2 rdim=1 cdim=1 text="Inter-regional Particleboard Trade Flows" SQ=N rng=pb_flow!a1 var=pb_flow.l rng=pb_flow!a2 rdim=1 cdim=1 text="Inter-regional Fibreboard Trade Flows" SQ=N rng=fb_flow!a1 var=fb_flow.l rng=fb_flow!a2 rdim=1 cdim=1 text="Inter-regional Pulp Trade Flows" SQ=N rng=pulp_flow!a1 var=pulp_flow.l rng=pulp_flow!a2 rdim=1 cdim=1 text="Inter-regional Pellet Trade Flows" SQ=N rng=pellet_flow!a1 var=pellet_flow.l rng=pellet_flow!a2 rdim=1 cdim=1 text="Inter-regional Residual Trade Flows" SQ=N rng=resid_flow!a1 var=resid_flow!a2 rdim=1 cdim=1'

Discussion

The model found in this Appendix is quite different from that in Appendices 4.A and 4.B, because the model covers eight products – sawlogs, lumber, plywood, pulp, particle board (oriented strand board, OSB), fiberboard (medium density fiberboard, MDF), wood pellets, and residuals – and 20 regions and/or countries (including five Canadian and three U.S. regions). The data to run the model are found in the file ‘TradeData.xlsx’. The bilateral trade matrices are important because they are used to calibrate the trade model. However, at this time, there may be too little information on the country-to-country trade flows of wood pellets to be included in the calibration component of the model. The GAMS results are then reported in two files, denoted ‘Results01.xlsx’ and ‘Results02.xlsx’, with production,
consumption and the various welfare measured reported in the former and trade flows in the latter.

**Appendix 4.D: R code**

In this Appendix, we provide the R code for allocating countries into regions. It calls data from Comtrade, which was downloaded into an Excel CSV file format and named ‘comtrade.csv’. The Comtrade file was then edited to remove columns that were empty. An example of the first 19 lines of the comtrade.csv file is provided in Fig. 4.D.1, which enables the user to better understand the R call to this file. Nonetheless, the user might need to modify the R code to suite their own needs.

The file in Fig. 4.D.1 is for HS 440710, defined as ‘Wood and articles of wood; wood charcoal. Wood sawn or chipped lengthwise, sliced or peeled, whether or not planed, sanded or end-jointed, of a thickness exceeding 6 mm. Coniferous’. For wood product trade (HS440710) in 2016, it includes the exports to and imports from each ‘reporter country’; the user will need to filter the data by imports or exports to obtain bilateral trade flows. An example of the type of output that one can expect from the R code below is found in Table 4.D.1.

![Fig. 4.D.1. Example of the modified CSV file downloaded from Comtrade (https://comtrade.un.org/data/) for Commodity HS4407](https://comtrade.un.org/data/)

### Table 4.D.1. Sample output from R file allocating countries into regions.

<table>
<thead>
<tr>
<th></th>
<th>AUS</th>
<th>CHL</th>
<th>CHN</th>
<th>USA</th>
<th>FIN</th>
<th>JPN</th>
<th>NZL</th>
<th>RUS</th>
<th>SWE</th>
<th>CAN</th>
<th>ROA</th>
<th>ROE</th>
<th>ROLA</th>
<th>ROW</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>-</td>
<td>-</td>
<td>56.2</td>
<td>0.1</td>
<td>-</td>
<td>2.8</td>
<td>0.6</td>
<td>-</td>
<td>-</td>
<td>0.0</td>
<td>104.7</td>
<td>0.0</td>
<td>-</td>
<td>18.6</td>
</tr>
<tr>
<td>CHL</td>
<td>22.0</td>
<td>-</td>
<td>735.5</td>
<td>358.6</td>
<td>-</td>
<td>256.1</td>
<td>5.2</td>
<td>-</td>
<td>-</td>
<td>3.2</td>
<td>745.0</td>
<td>51.2</td>
<td>647.3</td>
<td>450.9</td>
</tr>
<tr>
<td>CHN</td>
<td>1.5</td>
<td>-</td>
<td>696.4</td>
<td>-</td>
<td>-</td>
<td>243.9</td>
<td>0.7</td>
<td>0.6</td>
<td>0.1</td>
<td>875.4</td>
<td>282.9</td>
<td>108.5</td>
<td>1.9</td>
<td>3.0</td>
</tr>
<tr>
<td>USA</td>
<td>4.2</td>
<td>2.4</td>
<td>1,061.7</td>
<td>3.8</td>
<td>-</td>
<td>941.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>134.4</td>
<td>3,236.1</td>
<td>1.3</td>
<td>3,134.0</td>
<td></td>
</tr>
<tr>
<td>FIN</td>
<td>21.5</td>
<td>-</td>
<td>385.2</td>
<td>206.9</td>
<td>2.9</td>
<td>36.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>602.9</td>
<td>80.3</td>
<td>-</td>
<td>176.4</td>
</tr>
<tr>
<td>NZL</td>
<td>-</td>
<td>13,398.3</td>
<td>24.8</td>
<td>478.0</td>
<td>905.0</td>
<td>0.1</td>
<td>8.3</td>
<td>-</td>
<td>3,836.3</td>
<td>4,022.7</td>
<td>4.6</td>
<td>2,731.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RUS</td>
<td>42.9</td>
<td>-</td>
<td>733.9</td>
<td>250.1</td>
<td>16.5</td>
<td>778.4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.2</td>
<td>233.9</td>
<td>7,820.1</td>
<td>3.6</td>
<td>2,886.8</td>
</tr>
<tr>
<td>CAN</td>
<td>65.6</td>
<td>4.8</td>
<td>5,949.6</td>
<td>35,263.0</td>
<td>0.1</td>
<td>2,288.1</td>
<td>31.0</td>
<td>-</td>
<td>1.7</td>
<td>875.8</td>
<td>161.3</td>
<td>45.8</td>
<td>226.6</td>
<td></td>
</tr>
</tbody>
</table>
R-Code

# We thank Hugh Scorah for helping to write this code
# Creation of Bilateral Trade Flows
# The following code allocates countries to regions and determines the inter-
# regional trade flows.

library(tidyverse)
# if you do not have tidyverse, use the following command
# install.packages("tidyverse")

# -------------- Country listed by 3-digit ISO letters --------------

# Major exporting countries
core <- c("AUS", "CHL", "CHN", "USA", "FIN", "JPN", "NZL", "RUS", "SWE", "CAN")
output_regions <- c(core, "ROA", "ROE", "ROLA", "ROW")

rest_asia <- c("AFG", "BDG", "BTN", "HKG", "IDN", "IND", "KAZ", "KGZ", "KHM",
               "KOR", "LAO", "LKA", "MAC", "MDV", "MMR", "MNG", "MYS", "NPL", "PAK",
               "PHL", "PRK", "SGP", "THA", "TJK", "TKM", "TLS", "UZB", "VNM", "OAS")
rest_europe <- c("ALB", "ARM", "AUT", "AZE", "BEL", "BGR", "BIH", "BLR", "CHE",
               "CYP", "CZE", "DEU", "DNK", "ESP", "EST", "FRA", "FRO", "GBR", "GEO", "GRC",
               "NOR", "POL", "PRT", "ROU", "SRB", "SVK", "SVN", "TUR", "UKR")
rest_latin_am <- c("ABW", "AIA", "ARG", "ATG", "BHS", "BLZ", "BMU", "BRA",
                   "BRB", "COL", "CPV", "CRI", "CUB", "CUW", "CYM", "DMA", "DOM", "ECU",
                   "GTM", "GUY", "GRD", "HND", "HTI", "JAM", "KNA", "LCA", "MEX", "MSR", "NIC",
                   "PAN", "PER", "SLV", "TTO", "URY", "VCT", "VEN", "VGB")
rest_world <- c("AGO", "ARE", "ASM", "ATF", "BDI", "BHR", "BLM", "CCK", "CIV",
               "GAB", "GHA", "GIN", "GMB", "GNQ", "IRN", "IRQ", "ISR", "JOR", "KEN",
               "KIR", "KWT", "LBN", "MAR", "MDG", "MHL", "MNP", "MOZ", "MRT", "MUS",
               "PYF", "QAT", "SAU", "SDN", "SEN", "SHN", "SLB", "SLE", "SOM", "SPM", "SSD",
               "UMI", "VUT", "WLF", "WSM", "YEM", "ZAF", "ZMB", "ZWE")
         "NLD", "POL", "PRT", "ROU", "SRB", "SVK", "SVN", "SWE")
         "ITA", "LUX", "NLD", "PRT", "SWE")
# ----------------------------------COMTRADE---------------------------------------------------------
##functions##
trade_flow_sort <- function(trade_frame, core, output_regions, rest_asia, rest_europe, rest_latin_am, rest_world) {
  return_matrix <- data.frame(matrix(0, length(core), length(output_regions)))
  rownames(return_matrix) <- core
  colnames(return_matrix) <- output_regions
  for (i in 1:nrow(return_matrix)){
    current_reporter <- rownames(return_matrix)[i]
    c_table <- filter(trade_frame, reporter == current_reporter)
    # need to test for null table.
    if (nrow(c_table) > 0) {
      for (j in 1:nrow(c_table)){
        partner <- c_table$partner[j]
        qty <- c_table$qty[j]
        if (partner %in% core) {return_matrix[current_reporter, partner] <- qty}
        else if (partner %in% rest_asia) {
          return_matrix[current_reporter, "ROA"] <- return_matrix[current_reporter, "ROA"] + qty
        }
        else if (partner %in% rest_europe) {
          return_matrix[current_reporter, "ROE"] <- return_matrix[current_reporter, "ROE"] + qty
        }
        else if (partner %in% rest_latin_am) {
          return_matrix[current_reporter, "ROLA"] <- return_matrix[current_reporter, "ROLA"] + qty
        }
        else if (partner %in% rest_world) {
          return_matrix[current_reporter, "ROW"] <- return_matrix[current_reporter, "ROW"] + qty
        }
      }
    }
  }
  return (return_matrix)
}
#--------------------------------------------------------------------------------------------------
comtrade_raw <- read_csv("comtrade.csv")
comtrade <- select(comtrade_raw, reporter = 'Reporter ISO', partner = 'Partner ISO', trade_flow = 'Trade Flow Code', qty = 'Qty', value = 'Trade Value (US$)')
#trade_flow variable takes value ‘1’ for Import, ‘2’ for Export, ‘3’ for Re-export, ‘4’ for Re-import

####EXPORT####
comtrade_ex <- filter(comtrade, trade_flow==2)
exports <- trade_flow_sort(comtrade_ex, core, output_regions, rest_asia, rest_europe, rest_latin_am, rest_world)
write.csv(exports, ‘int_exports.csv’) # write bilateral trade exports to file

####RE-EXPORT####
comtrade_reex <- filter(comtrade, trade_flow==3)
reexports <- trade_flow_sort(comtrade_reex, core, output_regions, rest_asia, rest_europe, rest_latin_am, rest_world)
write.csv(exports, ‘int_exp_rexp.csv’) # write exports plus re-exports to file

####IMPORT####
comtrade_im <- filter(comtrade, trade_flow==1)
imports <- trade_flow_sort(comtrade_im, core, output_regions, rest_asia, rest_europe, rest_latin_am, rest_world)

####RE-IMPORT####
comtrade_reim <- filter(comtrade, trade_flow==4)
reimports <- trade_flow_sort(comtrade_reim, core, output_regions, rest_asia, rest_europe, rest_latin_am, rest_world)

# -------------- End R File -------------------------------------------------

References

BC Government (2019) Fibre and mill information. Available at: https://www2.gov.bc.ca/gov/content/industry/forestry/competitive-forest-industry/forest-industry-economics/fibre-mill-information (accessed November 7, 2019).


USDA Forest Service (2019) We are the nation’s forest census. Available at: https://www.fia.fs.fed.us/ (accessed October 16, 2019).

