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Log-Grade Volume Distribution Prediction Models for Tree Species in Red Oak-Sweetgum Stands on US Mid-South Minor Stream Bottoms

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Red oak (*Quercus section Labatae*)-sweetgum (*Liquidambar styraciflua* L.) stands growing on mid-south bottomland sites in the United States are well known for producing high-quality grade hardwood logs, but models for estimating the quantity and quality of standing grade wood in these stands have been unavailable. Prediction models to estimate total merchantable sawlog volume and volume by grade category within a standing tree are presented and discussed. Nonlinear regression, binary logistic regression, and probability distribution function techniques were used in the development of individual tree models for species groups based on 2,149 professionally graded trees. Species, dbh, total height, and merchantable height were input variables used to determine projected volumes for each of five volume units and five species groups: cherrybark oak (*Quercus pagoda* Raf.), other red oak, all red oak, sweetgum, and other commercial species. Total merchantable sawtimber volume models by species group provided logical and consistent predictions with nonlinear R^2 values ranging from 0.70 to 1.00 depending on species group and volume unit. The estimated volume by grade category varies by tree but will closely estimate average grade volume when applied across numerous stands. The resulting equations were incorporated into a growth and yield simulator (software available from the Forest and Wildlife Research Center, Mississippi State University website). The ability to predict the quantity and quality of merchantable volume for these bottomland tree species should greatly advance the valuation of bottomland hardwoods and encourage the establishment and improved management of these stands.

Keywords: log grade volume, bottomland hardwood, software download

Red oak (*Quercus* section *Labatae*)-sweetgum (*Liquidambar styraciflua* L.) stands growing on mid-south bottomland sites in the United States are well known for producing high-quality grade hardwood logs. Despite increased interest and understanding of this valuable hardwood forest type group (Hodges 1995, Meadows and Stanturf 1997, Lockhart et al. 2006), there has been little development of growth and yield systems or expected grade prediction models, making the estimation of quantity and quality of standing grade wood difficult. Publicly available growth and yield models for Southern species have concentrated on commercially important single-species conifer stands such as loblolly pine (*Pinus taeda* L.) (Matney and Farrar 1992, Burkhart et al. 2008), slash pine (*Pinus elliottii* Engelm.) (Zarnoch et al. 1991), and longleaf pine (*Pinus palustris* Mill.) (Farrar and Matney 1994), but there are few for bottomland hardwood forest complexes (McTague et al. 2008, Schultz et al. 2010). Tree grade predictions have been developed using discriminant analysis (Belli et al.1993) to provide information for the management of hardwood stands along the minor bottoms of Mississippi. Whereas tree grade prediction estimates the number of trees per grade, it lacks the ability to provide the volume by grade necessary for accurate tree and stand valuation. The addition of merchantable individual tree volume and grade prediction models to already developed stand-level models like those in Schultz et al. (2010) would create a more complete understanding of the dynamics of this forest type.

The development of a prediction tool that accurately distributes volume into grade categories would greatly benefit landowners and managers in making more informed decisions concerning the quality and health of hardwood forest complexes. The objective of this study was to develop individual tree volume prediction models for

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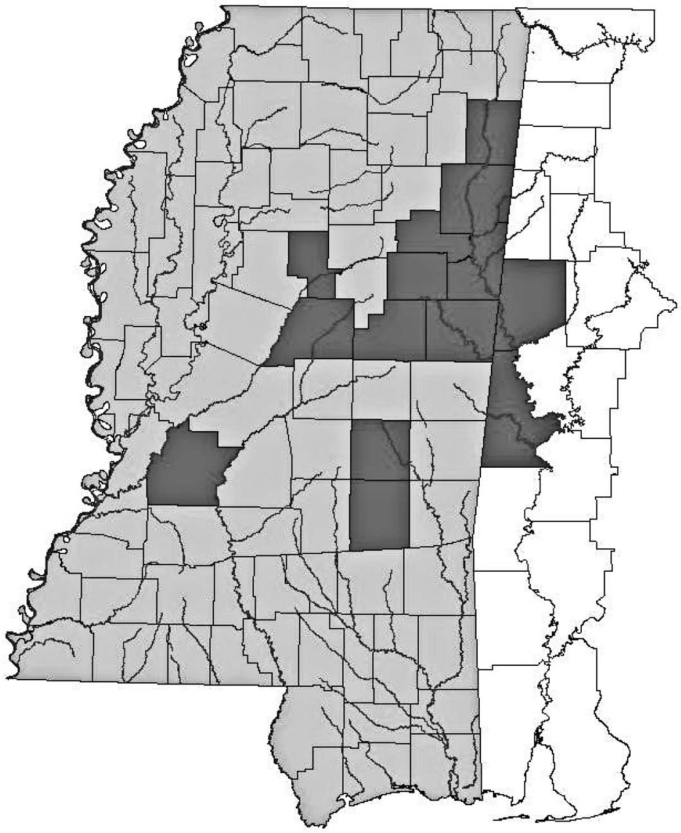

Figure 1. Mississippi (light gray) and Alabama (white) counties with red oak (*Quercus* section *Lobatae*)-sweetgum (*Liquidambar styraciflua* L.) forest mixture permanent growth and yield plots (dark gray) on minor stream bottoms. See Iles 2008 for related information. Note that the map is not to scale.

both total merchantable sawlog volume and volume by grade category. Models were developed to use input variables commonly collected during timber cruises, allowing ease of use by landowners, managers, and other decisionmakers. The log grade volume distribution models were designed to provide grade volume distributions for existing individual tree, stand level, and diameter distribution growth and yield models. The larger objective was to construct a red oak-sweetgum bottomland growth and yield system that predicts both total and grade volume on the stand and tree levels.

Data and Methods

Data

Beginning in 1981, a cooperative effort between the US Department of Agriculture (USDA) Forest Service Center for Bottomland Hardwoods Research, Stoneville, MS, and Mississippi State University, established a growth and yield study for the highly valued bottomland red oak-sweetgum forest mixture on mid-south minor stream bottoms (Schultz et al. 2010). Minor stream bottoms are defined as flood plains and terraces formed through the deposits of local soils (Hodges and Switzer 1979). One hundred fifty permanent plots were established in Mississippi and Alabama (Figure 1) over a wide range of site conditions, qualities, and ages in even-aged stands that had not received silvicultural treatment, harvest, or disturbance within the last 20 years. Circular plots were installed from 0.1 to 1.0 acre in size with a minimum of at least 20 years of age, 30% red oak component basal area per acre (BAPA), 50 measurable trees of ≥3.5 in. dbh, and 60 ft^2 total BAPA. Measurements were taken in 1981/1982, 1988, 1992/1993, and 2007. New plots were established over the 25-year period to replace harvested or damaged plots. There are currently 158 plots in the ongoing study, 86 of which are original (established in 1981). Measurements and remeasurements throughout the history of the study have established 29,244 tree records. Out of these tree records, 2,149 were professionally merchandized and graded to allow for the production of a growth and yield model capable of estimating the grade log volume of these valuable stands.

A target of 10, depending on availability, commercial species trees with minimum dbh of 9.6 in. (representing the range of 1-in. dbh classes and closest to plot center) were designated for grade distribution study measurements. Five species groups were assigned for utilization, value, frequency, and compatibility with previously developed stand-level models (Schultz et al. 2010). Species groups consisted of cherrybark oak (*Quercus pagoda* Raf.), other red oaks (water oak [*Quercus nigra* L.] and willow oak [*Quercus phellos* L.]), all red oaks (both cherrybark oak and other red oak groups), sweetgum, and other commercial species (tree species with commercial value but too few observations for a separate group, the most common of which were swamp chestnut oak (*Quercus michauxii* Nutt.), white oak (*Quercus alba* L.), overcup oak (*Quercus lyrata* Walt.), yellow-poplar (*Liriodendron tulipifera* L.), green ash (*Fraxinus pennsylvanica* Marsh.), sugarberry (*Celtis laevigata* Willd.), elm (*Ulmus* spp.), and hickory (*Carya* spp.).

Dbh, total height, merchantable sawlog height to an 8-in. top diameter, stump height, and gradable section (grade and length) were recorded on standing trees along with other variable measurements. Grades were taken on all logs throughout the entire merchantable length of each tree. USDA Forest Service log grades 1, 2, and 3 (Rast et al. 1973) were used to represent factory-grade sawtimber, and two additional grades of tie timber (grade 4) and cull (grade 5) were used to represent construction-grade and unmerchantable sawtimber, respectively. Grade is affected by not only diameter and length of the log but also by defects that cause reduction of clear wood from numerous factors such as bird peck, branch scars, butt swell, cankers, crooks/sweep, forks, holes, knots, limbs, overgrowths, rot, wounds (Carpenter et al. 1989), and stress reactions causing epicormic branches (Meadows 2001). The numbers of observations by species group and grade are given in Table 1. Measurements taken on study plots represent a wide range of data for both stand- and individual tree-level variables (Table 2), allowing the resulting models to be broadly applied.

Profile functions developed for southern tree species (Souter 2003) provided taper rates that were applied to each graded section to determine observed volume for the total merchantable stem and by grade category. Observed volume by grade category and total merchantable length were calculated for five volume units: international ¼-in. rule, Doyle, Scribner, cubic foot outside bark (Cvob), and cubic foot inside bark (Cvib).

Models

Model development methods were evaluated on the basis of precision of fit statistics for each grade category and compatibility with prior stand-level models (Schultz et al. 2010). A three-step method was selected for estimating total merchantable sawtimber and grade volumes. Steps 1 and 2 use nonlinear and binary logistic regression techniques, respectively, to predict the probability of each of the five log grades occurring in an individual tree. The final step uses a

Table 1. Number of observations by species and grade.

Species group	T[1]	Grade[2]				
		1	2	3	4	5
Cherrybark oak	614	323	327	507	116	223
Other red oak	763	245	348	647	218	292
All red oak	1,377	568	675	1,154	334	515
Sweetgum	590	115	234	524	219	180
Other commercial species	182	92	92	185	185	47

[1] Number of trees in total merchantable sawtimber volume calculations.
[2] Number of observations in USDA Forest Service grades (1, 2, and 3), tie timber (4), and cull (5) volume calculations.

Table 2. Range of observed data recorded across all measurement periods.

Variable	Minimum	Lower quartile	Mean	Upper quartile	Maximum
Age (yr)	15	44	57	68	92
Dbh (in.)	9.2	12.5	17.0	20.2	48.7
Merchantable height (ft)	9	25	36	47	83
Total height (ft)	38	89	100	111	163
Basal area (ft^2/ac)	44	124	141	158	245
Site index base age 50 (ft) for all red oaks	67	99	105	111	133
Trees ≥3.5 in. dbh (no. per acre)	88	150	229	280	742

weighted least-squares adjustment formula to ensure that estimated total merchantable sawtimber equals the sum of each estimated grade. Details of these steps are explained in the three sections that follow.

Total and Grade Volume Models

Weighted nonlinear equations (Draper and Smith 1981) were developed to predict total sawtimber merchantable volume and initial estimates for volume by grade category within a tree. The initial nonlinear equation predictions for each grade required the removal of zero volumes for that grade to properly converge and represent trends found in observed trees with grade index j. Zero volumes occurred because every tree did not possess all grades. The SAS 9.1 (SAS Institute, Inc. 2002) nonlinear regression procedure PROC NLIN was used to construct prediction equations (Equation 1) and obtain coefficients:

$$V_{ij} = a_{jk} + b_{jk}\text{dbh}_i^{\,c_{jk}} H_{ik}^{d_{jk}} + \sqrt{w_{ik}}\varepsilon_{ijk} \qquad (1)$$

where V_{ij} is the jth volume category for the ith tree, volume is total merchantable sawtimber (sum of all grade volumes) when $j = 0$, volume is one of the previously defined individual log grades when $j = 1, 2, 3, 4,$ and 5, dbh_i is dbh for the ith tree, and H_{ik} is height of the ith tree. If $k = 1$, the height is total tree height and merchantable height when $k = 2$, and w_{ik} is weight assigned to each tree chosen so that $\text{Var}(\sqrt{w_{ik}\varepsilon_{ijk}}) = \sigma^2_{jk}$ is constant for all levels of the independent variable and the regression satisfies the requirement for being a best linear unbiased estimator.

The results of Equation 1 predict total merchantable sawtimber volume but require additional steps for grade prediction. The weight function found to best create a homogeneous variance condition for all volume units and species was $w_{ik} = [(\text{dbh}_i^2 H_{ik})^2]^{-1}$.

Grade Occurrence Probability Model

The removal of zero-grade volumes results in overpredictions within grade volumes and requires an adjustment to better represent reality. Binary logistic regression equations (Menard 1995) were constructed to predict the probability of finding grade index j within tree index i and accounts for the removed zero values during the initial grade estimations. Minitab 13.2 (Minitab, Inc. 2000) was used for the development of the prediction equations and to obtain coefficients:

$$P_{ij} = \frac{1}{1 + e^{-S_{jk}}} \qquad (2)$$

where P_{ij} is the probability of the ith tree, $i = 1, 2, \ldots, n$, having grade volume $j = 1, 2, \ldots, 5$, and S_{jk} is the logistic function for heights $k = 1, 2$.

The best equation for the logistic function found was

$$S_{jk} = a_{jk} + b_{jk}\text{dbh}_i + c_{jk}\left(\frac{V_i}{H_{ik}}\right) \qquad (3)$$

where a_{jk}, b_{jk}, and c_{jk} are parameters to the estimate for each j and k, as i, j, and k are defined as in Equation 1, V_i is the total merchantable sawtimber volume in the ith tree, and H_{ik} is the kth height of the ith tree.

The probability calculated is then multiplied by the estimated maximum volume from Equation 1 for each grade index j to determine an adjusted volume:

$$\hat{V}_{ij}^{\text{est}} = \hat{P}_{ij}\hat{V}_{ij} \qquad (4)$$

where $\hat{V}_{ij}^{\text{est}}$ is the estimated volume of the ith tree having grade volume j, $\hat{P}_{ij}$ is the estimated probability of the ith tree having grade volume j, and $\hat{V}_{ij}$ is the weighted nonlinear predicted volume of the ith tree having grade volume j from Equation 1.

The results of this step greatly improve the precision of the prediction system, but require additional steps to maintain consistent behavior among all models combined. In effect, the procedure proportionally reduces the overestimated volume by the probability of the tree not having a grade present.

Weighted Least-Squares Adjustment

A weighted least-squares adjustment (Deming 1943) is applied to each $\hat{V}_{ij}^{\text{est}}$, forcing the sum of the grade index j volumes to equate to the best estimate of volume (total merchantable sawtimber). This procedure is needed for consistent behavior in application and presentation. It was found to be a flexible way to produce proportional adjustments, depending on the choice of weight assigned to each grade index *j* volume. The equation used to provide the adjustment is

$$\mathrm{Min}(\hat{V}_i^{\mathrm{adj}},\lambda)\ SS = \sum_i W_i(\hat{V}_i^{\mathrm{adj}} - \hat{V}_i^{\mathrm{est}})^2 + \lambda(\sum_i \hat{V}_i^{\mathrm{adj}} - \hat{V}_T) \quad (5)$$

where $\hat{V}_i^{\mathrm{est}}$ is predicted volume of grade index *i*, $\hat{V}_T$ is estimated total tree merchantable sawtimber volume, $\hat{V}_i^{\mathrm{adj}}$ is adjusted grade index *i* volume, λ is the Lagrangian multiplier for imposing the constraint, and W_i is weight assigned to grade volume index *i*.

Note that both *i* and *j* are indices for grade in the weighted least-squares adjustment procedure (Equations 5–8), where $i = 1, 2, \ldots, 5$ represents a single grade, and $j = 1, 2, \ldots, 5$ is used to sum across all grades.

Solving Equation 5 results in the adjustment equation:

$$\hat{V}_i^{\mathrm{adj}} = \hat{V}_j^{\mathrm{est}} - \frac{(\sum_i \hat{V}_j^{\mathrm{est}} - \hat{V}_T)W_i^{-1}}{\sum_j W_j^{-1}} \quad (6)$$

where $\hat{V}_i^{\mathrm{adj}}$ is adjusted grade index *i* volume, $\hat{V}_i^{\mathrm{est}}$ is predicted volume of grade index *i*, $\hat{V}_T$ is estimated total tree merchantable sawtimber volume, and W_i is weight assigned to grade volume index *i*. The weight, W_i, determines how much adjustment is applied to each $\hat{V}_i^{\mathrm{est}}$. Weights assigned to each $\hat{V}_i^{\mathrm{est}}$ were such that the lowest weight is allocated to the highest relative precision. That is, more error is applied to the grades with the worst estimates. As the relative variance of each grade volume tends to be inversely proportional to $\hat{V}_i^{\mathrm{est}}$, W_i was replaced with $\hat{V}_i^{\mathrm{est}}$ for application. For application, the following equation is used:

$$\hat{V}_i^{\mathrm{adj}} = \hat{V}_i^{\mathrm{est}} - \frac{(\sum_j V_j^{\mathrm{est}} - \hat{V}_T)(\hat{V}_i^{\mathrm{est}})^{-1}}{\sum_j (\hat{V}_j^{\mathrm{est}})^{-1}} \quad (7)$$

where $\hat{V}_i^{\mathrm{adj}}$ is adjusted grade index *i* volume, $\hat{V}_i^{\mathrm{est}}$ is estimated volume of grade index *i*, and $\hat{V}_T$ is estimated total tree merchantable sawtimber volume.

The result of summing the application of Equation 7 to each of the five predicted grade volumes is

$$\sum_i \hat{V}_i^{\mathrm{adj}} = \hat{V}_T \quad (8)$$

Negative estimates of a grade volume are allowed to occur in the system so that total grade volume will not be overpredicted. To obtain a set of grade volumes that sum to the total volume, negative grade volumes must be set to 0, followed by the readjustment of adjusted volumes. Adjusting grade volumes that are a small proportion of the total volume to zero is consistent with the weight $1/\hat{V}_i^{\mathrm{est}}$. The weight adjusts volume estimates with the greatest value and relative precision the least by throwing most of the error into the grade having the lowest proportional volume and greater relative error.

Model Assessment

Nonlinear models were assessed on the basis of the fit statistics root mean square error (RMSE), bias, and nonlinear R^2. RMSE is a measure of the average standard error of prediction for a single observation at the center of the data. Nonlinear R^2 was calculated as 1 minus the quantity of the error sum of squares divided by the total sum of squares, and bias was calculated as the average difference between observed and predicted values. The regression of observed on predicted was used to obtain an indication of how well the nonlinear system predicted the observed.

Binary logistic regression models were assessed on percentage concordance (Agresti 2007, p. 144). Concordance is a unitless index (0–100) that measures the ability of the model to discriminate cases.

Over a number of *n* predictions, the error of the mean prediction is approximately RMSE/$\sqrt{n}$. Thus, for a large number of future predictions, the error of the mean can be very small.

Results and Discussion

Separate regression equations for total merchantable sawlog volume (nonlinear regression), grade sawtimber volume (nonlinear regression), and probability of a grade occurring in a tree (binary logistic regression) were developed for each species group and volume unit. Parameter estimates and fit statistics for these equations are given in Supplemental Table S3[S] using dbh and total height predictors and in Supplemental Table S4 using dbh and merchantable height predictors. Because the slope and intercept coefficients for the regression of observed on predicted across all possible predictions were, respectively, close to 1 and zero, it was concluded that the nonlinear system was reasonably unbiased at all levels of the response variable. Tabular evaluations of the residuals by volume classes for all units and grades showed consistent unbiasness across volume levels. These evaluations are not shown because of space considerations.

Comparison of the RMSEs in Supplemental Tables S3 and S4 shows that merchantable height was a marginally better predictor variable than total height for most volume units and grades. Merchantable sawlog Cvib and Cvob were generally the least biased and most precise among the five volume categories modeled across the different species groups for merchantable sawtimber and grade volumes. Because of inconsistencies in board foot volume rule estimators, board foot volume estimates are much less precise and have higher bias than cubic foot volumes.

Total merchantable sawtimber volume models by species group provide logical and consistent predictions compared to those for observed volumes when applied on an average tree basis across a number of stands (Figure 2). RMSE, bias, and nonlinear R^2 were calculated to determine suitability. RMSE ranges depended on the volume unit. Bias ranged from −1.37 to 52.36, and nonlinear R^2 ranged from 0.70 to 1.00 (Supplemental Tables S3 and S4, rows labeled *T* under Grade column).

The performance of the modeling system for individual grades was also evaluated by examining RMSE, bias, and nonlinear R^2. Bias ranged from −2.42 to 36.78 for grade 1, from −0.09 to 74.79 for

[S] Supplementary data are available with this article at http://dx.doi.org/10.5849/forsci.15-138.

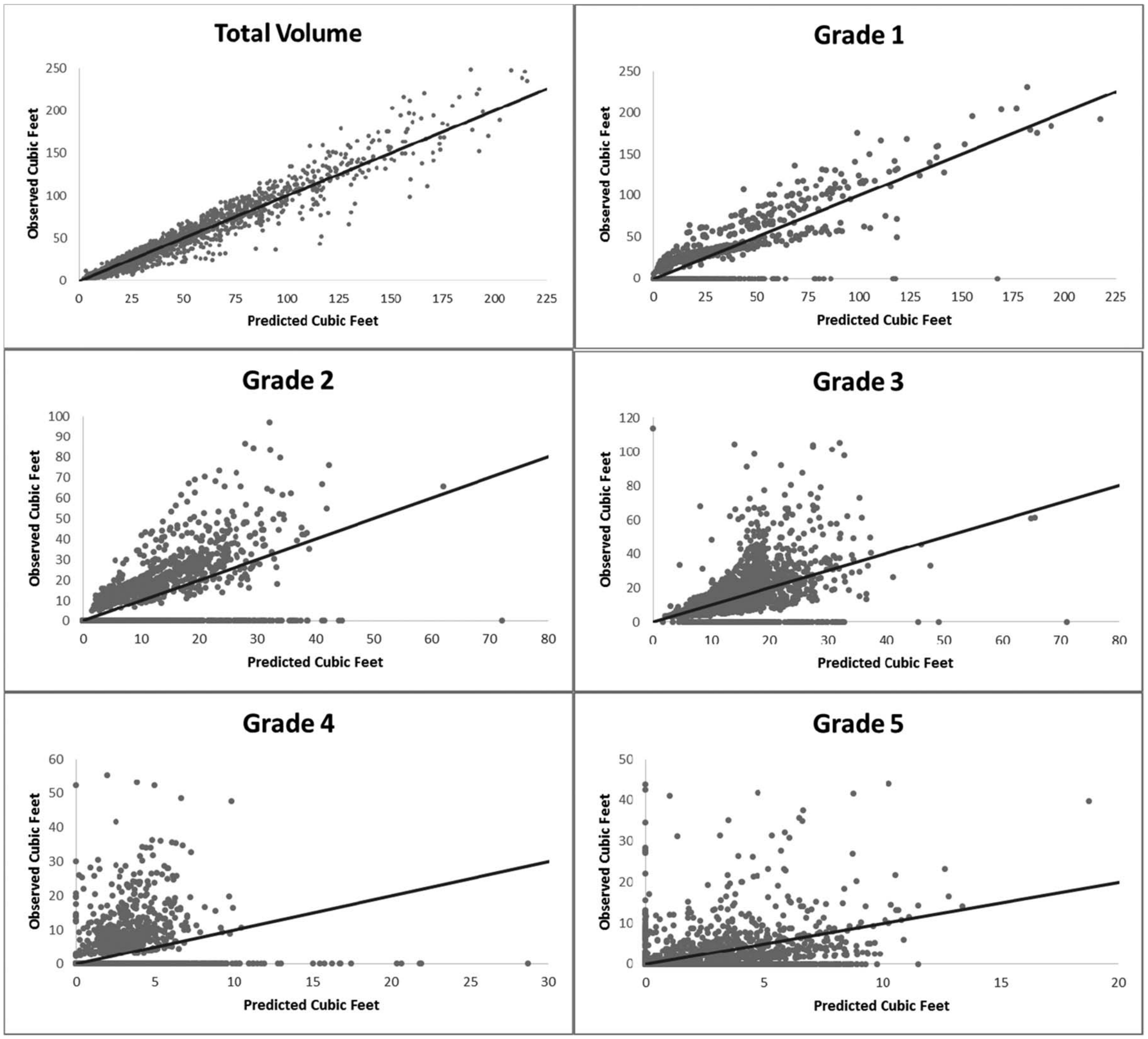

Figure 2. Observed versus predicted total merchantable (Equation 1) and grade 1–5 (Equations 1–7) cubic foot inside bark volume for all species combined calculated from dbh and total height. The solid line is the line with 0 intercept and slope of 1 (the 1:1 line).

grade 2, from −1.0 to 106.51 for grade 3, from −207.85 to 43.74 for grade 4, and from −2.75 to 38.55 for grade 5. All coefficients and fit statistics are given in Supplemental Tables S3 and S4, and observed versus predicted plots for all species groups combined are shown in Figure 2. Similar graphs were obtained for individual species groups (all red oaks, sweetgum, cherrybark oak, other red oak, and other commercial species) whose RMSEs and indices of fit can be found in Supplemental Tables S3 and S4. Variation about the regression line depends on species group and volume unit. As would be expected, the other commercial species group has a much higher variance because of the inclusion of a large number of different species and their small sample sizes. Because of the discrete nature of grading rules, however, trees of similar diameter and height can have widely varying grade volume distributions as can be graphically observed (Figure 2). Thus, grade models should be applied on an average tree basis (trees of equal dimensions) across a number of stands. The regression lines in most cases do not balance on the mass of nonzero grade volume points, because of the large number of observed zero grade volumes. However, in all cases, the errors are balanced about zero.

Tie timber (grade 4) indices of fit were in the negative range in many cases because the occurrence of tie timber in an individual tree is relatively unrelated to diameter and height. Tie timber may occur in all sawlog tree sizes because they are taken from small dimension trees and from the tops of trees that contain higher-grade logs.

Variances and biases for some species and grade combinations that occurred infrequently in the study are sometimes large and may be unacceptable for use, for example, grade 1 logs within the other commercial species group.

Some individual grade modeling improvement might be made by including other variables such as live crown ratio and the height to the first live limb. However, these variables are not commonly collected in forest inventories and would greatly reduce the application of the modeling system.

Computer Program

The log grade volume distribution prediction models were incorporated into a larger computerized red oak-sweetgum mixture growth and yield system that uses stand-level models reported by Schultz et al. (2010) and diameter distribution and mortality models reported by Jeffreys (2014). A Microsoft Excel Visual Basic software application is available.[1] Stand and individual tree predictions can be obtained from tabbed worksheets that implement different levels of user input for two hypothetical and two existing inventory

Table 3. Parameter estimates and fit statistics for individual tree merchantable sawtimber and grade volume models using dbh and total height as predictor variables used in the sample application.

			Nonlinear model (Equation 1)							Binary logistic regression (Equations 2 and 3)			
			Coefficients				Fit statistics			Coefficients			
Sp[1]	Grd[2]	Unit[3]	*a*	*b*	*c*	*d*	RMSE	Absolute bias	Nonlin R^2	*a*	*b*	*c*	Concord (%)[4]
ARO	*T*	Doyl	−40.08	0.00753	2.424	0.734	97.30	35.24	0.93				
	1		−5.26	0.00061	3.131	0.649	113.90	14.56	0.79	−7.68	0.43136	−0.271	88.2
	2		−80.47	1.80340	1.218	0.228	88.37	7.92	0.32	−4.07	0.27351	−0.388	71.3
	3		−47.85	0.50200	1.312	0.401	93.52	6.90	0.27	8.30	−0.46215	0.833	73.7
	4		−79.91	7.08800	0.782	0.153	41.99	−0.60	0.25	1.51	−0.19449	0.343	62.4
	5		−7.64	1.17490	1.782	−0.350	40.07	−0.78	0.25	−2.80	0.08384	0.274	76.8

Complete lists of coefficients for all species groups and volume units are given in Supplemental Tables S3 and S4.
[1] Species group: ARO, all red oak.
[2] Grade: *T*, total merchantable sawlog volume, 1–3, USDA Forest Service grades 1, 2, and 3 volumes; 4, tie timber volume; 5, cull volume.
[3] Doyl, Doyle board foot volume log rule.
[4] Concord, concordance.

scenarios. Predicted trees per acre, arithmetic mean diameter, quadratic mean diameter, basal area, and volume by species group are reported in both tabular and graphical forms. Volumes are produced in board feet and cubic feet by log grade, allowing realistic current and future valuation of stands. Estimates of the distribution of grade volume for individual trees by species group can be obtained by installing the individual tree calculator.[2]

Application

Steps for calculating board feet (bf) Doyle log volume, by grade, in a 30-in. dbh red oak tree 125 ft in total height are enumerated below.

1. Calculate the total and individual grade Doyle volumes with Equation 1 using the coefficients for Doyle volume of all red oak (ARO) from Table 3 (an excerpt from Supplemental Table S3). This step provides initial, unadjusted volumes for each grade.

$$\hat{V}_T = -40.08 + 0.00753(30^{2.424})(125^{0.734}) = 951.8\text{bf}$$

$$\hat{V}_1 = -5.26 + 0.00061(30^{3.132})(125^{0.649}) = 587.1\text{bf}$$

$$\hat{V}_2 = -80.47 + 1.80340(30^{1.218})(125^{0.228}) = 261.0\text{bf}$$

$$\hat{V}_3 = -47.85 + 0.50200(30^{1.312})(125^{0.401}) = 253.8\text{bf}$$

$$\hat{V}_4 = -79.91 + 7.08800(30^{0.782})(125^{0.153}) = 132.2\text{bf}$$

$$\hat{V}_5 = -7.64 + 1.17490(30^{1.782})(125^{-0.350}) = 85.3\text{bf}$$

2. Find the probabilities of grade 1, 2, 3, 4, and 5 occurring in the tree by applying Equation 2 with the logistic coefficients for ARO Doyle volume from Table 3.

$$\hat{P}_1 = \frac{1}{1 + e^{-\left[-7.68+0.43136(30)-0.271\left(\frac{951.8}{125}\right)\right]}} = 0.9607$$

$$\hat{P}_2 = \frac{1}{1 + e^{-\left[-4.07+0.27351(30)-0.388\left(\frac{951.8}{125}\right)\right]}} = 0.7651$$

$$\hat{P}_3 = \frac{1}{1 + e^{-\left[8.30-0.46215(30)+0.833\left(\frac{951.8}{125}\right)\right]}} = 0.6853$$

$$\hat{P}_4 = \frac{1}{1 + e^{-\left[1.51-0.19449(30)+0.343\left(\frac{951.8}{125}\right)\right]}} = 0.1528$$

$$\hat{P}_5 = \frac{1}{1 + e^{-\left[-2.80+0.08384(30)+0.274\left(\frac{951.8}{125}\right)\right]}} = 0.8583$$

3. Multiply each grade volume from Step 1 by its probability of occurrence from Step 2 (Equation 4) to obtain the unadjusted grade volume estimates.

$$\hat{V}_1^{\text{est}} = 0.9607(587.1) = 564.0\text{bf}$$

$$\hat{V}_2^{\text{est}} = 0.7651(261.0) = 199.7\text{bf}$$

$$\hat{V}_3^{\text{est}} = 0.6853(253.8) = 173.9\text{bf}$$

$$\hat{V}_4^{\text{est}} = 0.1528(132.2) = 20.2\text{bf}$$

$$\hat{V}_5^{\text{est}} = 0.8583(98.2) = 84.3\text{bf}$$

4a. Adjust the estimated volumes from Step 3 using Equation 7 to sum to the total tree volume estimated in Step 1.

$$\sum_j V_j^{\text{est}} - \hat{V}_T = (564.0 + 199.7 + 173.9 + 20.2 + 84.3) - 951.8 = 90.2$$

$$\sum_j (\hat{V}_j^{\text{est}})^{-1} = \frac{1}{564.0} + \frac{1}{199.7} + \frac{1}{173.9} + \frac{1}{20.2} + \frac{1}{84.3} = 0.073898$$

$$\hat{V}_1^{\text{adj}} = 564.0 - \frac{90.2\left(\frac{1}{564.0}\right)}{0.073898} = 561.8\text{bf}$$

$$\hat{V}_2^{\text{adj}} = 199.7 - \frac{90.2\left(\frac{1}{199.7}\right)}{0.073898} = 193.6\text{bf}$$

$$\hat{V}_3^{\text{adj}} = 173.9 - \frac{90.2\left(\frac{1}{173.9}\right)}{0.073898} = 166.9\text{bf}$$

$$\hat{V}_4^{\text{adj}} = 20.2 - \frac{90.2\left(\frac{1}{20.2}\right)}{0.073898} = -40.7\text{bf}$$

$$\hat{V}_5^{\text{adj}} = 84.3 - \frac{90.2\left(\frac{1}{84.3}\right)}{0.073898} = 69.8\text{bf}$$

4b. The adjusted estimate for grade 4 was negative. To obtain a set of nonnegative volumes that sum to the total volume, set grade volume 4 to 0, and repeat Step 4 with grade volume 4 excluded and substitute the adjusted volumes for the estimated volumes (i.e., the adjusted volumes are readjusted).

$$\sum_j V_j^{\text{est}} - \hat{V}_T = (561.8 + 193.6 + 166.9 + 69.8) - 951.8 = 40.3$$

$$\sum_j (\hat{V}_j^{\text{est}})^{-1} = \frac{1}{561.8} + \frac{1}{193.6} + \frac{1}{166.9} + \frac{1}{69.3} = 0.027367$$

$$\hat{V}_1^{\text{adj}} = 561.8 - \frac{40.3\left(\frac{1}{561.8}\right)}{0.027367} = 559.26\text{bf}$$

$$\hat{V}_2^{\text{adj}} = 193.6 - \frac{40.3\left(\frac{1}{193.6}\right)}{0.027367} = 186.0\text{bf}$$

$$\hat{V}_3^{\text{adj}} = 166.9 - \frac{40.3\left(\frac{1}{166.9}\right)}{0.027367} = 158.1\text{bf}$$

$$\hat{V}_5^{\text{adj}} = 69.8 - \frac{40.3\left(\frac{1}{69.8}\right)}{0.027367} = 48.7\text{bf}$$

If any of the adjusted volumes are still negative (although not the case in this example), set them to zero and repeat the adjustment procedure (Step 4b) until all volumes are nonnegative.

Adjusting grade volumes that are a small proportion of the total volume to zero is consistent with the weight $1/\hat{V}_i^{\text{est}}$. The weight adjusts volume estimates with the greatest value and relative precision the least by throwing most of the error into the grade having the lowest proportional volume and greater relative error.

Conclusions

Individual tree total merchantable sawtimber volume models for five species groups within recently undisturbed, even-aged red oak-sweetgum forest mixtures in mid-south minor stream bottoms produced logical and consistent predictions. Nonlinear R^2 ranged from 0.70 to 1.00. Volume distribution models for five log grades predicted the expected average grade volume of trees by species group and produced reasonable estimates. These models are not intended to precisely estimate the grade volume in an individual tree but rather provide good average grade volume across a number of stands.

Parameters are presented for two sets of model predictor variables (dbh plus total height and dbh plus merchantable height) commonly recorded in forest inventories and affording the greatest utility. Merchantable height was a marginally better predictor variable than total height. Merchantable sawlog Cvib and Cvob were generally the least biased and most precise among the five volume units modeled across the different species groups and log grades.

The single-tree grade distribution models are used within a red oak-sweetgum mixture individual tree growth and yield simulator to estimate the per unit land area distribution volume by dbh class. The interface allows the user to select from five species groups, five volume units, total or merchantable sawlog height, and input dbh to return the estimated average volume by log grade for the given input selections. The ability to quantify standing timber by grade for recently undisturbed, even-aged red oak-sweetgum forest mixtures in mid-south minor stream bottoms should lead to a better understanding of the following: how these dynamic forest complexes develop; what their potential value is; and how to improve management strategies. The prediction system should only be used on trees in forest conditions described in this article.

Endnotes

1. Downloadable from fwrc.msstate.edu/software.asp or timbercruise.com/Downloads/GYModels/BLHWGYSetup.exe.
2. Downloadable from timbercruise.com/Downloads/GYModels/MSU_USFS_GradeVolumeCalculatorSetup.exe.

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