J. For. 112(1):51-54
http://dx.doi.org/10.5849/jof.13-065

Accurately Measuring the Height of (Real) Forest Trees

Don C. Bragg

Quick and accurate tree height[1] measurement has always been a goal of foresters. The techniques and technology to measure height were developed long ago—even the earliest textbooks on mensuration showcased hypsometers (e.g., Schlich 1895, Mlodziansky 1898, Schenck 1905, Graves 1906), and approaches to refine these sometimes remarkable tools appeared in the first issues of *Forestry Quarterly*, *Proceedings of the Society of American Foresters*, and the *Journal of Forestry*. For example, one such hypsometer based on the geometric principle of similar triangles (top of Figure 1) employed rotary mirrors to allow the user to simultaneously see the top and bottom of the tree in "proper parallax" (Tieman 1904). Other early hypsometers applied different approaches that used angles and distance (e.g., Graves 1906, Detwiler 1915, Noyes 1916, Krauch 1918). Of these trigonometric hypsometers, those that calculated total tree height (*HT*) as a function of the tangent of the angles to the top (B_2) and bottom (B_1) of the tree and a baseline horizontal distance (b) to the stem were most common (Figure 1).

Because they are easy to apply and required only simple technology, these approaches (hereafter, the similar triangles and tangent methods) have dominated tree height measurement. That is not to say the challenge of accurate tree height measurement was solved—many early foresters reported problems with getting consistent data in uneven terrain or dense understories or with the use of different types of hypsometers. Good measurement practices usually mitigated these issues and became standard components of forester training programs. Others addressed these challenges by designing new techniques based on different trigonometric relationships. As an example, Haig (1925) proposed a slide rule solution that calculated tree height using the sine law and slope distance from the observer to the tree base (s_1)

$$HT = \frac{s_1 \sin(B_2 + B_1)}{\sin A_1}$$

(see the trigonometric example in Figure 1 for the angles involved). In very steep terrain, the ability to use a slope distance rather than horizontal distance (b) made height determination much simpler. McArdle and Chapman (1927) applied a different trigonometric solution for sloping ground when determining tree heights. Using a fixed slope distance ($s_1 = 100$ ft) from the eye of the observer to the base of the tree, they proposed the following: $HT = 100 \sin B_1 + 100 \cos B_1 \tan B_2$.

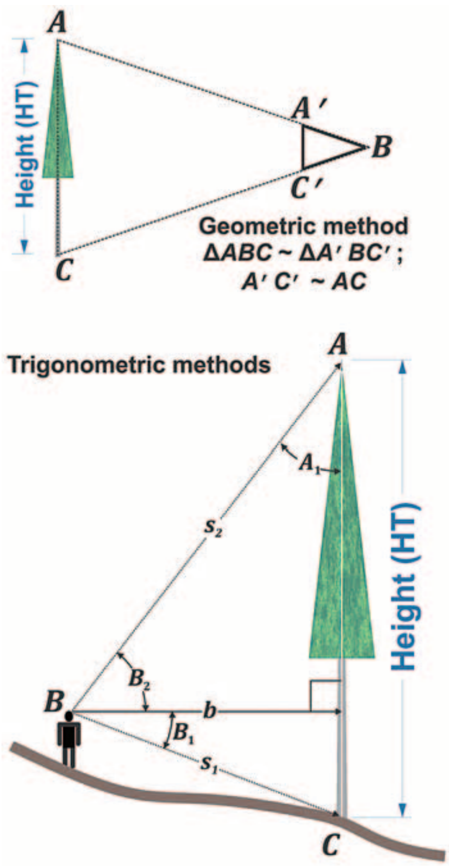

Figure 1. Graphical representation of tree height measurement on sloping ground (the same mathematics apply for a level surface) using the similar triangles and tangent methods. Note that all historic equations and figures referenced in this work follow the notation of these figures.

Yet such corrections, although they improved the reliability of indirect height measurement, failed to address other potential sources of error. For these adjustments to work, the trees had to be truly vertical, i.e., the point at their base measured for the horizontal distance was directly below the highest point at the top of the tree. As an example, if the subject tree happened to be leaning, the determination of A_1 in Haig's correction is less straightforward and required additional measurements. Hypsometers based on similar triangles could be used on leaning stems, but the geometry of this approach requires the user to exactly match the angle of lean, a daunting and imprecise task in most forested settings. For tree height measuring devices that used the tangent method, adjusting for lean also presented a serious challenge. Falconer (1931, p. 744) succinctly addressed this quandary:

Received August 19, 2013; accepted November 18, 2013.

Affiliations: *Don C. Bragg (braggd@safnet.org) is a research forester with the USDA Forest Service, Monticello, AR, and editor-in-chief of the* Journal of Forestry.

Acknowledgments: *I recognize the contributions of the Native Tree Society for their efforts to develop and promote the sine method. Mike Shelton, Nancy Koerth, and Jim Guldin (all of the USDA Forest Service), Bob Leverett, and several anonymous reviewers all contributed to the development of this article.*

> The incorrect location or choice of the point to which the measurement for distance is taken is the only common and continued source of error in measuring the height of leaning trees. The error is due, not to the actual chainage of the distance, but in the location of the point to which the distance is measured. The distance can be accurately measured only when the point is located on the ground vertically below the tip of the tree.

In other words, because the tangent method projects height as a function of the horizontal distance from the observer to the apex of the crown, if a tree leans away from or toward the observer, the proper baseline distance must be adjusted for accordingly. Think of how tree height measurement is often portrayed: the images are almost always straight, vertical, pointed crown conifers growing on level ground—an all-too-often inadequate representation of "real" trees in a more complicated forest environment.

Hypsometers based on the mathematical principles of similar triangles and tangents incorporate assumptions that are only sometimes (rarely?) met in the field. Thick undergrowth could be cleared and sloping ground adjusted for, but tree lean and crown irregularities proved much harder to address. A fix for these was critical: Chapman (1921, p. 246) noted that "(t)he error from the measurement of broad-crown trees [unless corrected] is cumulative and tends to over-estimate their heights." To this end, Graves (1906) recognized the challenges of measuring leaning and wide-crowned trees and proposed corrections based on measuring heights perpendicular to the lean and moving further away to ensure that subordinate branches are not mistaken for the highest point of the crown. Likewise, Krauch (1918, 1922) suggested a number of adjustments for the USDA Forest Service standard hypsometer to improve its accuracy and consistency in determining height.

Such ad hoc adjustments, designed for expedience, also came with their own assumptions, for example, failure to identify the actual nadir of the highest point of a leaning or wide-crowned tree (a distinct possibility under many circumstances) means that the horizontal baseline distance remained erroneous, and hence the estimate of total height remained inaccurate. Researchers continued to offer improvements: Falconer (1931) recommended using plumb bobs to identify the point on the ground directly below the highest point of a leaning tree using a three-person crew (two to handle the plumb lines and the third to identify the point on which these two vertical planes intersect). Unfortunately, such an approach required further training of field crews, significantly increased the amount of time needed to measure tree heights, especially in heavy timber or rough terrain, and may have necessitated adding staff for proper execution.

Such additional burdens did not endear themselves to forestry operations wanting to streamline inventory procedures. Not surprisingly, then, less rigorous corrections for tree lean and wide crowns became ingrained in mensuration textbooks and professional curricula. The study of field-based tree height measurement focused on optimizing the efficiency of existing techniques (e.g., Barrett 1929, Morey 1931, Hunt 1959, Bruce 1975, Rennie 1979, Long and Mohai 1986, Williams et al. 1994) and the reintroduction and/or modification of largely unused tools (e.g., Curtis and Bruce 1968, Anunchin 1971, Buckner et al. 1977, Larsen et al. 1987). Overall, most foresters were satisfied with the degree of accuracy possible from existing approaches (when the proper degree of fealty to adjusting for atypical trees was observed) and that continuity in technique over time trumped any possible impacts of systemic bias. To borrow a judicial phrase, it seems that height measurement was considered "settled law."

However, in the latter half of the 20th century, a new approach to tree height measurement that could address the complexity of tree architecture appeared. Starting in the 1950s, forest mensurationist Lewis R. Grosenbaugh developed and refined dendrometers capable of measuring tree heights, stem diameters, and bole volume under many different conditions (e.g., Grosenbaugh 1954, 1963, 1980, 1981, 1991). Fundamentally, Grosenbaugh (1963, p. 27) rejected the notion that it was not possible to increase the precision and accuracy of tree measurement, faulting "[g]roup pessimism after unsatisfactory trials of low-potential dendrometers [that] has fostered a defeatist attitude that has rejected dendrometers as impractical but *accepted the improper use of biased volume tables as inevitable*" (emphasis added). In making this statement, Grosenbaugh recognized an inherent tendency of many (perhaps most) foresters to embrace a correctable source of error. Although his work focused primarily on measuring diameter and estimating volume, accurately determining height was critical to this effort. His seminal *Forest Science* monograph provided such an insight: in his Figure 3, Grosenbaugh (1963, p. 8) provided the following equation for "exact" height, $H_0 = R'' \sin \theta''$, where the height to the diameter measurement location (H_0) was a function of a sloping baseline distance (R'') and angle between the observer and the measurement location (θ''). He used sine rather than tangent at this point because the slope distance (whose measurement was possible using a rangefinder-dendrometer) was required to get the exact height above the ground for that point along the trunk.

Grosenbaugh (1980, p. 204) later introduced further measures to avoid dendrometry bias for leaning trees because "…range, height, and (for optical forks) diameter biases attributed to neglected lean toward or away from observer can be much more serious and are *not* overcome by merely tilting dendrometer and hypsometer to match tree tilt in crosslevel." This assertion questioned the use of certain ad hoc corrections for leaning trees. His use of spherical trigonometry yielded a more complete version of the total tree height (Grosenbaugh 1980, p. 207), $s_2 \sin B_2 - s_1 \sin B_1$, representing the vertical difference between the crown high point (A) and the intersection of the bole with the ground (C),where B_1 and B_2 are the angles to the bottom and top of the tree, respectively, and s_1 and s_2 are the corresponding "slant range" (the sloping baseline distance from his 1963 article) estimates from the observer to those points (Figure 1).

For a truly vertical tree on level ground, this "sine method" yielded the exact same height as the tangent method (i.e., $HT = |b \tan B_2 - b \tan B_1| = |s_2 \sin B_2 - s_1 \sin B_1|$). Figure 2 highlights the fact that for a leaning tree, if the uncorrected distance to the stem (i.e., $b = b_2 + c_1$) is used to estimate height rather than the adjusted baseline length (b_2), the tangent method will overestimate height if the tree is leaning toward the observer and underestimate height if leaning away (not shown). Because the sine method directly measures slope distances s_1 and s_2, the proper height ($HT = H_1 + H_2$) will be calculated regardless of the direction or magnitude of tree lean (Figure 2). In fact, the sine method is also independent of ground slope, distance from the stem, angle of observation, and virtually all of the other assumptions of the similar triangles and tangent approaches (Blozan 2006, Bragg 2008, Bragg et al. 2011). This does not mean that foresters do

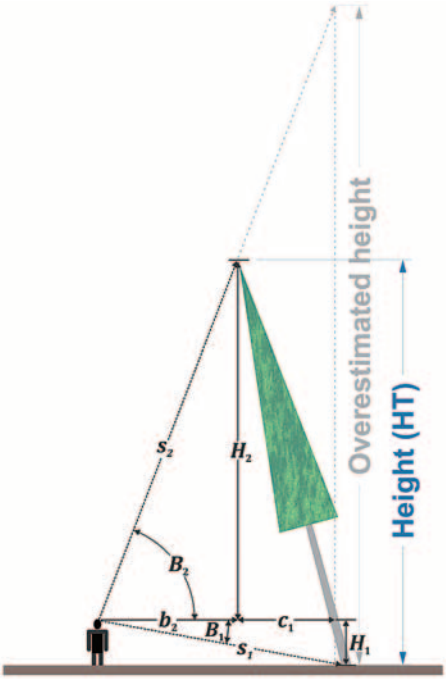

Figure 2. On level ground, it is easy to see the impact of an inappropriate baseline ($b_2 + c_1 = b$) on the estimated tree height using the tangent method: the proper baseline length (b_2) must be determined to accurately estimate tree height.

Figure 3. An example of an offset crown and how this may influence height estimation using the traditional approaches. In these examples of mature eastern white pine (*Pinus strobus*), their apparent high points (white arrows) are noticeably offset from the vertical axis of their boles, necessitating adjustments with the similar triangles and tangent approaches to ensure accurate height measurements. It is also quite possible with the pine on the left that the branch identified as the high point may actually represent a lower point in the crown than a different top (black arrow). With laser rangefinders, it is possible to use the sine method to directly calculate the actual height of each of these points, permitting a direct comparison of multiple leaders to determine which is the highest (the "top").

not face challenges in using the sine method to determine the actual total height of the tree—it is still possible to inadvertently select a subordinate point on a crown (yet the sine height would still be correct for that branch). Modern laser rangefinders that include the sine method can be used, however, to quickly scan multiple apparent high tops from the same vantage point (or even different viewpoints) to determine which is the maximum (i.e., total height) (Figure 3; see also discussion in Bragg et al. 2011). For trees with broad, multileader crowns (the deliquescent or decurrent growth form common to most hardwoods), it can be particularly hard to identify the highest point (the "top") using more traditional techniques, because each would have to be corrected for independently to ensure proper height estimates (Bragg 2007).

Although others had used elements of this approach (e.g., Haig 1925, McArdle and Chapman 1927), Grosenbaugh's application of sine and slope distance for tilted (leaning) tree hypsometry also translated to vertical trees growing on sloping ground and for trees with displaced tops (i.e., the highest point on their crowns not directly above the base of the bole; Figure 3) (Grosenbaugh 1981, 1991). However, his continued publication on these approaches years after their introduction suggested that they had failed to achieve much acceptance. Some of the barriers to acceptance probably rested with the instrumentation: the rangefinding dendrometer that he considered to be the most efficient was admittedly expensive and complicated (Grosenbaugh 1991). In addition, the modified technique introduced to address the need for expensive equipment remained mathematically complex and required two observations points (Grosenbaugh 1991), not exactly an efficient adjustment to make in the field, particularly in rough terrain or heavy forest cover.

Because Grosenbaugh's focus was on developing tools and techniques for foresters to calculate the dimensions and volumes of leaning (slanted) trees, rather than their vertical height, it can be understood that he did not emphasize the potential of the sine method for total height determination. After all, even though Grosenbaugh's solution using sine (rather than tangent) was remarkably simple and elegant, it was plagued by the fact that it was very hard to measure slope (slant) distance with the technology then available. Fortunately, more recent efforts led by canopy researcher Dr. Robert Van Pelt, champion tree expert Robert Leverett, arborist Will Blozan, forester Bill Carr, and numerous others have led to the adoption of accurate, relatively inexpensive, and portable laser distance measuring devices capable of performing the sine method. Note that laser-based hypsometers have been on the market for decades and have become an industry standard for many applications (e.g., Williams et al. 1994, Carr 1996, Bragg 2007, Farve 2010), but the default height measurement programs built into most of these devices still use the tangent method and all its inherent assumptions.

Although digital analysis of light detection and ranging (LiDAR) and other remotely sensed, high-resolution images offer new options for estimating total tree height, these tools will not be able to replace on-the-ground measurements of individual trees for many applications. Hence, research into the efficacy of different height measurement techniques, particularly field studies designed to compare various approaches across a range of species, stand conditions, and topographies, should continue. The sine method offers a good blend of accuracy, precision, and efficiency, and its popularity will probably grow as its utility is increasingly recognized.

Endnote

1. To avoid confusion, this article applies the classic definition of total tree height as the vertical distance between a horizontal plane that intersects the bole at groundline and a parallel horizontal plane tangent to the highest point of the crown (sensu Schlich 1895, p. 15–27, Avery and Burkhart 1983, p. 76, Philip 1994, p. 27, Husch et al. 2003, p. 99, and others). It is important to note that total tree height *does not* necessarily equal bole length or merchantable length under this definition, particularly in the case of leaning trees or specimens with broad, deliquescent crowns (see later discussion).

Literature Cited

ANUNCHIN, N.P. 1971. An optical hypsometer. P. 3–6 in *Advances in forest mensuration* (translated from Russian), Ovanesova, V.A., and V.S. Uchenkov (eds.). Israel Program for Scientific Translations, Jerusalem, Israel.

AVERY, T.E., AND H.E. BURKHART. 1983. *Forest measurements*, 3rd ed. McGraw-Hill Book Co., New York. 331 p.

BARRETT, L.I. 1929. Accuracy of Forest Service standard hypsometer. *J. For.* 27(5):587–588.

BLOZAN, W. 2006. Tree measuring guidelines of the Eastern Native Tree Society. *Bull. East. Native Tree Soc.* 1(1):3–10. Available online at www.nativetreesociety.org/bulletin/b1_1/bulletin1_1.htm; last accessed Dec. 5, 2013.

BRAGG, D.C. 2007. The sine method as a more accurate height predictor for hardwoods. P. 23–32 in *Proc. 15th Central Hardwoods Conference*, Buckley, D.S., and W.K. Clatterbuck (eds.). USDA For. Serv., Gen. Tech. Rep. SRS-101, Southern Research Station, Asheville, NC.

BRAGG, D.C. 2008. An improved tree height measurement technique tested on mature southern pines. *South. J. Appl. For.* 32:38–43.

BRAGG, D.C., L.E. FRELICH, R.T. LEVERETT, W. BLOZAN, AND D.J. LUTHRINGER. 2011. *The sine method: An alternative height measurement technique.* USDA For. Serv., Res. Note SRS-22, Southern Research Station, Asheville, NC. 11 p.

BRUCE, D. 1975. Evaluating the accuracy of tree measurements made with optical instruments. *For. Sci.* 21(4):421–426.

BUCKNER, E.R., J.L. RICHESON, J. RENNIE, AND W. BOYD. 1977. Modifying the Christen hypsometer for use in dense conifer stands. *J. For.* 75(3):139–140.

CARR, B. 1996. Using the Criterion 400 and Impulse 200 laser instruments to accurately measure trees. *Compiler* 14(4):6–12.

CHAPMAN, H.H. 1921. *Forest mensuration.* John Wiley & Sons, New York. 553 p.

CURTIS, R.O., AND D. BRUCE. 1968. Tree heights without a tape. *J. For.* 66(1):60–61.

DETWILER, S.B. 1915. Saving labor in measuring tree heights: A new principle applied to the hypsometer. *For. Q.* 13(4):442–444.

FALCONER, J.G. 1931. A method of accurate height measurement for forest trees. *J. For.* 29(5):742–746.

FARVE, R. 2010. *Evaluation of laser rangefinders.* USDA For. Serv., San Dimas Technology & Development Center, San Dimas, CA. 59 p.

GRAVES, H.S. 1906. *Forest mensuration.* John Wiley & Sons, New York. 458 p.

GROSENBAUGH, L.R. 1954. *New tree-measurement concepts: Height accumulation, giant tree, taper and shape.* USDA For. Serv., Occasional Pap. SO-134, Southern Forest Experiment Station, New Orleans, LA. 25 p.

GROSENBAUGH, L.R. 1963. *Optical dendrometers for out-of-reach diameters: A conspectus and some new theory.* Society of American Foresters, Forest Science Monograph 4, Bethesda, MD. 47 p.

GROSENBAUGH, L.R. 1980. Avoiding dendrometry bias when trees lean or taper. *For. Sci.* 26(2):203–215.

GROSENBAUGH, L.R. 1981. Measuring trees that lean, fork, crook, or sweep. *J. For.* 79(2): 89–92.

GROSENBAUGH, L.R. 1991. Tilted tree hypsometry. *For. Sci.* 37(6):1581–1590.

HAIG, I.T. 1925. Short cuts in measuring tree heights. *J. For.* 23(11):941–944.

HUNT, E.V. JR. 1959. A time and accuracy test of some hypsometers. *J. For.* 57(9):641–643.

HUSCH, B., T.W. BEERS, AND J.A. KERSHAW JR. 2003. *Forest mensuration,* 4th ed. John Wiley & Sons, Hoboken, NJ. 443 p.

KRAUCH, H. 1918. Some new aspects regarding the use of the Forest Service standard (gradimeter) hypsometer. *J. For.* 16(7):772–776.

KRAUCH, H. 1922. A new method of measuring tree heights on sample plots. *J. For.* 20(3): 220–222.

LARSEN, D.R., D.W. HANN, AND S.C. STEARNS-SMITH. 1987. Accuracy and precision of the tangent method of measuring tree height. *West. J. Appl. For.* 2(1):26–28.

LONG, J.N., AND P. MOHAI. 1986. A simplified procedure for measuring tree heights directly from slope distance. *West. J. Appl. For.* 1(2): 58–59.

MCARDLE, R.E., AND R.A. CHAPMAN. 1927. Measuring tree heights on slopes. *J. For.* 25(7): 843–847.

MLODZIANSKY, A.K. 1898. *Measuring the forest crop*. USDA For. Serv., Bull. 20, Forestry Division, Washington, DC. 71 p.

MOREY, H.F. 1931. A test of hypsometers on short trees. *J. For.* 29(2):233–237.

NOYES, D.K. 1916. Comparative test of the Klaussner and Forest Service standard hypsometers. *Proc. Soc. Am. For.* 11(4):417–424.

PHILIP, M.S. 1994. *Measuring trees and forests*, 2nd ed. CAB International, Wallingford, UK. 310 p.

RENNIE, J.C. 1979. Comparison of height-measurement techniques in a dense loblolly pine plantation. *South. J. Appl. For.* 3(4):146-148.

SCHENCK, C.A. 1905. *Forest mensuration.* Univ. Press of Sewanee Tennessee, Sewaneee, TN. 71 p.

SCHLICH, W. 1895. *A manual of forestry. Vol. III. Forest management.* Bradbury, Agnew, & Company, London, UK. 397 p.

TIEMAN, H.D. 1904. A new hypsometer. *For. Q.* 2(3):144–147.

WILLIAMS, M.S., W.A. BECHTOLD, AND V.J. LABAU. 1994. Five instruments for measuring tree height: An evaluation. *South. J. Appl. For.* 18(2):76–82.