

Stand-Level Growth and Yield Component Models for Red Oak–Sweetgum Forests on Mid-South Minor Stream Bottoms

Emily B. Schultz, J. Clint Iles, Thomas G. Matney, Andrew W. Ezell, James S. Meadows, Ted D. Leininger, W. Cade Booth, and J. Paul Jeffreys

ABSTRACT

Greater emphasis is being placed on Southern bottomland hardwood management, but relatively few growth and yield prediction systems exist that are based on sufficient measurements. We present the aggregate stand-level expected yield and structural component equations for a red oak (*Quercus* section *Lobatae*)-sweetgum (*Liquidambar styraciflua* L.) growth and yield model. Measurements from 638 stand-level observations on 258 distinct permanent growth and yield plots collected in 1981, 1988, 1994, and 2006 in minor stream bottoms in Mississippi and Alabama provided data for model development. Equations for average height of dominant and codominant red oaks, trees/ac, arithmetic mean diameter, quadratic mean diameter, and volume were selected on the basis of significance of independent variables, coefficient of determination, index of fit, and biological validity assessment. These models produce expected average yields for combined species or species groups in naturally developing stands and provide an average baseline for individuals managing their lands for the red oak–sweetgum complex. Models will be integrated with log grade volume and diameter distribution models that are in concurrent development to produce a growth and yield system capable of comparing management alternatives on a financial basis.

Keywords: source code, example calculations, diameter distribution moment recovery

Red oak (*Quercus* section *Lobatae*)-sweetgum (*Liquidambar styraciflua* L.) forest mixtures in the southeastern United States are important to wildlife habitat, water quality, and the production of grade hardwood for furniture, flooring, veneer, and other products (Banzhaf 2009). Cherrybark (*Quercus pagoda* Raf.), Shumard (*Quercus shumardii* Buckl.), and Nuttall (*Quercus texana* Buckl.) oaks are three of the forest's most highly desired and valued grade hardwood species. The red oak–sweetgum complex is the most widely distributed of the high value timber forests in the state of Mississippi, and models that describe growth and yield, log grade, and stand development are essential to its management and sustainability.

A majority of Southern pine growth and yield research has focused on a commercially important single species in even-aged stands, such as loblolly (*Pinus taeda* L.) (Amateis and Burkhart 1981, Matney and Farrar 1992), slash (*Pinus elliotii* Engelm.) (Zarnoch et al. 1991), and longleaf (*Pinus palustris* Mill.) (Farrar and Matney 1994, Meador 2002) pines. Mixed-species stands are more complex in structure, and predicting their growth is more difficult because of the varying composition and density of a number of species within a given stand.

This report presents the aggregate stand-level expected yield and structural estimation models for a red oak–sweetgum forest mixture. These models provide predictions for stand attributes by species groups and form the basis for diameter distributions and a future stand-level bounded growth and yield model constructed from the

same data. Arithmetic and quadratic mean dbh prediction equations allow the construction of diameter distribution moment recovery models (Matney and Farrar 1992, Farrar and Matney 1994). McTague et al. (2008) give a brief overview of existing individual tree growth models and present individual tree growth models for mixed-species Southern hardwood stands. Our objective was to estimate expected stand-level yields. Future articles will report growth equations and diameter distribution models for the data in a later article.

Data

The data available for model development were the 638 stand-level observations on 258 distinct permanent red oak–sweetgum growth and yield plots collected in 1981, 1988, 1994, and 2006 in minor stream bottoms (Hodges and Switzer 1979) formed from local soils in Mississippi and Alabama. Some plots were measured in all four data collection years and others in one, two, or three of the years. Only 160 of the 258 plots are now in existence. The geographic distribution of the plots is shown in Figure 1. Circular plots were established in even-aged unmanaged stands with minimums of 60 ft²/ac total basal area and 30% red oak basal area. Plot size ranged from a minimum of 0.1 ac to a maximum of 1.0 ac. No maximum age was imposed on stands, but the minimum selection age was 20 years and the minimum dbh was 3.6 in. Plot locations were selected to capture a wide range of site qualities and stand ages (Table 1) within criteria to ensure that models would be applicable to a variety

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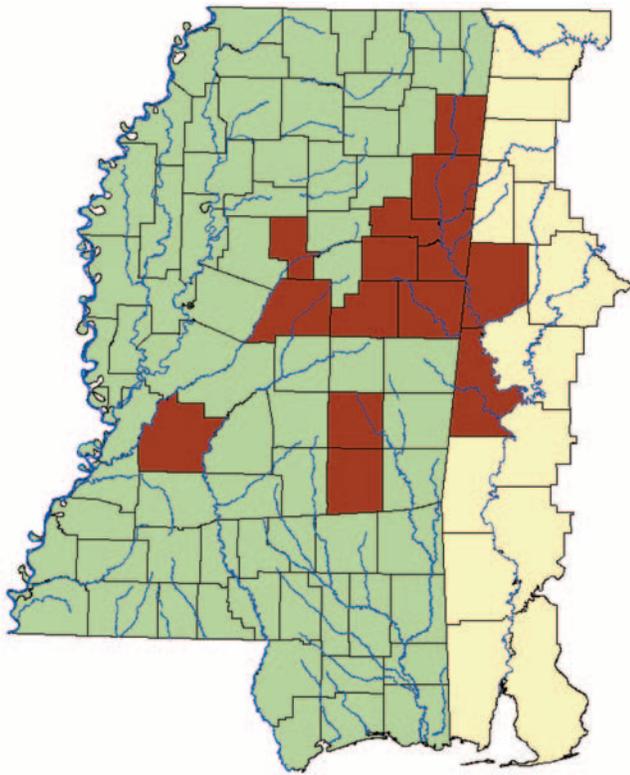


Figure 1. Mississippi and Alabama counties containing permanent growth and yield plots in red oak-sweetgum forest mixtures on minor stream bottoms (Iles 2008).

Table 1. Age class and red oak species site index (base age, 50 years) frequency distribution for observations in permanent growth and yield plots of red oak-sweetgum forest mixtures on minor stream bottoms in Mississippi and Alabama.

Age class	Site index (base age, 50 years)							All
	70	80	90	100	110	120	130+	
20	1	1	2	6	8	1	1	20
30	0	2	6	15	31	6	0	60
40	1	3	20	42	49	21	4	140
50	0	4	24	57	52	14	2	153
60	1	1	14	56	42	8	5	127
70	0	0	7	32	32	11	2	84
80	0	1	4	19	11	6	0	41
90+	0	0	2	6	5	0	0	1
All	3	12	79	233	230	67	14	638

of sites. Funding for plot establishment, remeasurement, and analysis was provided by the US Forest Service Center for Bottomland Hardwoods Research, Stoneville, MS.

Data were categorized into six species groups: red oak, white oak (*Quercus* section *Quercus*), sweetgum, hickory (*Carya* species), other commercial, and noncommercial, on the basis of commercial importance and frequency. Cherrybark oak, water oak (*Quercus nigra* L.), and willow oak (*Quercus phellos* L.) were the most frequently observed species in the red oak group, and swamp chestnut oak (*Quercus michauxii* Nutt.), white oak (*Quercus alba* L.), and overcup oak (*Quercus lyrata* Walt.) were the most frequently observed white oaks. Species that had commercial sawtimber value but did not occur frequently enough to form their own group were categorized as other commercial. Yellow-poplar (*Liriodendron tulipifera* L.), green ash (*Fraxinus pennsylvanica* Marsh.), and sugarberry (*Celtis*

laevigata Willd.) were the most common examples. Species with no commercial sawtimber value, such as American hornbeam (*Carpinus caroliniana* Walt.) and eastern hophornbeam (*Ostrya virginiana* [Mill.] K. Koch) were categorized as noncommercial.

Species, dbh, crown class, and azimuth and distance from plot center were recorded for all plot trees. Height to the first live limb and sawtimber merchantable height were recorded for sawtimber-sized trees. Total height, merchantable height, and heights to both a 4-in. and an 8-in. top were measured on 10 trees per plot selected randomly across the range of dbh on each plot. Log grade data were recorded only for trees measured for total height. Total height, dbh, and azimuth and distance from plot center were recorded on all ingrowth trees (trees not recorded in the last remeasurement) with dbh of 4 in. or greater.

Stand-level summary statistics for each species group are presented in Table 2. Stand density, across all species, ranged from 85 to 742 trees/ac, with an average of 262 ± 126 trees/ac. Basal area ranged from 44 to 245 ft²/ac, with an average of 138 ± 28 ft²/ac. Sweetgum was the most abundant species (Figure 2), whereas red oaks dominated stands in terms of average dbh (Figures 3 and 4), basal area (Figure 5), and board foot volume (Figure 6). Site index was predicted from measurements of age at dbh and total height on six dominant and codominant red oak trees per plot using an equation (Equation 2) developed for the red oak species group. Actual plot ages of red oaks ranged from 15 to 92 years, with an average of 52 ± 16 years. Red oak site indices ranged from 67 to 133 ft at age 50 years, with an average of 105 ± 10 ft.

Methods

Combined species and species group models were created for attributes integral to the estimation of stand-level yields and the creation of primary drivers for diameter distribution recovery models in a complete growth and yield system. Combined species models were constructed for trees per acre (TPA), arithmetic mean dbh (AD), and quadratic mean dbh (QD). Average height of dominants and codominants (HD) was modeled for the red oak species group only. Species group ratio equations were developed for TPA, AD, and QD to proportion out species group contributions from the total. Development of a stand-level volume prediction system based on predicted stand structural variables allowed calculation of volume (VOL) in cubic feet outside (ob) and inside (ib) bark, and in board feet (bd ft) for Doyle, Scribner, and International $\frac{1}{4}$ log rules for combined species and species groups. Board foot total stem volumes were calculated using equations developed in an associated study (Banzhaf 2009). A least-squares adjustment procedure was used to adjust species group estimates for TPA, AD, QD, and VOL so that they were logically consistent with estimated totals.

Stand Structure Variable Predictions

Combined Species Models

A weighted nonlinear Chapman-Richards function (Equation 1) was constructed to predict the height of dominant and codominant red oaks from red oak age. Red oak was the predominant overstory species group. Homogeneity of variance was enforced by using a weight of $1/\text{Age}^2$.

$$\widehat{\text{HD}} = a(1 - e^{-b\text{Age}})^c, \quad (1)$$

where HD = average height of the dominant and codominant red oaks in feet; Age = age of the dominant and codominant red oaks;

Table 2. Summary statistics for growth and yield plots in red oak–sweetgum forest mixtures on minor stream bottoms in Mississippi and Alabama.^a

Species groups	AD			QD			TPA		
	Minimum	Mean	Maximum	Minimum	Mean	Maximum	Minimum	Mean	Maximum
All species	4.86	9.27	15.9	5.16	10.63	17.77	85.0	261.9	741.5
Red oaks	4.78	15.11	34.5	5.04	15.75	34.50	2.9	62.7	213.4
White oaks	0.0	4.11	25.25	0.0	4.35	27.33	0.0	6.5	280.0
Sweetgum	0.0	8.78	21.38	0.0	9.31	22.49	0.0	133.4	581.1
Hickories	0.0	3.94	21.15	0.0	4.16	22.52	0.0	13.6	145.0
Other commercial	0.0	6.50	33.70	0.0	6.89	33.85	0.0	23.1	220.0
Noncommercial	0.0	4.49	13.60	0.0	4.66	14.38	0.0	22.6	220.0

^a AD, average plot arithmetic mean dbh; QD, average plot quadratic mean dbh; TPA, trees/ac.

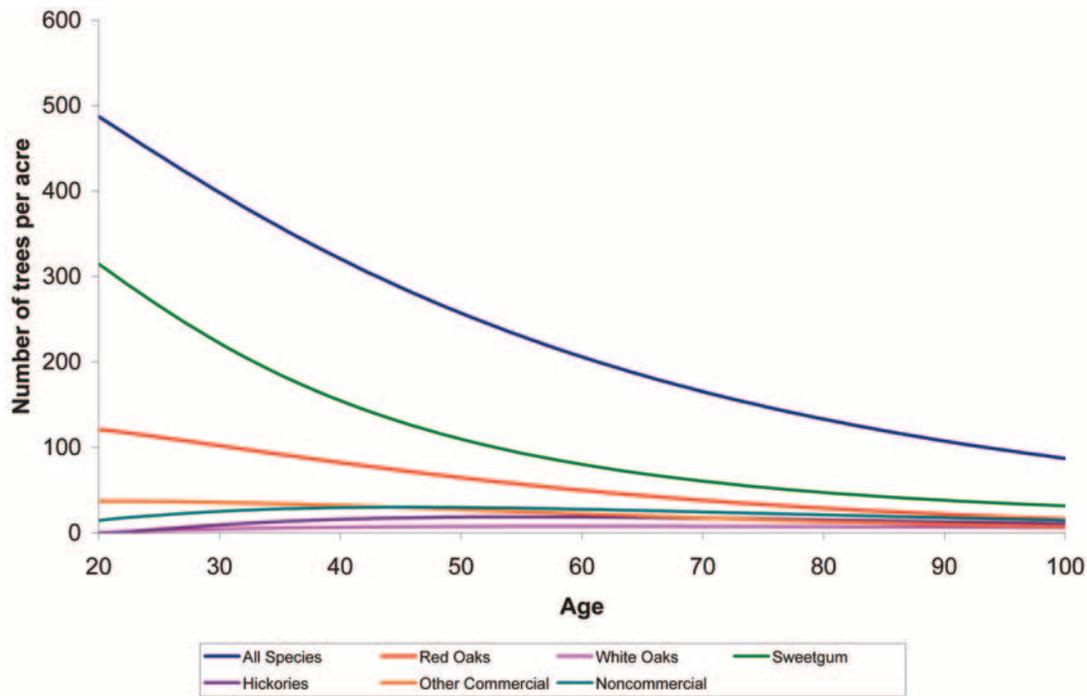


Figure 2. Expected trees per acre (total and species groups) for site index 105 ft (base age, 50 years) red oak–sweetgum forests on minor stream bottoms in Mississippi and Alabama.

e = base of natural logarithm; and a , b , and c are parameters to be estimated from the data.

The HD equation was converted into an anamorphic site index (base age, 50 years) as given in Equation 2.

$$\widehat{SI} = HD \left(\frac{1 - e^{b \text{BaseAge}}}{1 - e^{b \text{Age}}} \right)^c, \quad (2)$$

where SI = site index (base age, 50 years) of red oaks in feet; Base-Age = 50 years; and a , b , and c are parameters from Equation 1. HD is estimated from SI on rearrangement of Equation 2 as follows:

$$\widehat{HD} = SI \left(\frac{1 - e^{b \text{Age}}}{1 - e^{b \text{BaseAge}}} \right)^c. \quad (3)$$

TPA for all species was predicted from age and site index (Equation 4). A weight of Age^2 was applied in the nonlinear regression to create homogeneity of error variance across all levels of the independents,

$$\widehat{TPA}_{\text{all}} = \frac{aSI^b \text{Age}^c}{(1 + e^{d \text{Age}^e})^f}, \quad (4)$$

where TPA_{all} = combined species trees/ac; and a , b , c , d , e , and f are parameters to be estimated from the data.

Linear regression equations were constructed to predict combined species AD and QD (Equations 5 and 6). A number of different powers, such as $1/8$, $1/4$, $3/8$, and $2/3$, were tested for each independent variable in the regression models, and the equations that gave the best standard error of prediction, coefficient of determination, and residual pattern were selected for final testing in the complete system.

$$\widehat{\ln(AD_{\text{all}})} = a + b \ln(\text{Age}) + \frac{c}{\text{Age}} + d \text{SI}^{2/3} + \frac{e \text{SI}}{\text{Age}^{1/8}} + f \text{TPA}_{\text{all}}, \quad (5)$$

where AD_{all} = combined species arithmetic mean diameter in inches; \ln = natural logarithm; and a , b , c , d , e , and f are parameters

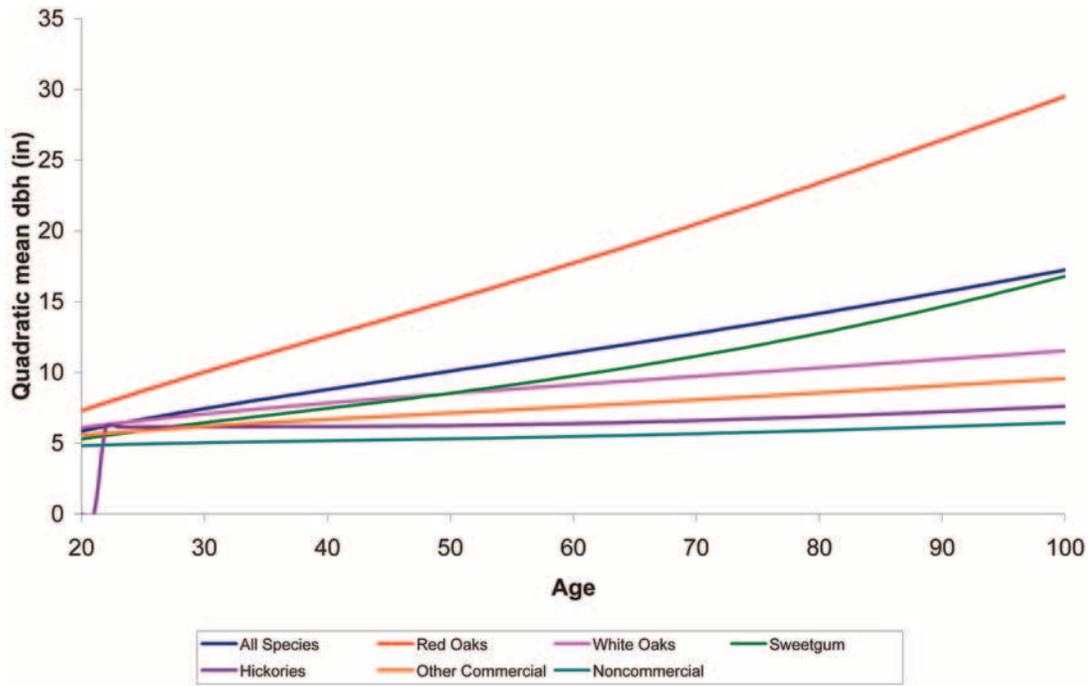


Figure 3. Expected quadratic mean dbh (in.) (total and species groups) for site index 105 ft (base age, 50 years) red oak-sweetgum forests on minor stream bottoms in Mississippi and Alabama.

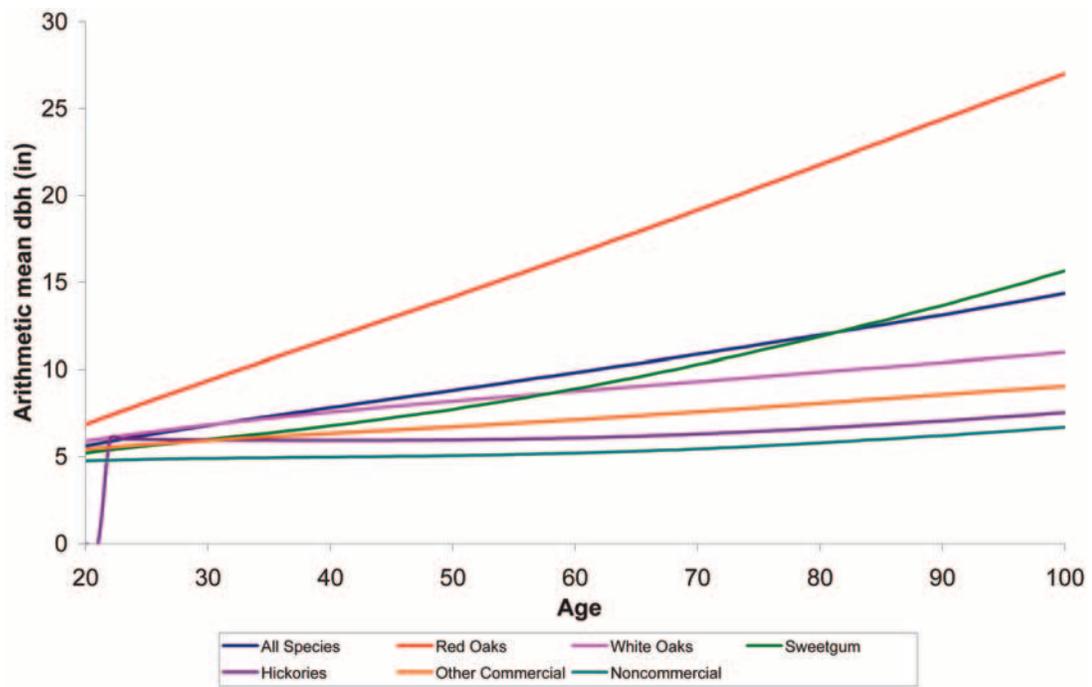


Figure 4. Expected arithmetic mean dbh (in.) (total and species groups) for site index 105 ft (base age, 50 years) red oak-sweetgum forests on minor stream bottoms in Mississippi and Alabama.

to be estimated from the data.

$$\widehat{\ln(QD_{all})} = a + b\ln(Age) + \frac{c}{Age} + dSI^{2/3} + \frac{eSI}{Age^{1/8}} + fTPA_{all} \quad (6)$$

where QD_{all} = combined species quadratic mean diameter in inches; and a , b , c , d , e , and f are parameters to be estimated from the data.

Species Group Models

A stand-level species group model was constructed for TPA_{all} (Equation 7), AD_{all} (Equation 8), and QD_{all} (Equation 9). Depen-

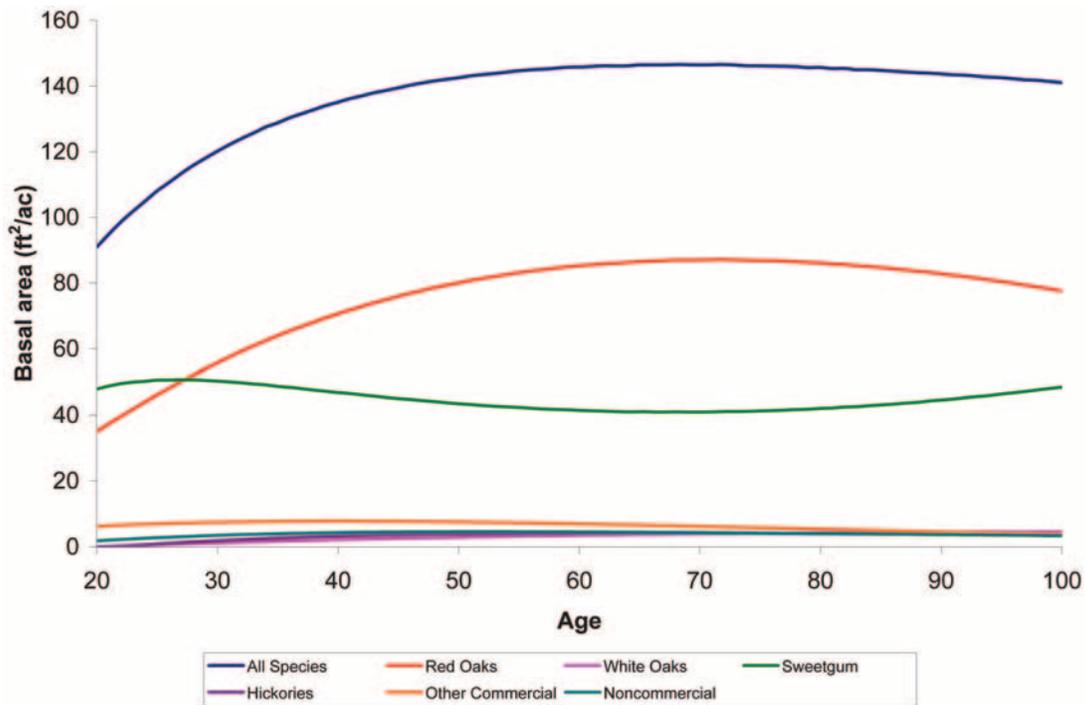


Figure 5. Expected basal area per acre (ft²) (total and species groups) for site index 105 ft (base age, 50 years) red oak-sweetgum forests on minor stream bottoms in Mississippi and Alabama.

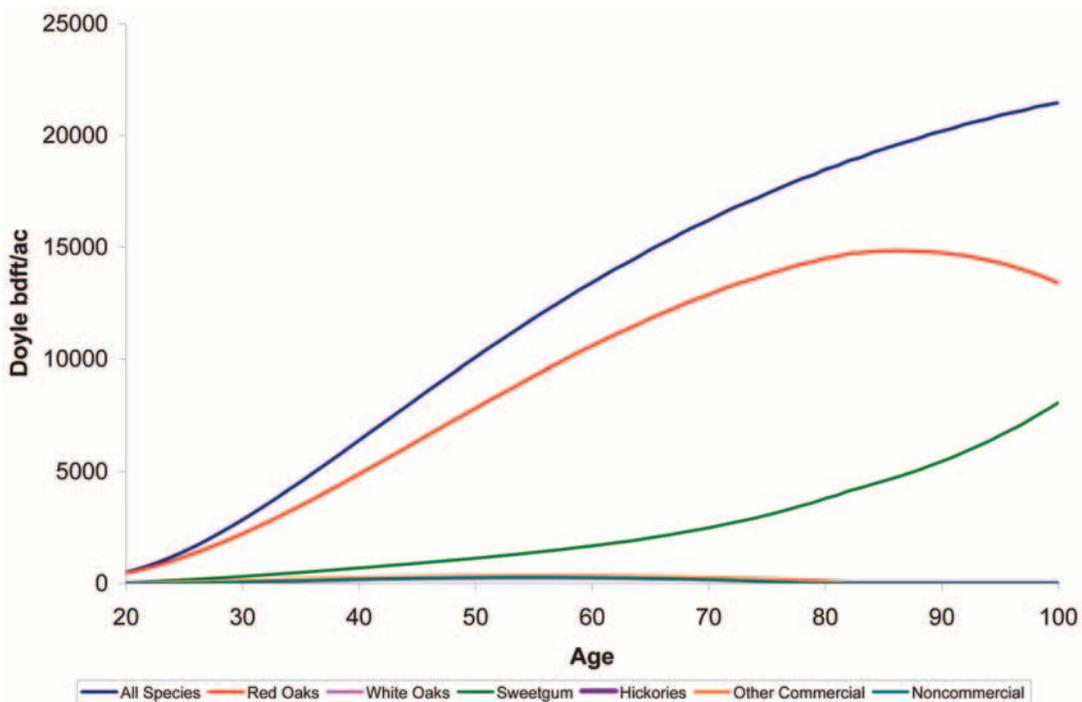


Figure 6. Expected Doyle board feet (total and species groups) for site index 105 ft (base age, 50 years) red oak-sweetgum forests on minor stream bottoms in Mississippi and Alabama.

dent variables were ratios of species group stand-level attributes to combined species stand-level attributes. All independent variables were combined species level attributes. For each species group, TPA_{sp} percentage of the total composition of the stand was plotted over AD_{all} , QD_{all} , and TPA_{all} . Plots indicated no trends that sug-

gested inclusion of species group variables in the model. Johnson and Krinard (1988) observed this same result over 29 years in two cutover red oak-sweetgum stands. They found that initial red oak-sweetgum stand composition varied widely but stands typically had very similar composition by the end of the study period. On the

basis of data inspection and the Johnson and Krinard (1988) findings, it was concluded that regardless of current species composition, future species composition can be predicted by combined species stand-level variables. This determination resulted in simpler models requiring fewer inputs.

$$\widehat{\frac{TPA_{sp}}{TPA_{all}}} = a + bAge + cQD_{all} + dTPA_{all}, \quad (7)$$

where TPA_{sp} = trees/ac for a species group; and a , b , c , and d are parameters to be estimated from the data. The subscript sp on a variable designates the species group associated with the variable (sp = ro for red oaks, wo for white oaks, sg for sweetgum, hk for hickories, oc for other commercial, and nc for noncommercial).

$$\ln\left(\frac{\widehat{AD}_{sp}}{\widehat{AD}_{all}}\right) = a + b\ln(Age) + c\ln(AD_{all}) + dSI, \quad (8)$$

where AD_{sp} = arithmetic mean diameter for a species group in inches; and a , b , c , and d are parameters to be estimated from the data.

$$\ln\left(\frac{\widehat{QD}_{sp}}{\widehat{QD}_{all}}\right) = a + b\ln(Age) + c\ln(QD_{all}) + dSI, \quad (9)$$

where QD_{sp} = quadratic mean diameter for a species group in inches; and a , b , c , and d are parameters to be estimated from the data.

Stand-Level Volume Prediction System

Volume models (Equation 10) were constructed for combined species and unadjusted species group levels, based on age and the corresponding combined species or species group predicted HD, TPA, and QD.

$$\widehat{\ln(VOL)} = a + b\ln(TPA) + c\ln(QD) + d\ln(Age) + \frac{e\ln(TPA)}{Age} + \frac{f\ln(QD)}{Age} + \frac{g\ln(HD)}{Age}, \quad (10)$$

where VOL = total merchantable volume/ac in the units associated with the equation parameters; TPA = trees/ac of all trees for combined species volume or trees/ac for a specific species group volume; QD = quadratic mean dbh of all trees for combined species volume or quadratic mean dbh for a specific species group volume; Age = average age of the red oak component; HD = average height of dominant and codominant red oaks; and a , b , c , d , e , f , and g are parameters to be estimated from the data.

Least-Squares Adjustment

The equations to predict TPA, AD, QD, and VOL for all species and the individual species groups are independently fitted to the data without consideration of the logical relations that must exist between the total and species groups estimates. Independently estimated equations have the lowest bias and greatest precision, but some downgrade of bias and precision must be allowed to produce a logically related prediction system. For the system presented, the sum of the TPA and VOL for each species group must sum to their corresponding totals, and the species TPA weighted arithmetic mean of AD and QD must equal the estimated AD and QD of all

species combined. Expressed mathematically, these logical constraints are

$$\sum_{sp} TPA_{sp}^{adj} = TPA_{all}, \quad (11)$$

$$\sum_{sp} TPA_{sp}^{adj} AD_{sp}^{adj} = TPA_{all} AD_{all}, \quad (12)$$

$$\sum_{sp} TPA_{sp}^{adj} (QD_{sp}^{adj})^2 = TPA_{all} (QD_{all})^2, \quad (13)$$

and

$$\sum_{sp} VOL_{sp}^{adj} = VOL_{all}, \quad (14)$$

where TPA_{all} is the regression estimated combined species TPA; TPA_{sp}^{adj} is the adjusted value of the regression estimated TPA_{sp} of species sp ; AD_{all} is the regression estimated combined species AD; AD_{sp}^{adj} is the adjusted value of the regression estimated AD_{sp} of species sp ; QD_{all} is the regression estimated combined species QD; QD_{sp}^{adj} is the adjusted value of the regression estimated QD_{sp} of species sp ; VOL_{all} is the regression estimated combined species VOL; VOL_{sp}^{adj} is the adjusted value of the regression estimated VOL_{sp} of species sp ; and \sum_{sp} denotes the summation across all six species groups.

A simple and effective way of implementing the constraints is to apply a weighted least-squares procedure minimizing the weighted sum of squared differences between the regression estimated and the adjusted values. The weighting variable is used to control the amount of adjustment with estimates of higher precision receiving proportionally the least amount of adjustment. The following adjustment procedures were found best in terms of minimizing the bias and maximizing the precision of prediction across all 638 plot measurements. The adjustment equations derived for TPA and VOL are simple, but those for AD and QD require the solution of a nonlinear equation for a constant (λ) called the Lagrangian multiplier. The computer code for finding λ is arduous and the Microsoft Visual C++ dynamic link library (dll) project developed to calculate λ is included with the Microsoft Excel spreadsheet implementation of the model found at www.timbercruise.com (Download Center, Growth and Yield Models). The dll exports all of its functions to be called from inside Microsoft Excel's Visual Basic Editor. The implementation of the model is completely implemented inside the Visual Basic Editor via the dll.

Trees per Acre

The weighted least-squares adjustment equation imposing the logical constraint specified by Equation 11 ($\sum_{sp} TPA_{sp}^{adj} = TPA_{all}$) is

$$TPA_{sp}^{adj} = TPA_{sp} - \lambda TPA_{sp}^{-P}, \quad (15)$$

where TPA_{sp}^{adj} is the adjusted value of TPA_{sp} , and

$$\lambda = \frac{\sum_{sp} TPA_{sp} - TPA_{all}}{\sum_{sp} TPA_{sp}^{-P}} \quad (16)$$

is the constraint (Lagrangian) multiplier. This is the solution of the following Lagrangian weighted least-squares equation adjustment (Equation 16) with weight $W_{t_{sp}} = TPA_{sp}^P$ and $P = 0.1$.

$$\begin{aligned} \text{Min } L(TPA_{sp}^{adj}, \lambda) = & \sum_{sp} W_{t_{sp}} (TPA_{sp}^{adj} - TPA_{sp})^2 \\ & + 2\lambda \left(\sum_{sp} TPA_{sp}^{adj} - TPA_{all} \right) \end{aligned} \quad (16)$$

Equation 16 enters the Lagrangian multiplier as 2λ to simplify the algebra of solution by allowing the cancellation of 2 from both sides of the characteristic equation. This insertion is used throughout this article to make algebraic manipulations easier.

Arithmetic Mean Diameter

The TPA_{sp} weighted mean of AD_{sp} must be equal to AD_{all} . That is, the weighted average constraint is

$$\frac{\sum_{sp} TPA_{sp}^{adj} AD_{sp}}{\sum_{sp} TPA_{sp}^{adj}} = AD_{all} \quad (17)$$

The least-squares adjusted AD_{sp} values satisfying the above constraint are calculated using

$$AD_{sp}^{adj} = \frac{AD_{sp}^P AD_{sp} - \lambda TPA_{sp}^{adj}}{AD_{sp}^P}, \quad (18)$$

where λ is the solution to the nonlinear equation

$$\lambda = \frac{\sum_{sp} TPA_{sp}^{adj} AD_{sp} - TPA_{all}^{adj} AD_{all}}{\sum_{sp} \frac{(TPA_{sp}^{adj})^2}{AD_{sp}^P}}; \quad (19)$$

and $P = 0.1$ is the power on the weighting function $W_{t_{sp}} = AD_{sp}^P$. This is the solution of the following Lagrangian weighted least-squares adjustment Equation 20, with $W_{t_{sp}} = AD_{sp}^P$.

$$\begin{aligned} \text{Min } L(AD_{sp}^{adj}, \lambda) &= \sum_{sp} W_{t_{sp}} (AD_{sp}^{adj} - AD_{sp})^2 \\ &+ 2\lambda \left(\sum_{sp} TPA_{sp}^{adj} AD_{sp}^{adj} - TPA_{all}^{adj} AD_{all} \right). \quad (20) \end{aligned}$$

Quadratic Mean Diameter

The TPA_{sp} weighted mean of QD_{sp}^2 must be equal to QD_{all}^2 . That is, the weighted average constraint is

$$\frac{\sum_{sp} TPA_{sp} QD_{sp}^2}{\sum_{sp} TPA_{sp}} = QD_{all}^2; \quad (21)$$

The least-squares adjusted QD_{sp} satisfying the above constraint are calculated using the adjustment Equation 22.

$$QD_{sp}^{adj} = \frac{QD_{sp}^P QD_{sp}}{QD_{sp}^P + \lambda TPA_{sp}^{adj}}, \quad (22)$$

where λ is the solution to the nonlinear equation

$$\sum_{sp} TPA_{sp}^{adj} \left(\frac{QD_{sp}^P QD_{sp}}{QD_{sp}^P + \lambda TPA_{sp}^{adj}} \right) - TPA_{all}^{adj} QD_{all}^2 = 0; \quad (23)$$

$P = 0.1$ is the power on the weighting function $W_{t_{sp}} = QD_{sp}^P$, and $TPA_{all}^{adj} = \sum_{sp} TPA_{sp}$. This is the solution of the following Lagrangian weighted least-squares adjustment Equation 24 with $W_{t_{sp}} = QD_{sp}^P$.

$$\begin{aligned} \text{Min } L(QD_{sp}^{adj}, \lambda) &= \sum_{sp} W_{t_{sp}} (QD_{sp}^{adj} - QD_{sp})^2 \\ &+ 2\lambda \left(\sum_{sp} TPA_{sp}^{adj} (QD_{sp}^{adj})^2 - TPA_{all}^{adj} (QD_{all})^2 \right). \quad (24) \end{aligned}$$

Because the TPA_{sp}^{adj} weighted average of $(QD_{sp}^{adj})^2$ equals QD_{all}^2 ,

$$\sum_{sp} 0.005454 TPA_{sp}^{adj} QD_{sp}^2 = \sum_{sp} BA_{sp}^{adj} = BA_{all}. \quad (25)$$

The values for TPA_{sp}^{adj} were used in Equations 20 and 24 as opposed to actual/unadjusted TPA_{sp} to make the adjustments for AD_{sp}^{adj} and QD_{sp}^{adj} logically consistent with the adjusted TPA (TPA_{sp}^{adj}).

Volume

The weighted least-squares adjustment of volume imposing the constraint specified by Equation 14 ($\sum_{sp} VOL_{sp} = VOL_{all}$) is

$$VOL_{sp}^{adj} = VOL_{sp} - \lambda VOL_{sp}^{-P}, \quad (26)$$

where

$$\lambda = \frac{\sum_{sp} VOL_{sp} - VOL_{all}}{\sum_{sp} VOL_{sp}^{-P}} \quad (27)$$

is the constraint (Lagrangian) multiplier and $P = 0.1$ is the power scale factor on the weighting function $W_{t_{sp}} = VOL_{sp}^P$. This is the solution of the following Lagrangian weighted least-squares equation adjustment Equation 28 with $W_{t_{sp}} = VOL_{sp}^P$.

$$\begin{aligned} \text{Min } L(VOL_{sp}^{adj}, \lambda) &= \sum_{sp} W_{t_{sp}} (VOL_{sp}^{adj} - VOL_{sp})^2 \\ &+ 2\lambda \left(\sum_{sp} VOL_{sp}^{adj} - VOL_{all} \right). \quad (28) \end{aligned}$$

Weighting Schemes

The key to successful adjustment of values is to choose the correct weighting function, one that minimizes the bias and root mean square error of prediction across the observed plot values of an adjusted variable. The least-squares adjustment weights chosen in this study were the best from among many possible weighting schemes evaluated. Other weighting schemes predicated on the idea that estimates with bigger values are less biased and have greater relative precision should work approximately as well. Under this predicate, larger values are given bigger weights which results in relatively smaller adjustments being made to the larger more precise and typically smaller biased estimates. We do not enumerate and discuss all of the weighting schemes evaluated, here, because of their large number.

Results and Discussion

Regression parameters and fit statistics for all the models developed are shown in Tables 3 and 4.

Sensitivity analyses of the complete system of models and comparisons of the predicted and observed data trends of TPA, AD, QD, BA, and VOL were conducted to determine whether the model system was sensitive to the input variables or exhibited any illogical behavior within the range of data. Basal area/ac (BA) was calculated from the mathematical relation equation

$$BA = 0.005454 TPA (QD^2), \quad (29)$$

where 0.005454 is the constant for converting square inches to square feet.

These analyses did not indicate any logical flaws in the relationships between model inputs and outputs. The model was most sensitive to Age but was also more sensitive to SI than was expected.

Table 3. Parameter estimates and fit statistics for combined species (all species) and species group stand level structural attribute equations for red oak-sweetgum forest mixtures on minor stream bottoms in Mississippi and Alabama.^a

Variable/species (Equation no.)	Stand structure attribute variable parameter estimates						Fit statistics	
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	SE	R ² (%)
Combined species								
TPA (4)	4,716.400000	-0.33730000	0.26680000	0.16820000	0.66510000	1.00710000	9.5800	51.5
AD (30)	-1.51160000	0.60330000	8.19300000	0.10370000	-0.01739000		0.1533	59.4
QD (31)	-2.07000000	0.65780000	10.19000000	0.15217000	-0.02735000		0.1481	67.9
AD (5)	1.15560000	0.11470000	0.34900000	0.08021000	-0.01350000	-0.00123794	0.1061	80.6
QD (6)	0.69360000	0.15160000	2.06100000	0.12784000	-0.02332000	-0.00128267	0.0939	87.2
SI guide curve								
RO (1)	127.200000	0.03170000	0.84810000				9.4891	0.61
TPA								
RO (7)	0.40665000	-0.00458900	0.01555000	-0.00032280			0.1085	18.0
WO (7)	-0.01451000	0.00032510	0.00318700	-0.00001951			0.0567	13.2
SG (7)	-0.12918000	0.00256650	0.00651700	0.00140830			0.1560	44.2
HK (7)	0.20216000	0.00213440	-0.01522000	-0.00032604			0.0993	11.8
OC (7)	0.28415000	-0.00083830	-0.00509000	-0.00033056			0.1049	5.1
NC (7)	0.25068000	0.00039770	-0.00491500	-0.00040944			0.1138	13.2
AD								
RO (8)	-1.38920000	0.72239000	-0.79404000	0.00742360			0.1893	41.2
WO (8)	1.05300000	0.33640000	-0.94960000	-0.00354600			0.4219	14.3
SG (8)	-0.14570000	-0.04904000	0.19161000	-0.00156850			0.2246	2.5
HK (8)	2.24400000	-0.45493000	-0.07000000	-0.00658700			0.3542	16.1
OC (8)	0.65780000	0.05641000	-0.59260000	0.00146700			0.3715	10.4
NC (8)	1.74450000	-0.10989000	-0.59106000	-0.00535270			0.2077	42.9
QD								
RO (9)	-1.14700000	0.59031000	-0.62780000	0.00661470			0.1718	31.7
WO (9)	1.20310000	0.23040000	-0.77640000	-0.00457200			0.4457	11.7
SG (9)	0.19370000	-0.17042000	0.23990000	-0.00231490			0.2289	2.6
HK (9)	2.65900000	-0.62130000	0.08770000	-0.00866300			0.3781	18.5
OC (9)	0.84790000	-0.05403000	-0.43850000	0.00030000			0.3900	9.7
NC (9)	1.92070000	-0.15508000	-0.55690000	-0.00633900			0.2276	42.4

^a SI, site index (base age, 50 years) for red oak species; TPA, trees/ac; AD, arithmetic mean dbh; QD, quadratic mean dbh; RO, red oaks; WO, white oaks; SG, sweetgum; HK, hickories; OC, other commercial; NC, noncommercial. The number in parenthesis is the equation number in the text applicable to the coefficient values in the table row.

Figures 2 through 6 show the behavior of the model for an average SI of 105. In Figures 3 and 4, the AD and QD predictions for hickory rapidly increase from 0 to about 6 between the ages of 20 and 23. This occurs because the hickories do not grow past the submerchantable threshold of 3.6 in. until around age 21. Figure 7 shows the expected Doyle board foot volume/ac for all species combined over a range of SI values from 70 to 130.

Because the red oak-sweetgum mixtures on minor stream bottoms are very site-specific and have a very narrow site index range on or before the age of 20, these stands begin to follow nearly the same development pattern. As a result of this narrow development pattern, the relations between the primary independent variables TPA, AD, and QD are rather simple functions of Age and TPA, with Age being the most significant variable, overshadowing the effects of TPA. These functions also exhibit low response curvature with respect to the independent variables (Figures 3 and 4). This observed linearity trend is a result of the longevity of the primary species and the fact that the study age range (15-92 years) lies entirely in the essentially linear portion of the sigmoid growth curve for the predominant species between the first and second inflection points on the curve. Pines such as loblolly show a high degree of curvature because they develop quickly and have a very narrow range of linear growth between the first and second growth curve inflection points. The length of the essentially linear portion of a growth curve is a function of the dependent variable of interest. For example, the variable merchantable cubic foot volume will have a narrower range than Doyle board feet volume. The age of the first and second

inflection points for merchantable cubic volume will also occur earlier than those for Doyle board feet volume.

The TPA data included ingrowth, and thus, it is possible for TPA to increase between two successive ages. Likewise, AD and QD also include ingrowth, so these quantities can decrease in a time period. Because VOL and BA are driven by TPA, AD, and QD, they are influenced by the ingrowth but to a lesser extent.

Some of the regression equations are ratios or percentages (Equations 7, 8, and 9). The R² values for these dependents are low because ratios eliminate the strength of relations between the dependent and independent variables when these ratios are nearly a constant, as in this study. This is a positive characteristic as opposed to a negative characteristic. By eliminating or reducing the strength of relations, variable ratios make model building simpler and result in less complex regression models. Less complex models tend to behave better in a system of equations forming a model. Complex models always have unforeseen bad behavior patterns when the inevitable extrapolation of a model occurs.

The least-squares procedures presented can result in adjusted values less than zero. When this condition occurs, the strategy that works best is to set the value to zero and adjust the remaining values, repeating the adjustment procedure until no adjusted values are less than zero.

The authors and others have voiced the opinion that hardwood stands are more difficult to model than single-species stands. However, the species group volume models in this study were relatively simple models based on combined species variables of TPA, HD,

Table 4. Merchantable volume equation parameter estimates and fit statistics for Equation 10 derived from stand level attributes of red oak-sweetgum forest mixtures in Mississippi and Alabama minor stream bottoms.

Unit ^b	Merchantable volume ^a parameter estimates							Fit statistics	
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	SE	R ² (%)
Combined species									
CVOB	-11.89640	1.52356	2.25400	1.27310	-30.54900	29.67900	33.82700	0.13644	95.0
CVIB	-12.56960	1.55095	2.31610	1.32030	-30.77300	28.86800	34.89400	0.13637	95.1
Doyle	-15.82800	1.78576	2.83790	1.81490	-45.37500	10.26200	64.89000	0.23769	92.1
Scribner	-13.27700	1.69135	2.61150	1.54020	-39.02100	15.64200	52.86000	0.17762	93.9
Int ¼	-12.27750	1.62378	2.50270	1.48200	-35.54600	22.88100	44.54300	0.17147	94.0
Red oaks									
CVOB	-10.53360	1.41423	2.24920	1.24920	-17.69300	33.51200	9.33100	0.18002	94.6
CVIB	-10.78790	1.42381	2.31500	1.22460	-18.05200	31.81100	10.36800	0.17911	94.8
Doyle	-10.37300	1.55456	3.21490	0.86490	-23.76500	13.19300	21.48000	0.22857	94.4
Scribner	-15.02300	1.62848	2.51440	2.15760	-26.74100	41.57000	22.66000	0.40981	82.4
Int ¼	-11.56500	1.52055	2.58130	1.53250	-21.84600	33.19800	16.28200	0.25485	91.8
White oaks									
CVOB	-11.25400	1.22440	3.13680	1.07840	-2.03700	45.31000	-10.37000	0.49500	88.8
CVIB	-8.23200	1.18280	3.13850	0.45930	-0.84900	44.68000	-18.07000	0.45462	90.2
Doyle	-14.17000	1.82990	3.57970	1.41200	-31.91200	55.17000	2.85000	0.49440	89.4
Scribner	-3.62700	1.53390	4.08710	-0.78110	-16.54800	8.35000	-3.24000	0.50089	89.3
Int ¼	-8.62500	1.43880	3.59040	0.53200	-8.92000	49.54000	-14.86000	0.70423	82.2
Sweetgum									
CVOB	-18.95600	1.53986	3.53010	2.07830	-26.88800	57.02000	21.07000	0.56745	79.3
CVIB	-18.05100	1.47933	3.49840	1.94280	-24.86000	52.40000	19.77000	0.50643	81.9
Doyle	-22.94600	1.61780	4.44180	2.65520	-29.91600	38.45000	37.26000	0.65572	78.8
Scribner	-19.40500	1.57920	4.25830	2.11200	-28.58700	26.88000	37.49000	0.56183	80.5
Int ¼	-18.04400	1.56080	3.90640	2.03960	-29.29300	42.13000	29.86000	0.57355	79.7
Hickories									
CVOB	-12.51200	1.15930	5.38710	0.30900	1.01400	-30.87000	18.14000	0.70569	81.3
CVIB	-12.00600	1.22020	5.60820	0.02200	-0.39000	-30.68000	15.77000	0.81854	77.9
Doyle	-10.85400	1.27890	5.93480	-0.02100	3.48300	-45.00000	15.11000	0.66333	83.8
Scribner	-20.18600	1.09440	5.74790	2.09800	9.31000	-39.96000	33.08000	0.78523	78.8
Int ¼	-10.72100	1.36560	6.30100	-0.28500	-3.89000	-67.76000	33.58000	0.75591	80.3
Other commercial									
CVOB	-14.20200	1.47740	4.68210	0.72680	-8.48600	-4.09000	17.49000	0.80735	79.3
CVIB	-11.15800	1.53530	4.84910	-0.05640	-12.31300	-15.71000	18.13000	0.77745	80.1
Doyle	-14.46100	1.74480	5.08150	0.73900	-17.56900	-13.56000	25.07000	0.63383	85.2
Scribner	-8.38400	1.60130	4.58610	-0.08400	-9.94800	9.19000	-1.75000	0.85838	77.1
Int ¼	-11.10900	1.68110	4.84050	0.29590	-14.16700	-3.39000	12.59000	0.79554	80.0
Noncommercial									
CVOB	-15.29600	1.63030	7.81580	-0.42000	-18.22000	-69.40000	42.17000	0.88775	66.4
CVIB	-15.20800	1.81760	7.61600	-0.49000	-22.60000	-52.98000	35.32000	0.84430	69.8
Doyle	-11.24000	1.58790	8.54800	-1.33300	-15.31000	-102.30000	45.29000	0.96274	58.5
Scribner	-6.67200	1.84420	7.33590	-1.75800	-26.80000	-29.02000	12.32000	0.91011	64.7
Int ¼	-7.31600	1.91370	6.91460	-1.44400	-29.27000	-12.67000	8.30000	0.92854	63.3

$$^a \text{Ln(MVol)} = a + b\text{Ln(TPA)} + c\text{Ln(QD)} + d\text{Ln(Age)} + \frac{e\text{Ln(TPA)}}{\text{Age}} + \frac{f\text{Ln(QD)}}{\text{Age}} + \frac{g\text{Ln(HD)}}{\text{Age}},$$

where MVol is merchantable volume/ac in the units associated with the equation parameters *a*, *b*, *c*, *d*, *e*, *f*, and *g*; TPA is trees/ac of all trees for total volume or trees/ac of a specific species for the volume of the species; QD is quadratic mean dbh of all trees for total volume or quadratic mean dbh of a specific species for the volume of the species; Age is average age of red oak component; and HD is average height of dominant and codominant red oaks.

^b CVOB, cubic foot volume outside bark; CVIB, cubic foot volume inside bark; Doyle, Scribner, and Int ¼ indicate Doyle, Scribner, and International ¼-inch board foot log rules, respectively.

Age, and QD. The primary difficulty with multiple-species models may be that they require more data because of the failure of all species to occur in all plots.

The 638 stand-level observations made on 258 distinct permanent plots in this study provided more than sufficient data to establish relationships between dependent and independent variables. However, because of the species diversity complexity, we did not have sufficient data to comprise both fitting and validation data sets, and no attempt was made to split the data set. Plans are being made to develop an independent validation data set, as well as data sets that will allow for the assessment of responses to management practices: thinning and partial harvest.

Sometimes it is desirable to allow for bare land estimates of AD and QD to be made from SI and Age without knowledge of TPA. The equations developed for this purpose are

$$\widehat{\text{Ln(AD}_{\text{all}})} = a + b\text{Ln(Age)} + \frac{c}{\text{Age}} + d\text{SI}^{2/3} + \frac{e\text{SI}}{\text{Age}^{1/8}} \quad (30)$$

and

$$\widehat{\text{Ln(QD}_{\text{all}})} = a + b\text{Ln(Age)} + \frac{c}{\text{Age}} + d\text{SI}^{2/3} + \frac{e\text{SI}}{\text{Age}^{1/8}} \quad (31)$$

Equations 30 and 31 were derived by dropping the independent variable TPA_{all} from Equations 5 and 6.

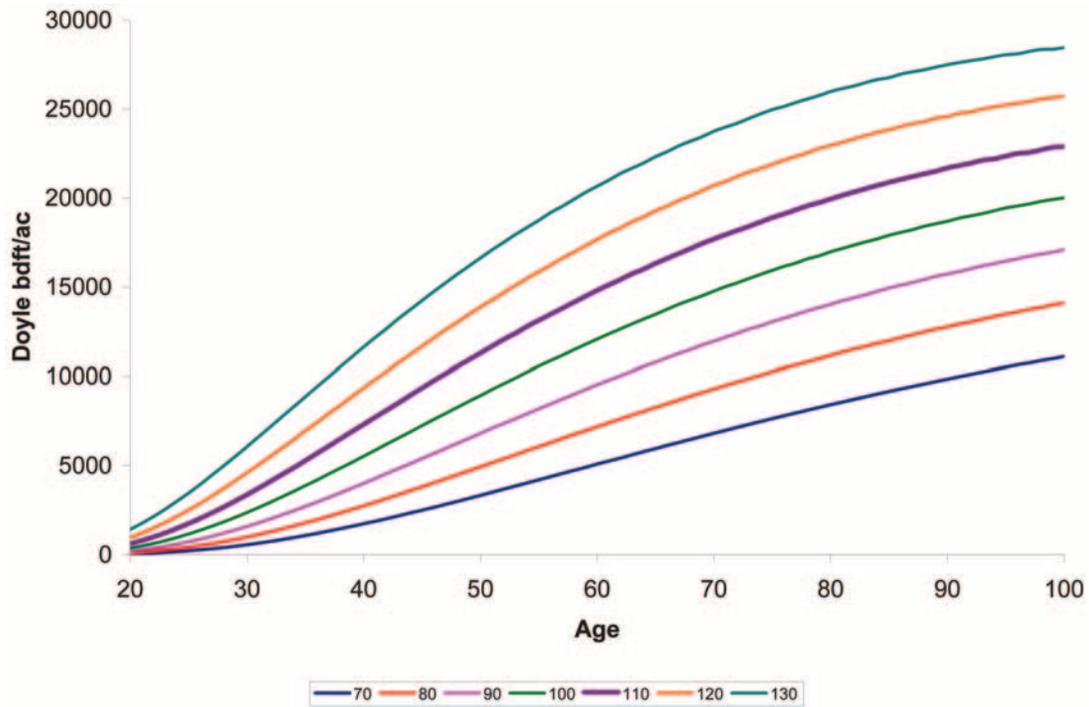


Figure 7. Expected Doyle board ft/ac for all species combined for site indices (base age, 50 years) of 70, 80, 90, 100, 110, 120, and 130 ft for red oak-sweetgum forests on minor stream bottoms in Mississippi and Alabama.

Application

Consider the estimation of expected stand structural and volume variables for an age 60 stand with a site index of 120 ft (base age, 50 years). The first step in the process is to estimate stand-level values of HD, TPA_{all} , AD_{all} , QD_{all} , and BA_{all} applying Equations 3, 4, 5, and 6 and the regression coefficients given in Table 3. BA was calculated using Equation 29. The estimates TPA_{all} , AD_{all} , and QD_{all} of all merchantable trees and species in the stand have the highest relative precision and least bias and are assumed to be base values to reconcile the estimates of equivalent variable values estimated for each species group from Equations 4, 5, and 6 and the coefficients in Table 3. They are

$$HD = 120 \left(\frac{1 - e^{-0.0317(60)}}{1 - e^{-0.0317(50)}} \right)^{0.8481} = 127.1 \text{ ft,}$$

$$TPA_{all} = \frac{4716.40000000(120^{-0.33730000})(60^{0.26680000})}{(1 + e^{0.16820000(60^{0.66510000})})^{1.00710000}} = 196.74 \text{ Trees/ac,}$$

$$AD_{all} = e^{1.15560000 + 0.11470000 \ln(60) + \frac{0.34900000}{60} + 0.08021000(120^{2/3}) - 0.01350000 \frac{120}{60^{1/8}} - 0.00123794(196.74)} = 10.673 \text{ in.,}$$

$$QD_{all} = e^{0.69360000 + 0.15160000 \ln(60) + \frac{2.06100000}{60} + 0.12784000(120^{2/3}) - 0.02332000 \frac{120}{60^{1/8}} - 0.00128267(196.74)} = 12.54 \text{ in.,}$$

and $BA_{all} = 0.005454(196.74)(12.542^2) = 168.79 \text{ ft}^2/\text{ac}$.

To obtain estimates of TPA_{sp} , AD_{sp} , QD_{sp} , and BA_{sp} for each of the six species groupings, the ratio estimation Equations 7, 8, and 9 must be applied to the combined species estimates of TPA_{all} , AD_{all} , and QD_{all} :

$$TPA_{ro} = (196.74)(0.40665000 - 0.00458900(60) + 0.01555000(12.542) - 0.00032280(196.74)) = 51.71 \text{ Trees/ac,}$$

$$TPA_{wo} = (196.74)(-0.01451000 + 0.00032510(60) + 0.00318700(12.542) - 0.00001951(196.74)) = 8.09 \text{ Trees/ac,}$$

$$TPA_{sg} = (196.74)(-0.12918000 + 0.00256650(60) + 0.00651700(12.542) + 0.00140830(196.74)) = 75.47 \text{ Trees/ac,}$$

$$TPA_{hik} = (196.74)(0.20216000 + 0.00213440(60) - 0.01522000(12.542) - 0.00032604(196.74)) = 14.79 \text{ Trees/ac,}$$

$$TPA_{ac} = (196.74)(0.28415000 - 0.00083830(60) - 0.00509000(12.542) - 0.00033056(196.74)) = 20.65 \text{ Trees/ac,}$$

and

$$TPA_{nc} = (196.74)(0.25068000 + 0.00039770(60) - 0.00491500(12.542) - 0.00040944(196.74)) = 26.04 \text{ Trees/ac.}$$

Adjustment procedures for AD_{sp} and QD_{sp} are similar to those for TPA_{sp} :

$$\begin{aligned}
 AD_{ro} &= 10.673e^{-1.38920000+0.72239000\text{Ln}(60)-0.79404000\text{Ln}(10.673)+0.00742360(120)} = 19.049 \text{ in.}, \\
 AD_{wo} &= 10.673e^{1.05300000+0.33640000\text{Ln}(60)-0.94960000\text{Ln}(10.673)-0.00354600(120)} = 8.366 \text{ in.}, \\
 AD_{sg} &= 10.673e^{-0.14570000-0.04904000\text{Ln}(60)+0.19161000\text{Ln}(10.673)-0.00156850(120)} = 9.844 \text{ in.}, \\
 AD_{hk} &= 10.673e^{2.24400000-0.45493000\text{Ln}(60)-0.07000000\text{Ln}(10.673)-0.00658700(120)} = 6.008 \text{ in.}, \\
 AD_{oc} &= 10.673e^{0.65780000+0.05641000\text{Ln}(60)-0.59260000\text{Ln}(10.673)+0.00146700(120)} = 7.610 \text{ in.}, \\
 AD_{nc} &= 10.673e^{1.74450000-0.10989000\text{Ln}(60)-0.59106000\text{Ln}(10.673)-0.00535270(120)} = 5.056 \text{ in.}, \\
 QD_{ro} &= 12.542e^{-1.14700000+0.59031000\text{Ln}(60)-0.62780000\text{Ln}(12.542)+0.00661470(120)} = 20.187 \text{ in.}, \\
 QD_{wo} &= 12.542e^{1.20310000+0.23040000\text{Ln}(60)-0.77640000\text{Ln}(12.542)-0.00457200(120)} = 8.700 \text{ in.}, \\
 QD_{sg} &= 12.542e^{0.19370000-0.17042000\text{Ln}(60)+0.23990000\text{Ln}(12.542)-0.00231490(120)} = 10.527 \text{ in.}, \\
 QD_{hk} &= 12.542e^{2.65900000-0.62130000\text{Ln}(60)+0.08770000\text{Ln}(12.542)-0.00866300(120)} = 6.212 \text{ in.}, \\
 QD_{oc} &= 12.542e^{0.84790000-0.05403000\text{Ln}(60)-0.43850000\text{Ln}(12.542)+0.00030000(120)} = 8.027 \text{ in.},
 \end{aligned}$$

and

$$QC_{nc} = 12.542e^{1.92070000-0.15508000\text{Ln}(60)-0.55690000\text{Ln}(12.542)-0.00633900(120)} = 5.185 \text{ in.}$$

The first step in the least-squares adjustment is to adjust the TPA_{sp} estimates. The adjusted TPA_{sp}^{adj} estimates are required to apply the least-squares adjustment procedures to the AD_{sp} and QD_{sp} estimates. The adjusted TPA_{sp}^{adj} values are not necessary for calculating adjusted values for VOL_{sp} . The least-squares adjustment of TPA_{sp} function presented in Equation 15 first requires the calculation of λ from Equation 16.

$$\begin{aligned}
 \sum_{sp} TPA_{sp} &= 51.71 + 8.09 + 75.47 + 14.79 + 20.65 + 26.04 = 196.75, \\
 \sum_{sp} TPA_{sp}^{-p} &= 51.71^{-0.1} + 8.09^{-0.1} + 75.47^{-0.1} + 14.79^{-0.1} + 20.65^{-0.1} + 26.04^{-0.1} = 4.35868, \\
 \lambda &= \frac{196.75 - 196.74}{4.35873} = 0.002294.
 \end{aligned}$$

Applying Equation 15 with $\lambda = 0.002294$ results in the adjusted TPA_{sp}^{adj} values:

$$\begin{aligned}
 TPA_{ro}^{adj} &= 51.71 - 0.002294(51.71^{-0.1}) = 51.708 \text{ Trees/ac}, \\
 TPA_{wo}^{adj} &= 8.09 - 0.002294(8.09^{-0.1}) = 8.088 \text{ Trees/ac}, \\
 TPA_{sg}^{adj} &= 75.47 - 0.002294(75.47^{-0.1}) = 75.468 \text{ Trees/ac}, \\
 TPA_{hk}^{adj} &= 14.79 - 0.002294(14.79^{-0.1}) = 14.788 \text{ Trees/ac}, \\
 TPA_{oc}^{adj} &= 20.65 - 0.002294(20.65^{-0.1}) = 20.648 \text{ Trees/ac},
 \end{aligned}$$

and

$$TPA_{nc}^{adj} = 26.04 - 0.002294(26.04^{-0.1}) = 26.038 \text{ Trees/ac.}$$

After making the adjustments to TPA_{sp} , the adjustments to AD_{sp} and QD_{sp} can be made. By Equation 19, the value of λ required in

Equation 18 is

$$\lambda = \frac{[(51.708)(19.049) + \dots + (26.038)(5.056)] - (196.74)(10.673)}{\frac{51.708^2}{19.049^{0.1}} + \dots + \frac{26.038^2}{5.056^{0.1}}} = 0.0096055,$$

and by Equation 18, the adjusted AD_{sp} values are

$$AD_{ro}^{adj} = \frac{(19.049^{0.1})(19.049) - (0.0096055)(51.708)}{19.049^{0.1}} = 18.677 \text{ in.},$$

$$AD_{wo}^{adj} = \frac{(8.366^{0.1})(8.366) - (0.0096055)(8.088)}{8.366^{0.1}} = 8.303 \text{ in.},$$

$$AD_{sg}^{adj} = \frac{(9.844^{0.1})(9.844) - (0.0096055)(75.468)}{9.844^{0.1}} = 9.264 \text{ in.},$$

$$AD_{hk}^{adj} = \frac{(6.008^{0.1})(6.008) - (0.0096055)(14.788)}{6.008^{0.1}} = 5.889 \text{ in.},$$

$$AD_{oc}^{adj} = \frac{(7.610^{0.1})(7.610) - (0.0096055)(20.648)}{7.610^{0.1}} = 7.447 \text{ in.},$$

and

$$AD_{nc}^{adj} = \frac{(5.056^{0.1})(5.056) - (0.0096055)(26.038)}{5.056^{0.1}} = 4.842 \text{ in.}$$

For adjusting QD_{sp} , a nonlinear equation (Equation 23) must be solved for λ to apply Equation 22 to calculate the adjusted QD_{sp}^{adj} . The solution to

$$51.708 \left(\frac{(20.187^{0.1})(20.187)}{20.187^{0.1} + \lambda 51.708} \right) + \dots + 26.038 \left(\frac{(5.185^{0.1})(5.185)}{5.185^{0.1} + \lambda 26.038} \right) - (196.74)(12.542^2) = 0$$

is $\lambda = 0.00065598$, and by Equation 22, the adjusted QD_{sp}^{adj} values are

$$QD_{ro}^{adj} = \frac{(20.187^{0.1})(20.187)}{20.187^{0.1} + (0.00065598)(51.708)} = 19.692 \text{ in.},$$

$$QD_{wo}^{adj} = \frac{(8.700^{0.1})(8.700)}{8.700^{0.1} + (0.00065598)(8.088)} = 8.663 \text{ in.},$$

$$QD_{sg}^{adj} = \frac{(10.527^{0.1})(10.527)}{10.527^{0.1} + (0.00065598)(75.468)} = 10.131 \text{ in.},$$

$$QD_{hk}^{adj} = \frac{(6.212^{0.1})(6.212)}{6.212^{0.1} + (0.00065598)(14.788)} = 6.162 \text{ in.},$$

$$QD_{oc}^{adj} = \frac{(8.027^{0.1})(8.027)}{8.027^{0.1} + (0.00065598)(20.648)} = 7.940 \text{ in.},$$

and

$$QD_{nc}^{adj} = \frac{(5.185^{0.1})(5.185)}{5.185^{0.1} + (0.00065598)(26.038)} = 5.111 \text{ in.}$$

The method of bisection (Burden et al. 1981) for solving nonlinear equations is embedded in the Microsoft Visual C++ dll project installed with the Microsoft Excel spreadsheet implementing the model presented in this article. Readers interested in the solution algorithm should consult the C++ source code of the dll (www.timbercruise.com, Download Center, Growth and Yield Models).

Following adjustments of TPA, AD, and QD, volumes can be calculated and adjusted. The unadjusted Doyle board feet volumes based on Equation 10 and regression parameters from Table 4 are

$$\text{VOL}_{\text{all}} = \exp \left[-15.82800 + 1.78576 \text{Ln}(196.74) + 2.83790 \text{Ln}(12.542) + 1.81490 \text{Ln}(60) \right. \\ \left. - \frac{45.37500 \text{Ln}(196.74)}{60} + \frac{10.26200 \text{Ln}(12.542)}{60} + \frac{64.89000 \text{Ln}(127.1)}{60} \right] = 19736.18 \text{ Bdft},$$

$$\text{VOL}_{\text{no}} = \exp \left[-10.37300 + 1.55456 \text{Ln}(51.708) + 3.21490 \text{Ln}(19.692) + 0.86499 \text{Ln}(60) \right. \\ \left. - \frac{23.76500 \text{Ln}(51.708)}{60} + \frac{13.19300 \text{Ln}(19.692)}{60} + \frac{21.48000 \text{Ln}(127.1)}{60} \right] = 16481.89 \text{ Bdft},$$

$$\text{VOL}_{\text{wo}} = \exp \left[-14.17000 + 1.82990 \text{Ln}(8.088) + 3.57970 \text{Ln}(8.663) + 1.41200 \text{Ln}(60) \right. \\ \left. - \frac{31.91200 \text{Ln}(8.088)}{60} + \frac{55.17000 \text{Ln}(8.663)}{60} + \frac{2.85000 \text{Ln}(127.1)}{60} \right] = 71.43 \text{ Bdft},$$

$$\text{VOL}_{\text{sg}} = \exp \left[-22.94600 + 1.61780 \text{Ln}(75.468) + 4.44180 \text{Ln}(10.131) + 2.65520 \text{Ln}(60) \right. \\ \left. - \frac{29.91600 \text{Ln}(75.468)}{60} + \frac{38.45000 \text{Ln}(10.131)}{60} + \frac{37.26000 \text{Ln}(127.1)}{60} \right] = 1886.64 \text{ Bdft},$$

$$\text{VOL}_{\text{hk}} = \exp \left[-10.85400 + 1.27890 \text{Ln}(14.788) + 5.93480 \text{Ln}(6.162) - 0.02100 \text{Ln}(60) \right. \\ \left. + \frac{3.48300 \text{Ln}(14.788)}{60} - \frac{45.00000 \text{Ln}(6.162)}{60} + \frac{15.11000 \text{Ln}(127.1)}{60} \right] = 27.39 \text{ Bdft},$$

$$\text{VOL}_{\text{ac}} = \exp \left[-14.46100 + 1.74480 \text{Ln}(20.648) + 5.08150 \text{Ln}(7.940) + 0.73900 \text{Ln}(60) \right. \\ \left. - \frac{17.56900 \text{Ln}(20.648)}{60} - \frac{13.56000 \text{Ln}(7.940)}{60} + \frac{25.07000 \text{Ln}(127.1)}{60} \right] = 155.26 \text{ Bdft},$$

and

$$\text{VOL}_{\text{nc}} = \exp \left[-11.24000 + 1.58790 \text{Ln}(26.038) + 8.54800 \text{Ln}(5.111) - 1.33300 \text{Ln}(60) \right. \\ \left. - \frac{15.31000 \text{Ln}(26.038)}{60} - \frac{102.30000 \text{Ln}(5.111)}{60} + \frac{45.29000 \text{Ln}(127.1)}{60} \right] = 11.78 \text{ Bdft}.$$

To adjust the unadjusted Doyle volumes, first calculate λ from Equation 27 and then use Equation 26 to determine the final adjusted volumes.

The two sums required by Equation 27 are

$$\sum_{sp} VOL_{sp} = 16481.89 + 71.43 + 1886.64 + 27.39 + 155.26 + 11.78 = 18634.39,$$

and

$$\sum_{sp} VOL_{sp}^{-p} = 16481.89^{-0.1} + 71.43^{-0.1} + 1886.64^{-0.1} + 27.39^{-0.1} + 155.26^{-0.1} + 11.78^{-0.1} + 3.60518.$$

With the summation values determined, Equation 27 yields

$$\lambda = \frac{18634.39 - 19736.18}{3.605018} = -305.6295,$$

and from Equation 26, the adjusted volumes are

$$VOL_{ro}^{adj} = 16481.89 + 305.6295(16481.89^{-0.1}) = 16597.63 \text{ Bdft/ac},$$

$$VOL_{wo}^{adj} = 71.43 + 305.6295(71.43^{-0.1}) = 270.87 \text{ Bdft/ac},$$

$$VOL_{sg}^{adj} = 1886.64 + 305.6295(1886.64^{-0.1}) = 2030.40 \text{ Bdft/ac},$$

$$VOL_{hk}^{adj} = 27.39 + 305.6295(27.39^{-0.1}) = 246.89 \text{ Bdft/ac},$$

$$VOL_{oc}^{adj} = 155.26 + 305.6295(155.26^{-0.1}) = 339.80 \text{ Bdft/ac},$$

and

$$VOL_{nc}^{adj} = 11.78 + 305.6295(11.78^{-0.1}) = 250.61 \text{ Bdft/ac}.$$

In this hypothetical example of a 60-year-old stand growing on a site with site index of 120 ft at 50 years, our model estimates that stand density is 197 trees/ac with a basal area of 169 ft²/ac. Arithmetic mean diameter is 10.7 in., and quadratic mean diameter is 12.5 in. Total stand volume is 19,736 bd ft (Doyle)/ac. Red oaks clearly dominate this hypothetical stand, with a quadratic mean diameter among red oaks of 19.7 in. Although red oaks account for only 26% of the trees in the stand, they account for 84% of the sawtimber volume.

All calculations were performed in a C++ program using double precision arithmetic, and thus hand calculations from the equations using rounded values of the inputs will differ slightly because of rounding errors.

Equations 5, 6, 10, 30, and 31 predict the logarithm of the dependent variable and can produce biased estimates when untransformed (Flewelling and Pienaar 1981). When there is evidence that the untransformed estimates have significant bias, a bias correction can be applied or the equations can be fitted using weighted nonlinear least squares. For these equations, comparison of the observed and predicted across all levels of the observed indicated that no biases would warrant a bias correction or nonlinear refits. All regressions of observed on predicted were straight lines and not significantly different from a line with intercept 0 and slope 1 at a probability level of 0.05.

Conclusion

Aggregate stand-level expected yield and structural estimation models were constructed for a red oak–sweetgum forest mixture growth and yield simulator. The models will allow users to estimate expected average yield by species in a naturally developing stand. These yields provide an average baseline for individuals who may be considering managing their lands for the red oak–sweetgum complex. Future models to be developed from the data will yield projections for stands with existing inventories. Because all of the models estimate arithmetic and quadratic mean dbh, diameter distribution recovery methods can be used to display yields by dbh class. As a separate approach to modeling, an individual tree model (Daniels and Burkhart 1975, Ek and Randall 1985) using least-squares re-

covery and adjustment is being developed that will be bounded by the stand-level models presented here. The diameter distribution recovery model is, by default, bounded by the stand-level model.

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