

Modeling the longitudinal variation in wood specific gravity of planted loblolly pine (*Pinus taeda*) in the United States

F. Antony, L.R. Schimleck, R.F. Daniels, A. Clark III, and D.B. Hall

Abstract: Loblolly pine (*Pinus taeda* L.) is a major plantation species grown in the southern United States, producing wood having a multitude of uses including pulp and lumber production. Specific gravity (SG) is an important property used to measure the quality of wood produced, and it varies regionally and within the tree with height and radius. SG at different height levels was measured from 407 trees representing 135 plantations across the natural range of loblolly pine. A three-segment quadratic model and a semiparametric model were proposed to explain the vertical and regional variations in SG. Both models were in agreement that a stem can be divided into three segments based on the vertical variation in SG. Based on the fitted models, the mean trend in SG of trees from the southern Atlantic Coastal Plain and Gulf Coastal Plain was observed to be higher than in other physiographical regions (Upper Coastal Plain, Hilly Coastal Plain, northern Atlantic Coastal Plain, and Piedmont). Maps showing the regional variation in disk SG at a specified height were also developed. Maps indicated that the stands in the southern Atlantic Coastal Plain and Gulf Coastal Plain have the highest SG at a given height level.

Résumé : Le pin à encens (*Pinus taeda* L.), dont le bois a de multiples usages tels que la pâte et le bois de sciage, est une espèce de grande importance dans les plantations du sud des États-Unis. Le poids spécifique (PS) est une propriété importante utilisée pour mesurer la qualité du bois qui est produit; il varie selon la région ainsi que verticalement et radialement à l'intérieur de l'arbre. Le PS a été mesuré à différentes hauteurs sur 407 arbres représentant 135 plantations couvrant l'ensemble de l'aire de distribution naturelle du pin à encens. Un modèle quadratique à trois segments et un modèle semi paramétrique ont été proposés pour expliquer la variation verticale et régionale du PS. Les deux modèles s'accordent sur le fait qu'il est possible de diviser la tige en trois segments selon la variation verticale du PS. Selon les modèles ajustés, le PS moyen des arbres provenant de la plaine côtière de l'Atlantique Sud et de la plaine côtière du Golfe a tendance à être plus élevé que dans d'autres régions physiographiques (haute plaine côtière, plaine côtière accidentée, plaine côtière de l'Atlantique Nord et piedmont). Des cartes montrant la variation régionale du PS de disques à une hauteur déterminée ont aussi été développées. Ces cartes indiquent que les peuplements de la plaine côtière de l'Atlantique Sud et de la plaine côtière du Golfe ont le PS le plus élevé pour une hauteur donnée.

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Introduction

Pine plantations occupy approximately 13 million ha of the southern United States (US), which carries 680 million m³ of timber, with a projected increase in area of 67% (22 million ha) by 2040 (Wear and Greis 2002). A twofold increase in productivity and a 50% reduction in rotation length of pine plantations during the last few decades have turned the southern US into the wood basket of the US (Fox et al. 2007). Currently, the southern US produces around 58% of the total wood supply in the US and 16% of world's industrial wood supply (Wear and Greis 2002). Loblolly pine (*Pinus taeda* L.), with more than half of the

standing pine volume, is the most important plantation species in the southern US. Wood from loblolly pine is a principal source of raw material for the pulp and paper industry and is desirable for the production of lumber and composite wood products. The quality of wood produced from a loblolly pine tree is defined by its physical and mechanical properties. Of these, specific gravity (SG) is considered as the most important wood quality indicator because of its strong correlation with the strength of solid wood products, as well as the yield and quality of pulp (Panshin and de Zeeuw 1980).

The wood properties of loblolly pine vary considerably across its growing range, between stands within a region,

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between trees within stands, and within the trees. Significant variation in wood properties within trees occurs from pith-to-bark, from stump-to-tip, and also within annual rings between earlywood and latewood. Clark and Saucier (1989) divided the radial section of a pine stem into three zones: core wood and transition wood, which together can be referred to as juvenile wood, and mature wood. Juvenile wood is the wood that is formed in the vicinity of the crown forming a core near the center of the stem having low SG, short tracheids with large microfibril angles (Larson et al. 2001). Zobel (1972) reported an average SG ranging from 0.36 to 0.45 for juvenile wood and from 0.42 to 0.64 for mature wood in loblolly pine.

According to Burdon et al. (2004), the concept of juvenile to mature wood progression from pith to bark is inadequate to represent the pattern of variation within a tree and is an oversimplification of the physiological process. They advocated the use of two separate concepts, corewood versus outerwood in the radial direction and juvenile versus mature wood in the longitudinal direction, to explain the within-tree variation in wood properties. Based on the proposed classification, juvenile wood occurs in the lower butt log with height < 3 m, transition wood occurs between 3 and 5 m in height, and mature wood occurs at heights > 5 m.

The longitudinal variation in SG of loblolly pine was reported in several studies. Early studies reported a decrease in SG from stump-to-tip of loblolly pine (Megraw 1985; Zobel and van Buijtenen 1989). Tasissa and Burkhart (1998b) modeled the within-tree variation (stump-to-tip and pith-to-bark) in SG of loblolly pine using a linear function of physiological age, relative height, percent latewood, latewood width, and ring width. Phillips (2002) and He (2004) modeled the longitudinal variation in disk SG of loblolly pine using subject-specific nonlinear models.

Marked geographical variation in SG has been reported for loblolly pine by Tasissa and Burkhart (1998a), Clark and Daniels (2002), and Jordan et al. (2008). SG was significantly higher in trees from the Coastal Plains compared with those from inland areas. Higher SG for trees from the Coastal Plains might be due to the increased latewood production of these trees, which has been attributed to increased moisture availability from frequent summer rainfall in the area (Clark and Daniels 2002; Jordan et al. 2008). Jordan et al. (2008) reported a higher whole-core average SG (of 0.49) for trees from the southern Atlantic Coastal Plain compared with other regions (Gulf Coastal Plain, Hilly Coastal Plain, northern Atlantic Coastal Plain, Upper Coastal Plain, and Piedmont), which averaged 0.455 using breast height cores collected from trees. They also produced maps showing regional variation in SG at different stand ages at breast height. However, the maps showing the regional variation in SG at different height levels within a tree were lacking and are important for maximizing product utilization.

Both parametric and semiparametric regression methods are well known and potentially can be applied in forestry to explain and analyze the nonlinear trend in a property (Max and Burkhart 1976; Ruppert et al. 2003; Jordan et al. 2008) with respect to some explanatory variable(s). Parametric models are parsimonious, have meaningful parameter interpretations, and are more suitable for making predictions, but the curve shapes are constrained by the functional form of

the model, which can restrict inferences drawn from it. Semiparametric regression is a flexible method of defining nonlinear trends in any property of interest and its analysis (Ruppert et al. 2003). In addition, semiparametric regression can be extended easily to define the variation in a property at higher dimensions (e.g., spatial variation in SG in this study) along with its nonlinear trend with any explanatory variable(s). The present study uses both methods to draw inferences about SG variation at different disk heights within trees and regional variation in mean SG trends within trees. The primary objectives of the present study are (i) to examine and model the longitudinal variation in disk SG, (ii) to examine regional variation of disk SG, and (iii) to develop maps depicting the regional variation of disk SG across the southern US.

Data and methods

Data

The Wood Quality Consortium at the University of Georgia and the United States Department of Agriculture (USDA) Forest Service Southern Research Station sampled planted loblolly pine trees across its natural range to study the vertical variation in wood SG. Trees were sampled from 135 stands from six physiographic regions across the southeastern US. Regions sampled included (1) southern Atlantic Coastal Plain, (2) northern Atlantic Coastal Plain, (3) Upper Coastal Plain, (4) Piedmont, (5) Gulf Coastal Plain, and (6) Hilly Coastal Plain. A minimum of 12 plantations from each of the six physiographic regions were sampled. The stands selected for sampling included 20- to 25-year-old loblolly pine plantations planted at 1250 or more trees per hectare and contained 625 trees per hectare or more after thinning. Only stands that were conventionally managed with no fertilization (except phosphorus at planting on phosphorus-deficient sites) and no competition control were sampled. Three trees from each stand were felled and 3.8 cm thick, cross-sectional disks were collected at 0.15 and 1.37 m and then at 1.52 m intervals along the stem up to a diameter of 50 mm outside bark. The disks were sealed in plastic bags and shipped to the USDA Forest Service laboratory for physical property analysis. Disk SG based on green volume and oven-dry weight was measured for each disk collected at different heights. A map showing the sampled locations is presented in Fig. 1. A summary of the stand characteristics along with the number of stands and trees sampled from each region is presented in Table 1.

Parametric model

Disk SG follows a nonlinear decreasing trend from stump-to-tip in loblolly pine. Relative height, the ratio of height at any point to the total height of the tree, has explained the maximum amount of variation in SG and possesses the property of homogeneous variance. Relative height was used as a potential variable to explain the change in disk SG from stump-to-tip in this study. Large tree-to-tree variation was also evident in the observed disk SG profiles. Because the rate of change of SG varies at different parts of the stem (at least two inflection points are present in most of the individual tree profiles), it was difficult to explain the phenomenon using a single function.

Fig. 1. Locations of 135 sampled stands (*) and a subset of 34 stands (○) used for reduced knot kriging.



Table 1. Mean (standard deviation in parenthesis) stand attributes collected from six regions.

Region	No. of stands	No. of trees	Age (years)	DBH (cm)	Total height (m)	Disk SG
Southern Atlantic Coastal	39	117	22.73 (1.82)	24.07 (4.58)	20.86 (2.50)	0.45 (0.06)
Northern Atlantic Coastal	7	20	22.46 (1.61)	24.56 (3.74)	18.89 (2.48)	0.41 (0.05)
Upper Coastal	17	51	23.00 (1.46)	24.07 (4.87)	19.39 (3.08)	0.43 (0.05)
Piedmont	26	78	23.08 (2.01)	23.90 (4.54)	18.19 (2.11)	0.42 (0.05)
Gulf Coastal	17	54	23.22 (3.26)	21.16 (3.79)	19.54 (2.58)	0.46 (0.05)
Hilly Coastal	29	87	23.86 (3.58)	23.39 (4.12)	19.59 (2.75)	0.43 (0.05)

Note: DBH, diameter at breast height; SG, specific gravity.

A segmented regression model proposed by Gallant and Fuller (1973) was used to explain the change in disk SG with relative height in this study. Let y_{ijk} represent the SG measurement from disk k from tree j in stand i ; the general form of the segmented regression model (Gallant and Fuller 1973) for tree j in stand i can be represented as

$$[1] \quad y_{ijk} = g_r(x_{ijk}, \beta_r) + \varepsilon_{ijk}$$

where x_{ijk} is the k th relative height measurement from tree j in stand i , h_{ijk}/H_{ij} , where h_{ijk} is the k th height above ground, and H_{ij} is the total height of tree j in stand i , $g_r(x_{ijk}, \beta_r)$ is a sequence of r -grafted submodels with $r = 1, 2, \dots, R$, where R is the number of segments and $R - 1$ is the number of knots,

$$\begin{aligned}
 g(x_{ijk}, \beta) &= g_1(x_{ijk}, \beta_1), & 0 \leq x_{ijk} \leq \alpha_1 \\
 &\vdots \\
 &= g_r(x_{ijk}, \beta_r), & \alpha_{r-1} < x_{ijk} \leq \alpha_r \\
 &\vdots \\
 &= g_R(x_{ijk}, \beta_R), & \alpha_{R-1} < x_{ijk} \leq 1
 \end{aligned}$$

where β_r represents the parameter specific to segment r joined to the previous segment at the $(r - 1)$ knot location, α_{r-1} , and $\varepsilon_{ijk} \sim N(0, \sigma^2)$. Each of these submodels is subjected to continuity and smoothness constraints at the $(r - 1)$ knot point as

$$\begin{aligned}
 [2] \quad g_{r-1}(\alpha_{r-1}, \beta_{r-1}) &= g_r(\alpha_{r-1}, \beta_r) \\
 \frac{\partial}{\partial x} g_{r-1}(\alpha_{r-1}, \beta_{r-1}) &= \frac{\partial}{\partial x} g_r(\alpha_{r-1}, \beta_r), \quad \text{for } r > 1
 \end{aligned}$$

Following Gallant and Fuller (1973), a segmented model formed after splicing three-quadratic submodels was used to explain changes in disk SG with relative height for tree j and had the following form:

$$\begin{aligned}
 [3] \quad g(x_{jk}, \beta) &= \beta_0 + \beta_1 x_{jk} + \beta_2 x_{jk}^2 + \beta_3 (\alpha_1 - x_{jk})_+^2 \\
 &\quad + \beta_4 (\alpha_2 - x_{jk})_+^2
 \end{aligned}$$

where $[\beta_0 \ \beta_1 \ \beta_2 \ \beta_3 \ \beta_4 \ \alpha_1 \ \alpha_2]^T$ are parameters to be estimated, with $[1 > \alpha_1 > \alpha_2 > 0]$. The $(\alpha_r - x_{jk})_+^2$ terms indicate the positive part of the function $\alpha_r - x_{jk}$, where “+” sets it to zero for those values of relative heights where $\alpha_r - x_{jk}$ is negative (here $x_{jk} > \alpha_r$). The above model is equivalent to the standard form of the taper model proposed

by Max and Burkhardt (1976), which is not constrained to have a value of zero at the tip of the tree. If the knot points $[\alpha_1 \alpha_2]^T$ are known, then the model becomes a simple linear model and the estimates of $[\beta_0 \beta_1 \beta_2 \beta_3 \beta_4]^T$ can be obtained through an ordinary least-squares solution. However, if the knot points are unknown, a solution for the parameters can be estimated using a nonlinear least-squares procedure. In this study, we are proceeding under the assumption that the knot points are unknown and need to be estimated from the data.

Because the data follow a hierarchical structure by design (stands and trees within stands), a nonlinear mixed model was used to account for the heterogeneity between stands and trees within stands. The nonlinear mixed model can be represented as

$$[4] \quad y_{ijk} = \beta_{ij,0} + \beta_{ij,1}x_{ijk} + \beta_{ij,2}x_{ijk}^2 + \beta_{ij,3}(\alpha_{ij,1} - x_{ijk})_+^2 + \beta_{ij,4}(\alpha_{ij,2} - x_{ijk})_+^2 + \varepsilon_{ijk}$$

Following Vonesh and Chinchilli (1997), the vector of mixed-effect parameters β_{ij} in the model can be represented as

$$[5] \quad \beta_{ij} = \mathbf{A}_{ij}\boldsymbol{\beta} + \mathbf{B}_{ij,1}\mathbf{b}_i + \mathbf{B}_{ij,2}\mathbf{b}_{ij}$$

where

$$\mathbf{A}_{ij} = \mathbf{B}_{ij,1} = \mathbf{B}_{ij,2} = \mathbf{I}_7$$

$$\boldsymbol{\beta} = [\beta_0 \beta_1 \beta_2 \beta_3 \beta_4 \alpha_1 \alpha_2]^T$$

$$\mathbf{b}_i = [b_i^{(0)} b_i^{(1)} b_i^{(2)} b_i^{(3)} b_i^{(4)} b_i^{(5)} b_i^{(6)}]^T$$

$$\mathbf{b}_{ij} = [b_{ij}^{(0)} b_{ij}^{(1)} b_{ij}^{(2)} b_{ij}^{(3)} b_{ij}^{(4)} b_{ij}^{(5)} b_{ij}^{(6)}]^T$$

where b_i and b_{ij} are the stand- and tree-level random effects; $\mathbf{B}_{ij,1}$ and $\mathbf{B}_{ij,2}$ are the associated random-effect design matrices; and \mathbf{A}_{ij} and $\boldsymbol{\beta}$ are the fixed-effect design matrix and parameter vector, respectively. \mathbf{I}_7 is a 7×7 identity matrix with all the diagonal elements equal to 1.

The random effects and within-tree error terms were assumed to be distributed normally as $\mathbf{b}_i \sim N(0, \boldsymbol{\Psi}_1)$, $\mathbf{b}_{ij} \sim N(0, \boldsymbol{\Psi}_2)$, and $\varepsilon_{ij} \sim N(0, \sigma^2 \mathbf{A}_{ij})$ and are independent of each other. Here $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$ are variance-covariance matrices representing different levels of stand and tree random effects and \mathbf{A}_{ij} is a positive-definite matrix representing the within-tree error structure. A full model with random effects associated with all of the parameters in the model is considered first by assuming a diagonal variance-covariance matrix structure for random effects and an independent structure for within-tree error ($\mathbf{A}_{ij} = \mathbf{I}_{ij}$). These assumptions were relaxed in the later stages of fitting by assuming different variance-covariance structures for the random effects. Several reduced models were also fitted by dropping the random-effect terms associated with the parameters. The best of these models was selected by comparing the fitted models using Akaike's information criterion (AIC).

The next step in the model building process was to incorporate any covariates. Based on the results from preliminary analysis, no significant improvement in model performance was observed after adding the covariates age and diameter

at breast height. One of our objectives was to identify the regional variation in mean trend of SG, and it is incorporated into appropriate parameters in the model using indicator variables. As we had six distinct physiographical regions in the study, we assumed different fixed-effect parameters for each region with the southern Atlantic Coastal Plain as the reference region and all other regions having their own parameters, which are deviations from the reference (effect version of parameterization). After assuming all of the parameters in the model to be region-specific, the fixed-effect design matrix and parameter vector for parameter l in $\boldsymbol{\beta}$ can be represented as

$$\mathbf{A}_l = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\beta}_l = [\beta_{l1} \beta_{l2} \beta_{l3} \beta_{l4} \beta_{l5} \beta_{l6}]^T$$

After formulating the appropriate mean model and random-effect structure, the homogeneity and independence assumptions on the error terms were relaxed by adding appropriate variance function and correlation structures to the model. This was done to enable us to explain the heteroscedasticity in the data and serial correlation across measurements successfully. Different variance functions usually used in growth modeling such as the power model ($\text{Var}(\varepsilon_{ijk}) = \sigma^2 |v_{ijk}|^{2\delta}$), the exponential model ($\text{Var}(\varepsilon_{ijk}) = \sigma^2 e^{2\delta v_{ijk}}$), and the constant power model ($\text{Var}(\varepsilon_{ijk}) = \sigma^2 (\delta_1 + |v_{ijk}|^{\delta_2})^2$) were used to define any non-constant variance within the data. The autoregressive models (AR(p)), moving-average models (MA(q)), and autoregressive with moving average models (ARMA(p, q); $\varepsilon_t = \sum_{m=1}^p \varphi_m \varepsilon_{t-m} + \sum_{n=1}^q \theta_n a_{t-n} + a_t$, where ε_t is the current error term, φ_m are the autoregressive parameters with $m = 1, 2, \dots, p$, θ_n are the moving average parameters with $n = 1, 2, \dots, q$, and a_t is the homoskedastic noise term with $E[a_t] = 0$) were used with the data to account for dependence across repeated measurements within each tree. AIC criterion was used for checking significant changes in performance of the models. The nonlinear mixed models were fitted using the nlme package available in R (Pinheiro et al. 2009).

Semiparametric model

A more flexible approach to explain the nonlinear trend in disk SG with relative height is by semiparametric regression. Semiparametric regression can model nonlinear relationships, here the change in disk SG with relative height, without having any parametric restriction. The advantage is that these models can be formulated in a linear mixed-model framework (Ngo and Wand 2004), allowing the use of estimation and inferential tools available in mixed-model methodology.

Let y_{jk} represents the disk SG observed at disk k of tree j . A simple model form to explain disk SG with relative height is

$$[6] \quad y_{jk} = f(x_{jk}) + \varepsilon_{jk}$$

where $\varepsilon_{jk} \sim N(0, \sigma_\varepsilon^2)$ and f is a smooth function describing the trend in disk SG with relative height. We utilized penalized smoothing splines, curves that are formed by splicing low-order polynomials at known knot locations, to model the change in disk SG with relative height. A truncated quadratic basis was used to model the function $f(x_{jk})$. Model 6 (eq. 6) can be represented as

$$[7] \quad y_{jk} = \beta_0 + \beta_1 x_{jk} + \beta_2 x_{jk}^2 + \sum_{p=1}^P u_p (x_{jk} - \kappa_p)_+^2 + \varepsilon_{jk}$$

where $u_p \sim N(0, \sigma_u^2)$. Here, $\kappa_1, \dots, \kappa_p$ are distinct knot locations within the range of x_{jk} s and $(x_{jk} - \kappa_p)_+$ is the positive function where “+” sets it to zero for those values of x_{jk} where $x_{jk} < \kappa_p$ is negative (here $x_{jk} < \kappa_p$). According to Ruppert et al. (2003), a reasonable choice for selecting knots is that there should be four to five unique data points between two knots, with 35 knots as the maximum number of allowable knots. They proposed a simple method for knot selection such that knot κ_p equals sample location $(p + 1)/(p + 2)$ of the unique x_{jk} s, $p = 1, \dots, P$, where $P = \max(5, \min(1/4 \times \text{number of unique } x_{jk}\text{s}, 35))$. Use of the default knot selection procedure resulted in selecting 35 knots in this study, the maximum allowable knots based on the above procedure. Evenly spaced knots were also recommended and practiced in fitting the semiparametric regression (Ruppert et al. 2003; Jordan et al. 2008). Here, we used eight evenly spaced knots at an interval of 0.1 between the minimum and maximum relative height from the available data.

An estimate of $[\beta, \mathbf{u}]$ can be obtained by formulating model 7 as a linear mixed model as follows:

$$[8] \quad \mathbf{y} = \mathbf{X}\beta + \mathbf{Z}\mathbf{u} + \boldsymbol{\varepsilon}$$

Here,

$$\mathbf{y} = \begin{bmatrix} y_{11} \\ \vdots \\ y_{1m_1} \\ \vdots \\ y_{nm_n} \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{11}^2 \\ \vdots & \vdots & \vdots \\ 1 & x_{1m_1} & x_{1m_1}^2 \\ \vdots & \vdots & \vdots \\ 1 & x_{nm_n} & x_{nm_n}^2 \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} (x_{11} - \kappa_1)_+^2 & \dots & (x_{11} - \kappa_P)_+^2 \\ \vdots & \vdots & \vdots \\ (x_{1m_n} - \kappa_1)_+^2 & \dots & (x_{1m_n} - \kappa_P)_+^2 \\ \vdots & \vdots & \vdots \\ (x_{nm_n} - \kappa_1)_+^2 & \dots & (x_{nm_n} - \kappa_P)_+^2 \end{bmatrix}$$

$$\mathbf{u} = [u_1 \quad \dots \quad u_P]$$

The maximum likelihood estimate (MLE) of $\hat{\beta}$ and an empirical best linear unbiased predictor for $\hat{\mathbf{u}}$ can be obtained by fitting the above model form in any standard mixed-model software (e.g., lme in S-PLUS and R, PROC MIXED in SAS). The smoothness of the curve is controlled by the parameter $\lambda = \sigma_\varepsilon^2/\sigma_u^2$, which is calculated automatically using the restricted MLEs of σ_u^2 and σ_ε^2 .

One of the major objectives of this study was to understand the regional variation in the mean trend of disk SG with relative height. The addition of the interaction term in model 7 was used to examine regional differences. Model 7 with an interaction term can be represented as

$$[9] \quad y_{jk} = \beta_0 + \beta_1 x_{jk} + \beta_2 x_{jk}^2 + \sum_{p=1}^P u_p (x_{jk} - \kappa_p)_+^2 + \sum_{\ell=2}^L z_{j\ell} (\gamma_{0\ell} + \gamma_{1\ell} x_{jk} + \gamma_{2\ell} x_{jk}^2) + \sum_{\ell=2}^L z_{j\ell} \left(\sum_{p=1}^P v_p^\ell (x_{jk} - \kappa_p)_+^2 \right) + \varepsilon_{jk}$$

Here $z_{j\ell}$ is a regional indicator variable with $z_{j\ell} = 1$ if tree j is in region ℓ and 0 otherwise for $\ell = 2, \dots, L$ ($L = 6$), and $v_p^\ell \sim N(0, \sigma_{v\ell}^2)$. The parameters $[\beta_0 \quad \beta_1 \quad \beta_2 \quad u_p]$ in eq. 9 represent the southern Atlantic Coastal Plain ($\ell = 1$), and extra terms (γ and v) represent deviation of other regions from the mean trend of the southern Atlantic Coastal Plain.

To account for the heterogeneity between stands and trees within stands from the design, we used random stand (b_i) and tree (b_{ij}) effects in the model. Let y_{ijk} represent the SG of disk k in tree j in stand i ; model 9 with random stand and tree effects can be represented as

$$[10] \quad y_{ijk} = \beta_0 + \beta_1 x_{ijk} + \beta_2 x_{ijk}^2 + \sum_{p=1}^P u_p (x_{ijk} - \kappa_p)_+^2 + \sum_{\ell=2}^L z_{ij\ell} (\gamma_{0\ell} + \gamma_{1\ell} x_{ijk} + \gamma_{2\ell} x_{ijk}^2) + \sum_{\ell=2}^L z_{ij\ell} \left(\sum_{p=1}^P v_p^\ell (x_{ijk} - \kappa_p)_+^2 \right) + b_i + b_{ij} + \varepsilon_{ijk}$$

where $b_i \sim N(0, \sigma_{b_i}^2)$ and $b_{ij} \sim N(0, \sigma_{b_{ij}}^2)$.

It is interesting to know the rate at which SG changes along the length of a loblolly pine stem. The derivative of model 10 could potentially be used to explore the rate of

change of disk SG with height. This can answer questions such as how fast does SG change along a stem, does it approach a plateau, at what height does the rate of change in SG approach a plateau. Differentiating model 10 with respect to x_{ijk} gives

$$[11] \quad \frac{\partial y_{ijk}}{\partial x_{ijk}} = \beta_1 + 2\beta_2 x_{ijk} + \sum_{p=1}^P 2u_k(x_{ijk} - \kappa_p)_+ + \sum_{\ell=2}^L z_{ij\ell} (\gamma_{1\ell} + 2\gamma_{2\ell} x_{ijk}) + \sum_{\ell=2}^L z_{ij\ell} \left(\sum_{p=1}^P 2v_k^\ell (x_{ijk} - \kappa_p)_+ \right)$$

A solution to the mixed-model equation can be utilized to get predicted values and standard errors from models 10 and 11. For more details of model formulation, fitting, and prediction using this procedure, readers are referred to Ruppert et al. (2003). All of the models in the above sections were fitted using S-PLUS software (Pinheiro and Bates 2000).

Specific gravity maps

Maps explaining the variation in whole-disk SG across the geographical range of loblolly pine at a given height are useful for making decisions in product categorization and utilization. The spatial variation in a particular entity is usually explained using a method known as kriging (Cressie 1993), which has been widely applied in geostatistics. Kriging is an interpolation method that predicts the value of a variable (here disk SG) at an unknown spatial point using the spatial covariance information calculated from the available data. Because SG data in this study were collected across space (latitude and longitude) and tree height, it is important to model the variation in SG across height and space simultaneously to understand the spatial variation in SG at a particular height level. Because the variation in SG across height was highly nonlinear, a geoadditive approach proposed by Kamman and Wand (2003) was used in this study. The geoadditive approach is a combination of geostatistical and additive models and accounts for the nonlinear covariate effect (here tree height) under the assumption of additivity (Kamman and Wand 2003). These models can be implemented using the mixed-model framework.

The covariate in the present study was relative height of a tree and the geographical locations are represented by latitude and longitude of the stand from which SG was measured. Following Kamman and Wand (2003) and Ruppert et al. (2003), the geoadditive model can be formulated as follows. The additive model component for explaining the change in disk SG with continuous variable relative height is given as

$$[12] \quad y_{ik} = \beta_0 + f(x_{ik}) + \varepsilon_{ik}$$

where y_{ik} is the SG measurement from disk k in stand i and f is a smoothing function of relative height x_{ik} . Model 12 is equivalent to model 7 with a truncated quadratic basis. The random intercept for stand and tree variables was omitted from model 12.

Given the data of form $(\mathbf{D}_{ik}, y_{ik})$, where y_{ik} is a scalar and $\mathbf{D}_{ik} \in \mathfrak{R}^2$ represents geographical locations, a simple universal Kriging model with linear covariate is

$$[13] \quad y_{ik} = \beta_0 + \boldsymbol{\beta}_1^T \mathbf{D}_{ik} + S(\mathbf{D}_{ik}) + \varepsilon_{ik}$$

where $\{S(\mathbf{D}_{ik}) : \mathbf{D} \in \mathfrak{R}^2\}$ is a stationary mean zero stochastic process. Prediction to a new location $\mathbf{D}_0 \in \mathfrak{R}^2$ within the sampling space is done by substituting the estimates of $\hat{\beta}_0$ and $\hat{\beta}_1$ and an empirical best linear predictor $\hat{S}(\mathbf{D}_0)$ for an estimated covariance structure for S into model 13. The geographical component was fitted as a linear mixed model by using a bivariate thin plate spline to a geographic location (Ruppert et al. 2003). The covariance for S is assumed to be isotropic, i.e., the covariance between two stands that are $\|h\|$ units apart is the same regardless of direction and location of the stand.

The final geoadditive model can be obtained by merging models 12 and 13 as

$$[14] \quad y_{ik} = \beta_0 + f(x_{ik}) + \boldsymbol{\beta}_1^T \mathbf{D}_{ik} + S(\mathbf{D}_{ik}) + \varepsilon_{ik}$$

which can be expressed as a linear mixed model as

$$[15] \quad \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \boldsymbol{\varepsilon}$$

where \mathbf{y} is the vector of response (here SG), $\mathbf{X} = [1 \ x_{ik} \ x_{ik}^2 \ \mathbf{D}_{ik}]$, and \mathbf{Z} corresponds to the basis functions for f and S . The additive component in the model allows us to appropriately explain the nonlinear trend in SG with relative height. The geographical component in the model was fitted using reduced knot kriging, where $\{\kappa_1, \dots, \kappa_p\}$ is a subset of knots selected from sample space $\mathbf{D}_{ik} \in \mathfrak{R}^2$. The knots were selected using the space-filling algorithm discussed by Kamman and Wand (2003) and Ruppert et al. (2003). Readers are referred to Ruppert et al. (2003) and Kamman and Wand (2003) for more details on geoadditive model formulation, fitting, and prediction. Maps were produced by fitting the geoadditive model to the data. Model 14 was fitted using the SemiPar library in R (Wand et al. 2005).

Results

Parametric model

The model with stand- and tree-level random effects on parameters β_0 , β_1 , and β_2 was selected as the best random-effects model (AIC = -24447.77). After identifying the mixed-effect parameters, all parameters (except the knot parameters α_1 and α_2) were allowed to vary from region to region (AIC = -24538.6). The heteroskedasticity in residuals was accounted for by using a power-of-the-mean variance function (AIC = -24609.94). The correlation across repeated measurement taken from each tree was best represented using an ARMA(1, 1) model (AIC = -25007.74).

The difference between mean trends in disk SG among regions was addressed using likelihood ratio tests (LRTs) by dropping nonsignificant region-specific fixed-effect parameters from the full model fitted above. The final model was selected based on improvement in AIC criterion. We also allowed the knot parameters α_1 and α_2 to vary from region to region at this stage. The final model (AIC = -25057.16) was developed through "stepwise" procedure by dropping

Table 2. Estimated parameters from the three-segmented quadratic model.

Parameter	Estimate	SE	<i>t</i> value	<i>p</i> value
$\beta_{0,intercept}$	0.4678	0.0044	106.98	0.0000
$\beta_{0,northern Atlantic}$	-0.0477	0.0096	-4.96	0.0000
$\beta_{0,Upper Coastal}$	-0.0197	0.0055	-3.57	0.0004
$\beta_{0,Piedmont}$	-0.0251	0.0047	-5.30	0.0000
$\beta_{0,Hilly Coastal}$	-0.0135	0.0046	-2.95	0.0031
$\beta_{1,intercept}$	-0.0437	0.0121	-3.60	0.0003
$\beta_{1,Gulf Coastal}$	0.0694	0.0147	4.72	0.0000
$\beta_{2,intercept}$	-0.0493	0.0105	-4.70	0.0000
$\beta_{2,northern Atlantic}$	0.0344	0.0122	2.82	0.0048
$\beta_{2,Gulf Coastal}$	-0.0636	0.0174	-3.65	0.0003
$\beta_{3,intercept}$	1.2199	0.0946	12.89	0.0000
$\beta_{3,Upper Coastal}$	-0.5382	0.1514	-3.56	0.0004
$\beta_{3,Piedmont}$	-0.1169	0.0423	-2.77	0.0057
$\beta_{4,intercept}$	-5.4250	1.6637	-3.26	0.0011
$\beta_{4,northern Atlantic}$	2.4207	1.2575	1.92	0.0543
$\alpha_{1,intercept}$	0.2878	0.0095	30.35	0.0000
$\alpha_{1,Upper Coastal}$	0.0512	0.0257	1.99	0.0464
$\alpha_{1,Hilly Coastal}$	-0.0171	0.0049	-3.45	0.0006
$\alpha_{2,intercept}$	0.0800	0.0127	6.31	0.0000
$\alpha_{2,Upper Coastal}$	-0.0219	0.0107	-2.05	0.0403
Random parameters				
$\sigma_{b_{0,i}}$	0.0187			
$\sigma_{b_{3,i}}$	0.0234			
$\sigma_{b_{03,i}}$	-0.0003			
$\sigma_{b_{0,ij}}$	0.0151			
σ	0.0362			
Heteroskedasticity and autocorrelation parameters				
φ	0.8737			
θ	-0.3424			
δ	0.5122			

Note: SE, standard error.

the nonsignificant parameters from the full model. The parameter estimates from the final fitted model are presented in Table 2.

Based on the final model, estimates of β_0 from the southern Atlantic Coastal Plain and Gulf Coastal Plain were not significantly different. The estimated β_0 parameters from other regions were found to be significantly different from these two regions. The estimate of the β_0 parameter was highest for the southern Atlantic Coastal Plain and Gulf Coastal Plain (0.4678) and lowest for the northern Atlantic Coastal Plain (0.4201). The estimate of the β_0 parameter for the other three regions was between these two groups (Piedmont = 0.4427; Upper Coastal Plain = 0.4481; Hilly Coastal Plain = 0.4543). The estimate of the β_1 parameter was not significantly different for all regions except the Gulf Coastal Plain. Similarly, the estimated β_2 parameter was not significantly different for all regions except the northern Atlantic Coastal Plain and Gulf Coastal Plain. The estimated β_3 parameter from the Upper Coastal Plain and Piedmont was significantly different from all other regions. The β_4 parameter was significantly different for the northern Atlantic Coastal Plain compared with all other regions. The first knot from the tip of the tree, α_1 , was estimated to be at 0.2878 for all regions except the Upper Coastal Plain (0.3390) and Hilly Coastal Plain (0.2707). The estimate of the second knot pa-

rameter from the tip of the tree, α_2 , was at a relative height of 0.08 for all regions except Upper Coastal Plain, where the estimate was at a relative height of 0.0581.

Plots of mean predicted disk SG are presented in Fig. 2. Based on the three-segmented quadratic model, the mean disk SG trends of trees from the southern Atlantic Coastal Plain and Gulf Coastal Plain were higher than all other regions, with the mean trend of Gulf Coastal Plain above that of the southern Atlantic Coastal Plain. The mean trend in disk SG was lowest for trees from the northern Atlantic Coastal Plain. Mean disk SG trend of the other regions fell between these two limits, with the Hilly Coastal Plain having the highest SGs followed by the Upper Coastal Plain and then Piedmont. It was also observed that the mean trend in disk SG of trees from the northern Atlantic Coastal Plain merged with the mean SG trend of the Hilly Coastal Plain, Upper Coastal Plain, and Piedmont above a relative height of 0.8.

Semiparametric model

The nonlinear trend in disk SG with tree height and the regional variation in mean trend was explained in a more flexible way by using a semiparametric model. A model with a common smoothing parameter for all regions ($\sigma_u^2 = \sigma_{ve}^2$) was favored and fitted based on preliminary

Fig. 2. Mean predicted disk specific gravity for six regions from the three-segmented (two knots) parametric model.

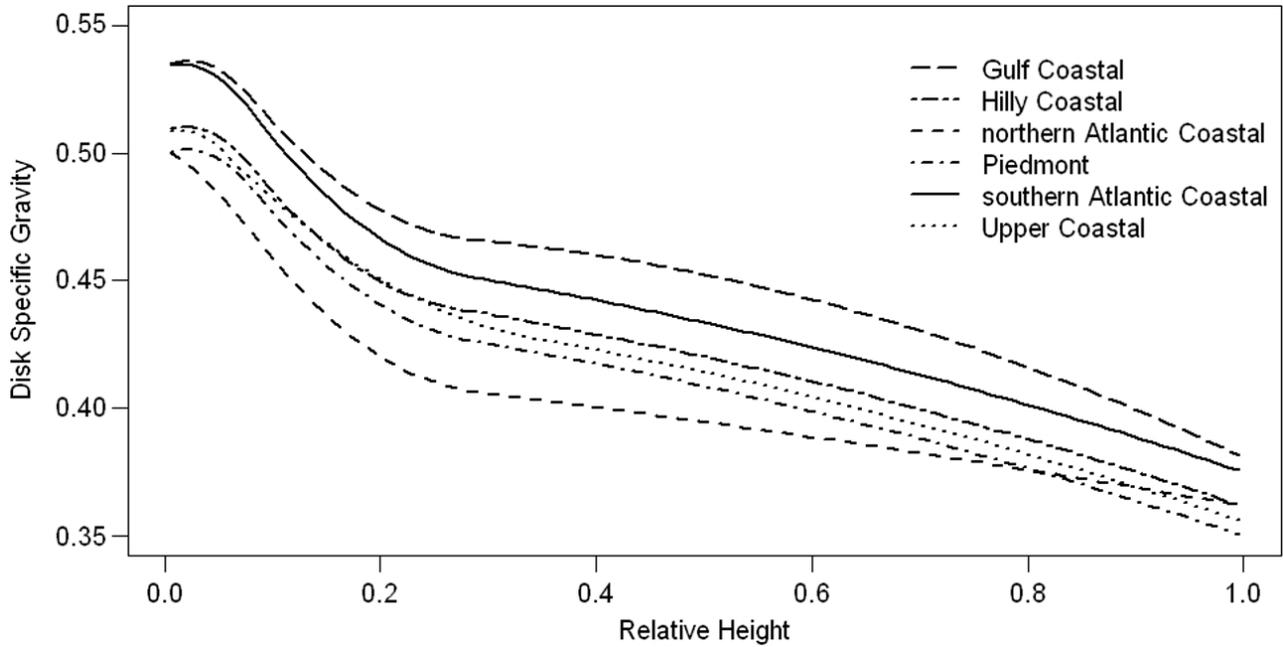
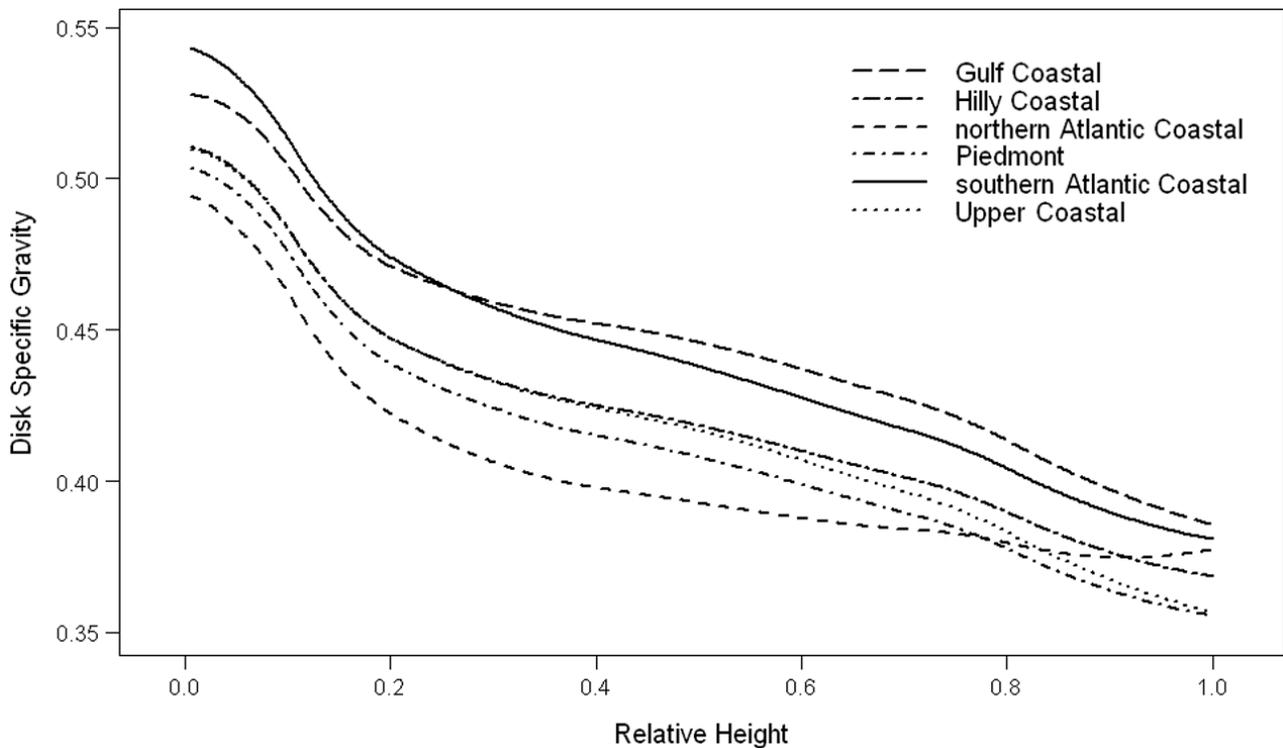


Fig. 3. Mean predicted disk specific gravity for six regions from the semiparametric model (eight known knots).



analysis. Based on the fitted model, disk SG follows a decreasing trend with relative height. A test of regional variation on the mean trend of disk SG was addressed by using a LRT test by fitting the full model (eq. 10 with assumed common smoothing parameters for all regions) and a reduced model with $H_0: \gamma_{p\ell} = 0$, where $p = 0, 1, \text{ or } 2$ and $\ell = 2, 3, 4, 5, \text{ or } 6$. Based on the LRT, significant differences between regions were found with a test statistic of 103.94 (p value < 0.0001), which follows an asymptotic $\chi^2_{[15]}$.

A plot of predicted SG from the model is presented in Fig. 3. Mean trends of SG for trees from the southern Atlantic Coastal Plain and Gulf Coastal Plain were higher than all other regions. It was observed from the mean plot of these two regions that at the base of the tree, the mean trend of disk SG for the southern Atlantic Coastal Plain was above that of the Gulf Coastal Plain up to a relative height ~ 0.25 , at relative heights > 0.25 , the trend was reversed. The predicted disk SG of trees from the northern Atlantic Coastal

Fig. 4. Mean predicted disk specific gravity, with 95% confidence intervals, from the semiparametric model (eight known knots) by region.

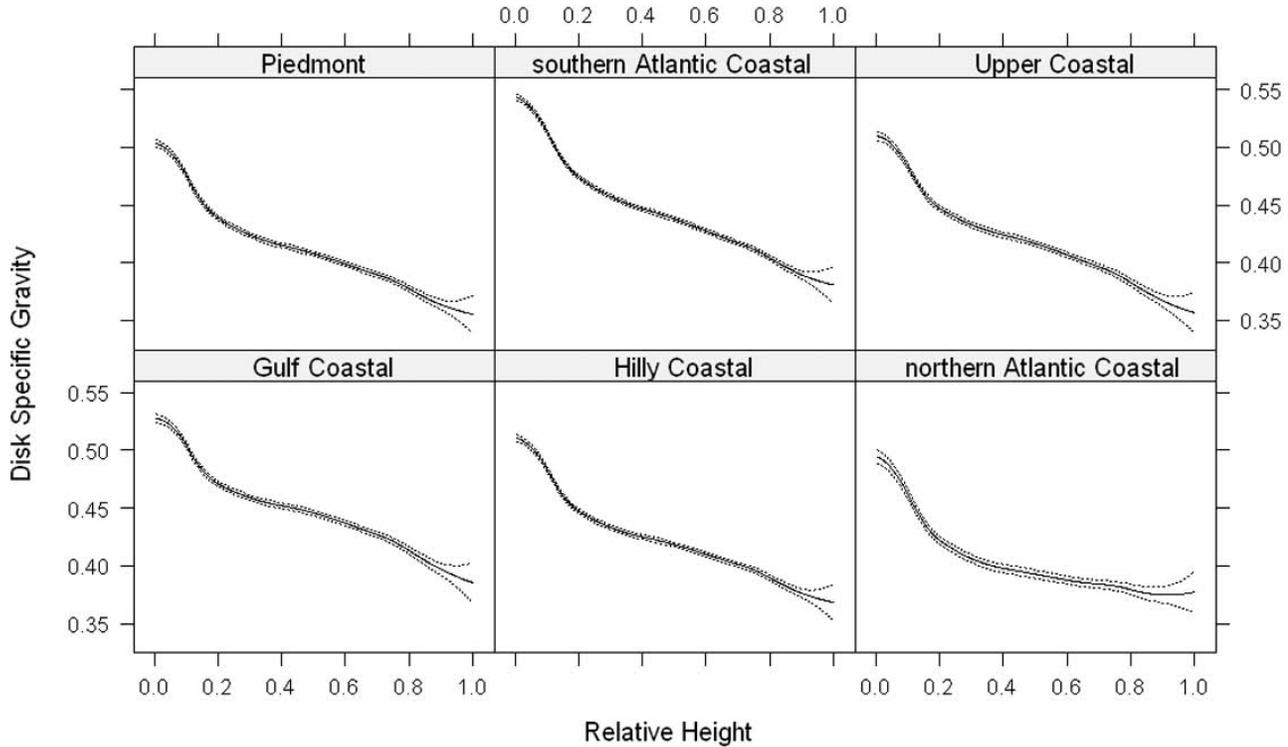
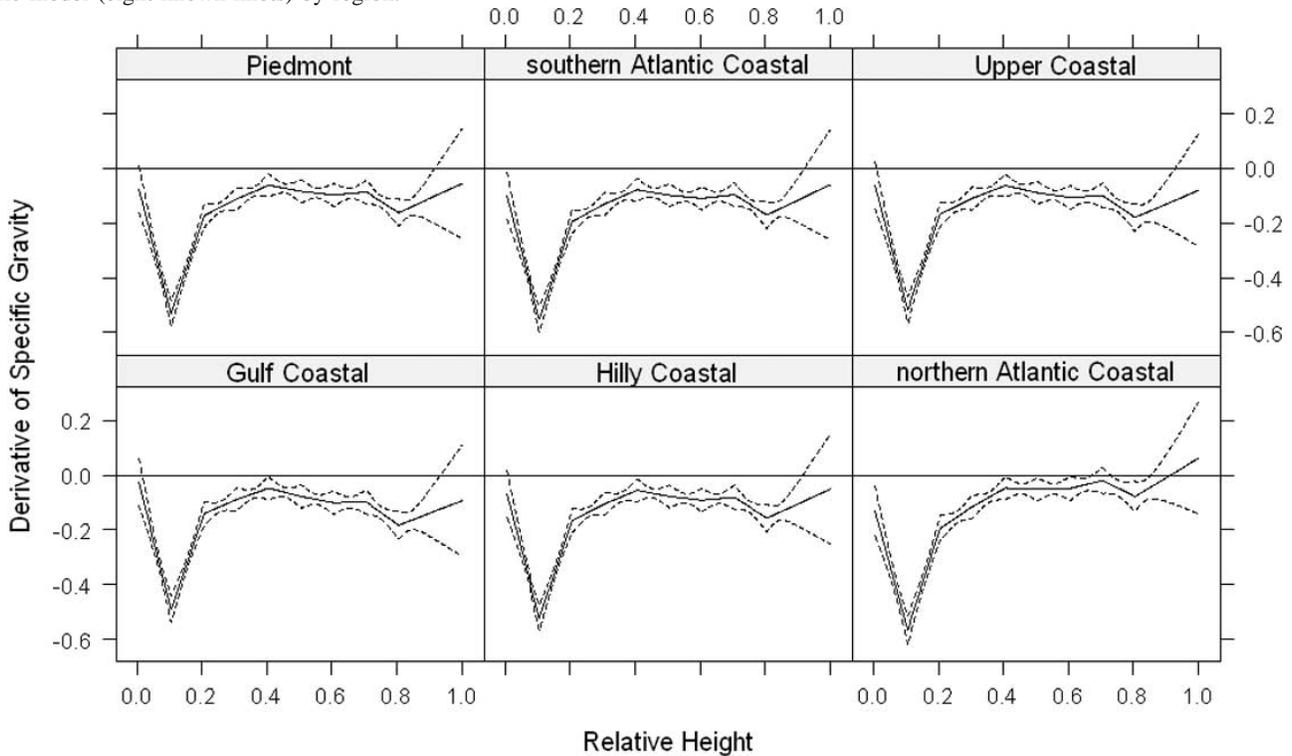


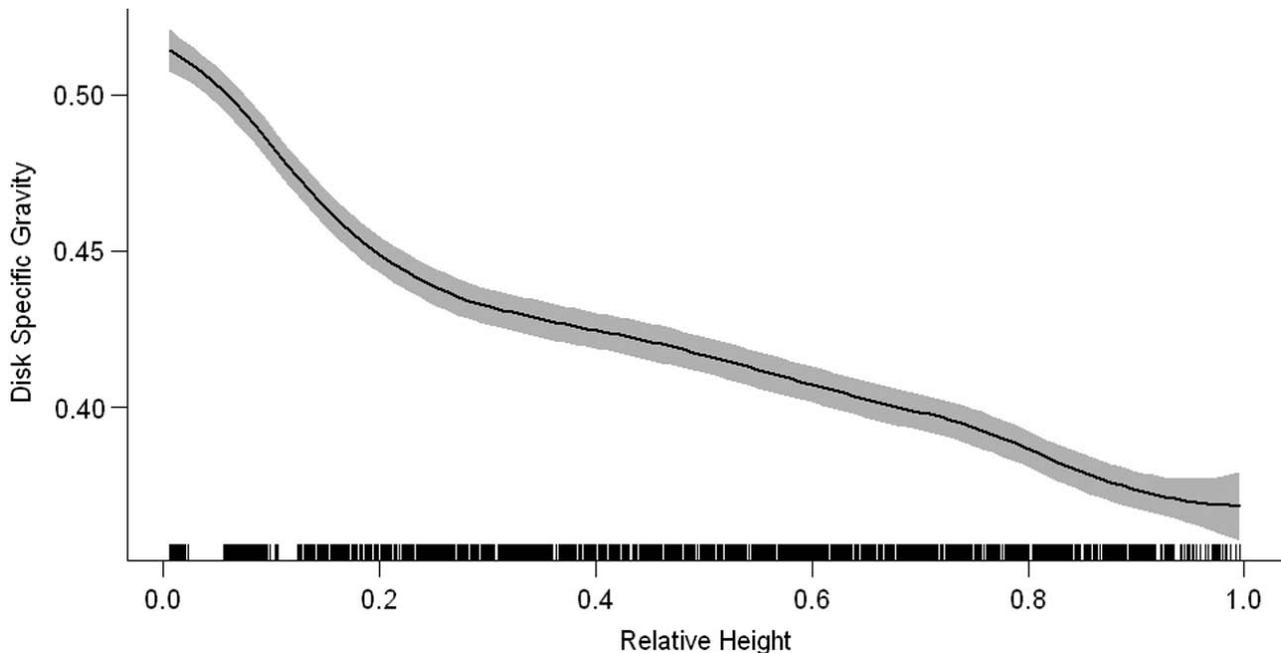
Fig. 5. Mean predicted rate of change in specific gravity, with 95% confidence intervals, with respect to relative height from the semiparametric model (eight known knots) by region.



Plain was the lowest. The predicted SG of other regions again fell between these two groups, with the Hilly Coastal Plain having the highest predicted disk SGs, followed by the Upper Coastal Plain and Piedmont. The predicted SG of trees from the northern Atlantic Coastal Plain again merged

with the mean SG trend of the Hilly Coastal Plain, Upper Coastal Plain, and Piedmont above a relative height of 0.8. A plot of the mean predicted disk SG with 95% prediction intervals is presented in Fig. 4. A plot of the derivatives of mean predicted disk SG along with 95% point-wise confi-

Fig. 6. Effect of relative height on whole-stand disk specific gravity, with 95% variability bar from geoadditive model fitting. The tick marks on horizontal axis represent the observed data points.



dence bands is presented in Fig. 5. Based on the plot, SG decreases very rapidly near the base of the tree to a relative height of ~ 0.1 , decreases at a decreasing rate from a relative height of ~ 0.1 to ~ 0.3 , and then decreases at a constant rate above a relative height ~ 0.3 to the top of the tree.

Specific gravity maps

After fitting the geoadditive model (eq. 14; Fig. 6), it was observed that the stand average disk SG followed a pattern similar to that described based on a semiparametric model (Fig. 5).

Maps showing the geographical variation in whole-stand disk SG along with the standard error of predictions at specific relative heights (0.05, 0.15, 0.5, and 0.8) are presented in Fig. 7. These maps were made under the assumption that all of the stands sampled are of the same age (average age of ~ 23 years). The primary reason for making such an assumption is that the stands sampled were from a narrow range of ages (Table 1). Based on the maps, a decreasing trend in disk SG was observed from south to north and from east to west. Whole-stand disk SG was higher near the Coastal Plain, with high SG bands in the southern Atlantic Coastal Plain and Gulf Coastal Plain. Disk SG was high in southern Georgia, southwestern Alabama, and the western edge of Texas. The Upper Coastal Plain, Hilly Coastal Plain, and Piedmont formed a band of lower SG wood, whereas the lowest SG wood was from the northern Atlantic Coastal Plain and parts of Piedmont. Areas with low sampling intensity such as Tennessee, northern Arkansas, Alabama, Virginia, and southern parts of Mississippi and Louisiana can be identified from the large standard errors of prediction.

Discussion

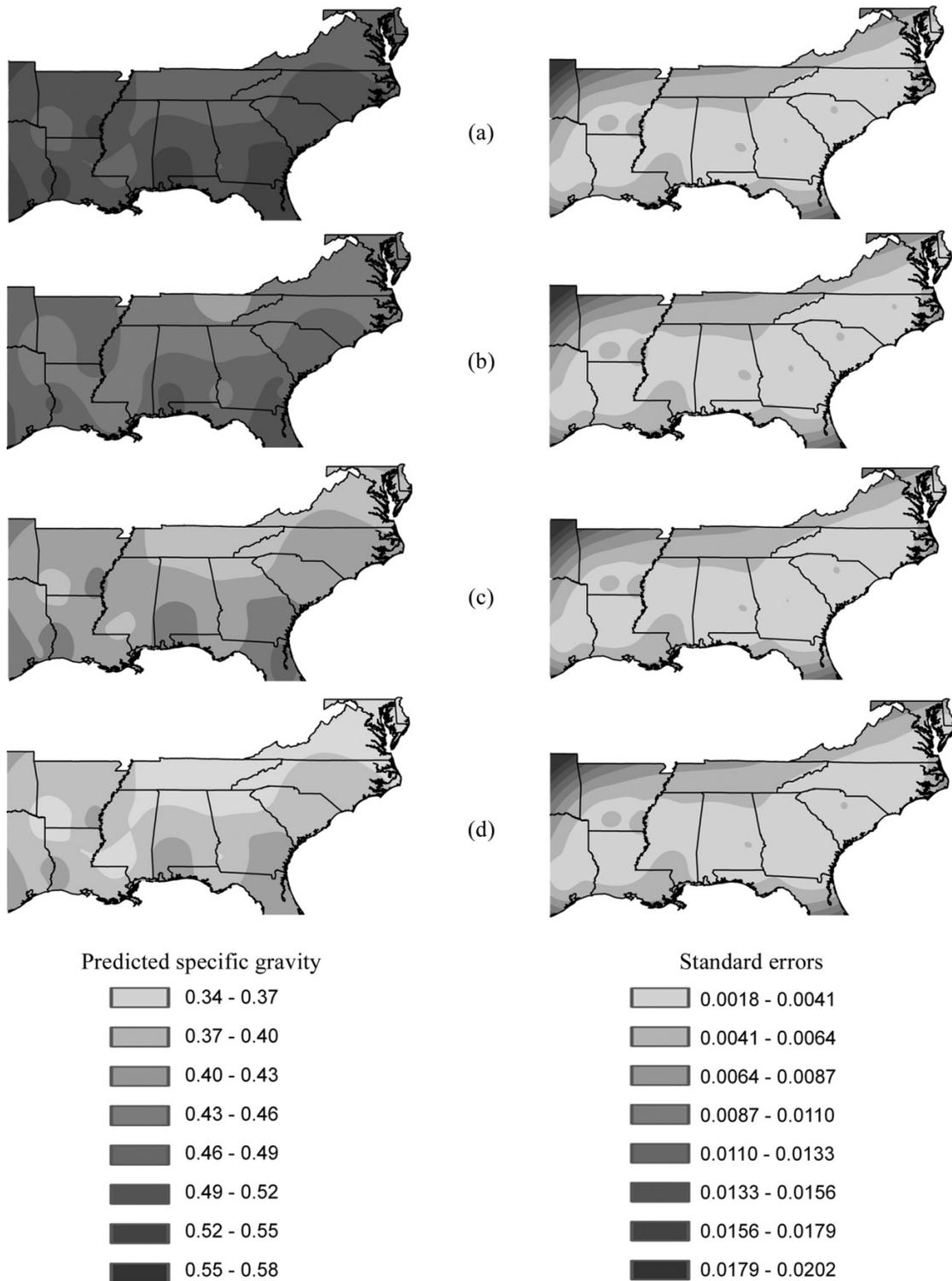
Disk SG of loblolly pine trees decreases in a nonlinear fashion with tree height. Both parametric and semiparamet-

ric approaches were used to explain the longitudinal and regional variation in disk SG along the stem. A geoadditive approach was used to describe the regional variation in disk SG at a specific disk height, and significant regional variation in mean SG trends was observed. Generally, mean SG trends were highest for trees from the southern Atlantic Coastal Plain and Gulf Coastal Plain and lowest for trees from the northern Atlantic Coastal Plain, with trees from the Hilly Coastal Plain, Upper Coastal Plain, and Piedmont between these extremes. Both parametric and semiparametric modeling approaches agreed and resulted in similar conclusions.

Our study suggests that loblolly pine stems can be divided into three zones based on the longitudinal variation of disk SG. Based on the derivative plots from the semiparametric model (Fig. 5) for all regions, mean SG decreased rapidly from the base of the tree to a relative height of ~ 0.1 ; SG then decreased at a decreasing rate between relative heights of ~ 0.1 to ~ 0.3 ; for relative heights $> \sim 0.3$, SG decreases at constant rate. Results from the semiparametric model support the proposed parametric model in which a stem is represented as three segments with each segment represented by a quadratic function of relative height with two knot points that are unknown and estimated from the data. Based on the three-segment parametric model, the first change in curve shapes occurred at a relative height of ~ 0.08 . The second change in curve shape of mean disk SG was at a relative height of ~ 0.29 . These findings agree with the three-segmented classification of the stems of loblolly pine proposed by Burdon et al. (2004).

The mean trend of disk SG was highest for the southern Atlantic Coastal Plain and Gulf Coastal Plain. The overall mean SG observed for these two regions was 0.46, which was higher than the mean disk SG observed for the other regions (0.42) (Table 1). Both parametric and semiparametric models support this conclusion, with higher mean SG curves

Fig. 7. Maps of the predicted whole-stand disk specific gravity and standard error (SE) using the geoadditive model at the following specific relative heights: (a) 0.05; (b) 0.15; (c) 0.5; (d) 0.8.



of trees from the southern Atlantic Coastal Plain and Gulf Coastal Plain compared with all other regions (Figs. 2 and 3). The high SG of trees from the southern Atlantic Coastal Plain and Gulf Coastal Plain might be attributed to two major reasons: (i) reduced length of core wood formation and proportion of core wood formed in these two regions com-

pared with that formed in other inland regions (Clark and Saucier 1989; Clark and Daniels 2002; Jordan et al. 2008); and (ii) high percentage of latewood in the rings of trees growing in these regions (~40%) compared with those growing in other regions (~35%). The proportion of latewood formed is highly correlated with summer precipitation,

mean annual temperature, and number of growing days. The trees growing in the southern Atlantic Coastal Plain and Gulf Coastal Plain receive, on average, more summer precipitation and have a higher mean annual temperature and more growing days than those in the other regions (Clark and Daniels 2002).

Maps of mean stand disk SG showed similar trends of regional variation in SG as described based on the parametric and semiparametric models. A decreasing trend in disk SG was present from south to north and from east to west. The disk SG maps, depending on the specified height, divided the loblolly pine growing range into three major regions: a high SG band that mainly included parts of the southern Atlantic Coastal Plain and Gulf Coastal Plain; a medium SG band that included the northern parts of the southern Atlantic Coastal Plain and Gulf Coastal Plain and parts of the Upper Coastal Plain and Hilly Coastal Plain; and a low SG band that included the Piedmont and Hilly Coastal Plain and northern Atlantic Coastal Plain. The above findings are in accordance with earlier results by Clark and Daniels (2002) and Jordan et al. (2008), who reported a decrease in SG with increase in latitude and increase in SG with increase in longitude based on the ring-by-ring data collected from breast height of trees. It should be noted that the maps were produced with the assumption that stands were the same age when sampled (approximately 23 years).

Significant longitudinal and regional variation in SG was observed in loblolly pine. For forest product industries, an understanding of both longitudinal and regional variation in SG is important as it allows raw material segregation and optimization of manufacturing processes. SG is an important wood quality index, is highly correlated with the strength and stiffness of wood, and determines the pulp yield and quality. An increase in SG of 0.02 units will result in a 22.7 kg increase in dry pulp per tonne of round wood (Mitchell 1964) and (or) an increase in modulus of elasticity ($31.15 \text{ kg}\cdot\text{cm}^{-2}$) and modulus of rupture ($35.16 \text{ kg}\cdot\text{cm}^{-2}$) (Wahlgren and Schumann 1975). Hence the strength of lumber or yield of pulp from a tree harvested from the southern Atlantic Coastal Plain and Gulf Coastal Plain will generally be greater than that of trees harvested from other regions at an equivalent age.

The primary objective of this study was to understand the within-tree and regional variation in SG of wood produced from conventionally managed (no fertilization apart from P at establishment on P-deficient sites and no competition control) loblolly pine plantations across the southeastern US. One of the limitations was that the trees utilized in this study came from stands within a narrow range of ages (20–25 years). However, this is a typical representation of the age range of short-rotation loblolly pine plantations throughout the southeastern US. As indicated earlier in the manuscript, the user should restrict the application of models and maps presented in this study to trees within the range of 20–25 years and subject to conventional management.

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References

- Burdon, R.D., Kibblewhite, R.P., John, C.F.W., Robert, A.M., Robert, E., and David, J.C. 2004. Juvenile versus mature wood: a new concept, orthogonal to corewood versus outerwood, with special reference to *Pinus radiata* and *P. taeda*. *For. Sci.* **50**: 399–415.
- Clark, A., and Daniels, R.F. 2002. Modeling the effect of physiographic region on wood properties of planted loblolly pine in southeastern United States. *In* Connection between Forest Resources and Wood Quality: Modeling Approaches and Simulation Software. Fourth Workshop IUFRO Working Party S5.01-04. INRA – Centre de Recherches de Nancy, France, Harrison Hot Springs, B.C. pp. 54–60.
- Clark, A., and Saucier, J.R. 1989. Influence of initial planting density, geographic location, and species on juvenile wood formation in southern pine. *For. Prod. J.* **39**(7–8): 42–48.
- Cressie, N. 1993. *Statistics for spatial data*. Wiley, New York.
- Fox, T.R., Jokela, E.J., and Allen, H.L. 2007. The development of pine plantation silviculture in the southern United States. *J. For.* **105**(7): 337–347.
- Gallant, A.R., and Fuller, W.A. 1973. Fitting segmented polynomial regression models whose join points have to be estimated. *J. Am. Stat. Assoc.* **68**(341): 144–147.
- He, R. 2004. Mixed effects modeling of wood properties of loblolly pine in the southeastern United States. Ph.D. dissertation, University of Georgia, Athens, Georgia.
- Jordan, L., Clark, A., Schimleck, L.R., Hall, D.B., and Daniels, R.F. 2008. Regional variation in wood specific gravity of planted loblolly pine in the United States. *Can. J. For. Res.* **38**: 698–710.
- Kamman, E.E., and Wand, M.P. 2003. Geoadditive models. *Appl. Stat.* **52**: 1–18.
- Larson, P.R., Kretschmann, D.E., Clark, A., and Isebrands, J.G. 2001. Formation and properties of juvenile wood in southern pines: a synopsis. USDA Forest Service, General Technical Report FPL-GTR-129.
- Max, T.A., and Burkhart, H.E. 1976. Segmented polynomial regression applied to taper equations. *For. Sci.* **22**: 283–289.
- Megraw, R.A. 1985. *Wood quality factors in loblolly pine*. TAPPI Press, Atlanta, Georgia.
- Mitchell, H. 1964. Patterns of variation in specific gravity of southern pines and other coniferous species. *TAPPI (Tech. Assoc. Pulp Pap. Ind.) J.* **47**: 276–283.
- Ngo, L., and Wand, M.P. 2004. Smoothing with mixed model software. *J. Stat. Softw.* **9**(1): 1–54.
- Panshin, A.J., and de Zeeuw, C. 1980. *Textbook of wood technology*. 4th ed. McGraw-Hill Book Co., New York.
- Phillips, K.M. 2002. Modeling within tree changes in wood specific gravity and moisture content for loblolly pine in Georgia. M.S. thesis, Warnell School of Forestry and Natural Resources, University of Georgia, Athens, Georgia.
- Pinheiro, C.J., and Bates, D.M. 2000. *Mixed effects models in S and S-PLUS*. Springer-Verlag, New York.
- Pinheiro, C.J., Bates, M.D., DebRoy, S., Sarkar, D., and R Core Team. 2009. nlme: Linear and nonlinear mixed effects models. R package version 3. Available at <http://cran.r-project.org/web/packages/nlme/index.html>.
- Ruppert, D., Wand, M.P., and Carroll, R.J. 2003. *Semiparametric regression*. 1st ed. Cambridge University Press, New York.

- Tasissa, G., and Burkhart, H.E. 1998a. Juvenile–mature wood demarcation in loblolly pine trees. *Wood Fiber Sci.* **30**(2): 119–127.
- Tasissa, G., and Burkhart, H.E. 1998b. Modeling thinning effects on ring specific gravity of loblolly pine (*Pinus taeda* L.). *For. Sci.* **44**(2): 212–223.
- Vonesh, E.F., and Chinchilli, V.M. 1997. Linear and nonlinear models for the analysis of repeated measurements. Marcel Dekker, New York.
- Wahlgren, H.D., and Schumann, D. 1975. Properties of major southern pines: Part I. Wood density survey. USDA Forest Service, Forest Product Laboratory, Madison, Wisconsin, Research Paper FPL-176.
- Wand, M.P., Coull, B., French, J., Ganguli, B., Kamman, E.E., Staudenmeier, J., and Zanobetti, A. 2005. SemiPar 1.0, R package [online]. Available at <http://cran.r-project.org>.
- Wear, D.N., and Greis, J.G. 2002. The Southern Forest Resource Assessment: summary report. USDA Forest Service, Southern Research Station Report No. 53.
- Zobel, B. 1972. Three rings-per-inch, dense southern pine — can it be developed? *J. For.* **70**: 333–336.
- Zobel, B.J., and van Buijtenen, J.P. 1989. Wood variation: its causes and control. Springer-Verlag, Berlin.