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Title: Building aggregate timber supply models from individual harvest choice

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Building aggregate timber supply models from individual harvest choice

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\textbf{Abstract}

Timber supply has traditionally been modelled using aggregate data. In this paper, we build aggregate supply models for four roundwood products for the US state of North Carolina from a stand-level harvest choice model applied to detailed forest inventory. The simulated elasticities of pulpwood supply are much lower than reported by previous studies. Cross price elasticities indicate a dominant influence of sawtimber markets on pulpwood supply. This approach allows predicting the supply consequences of exogenous factors and supports regular updating of supply models.

\textbf{Keywords:} Timber supply, harvest choice, conditional logit, elasticity, expectations, simulation.

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Introduction

Forecasting forest conditions requires insights into the effects of human activities, most especially timber harvesting. A number of timber supply models (Adams and Haynes 1980, Jackson 1980, Hyde 1980) have been estimated from aggregated inventory data for broad regions but few studies have explicitly linked aggregate timber supply models to observations of individual harvest behaviour (an exception is Prestemon and Wear 2000). The objective of this paper is to use harvest choice models applied to standard forest inventory data to derive complete aggregate supply models for a broad region.

Our goal is to provide a supply model that can link wood products market activities to timber harvest activities and forest inventories. Harvests can be viewed as withdrawals from a standing inventory of forests characterised by variable site qualities, species composition, and vintages, and future supply depends, not only on how much is harvested, but also on which types of stands are harvested. Given an initial inventory, production possibilities in any given period are intrinsically defined by all preceding harvest activity, biological growth, and other disturbances. Unlike other natural resources such as fisheries, where inventories might be adequately described in terms of total biomass, knowing the quality distribution of forest inventory is essential for defining future harvest possibilities.

To estimate harvest choice models, we use a two period formulation of the intertemporal choice problem (e.g., Max and Lehman 1988) applied to individual inventory records (plots). Predicted probabilities of harvests are then linked to plots, and the area-frame structure of the inventory is used to simulate regional supply responses.

We test our models using several panels of Forest Inventory Analysis (FIA) inventories for the state of North Carolina in the south-eastern United States (Miles et al. 2001). These ongoing inventories are the best available and only comprehensive data on forest conditions in the US and provide insights into management activities through regular re-measurement of plots. However, because these inventories are designed to provide precise estimates of variables that describe standing forests, they are not optimally designed for the study of harvest choice. As a result, we must design methods that are consistent with the general economic theory regarding harvest choice, yet adapted to the idiosyncrasies of survey methods. This approach is ultimately justified by our need to provide precise forecasts of FIA inventories to support multiple resource analysis within a national assessment framework.

Theory

Timber supply models summarize the production behaviour of forest managers in a market setting. Their conceptual foundation is the biological/physical production possibilities of timber growing and inventory adjustment, as well as information on the objectives of forest landowners. The choices of owners with heterogeneous objectives managing heterogeneous forestland then must be aggregated. In this section, we first describe the theory of harvest

1. These models are part of the US Forest Assessment System, built to support the decadal RPA Assessments mandated by the Renewable Resources and Rangelands… Act of 1974 which requires the USFS to deliver 50 year forecasts of resource supply demand and conditions every 5-10 years.
choice for a well-defined even-aged management problem and then how to adopt the theory to the more general cases.

Timber supply from even-aged management is described by a production function, which inputs generally include the age of the forest, $a$, the level of forest management effort, $E$, and the quality of the land, $q$ (e.g., Wear and Parks 1992, Binkley 1987). In the simple, even-aged case, merchantable timber volume per unit area, $V$, is given by the yield function:

$$ V = v(a, E; q) \quad (1) $$

The marginal physical products of age and management effort are both positive and decreasing in the relevant ranges of age and effort. Provided that the forest manager’s objective function and discount rate can be specified, then the forest yield function can be used to define if and when a forest stand would be harvested. For example, consider a manager, who faces prices $p$ for timber and $w$ for management effort (in this case, effort used to reforest the land after harvest). When the land is maintained indefinitely in forest use (i.e., forestry is the high-value use), the manager will maximize profit by selecting harvest ages and levels of effort $E$ to optimize:

$$ F^\pi = \max_{a, E} \left\{ \sum_{j=0}^{\infty} \left[ p v(a, E; q) e^{-ra} - wE \right] e^{-rj} \right\} \quad (2) $$

where $r$ is the interest rate and $j$ is the period. The optimum profit obtained, $F^\pi$, is the present net value for an infinite sequence of identical harvest ages. As long as the manager’s optimum timber profits are positive and greater than the value of land in alternative uses, then the manager’s solution to (2) will identify profit maximizing harvest dates, harvest volumes, and levels of regeneration effort. In a two-period model, where landowners simply determine whether to exercise or delay the harvest, harvests at the optimal age are revealed where the marginal benefits from delaying the harvest are just equal to the marginal opportunity costs of the delay (e.g., Max and Lehman 1988). However, the pure single-stand, even-aged management case rarely describes the actual management scenario. Instead, management is often driven by complex, multiple benefit objectives, forests are not even-aged, and harvests remove only a portion of the forest.

When forest management decisions are guided by utility rather than profit maximization, non-priced amenity services could be included in the manager’s objective function. We can calculate marginal benefits ($MBD$) and marginal opportunity costs ($MOC$) of delaying harvest that take into account non-priced amenity services. Instead of standard growth model (equation 1), we model harvest choice using a two period model where harvest occurs ($H=1$) when the $MBD$ equals the $MOC$ for a forest plot where these values depend exclusively on the attributes of the plot (which may or may not include a unique age record) and the ability to forecast end-of-period values:

$$ H = \begin{cases} 
1 & \text{if } MBD(q) \leq MOC(q) \\
0 & \text{otherwise} 
\end{cases} \quad (3) $$

The decision variable in this formulation is simply whether or not to harvest at the beginning of the analysis period (rather than the age at which a harvest might occur) and depends on the
benefits and opportunity costs of harvesting. It therefore depends on the ability to estimate net harvest benefits for the two periods being analysed.

Yet another complication arises when only a part of the stand is harvested (e.g., a third alternative of thinning or selective harvest). We can readily extend model (4) to allow for three choices: “partial harvest,” “complete harvest” and “no harvest”. If we view “marginal opportunity cost of delay” in model (4) as the “marginal benefit of harvest” and define $MB(h \mid q)$ as the marginal benefit of management decision $h$ conditional on $q$ (where $h$ could reflect any number of choices, including no action), then (4) can be expressed as:

$$H = \max\{h\} MB(h \mid q)$$  \hspace{1cm} (4)

This model could be generalized to any number of management decisions as long as we can predict growth of the stand and calculate the marginal benefits of each management decision.

Because timber inventories are heterogeneous in terms of vintage, species, and condition and timber is produced from forests allocated to a variety of uses with joint products, we construct timber supply from a systematic aggregation of individual harvest choices across the quality distributions defined by a forest inventory:

$$S_t = \sum_{j=1}^{J} A_j \times v(q_j) \times \Theta(h_j(q, p_t)) \hspace{1cm} (5)$$

where $A_j$ is the area of forest in quality class $j$ 2, $\Theta$ is the harvest intensity of management decision $h_j$ (from equation 4), which depends on the quality class of the stand as described above, and is a function of quality distribution of the forest existing at the beginning of the period (indexed by $t$) and price ($p$). Harvest volume ($v$) is indexed by quality classes that are defined by variables such as diameter, site index, and forest management type. For a clear felling, $v$ is simply equal to the standing merchantable volume at the beginning of the period. Harvest intensity is equal to 0 for “no harvest” and 1 for final harvest. In the case of partial harvests, it can be defined as a function of variables that describe the quality distribution of material on the plot, as well as on revenue and cost variables as found in the harvest choice equation. Each price yields an aggregate harvest response and the supply response can be approximated by simulating the harvest responses across a range of prices. The supply model can be extended to address $K$ multiple timber products by indexing the harvest volume by product class so that supply of product $k$ is defined as:

$$S_{k,t} = \sum_{j=1}^{J} A_j \times v_k(q_j) \times \Theta(h_j(q, p_t)) \hspace{1cm} \forall k \hspace{1cm} (6)$$

**Empirical Model**

An empirical application of the harvest choice model described in equation 4 requires observations of harvest decisions for a sample of forest plots along with estimates of the

2. Note that the area variables are the area expansion factors for each plot in a forest inventory.
benefits for each of all possible management decisions including no harvest. With forest plot measurements at times \( t \) and \( t + n \), the utility-maximizing landowner faces a choice among several management options, for example, no harvest, partial harvest (including thinning), or final harvest. Extending the two-period harvest choice model (Provencher 1997, Prestemon and Wear 1999) to multiple management decisions, the benefits of each choice \( h \in H \) can be expressed as follows:

\[
\pi_i(h) = u(h) + \mathbf{p}_i^\top \mathbf{v}_i(q | h) - c(q) + \Psi(q) + \rho \mathbf{E}[\mathbf{p}_{t+n}^\top \mathbf{v}_{t+n}(q | h) - c(q) + \Psi(q)]
\]

(7)

where \( u(h) \) is the non-timber utility associated with the stand under management decision \( h \), \( \mathbf{p}_i \) is the vector of prices of roundwood products, \( \mathbf{v}_i(q | h_i) \) is the vector of volumes of roundwood product harvested in period \( t \) with management decision \( h \) implemented in period \( t \), and \( \mathbf{v}_{t+n}(q | h_i) \) is the vector of roundwood volumes in period \( t+n \) if management decision \( h \) was implemented in period \( t \). \( c \) is the cost function which depends on site characteristics, \( \Psi(q) \) is the discounted residual value of the harvested stand (equal to the familiar bare land value if a clearcut is implemented), and \( \rho \) is the discount factor. If \( h \) = “no harvest”, \( \mathbf{v}_i(q | h_i) = 0 \) and \( \mathbf{v}_{t+n}(q | h_i) \) are the volumes of roundwood products in the stand grown for \( n \) years; if \( h \) = “partial harvest” \( \mathbf{v}_i(q | h_i) \) are the volumes of the removed roundwood products and \( \mathbf{v}_{t+n}(q | h_i) \) are the volumes of roundwood products in the retained part grown for \( n \) years; and if \( h \) = “final harvest” \( \mathbf{v}_i(q | h_i) \) are the volumes of roundwood products in the stand and \( \mathbf{v}_{t+n}(q | h_i) \) are the total volumes of roundwood products in the regenerated stand grown for \( n \) years.

Unobservable components of value may also accrue to management choices. Here we simply assume that total benefits have measurable and random components: \( (\pi_i(h) = \mu_i(h) + \epsilon_i(h)) \), and that benefits are a function of management decision, prices, and observable attributes of the stand such as volume and site characteristics, that affect growth, non-timber utilities, and management costs: \( \mu_i(h) = \mu_i(h, p, q) \). A rational landowner is expected to choose management decision with the greatest benefits. The probability of selecting management decision \( h \) is:

\[
\Pr(h | p, q) = \Pr(\mu_i(h, p, q) + \epsilon_i(h) > \mu_i(k, p, q) + \epsilon_i(k) \forall k \in H, k \neq h) = \Pr(\mu_i(h, p, q) - \mu_i(k, p, q) > \epsilon_i(k) - \epsilon_i(h) \forall k \in H, k \neq h)
\]

(8)

Assuming random components are independent and identically distributed (iid) with a type I extreme value distribution, the probability of choosing management decision \( h \) can be estimated using a conditional logit model (McFadden 1973):

\[
\Pr(h | p, q) = \frac{\exp(\mu_i(h, p, q))}{\sum_{k \in H} \exp(\mu_i(k, p, q))}
\]

(9)

The estimated discrete choice model can then be used to assign predicted probabilities of harvest to each plot within the inventory given a set of prices, and harvests can be simulated utilizing random number draws evaluated against the distributions of these predicted probabilities.
The harvest choice model as implemented above provides a means of predicting the probability of harvesting for each forest plot within a measured inventory and a given price level consistent with historical behaviour. While the price is constant for all plots across the inventory during the historical period, observed revenue levels and revenue changes vary due to considerable variability in the volume and volume growth estimated for each plot. We can therefore deduce the effects of a price change on harvesting activity through the revenue argument in equation 9 by simulating harvest outcomes for multiple price realizations.

Equation 9 can be used to generate a vector of harvest probabilities for any price scenario. Accordingly, by applying equation 9 to a forest inventory, we can generate a set of timber supply responses for a price scenario by aggregating harvested volume over probabilities of all modelled management decisions:

$$S_{k,t} = \sum_{j=1}^{J} \sum_{h \in H} A_j \times v_j(q_j) \times \Theta(h_j) \times \Pr(h_j|q,p_t) \quad \forall k$$

(10)

This defines the mean expected timber harvest response given the distribution of forest types and area expansion factors at the beginning of the period. Because of the error structure of the harvest probability model, equation 10 can generate multiple realizations of supply for a given price—that is, \(g(p)\) is a stochastic relationship. In order to summarize the full supply model, we generate a large number of estimates of timber supply across a broad range of prices using the harvest probability model applied to the measured inventory. We summarize these simulated data (pseudo-data), with \(K\) regression equations that defines the natural log of each timber output as a function of the natural log of all timber prices. Because prices are exogenous for the individual decision makers, this can be viewed as a pure model of timber supply conditioned on the existing inventory (i.e., supply is identified with respect to demand):

$$\ln(S_{k,t,t+1}) = \alpha_t + \sum_{i=1}^{K} \beta_i \ln(p_t) + \epsilon_k \quad \forall k$$

(11)

The \(I\) in the subscript of supply defines equation 11 as a set of timber supply functions conditioned on the inventory at the beginning of the period.

**Data**

With this general theoretical and empirical framework we investigate harvest choice and timber supply implications for the state of North Carolina. This state has been surveyed multiple times by the Forest Inventory and Analysis program of the US Forest Service. FIA data provide information on the overall plot characteristics, discrete landscape features, and measures associated with individual trees larger than an inch in diameter, respectively. Each plot represents a larger portion of the landscape to estimate the total inventory—the representative area is called the expansion factor.

Data on volumes by product classes, harvest choices, location and other site characteristics were compiled for matched plots for the \(t\) and \(t+1\) inventories. Volume of growing stock and volume of sawtimber volume were calculated from the plot records. We estimated the pulpwood volume as the difference between the sawtimber volume and the total growing
stock volume. Growing stock and trees per acre were delineated by broad species type, i.e. softwood and hardwood, using the species group variable recorded in the FIA database.

Several other variables were calculated for each plot by combining information from the plot, condition, and tree tables in the FIA database. Forest type and stand origin were combined to create a broad management class variable coinciding with the definition in published reports. The five broad management classes were: natural pine, planted pine, oak-pine (further referred to as mixed pine), upland hardwood, and lowland hardwood.

We determined whether the stand was harvested during the re-measurement period and identified the type of harvest using information about removals in the FIA data set. In order to calculate volume removed during the re-measurement period, annual removed volume is multiplied by the length of re-measurement period. The removals rate is defined as the ratio of removed volume to the sum of removed and retained volume. We define a final harvest if the removals rate is greater than 75%, and a partial harvest if the removals rate is between 5% and 75%. The removals rate for a partial harvest was calculated as the average removals rate from all stands that were identified as partially harvested.

To compute the revenue variables needed for the harvest choice model, we required (i) prices, (ii) volume of removals during the observation period, and (iii) volume of the retained part of the stand at the end of the observation period for four major products (softwood sawtimber, softwood pulpwood, hardwood sawtimber, and hardwood pulpwood) and for each of the possible management decisions (final harvest, partial harvest, no harvest).

Product prices were defined as the average of stumpage prices recorded during the observation period for each survey unit by Timber Mart South, a region-wide price reporting service (Norris Foundation). The volumes of the removals for the management decision “final harvest” were taken from the initial inventory. Total volume of removals for the management decision “partial harvest” is calculated by applying harvest intensity to the volume of growing stock. The proportions of softwood, softwood sawtimber, and hardwood sawtimber in the removed part of the stand are different from proportions in the original stand. For example, more sawtimber is extracted during selective harvest of natural pine stands. We model the proportion of roundwood removed using removals data of partially harvested stands and proportions of these products in the original stand as explanatory variables. The retained volumes of the four roundwood products after partial harvest are calculated by subtracting removed volumes from the volumes of these products in the original stand.

To calculate the expected revenue at the end of the period, we forecasted the volumes in each product class. The changes of softwood and hardwood growing stock volumes and changes of proportion of softwood and hardwood saw-timber during the re-measurement period was forecasted using regression using unharvested plots. Because of variation in the re-measurement period among individual FIA plots, especially in the states where FIA is in transition from periodic to annual inventory design, the change of softwood and hardwood growing stock was normalized to the average re-measurement period. The change in hardwood and softwood growing stock is a function of age, mean quadratic diameter at breast height (dbh) of the growing stock trees, volume of softwood and hardwood growing stock, site index, and basal area of softwood and hardwood trees with dbh < 12.7 cm (5”) at the beginning of the re-measurement period. The basal area of trees with dbh < 12.7 cm is included to account for in-growth, as volume of these trees is not recorded in the FIA database. As the stand grows, the proportion of saw-timber volume increases, especially in pine plantations. Change in the proportion of saw-timber in softwood and hardwood growing
stock is a function of proportion of saw-timber and mean quadratic dbh of the growing stock trees at the beginning of the period.

These models were applied to every stand to calculate the volumes of four round-wood products for each of three possible management decisions: (i) the stand is not harvested (models are applied to the parameters of the original stand); (ii) the stand is partially harvested (models are applied to the retained part of the stand, basal area is reduced proportional to the assumed harvesting intensity, and dbh and age are not changed); and (iii) the stand receives a final harvest (volumes, dbh, age, basal area reset to 0).

Following equation 5, the discounted revenue for a specific management decision was calculated as follows:

\[ R(q|h) = p_j^h v_{jp}(q|h) + \rho \left[ p_{j+n}^h v_{j+n}(q|h) \right]. \quad (12) \]

**Estimation and Results**

Harvest choice model was estimated using conditional logit model with forest management type-specific coefficients for discounted revenues and choice-specific constants. The re-measurement periods between consecutive inventories in our samples varied between 1 and 8 years with a mean re-measurement period of about 5 years. Since probabilities of harvest or partial harvest are proportional to the observation (re-measurement) period, we also incorporated log of the re-measurement period into the model:

\[ P_h = \frac{\exp[\alpha_h + \beta_j R(q|h) + \gamma_h S + \delta_h D + \omega_h O + \tau_h T]}{\sum_{k=H}^{f} \exp[\alpha_h + \beta_j R(q|k) + \gamma_h S + \delta_h D + \omega_h O + \tau_h T]} \quad (13) \]

where \( \alpha_h \) is the forest type-choice-specific constant (\( \alpha_h = 0 \forall h = H \)), \( \beta_j \) is the forest type specific coefficient for discounted revenue, \( S \) is the proxy for harvesting costs (slope), \( O \) is the ownership (private or public), \( \gamma_h \) and \( \omega_h \) are estimated coefficients (\( \gamma_h = 0 \), \( \omega_h = 0 \forall h = H \)), \( f \) is the forest type (pine plantations: PP, natural pine: NP, mixed pine: MP, upland hardwoods: UH, and bottomland hardwoods: BH), \( T \) is the log of re-measurement period, and \( \tau_h \) is the coefficient (\( \tau_h = 0 \forall h = H \)). The unit of observation was a “condition”, a part of the plot, and we used “Condition Proportion” as a weight in model estimation.

Estimation results are presented in Table 1. Based on the likelihood ratio test carried against the model with an intercept only, we reject the null hypothesis that the equation have no explanatory power (p=0.01) for all cases.

The forest type–choice–specific constants define a matrix of probabilities for management alternatives: the greater the value of a particular constant, the higher the probability of the corresponding alternative, *ceteris paribus*. Constants corresponding to “no harvest,” which have the highest probabilities, are restricted to zero for model identification, and constants for other alternatives (with lower probabilities) are all negative, as expected. We expect the probability of selecting each management alternative to be positively related to its discounted value of revenues. Four out of five coefficients for discounted revenue are positive (the exception is the coefficients for upland hardwoods).
### Table 1. Estimation results for harvest choice models for North Carolina

<table>
<thead>
<tr>
<th>Variables</th>
<th>Choice</th>
<th>Forest type</th>
<th>Coefficient</th>
<th>Std Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercepts</td>
<td>Final</td>
<td>Planted pine</td>
<td>-3.4278‡</td>
<td>(0.4540)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Natural Pine</td>
<td>-3.7623‡</td>
<td>(0.4634)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mixed pine</td>
<td>-4.3220‡</td>
<td>(0.4801)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upland hardwoods</td>
<td>-4.4656‡</td>
<td>(0.4772)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bottomland Hardwoods</td>
<td>-5.1123‡</td>
<td>(0.5843)</td>
</tr>
<tr>
<td></td>
<td>Partial</td>
<td>Planted pine</td>
<td>-2.8378‡</td>
<td>(0.4134)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Natural Pine</td>
<td>-4.4040‡</td>
<td>(0.4611)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mixed pine</td>
<td>-4.4481‡</td>
<td>(0.4600)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upland hardwoods</td>
<td>-4.9374‡</td>
<td>(0.4625)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bottomland Hardwoods</td>
<td>-5.0846‡</td>
<td>(0.5529)</td>
</tr>
<tr>
<td>Discounted Revenue</td>
<td></td>
<td>Planted pine</td>
<td>0.0008†</td>
<td>(0.0004)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Natural Pine</td>
<td>0.0004*</td>
<td>(0.0002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mixed pine</td>
<td>0.0004*</td>
<td>(0.0003)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upland hardwoods</td>
<td>0.0004</td>
<td>(0.0003)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bottomland Hardwoods</td>
<td>0.0008*</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>Public</td>
<td>Final</td>
<td>-2.3008‡</td>
<td>(0.7105)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Partial</td>
<td>-0.4315</td>
<td>(0.3410)</td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>Final</td>
<td>-0.0243†</td>
<td>(0.0108)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Partial</td>
<td>0.0101</td>
<td>(0.0077)</td>
<td></td>
</tr>
<tr>
<td>Log(Re-measurement, period)</td>
<td>Final</td>
<td>1.1992‡</td>
<td>(0.2399)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Partial</td>
<td>1.0066‡</td>
<td>(0.2325)</td>
<td></td>
</tr>
</tbody>
</table>

Number of observations: 2968
McFadden Pseudo-R2: 0.12
Log-Likelihood: -794

‡ significant at 1%, † significant at 5%, * significant at 10%

Public forests are less likely to be finally or partially harvested which is generally consistent with the assumption that public forests are managed primarily for environmental, aesthetic, and recreational uses. However, this result may obscure differences between management of state forests with more of a profit making mandate and national forests where recreation and other non-timber values are more dominant. Sample size precluded us from distinguishing between these different public ownership types.

We expect that the probability of final or partial harvest is negatively associated with the slope of the site due to higher harvesting costs on steep slopes. The positive coefficients for natural logarithm of re-measurement period for final and partial harvest outcomes is consistent with the expectation that the probability of an event occurring is proportional to the length of observation period.

We used the harvest choice models to simulate supply responses for each of the four products using the latest available inventory data (2006). We drew 100 quartets of random numbers drawn from a uniform distribution to generate price quartets within the range of ±50% of the observed prices for each state. For each price quartet and for each FIA plot, we calculated a discounted revenue term for each of the considered management decisions, estimated probabilities of these decisions, and calculated the harvest response based on plot characteristics. The harvest response of the entire inventory was aggregated using the area
expansion factors for the FIA plots. The area expansion factors were also used to calculate weighted average prices of roundwood products.

We then estimated the supply equations. The natural logs of total output for each of the four products (softwood pulpwood, softwood sawtimber, hardwood pulpwood, and hardwood sawtimber) were estimated as functions of the natural logs of all four product prices. Because the equations are using the same data, the errors may be correlated across the equations; therefore we estimate the system of regression equations using method of seemingly unrelated regression. The results of the estimation are presented in Table 2.

Table 2. Estimates of aggregate supply model for North Carolina. Because of the log-log form of the equations, estimated coefficients reveal the own and cross-price elasticities of supply for each product.

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>Softwood sawtimber</th>
<th>Softwood pulpwood</th>
<th>Hardwood sawtimber</th>
<th>Hardwood pulpwood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>10.740‡</td>
<td>11.521‡</td>
<td>9.827‡</td>
<td>10.235‡</td>
</tr>
<tr>
<td>Price of softwood sawtimber</td>
<td>0.260‡</td>
<td>-0.038‡</td>
<td>0.070‡</td>
<td>0.086‡</td>
</tr>
<tr>
<td>Price of softwood pulpwood</td>
<td>0.018‡</td>
<td>0.033‡</td>
<td>0.009</td>
<td>0.009‡</td>
</tr>
<tr>
<td>Price of hardwood sawtimber</td>
<td>0.032‡</td>
<td>0.012‡</td>
<td>0.317‡</td>
<td>0.160‡</td>
</tr>
<tr>
<td>Price of hardwood pulpwood</td>
<td>0.005</td>
<td>0.001</td>
<td>0.024‡</td>
<td>0.023‡</td>
</tr>
</tbody>
</table>

‡ significant at 1%, † significant at 5%, * significant at 10%
Own price elasticities are shown in bold.

For all estimated equations, we reject the null hypothesis that the equation has no explanatory power (likelihood ratio test, p=0.01). Because of the log-log functional form, all coefficients in these equations define price elasticities. All own price elasticities (for example, elasticity of supply of softwood sawtimber with respect to price of softwood sawtimber) as well as most of cross-price elasticities are significant (p=0.01). Among the cross-price elasticities, which are not significant are elasticities softwood pulpwood supply with respect to price of hardwood pulpwood and sawtimber. Economic theory indicates that the own-price elasticity of supply should be positive and this holds for all equations.

Previous studies of the U.S. stumpage market (e.g., Newman and Wear 1993, Adams and Haynes 1980, etc.), suggest that the short-run supply of timber should be inelastic (values less than one). In our study, sawtimber products are much more price elastic than pulpwood products, also consistent with previous studies (e.g., Newman and Wear 1993). The sign of the cross price elasticity indicates whether products are substitutes (negative) or complements (positive) in production. As expected for a short-run forest supply model, complementarity dominates. Except for negative softwood pulpwood supply elasticity with respect to price of softwood sawtimber, all cross price terms are positive.

**Conclusions**

This study develops method for building an aggregate timber supply model from detailed forest inventories and empirical models of harvest choice based on observed individual harvest decisions. It expands on the modelling approach developed by Prestemon and Wear (2000) by extending the analysis to address all forest types within a region, partial harvests in addition to final harvest, both hardwood and softwood forest products, and timber supply. Aggregate supply response equations using pseudo-data from the harvest choice predictions also provide an innovation for aggregating individual choices within a tractable regional model. While other studies (e.g., Teeter et al. 2006) have used simulation or optimisation
methods to build supply from individual choices, our models allow for validation against observed choices recorded in standard forest inventories and regular updating as new inventories are completed.

The model of harvest choice significantly explains variation of harvest decisions with the present value of alternative management decisions being a significant explanatory variable. The elasticities of softwood and hardwood sawtimber supply generally correspond with the findings of previous studies but the elasticities of both softwood and hardwood pulpwood supplies are lower than previous estimates (Newman 1987, Carter 1992, Polyakov et al. 2005). This finding is consistent with the structure of forest production where sawtimber and pulpwood are joint products in the short run and sawtimber prices are substantially higher than pulpwood prices—i.e., pulpwood supply is heavily influenced by sawtimber markets in the short run. Pulpwood inelasticity may also be related to substantial pulpwood thinning from young plantations. These thinnings are embedded within multiple period management schemes, making them costly to forego in the short run.

We found significant positive cross-price elasticities, consistent with the hypothesis of joint production of all four products. Furthermore, the prices of sawtimber have greater effects on the supply of pulpwood than the prices of pulpwood. The literature provides inconsistent estimates of these cross price effects, and our findings fall within the range of estimates produced by earlier studies. Complementarity of sawtimber in pulpwood supply in the US South was found by Newman (1987). However, contrary to our results, Newman (1987) found substitution of pulpwood in sawtimber supply, while Polyakov et al. (2005) found substitution of sawtimber in hardwood pulpwood supply.

Our modelling approach translates the heterogeneous and complex capital structure of forest inventories into effects on timer supply. It therefore provides a mechanism for examining the potential implications of exogenous shocks to inventory through simulation modelling. This is especially important for the conduct of broad scale natural resource assessments where policy relevant questions have to do with understanding the interactions between economic activity and the future structure of forested ecosystems.

**Literature Cited**


