A Height–Diameter Curve for Longleaf Pine Plantations in the Gulf Coastal Plain

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ABSTRACT

Tree height is a critical component of a complete growth-and-yield model because it is one of the primary components used in volume calculation. To develop an equation to predict total height from dbh for longleaf pine (*Pinus palustris* Mill.) plantations in the West Gulf region, many different sigmoidal curve forms, weighting functions, and ways of expressing height and diameter were explored. Most of the functional forms tried produced very similar results, but ultimately the form developed by Levakovic was chosen as best. Another useful result was that scaling diameters by the quadratic mean diameter on a plot and height by the average height of dominant and codominant trees in the target stand resulted in dramatically better fits than using these variables in their raw forms.

Keywords: *Pinus palustris*, nonlinear models, model evaluation

An important component of a growth-and-yield model is the relationship between dbh (4.5 ft above ground) and the total height of individual trees. The US Forest Service laboratory at Pineville, Louisiana, has collected over 65 years of longleaf pine plantation growth information from seven studies within the Gulf Coastal plain. The primary purpose of this collection is to develop a regional growth-and-yield model for thinned and unthinned plantations of longleaf pine (*Pinus palustris* Mill.). An important component of a regional growth-and-yield model is the ability to supply heights for trees where only diameter is known. Total height is both an important descriptive individual tree variable and also is necessary to calculate tree volume or other product yields. Height predictions can be used both in this regional model and in other situations where only diameters were measured.

Model development is frequently not as orderly as one would like because the true mathematical relationship is seldom known, and there are infinite possible approximations for the true model. Curtis (1967) examined many forms of height–diameter equations and ultimately suggested that "any reasonable and moderately flexible curve" will give similar results. Complex mathematical functions are much easier to fit now than in 1967, so a current evaluation of many model forms that were formerly used by other researchers was conducted. The necessary assumptions for valid regression models and the possible scaling of key variables were also evaluated.

Recently, several researchers, including Sharma and Parton (2007), Calama and Montero (2004), Trincado et al. (2007), and Jayaraman and Zakrzewski (2001), have begun to use mixed-effects models for height–diameter relationships. In particular, Calama and Montero (2004) mention that this is just one of several methods to capture the variability in different stands and the same stand over time. Mixed-effects models estimate fixed parameters for the general model and then use calibration data such as a few measured tree heights to achieve specificity for a given stand. This is a reasonable approach, but the main purpose of the model presented here is for height estimation in a comprehensive growth-and-yield model where there are no measured heights available. For this reason, the model fitted here relies on quadratic mean diameter and the height of dominant and codominant trees to achieve the required specificity.

Methods

Data

Two hundred sixty-seven sample plots of various sizes were distributed in southern Alabama, Mississippi, Louisiana and eastern Texas and repeatedly measured, resulting in 2,005 plot-age combinations, or 26,490 useable measured tree observations. The studies combined to create this data set are described in Table 1, and the means and ranges of important variables are shown in Table 2. The studies are further described in Goelz and Leduc (2001) and Goelz et al. (2004).

Model Development

To select the best model form, the literature was searched for sigmoidal and other curves that have been previously used to predict height from diameter. Table 3 shows the 41 curve forms found for testing and comparison, along with their sources. Regardless of any effect on goodness of fit, some modifications were made to make all of the equations more closely match the reality of height versus dbh models. The first modification was suggested by Meyer (1940), and it is setting the minimum height to be 4.5 ft. Furthermore, because any tree having a measurable dbh cannot have zero diameter at this point, in all models the diameter at 4.5 ft was set to 0.5 inches. This is a reasonable approximation for longleaf pine, which has a thick terminal bud.

All of the test equations were fit to diameter and height in their original units, but it was soon discovered that a dramatically improved fit could be obtained by using relative diameter and relative height. The relative dbh used was obtained by subtracting 0.5 from

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Table 1. Description of West Gulf planted longleaf studies used in this report. Observations are the number of data points obtained from the combination of all available trees on all plots for all of the times that a tree was measured.

Title	Citation	States	Observations
Burning, pruning, and thinning in a longleaf spacing plantation (unburned portion)	Enghardt (1966)	LA	3,634
Growth and yield of planted longleaf pine at Sunset Tower	Lohrey et al. (1987)	LA	1,124
Burning, pruning, and thinning in a longleaf spacing plantation (burned portion)	Enghardt (1966)	LA	6,489
Effect of age and residual basal area on growth and yield of planted longleaf pine on a good site	Lohrey (1971)	MS	1,938
Growth and yield of planted longleaf pine on medium and poor sites	Lohrey (1972)	TX	5,521
Yields of unthinned longleaf pine plantation on cutover sites in the west gulf region	Lohrey (1975)	TX, LA	5,311
Early longleaf plantation growth on machine-planted prepared sites	Boyer and Kush (2004)	AL, FL	2,473

Table 2. Descriptive statistics for the data used in model fitting. Stand density index is based upon an exponent of 1.605.

Variable	Mean	Range
Dbh (in.)	9.4	0.6–23.7
Total height (ft)	62.5	5.0–107.5
Age (years)	36.5	5–65
Basal area per acre (ft^2)	109.7	1.8–2566
Trees per acre	312.6	20–1550
Base age 50 site index (ft)	82.4	62.6–100.8
Stand density index	210.0	9.0–497.2

dbh and then dividing the result by the quadratic mean dbh of that plot. Similarly, 4.5 was subtracted from tree height and the result was divided by the average height of dominant and codominant trees on the plot. Thus, the height prediction equation is specified as

$$h = 4.5 + (H_d - 4.5) \times f[(d - 0.5)/D_q], \qquad (42)$$

where h is total tree height, d is dbh in inches, H_d is average height of dominant and codominant trees in feet, D_q is the quadratic mean dbh in inches for a plot, and $f(x)$ is the equation form being tested.

The standard regression assumptions of independence, normality, and homogeneous variance were examined. It is expected that the data used will be autocorrelated since they include many trees from the same plots and the same trees measured over time. Violation of this assumption will not affect the parameter estimates, but calculated variances are most likely underestimated (West 1995). All data were used in fitting, since it was felt that the benefits of using the available large and diverse data set outweighed the deficiencies, especially given that all equations were fit with the same data. However, results were confirmed with independent observations in subsets of the larger data set created by selecting only one tree at one time from each plot. The assumptions of normality and homogeneity could also not be shown statistically in the full data set, but they were shown to be true in most (all homogeneous and 7 of 10 normal) of the independent subsets.

Not all of the equations selected for evaluation are simple functions of x and y. Some include other stand variables and some include parameters that could be taken as population or stand variables such as the maximum and/or minimum value of dbh or height. These equations were tested with individual plot values and fixed population values. To maximize goodness of fit, the final versions of Equation 26 uses plot-specific values for $x_{\min}$ and $x_{\max}$, and Equation 41 uses fixed population values for $x_{\min}$, $x_{\max}$, and $y_{\min}$ (Table 3).

After each equation was fit, the parameters were examined to see whether they were significantly different from 0 in all parameters and 1 or -1 in those parameters that are multipliers, nonsignificant parameters at the $\alpha = 0.05$ level were dropped, and the equations were refitted. If more than one parameter was dropped, previously dropped parameters were checked again for inclusion.

In addition to removing nonsignificant parameters, additions of stand and tree descriptor variables were tried on the published form of the best equation to see whether it could be made better. The variables thinning (yes or no), stand density index, trees per acre, basal area, age, and crown class were tested by using simple additive modifications to the base models.

Most equations were fit using nonlinear regression in PROC NLIN from the SAS Institute (1994). Usually Newton's method was used to find parameters, but when difficult fits were encountered other techniques were used as needed. There was actually one equation that always failed to converge in SAS and could be fit only using the simplex method as coded in NONLIN (Leduc 1986).

Model Evaluation

The primary factor used to evaluate this set of equations was root mean squared error (RMSE). Fit index, a number comparable to r^2 in linear regression (Schlaegel 1981) is also presented, since it is easier to interpret. However, it will be shown that there is little difference between equations using just these criteria, so the five best equations were also checked for bias, mean squared error and absolute median difference with cross-validation, and mean squared error on repeated independent sets of data, as explained below. Even though relative values were used in fitting many of these equations, the results were always converted back to actual tree heights before deviations were calculated.

Because "splitting a sample into two pieces cannot substitute for true attempts at replication" (Hursch 1991), the data were not split into a set for fitting and a set for testing. All of the data were used for fitting the models. Although it is not a substitute for true replication, model stability was tested for the five best models by cross-validating based on study. This extra level of testing was done on only the best models since the purpose of this test was to determine whether any of these models is less stable than the others that are closely matched. In this test, each study was excluded from the fitting dataset and the models were fit with this reduced data. Deviations (predicted minus actual) of the excluded study data were recorded, and this process was repeated, excluding each study in turn. The RMSE, mean bias, and median absolute deviation were calculated for all tested observations.

Although the complete data set was used in all of the initial fits and for the final results, because it is known that our data are correlated within plots and across years, the best five equations were also fitted to a subset of data with only one tree from each plot. This fit was repeated 10 times for each equation, and the resulting mean squared errors were compared for each repetition.

Table 3. Equation forms investigated for longleaf pine height-diameter relationship. y is the total height, and x is dbh or relative dbh. Additional variables that appear in some equations are x_{min} for minimum dbh, x_{max} for maximum dbh, y_{min} for minimum height, s for site index, h for average height of dominant and codominant trees, a for age, t for trees per acre, and b for basal area. The estimated parameters are the Greek letters α, β, δ, λ, ϕ, and ψ.

Equation Number	Common Name*	Form	References
1	Polynomial	$y = \alpha x + \beta x^2 + \delta s$	Brender (1986)
2		$y = \alpha \beta^x x^\delta$	Staudhammer and Lemay (2000)
3	Power	$y = \alpha x^\beta$	Zeide and Vanderschaaf (2002)
4		$y = \alpha x^{\beta x^\delta}$	Sibbesen (1981)
5		$y = 10^\alpha x^\beta$	Larson (1986)
6	Schumacher	$y = \alpha e^{\beta/x}$	Huang and Titus (1992)
7		$y = e^{\alpha + \beta/x}$	Curtis (1967)
8		$y = e^{\alpha + \beta x^\delta}$	Curtis et al. (1981)
9		$y = e^{\alpha + \beta x^\delta + \lambda b}$	Parresol (1992)
10		$y = \alpha e^{\beta/(x+\delta)}$	Huang and Titus (1992)
11	Korf	$y = \alpha e^{\beta x^\delta}$	Zeide (1993)
12	Gompertz	$y = \alpha e^{\beta e^{\delta x}}$	Zeide (1993)
13		$y = \alpha e^{-e^{\beta(x+\delta)}}$	Fang and Bailey (1998)
14		$y = e^{\alpha + \beta/(x+1)}$	Huang and Titus (1992)
15	Sloboda	$y = \alpha e^{\beta e^{\delta x^\lambda}}$	Zeide (1993)
16		$y = \alpha(1 - e^{\beta x})$	Huang and Titus (1992)
17		$y = \alpha + \beta(1 - e^{\delta x})$	Fang and Bailey (1998)
18	Monomolecular	$y = \alpha(1 - \beta e^{\delta x})$	Zeide (1993)
19		$y = \alpha + \beta(1 - e^{\delta(x - x_{\min})})$	Fang and Bailey (1998)
20†		$y = \alpha(1 - \beta e^{\delta(x - e')})$	Fang and Bailey (1998)
21	Richards	$y = \alpha(1 - e^{\beta x})^\delta$	Huang and Titus (1992)
22	Chapman-Richards	$y = \alpha(1 - \beta e^{\delta x})^\lambda$	Huang and Titus (1992)
23	Weibull	$y = \alpha(1 - e^{\beta x^\delta})$	Yang et al. (1978)
24		$y = \alpha(1 - e^{\beta(x+\delta)^\lambda})$	Seber and Wild (1989)
25		$y = \alpha(1 - \beta e^{\delta x^\lambda})$	Fang and Bailey (1998)
26		$y = \alpha h^\beta e^{\delta/a + (1/x - 1/x_{\max})\,(\lambda + \phi(\ln(t)/a))}$	Zhang et al. (1997)
27	Michaelis-Menten	$y = \alpha x/(\beta + x)$	Huang and Titus (1992)
28		$y = \alpha + \beta/(x + \delta)$	Fang and Bailey (1998)
29	Vestjordet	$y = (\alpha + \beta/x)^3$	Staudhammer and LeMay (2000)
30		$y = x^2/(\alpha + \beta x + \delta x^2)$	Curtis (1967)
31		$y = x^2/(\alpha + \beta x)^2$	Huang and Titus (1992)
32		$y = \alpha/(1 + \beta x^\delta)$	Ratkowsky and Reedy (1986)
33		$y = \alpha x/(x + 1) + \beta x$	Huang and Titus (1992)
34	Yoshida I	$y = \alpha x^\delta/(\beta + x^\delta) + \lambda$	Zeide (1993)
35		$y = \alpha(x/(1 + x))^\beta$	Curtis (1967)
36	Levakovic I	$y = \alpha(x^\delta/(\beta + x^\delta))^\lambda$	Zeide (1993)
37	Hossfeld IV	$y = x^\delta/(\beta + x^\delta/\alpha)$	Zeide (1993)
38	Pearl-Reed	$y = \alpha/(1 + \beta e^{\delta x})$	Huang and Titus (1992)
39		$y = a/(1 + e^{\beta(x-\delta)})$	Seber and Wild (1989)
40		$\omega = e^{-\alpha(x-\beta)}$ $y = \delta + \lambda(e^{(\phi^{2}-1)\omega} - \phi\omega)^{\phi\psi+1}$	Grosenbaugh (1965)
41		$y = \left(y_{\min}^\beta + (\delta^\beta - y_{\min}^\beta)\dfrac{(1 - e^{\alpha(x - x_{\min})})}{(1 - e^{\alpha(x_{\max} - x_{\min})})}\right)^{1/\beta}$	Schnute (1981)

* Common names only apply to the single equation that they are aligned with.
† In Equation 20, when x is relative dbh, e' is the base of the natural logarithm divided by quadratic mean diameter. When x is actual dbh, e is just the base of the natural logarithm.

Results

As mentioned in the section on model development, all of the equations were fitted in actual and relative units. Because the use of relative units produced such overwhelmingly superior results, final comparisons were made only in terms of the equations fit with relative units. In fact, the best equation using actual units, Equation 26, was slightly worse than the 28th best equation using relative units. When Equation 26 was excluded from the comparison, all

Table 4. Statistics measuring the goodness of fit for equations using relative height and relative dbh. The results are ranked by root mean squared error (RMSE).

Model	Terms in model	RMSE	Fit index
36	4	4.14889	0.9561
40 reduced	3	4.15479	0.9560
22 reduced	4	4.15509	0.9560
15	4	4.15601	0.9559
12	3	4.15762	0.9559
13	3	4.15762	0.9559
25	4	4.16011	0.9559
34	4	4.16163	0.9558
38	3	4.16310	0.9558
39	3	4.16310	0.9558
24 reduced	3	4.16389	0.9558
23	3	4.16389	0.9558
41	3	4.17067	0.9556
21	3	4.17578	0.9555
26	5	4.19600	0.9551
30	3	4.20049	0.9550
8	3	4.20749	0.9548
32	3	4.20837	0.9548
37	3	4.20837	0.9548
19	3	4.23223	0.9543
17	3	4.23223	0.9543
18	3	4.23223	0.9543
20	3	4.23223	0.9543
4	3	4.25580	0.9538
9	4	4.26700	0.9536
2	3	4.28252	0.9532
11	3	4.29043	0.9530
10	3	4.34428	0.9519
6	2	4.37089	0.9513
7	2	4.37089	0.9513
28	3	4.41979	0.9502
16	2	4.43718	0.9498
31	2	4.50779	0.9482
1	3	4.66842	0.9444
27	2	4.69898	0.9437
29	2	4.80634	0.9411
35	2	4.86111	0.9397
33	2	5.19380	0.9312
3	2	5.37488	0.9263
5	2	5.37488	0.9263
14	2	5.64446	0.9187

relative-unit equations were better than all of the actual-unit equations. Equation 26 (in Table 3) is unique in that it includes H_d as a multiplier, making it somewhat similar to the relative-unit equations. RMSE and fit index of the tested models are shown in Table 4. There is very little difference in the goodness of fit of the equations using relative units, and all of these equations tested had fit indices that differed by less than 0.05. Nonetheless it is desirable to pick a single best equation, so ease of fit, stability, performance on independent data, and the use of auxiliary variables were used to conclude that model form 36, attributed to Levakovik by Zeide (1993), was best. With the modifications suggested earlier, the final model based on Equation 36 is as follows:

$$h = 4.5 + (H_d - 4.5) * \alpha\left(\frac{((d-0.5)/D_q)^\delta}{\beta + ((d-0.5)/D_q)^\delta}\right)^\lambda + \varepsilon, \tag{43}$$

where α, β, δ, and λ are the estimated parameters equal to 1.0491, 0.2172, 0.2541, and 4.0543, respectively, and ε is a random error, $\sim N(0, \sigma^2)$.

Table 5. Statistics that describe the estimated parameters for Equation 43.

Parameter	Estimate	Approximate standard error
α	1.0491	0.00184
β	0.2172	0.00472
δ	0.2541	0.00935
λ	4.0543	0.09710

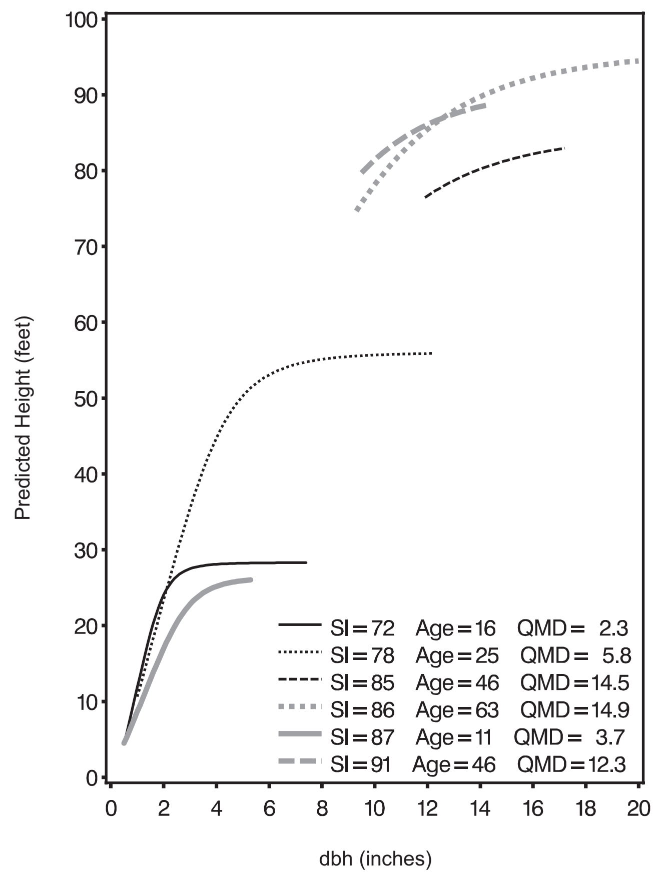

Figure 1. Example curves produced from the final four-parameter equation. Six curves are shown with different base age 50 site indices (SI) in feet, ages in years, and quadratic mean diameters (QMD) in inches.

Statistics resulting from the fit of the parameters are shown in Table 5, and Figure 1 is a graph of some of the height-dbh relationships possible with this model.

A numerical example will illustrate the ease of applying this equation. This example will assume that we have a stand with a quadratic mean diameter of 11.6 inches, an average height of dominant and codominant trees of 90.3 ft, and a tree with a dbh of 11 inches. Although the equation appears complex, solving this equation is simply a matter of plugging in the estimated parameters and desired values, which results in the following equation:

$$h = 4.5 + (90.3 - 4.5) * 1.0491\left(\frac{((11.0-0.5)/11.6)^{0.2541}}{0.2172 + ((11.0-0.5)/11.6)^{0.2541}}\right)^{4.0543}. \tag{44a}$$

Table 6. Statistics measuring the goodness of fit for some of the best equations when they were cross-validated by study. They are sorted from best to worst using root mean squared error as the ranking criterion. The bias is based on predicted minus actual values.

Equation	Root mean squared error	Bias	Median absolute difference
36	4.17238	0.0816	2.51
40 reduced	4.17772	0.0818	2.51
15	4.17971	0.0827	2.52
12	4.18162	0.0813	2.51
22 reduced	4.20107	0.0735	2.53

This simplifies to

$$h = 4.5 + 85.8 * 1.0491\left(\frac{(0.9052)^{0.2541}}{0.2172 + (0.9052)^{0.2541}}\right)^{4.0543} \quad (44b)$$

and further to

$$h = 4.5 + 85.8 * 1.0491\left(\frac{0.6677}{0.2172 + 0.6677}\right)^{4.0543} \quad (44c)$$

then

$$h = 4.5 + 85.8 * 1.0491 * 0.9309, \quad (44d)$$

with the final result being 88.3 ft. The actual tree from which these starting parameters came has a total height of 89 ft. Of course, one would not normally do this calculation by hand, but rather put the formula into a computer program for instant calculation.

The top five equations were cross-validated by leaving out one study at a time and then applying the equation to the left-out study, and the results are presented in Table 6. Equation 36 remained the best model by a very small margin in root mean square error and median absolute difference, but it had a slightly higher bias than model forms 12 and 22.

As mentioned above, the full set of data very likely has many correlated observations. Using only a single observation from each plot and repeating this exercise 10 times, the top five equations were refit, resulting in model form 36 having the lowest RMSE in 5 of the 10 fits. Although model form 36 was not always best, it is, on average, slightly better than the other forms. This in shown in Table 7, where all of the RMSE values calculated are standardized by dividing by the RMSE of model form 36. Values above 1.0 are lesser fits, and values below 1.0 are better. Whereas other forms are sometimes better for a given sample, on average, form 36 is best. However, the difference is very slight, as shown by the nearness of all values to 1.0. Since the variance of the estimate made with the full data set is potentially too low, it is useful to note that the highest estimate of RMSE for model form 36 using these independent data sets is 4.66 and the corresponding fit index is 0.943, still a high-quality fit.

As mentioned above, thinning, stand density index, trees per acre, basal area, age, and crown class were tested to see whether their addition improved the base model. In all cases, there was a significant improvement ($\alpha = 0.05$) in fit. However, when the extended equations were compared with the base equations, there was little practical improvement. The largest total height difference observed with the addition of thinning, stand density index, trees per acre, basal area, or age is 2.2 ft, but the average is only 0.09 ft. Crown class produced some differences as large as 9.4 ft, with an average difference of 0.02 ft, but results were about equally likely to be better or worse (Figure 2). In the interest of parsimony, it was decided to avoid these model extensions.

Table 7. Root mean squared error (RMSE) standardized by dividing by the RMSE of equation 36 for the best five models using 10 randomly selected independent subsets of data.

Sample	Standardized RMSE by equation form: 12	15	22	36	40
1	1.000	1.001	0.999	1.000	0.999
2	0.999	1.001	0.999	1.000	0.999
3	1.002	1.000	1.000	1.000	0.999
4	1.003	1.002	1.006	1.000	1.008
5	0.999	1.001	0.999	1.000	0.999
6	1.003	1.004	1.003	1.000	1.002
7	1.007	1.004	1.010	1.000	1.013
8	0.999	1.001	1.000	1.000	0.999
9	1.000	1.002	1.000	1.000	1.000
10	1.001	1.001	1.004	1.000	1.005
Average	1.001	1.002	1.002	1.000	1.003

Discussion and Conclusions

Equation 43 is the best equation that we could find for predicting the total height of longleaf pine trees from their dbh. However, there is very little difference between any of the equations evaluated that used relative diameter and height.

The equations were all fit with and without weighting. Initial work appeared to indicate that weighting would be required to get a correct fit for these equations, but further evaluation revealed that this was not necessary. A review of the literature that mentions heteroscedasticity in height–diameter equations revealed only Huang and Titus (1992) using a weight of 1/dbh, whereas many others, including Parresol (1992), Peng (1999), Zhang et al. (1997), Fang and Bailey (1998), Moore et al. (1996), and Staudhammer and LeMay (2000), found no need for weighting in height–diameter relationships.

This model was developed to answer the need for height estimates in a growth-and-yield projection system that is being developed for thinned and unthinned longleaf pine. The work was not based on any new theories or methods, but it did reveal several interesting observations.

The most obvious of these is that scaling dbh by the quadratic mean dbh in the stand and total height by the average height of dominant and codominant trees brought about dramatic improvements in model fit. Staudhammer and LeMay (2000) discovered a similar result, but they scaled only the dbh and not the total height, and they used maximum diameter rather than quadratic mean diameter. In addition to dividing dbh by the quadratic mean diameter on a plot, maximum diameter was also tried as a divisor, but it was not as good with these data. In the best equation, the RMSE using maximum diameter was only 4.1972 compared with 4.1489 for quadratic mean diameter.

As mentioned above, adding supplemental variables to the base equation provided no practical advantage, but previous authors have emphasized the importance of some supplemental variables. Two of them, density and thinning, are worth discussing further.

Zeide and Vanderschaaf (2002) stressed the importance of including a density term in a height–diameter model. Parresol (1992) found that adding a basal area term to his height–diameter function

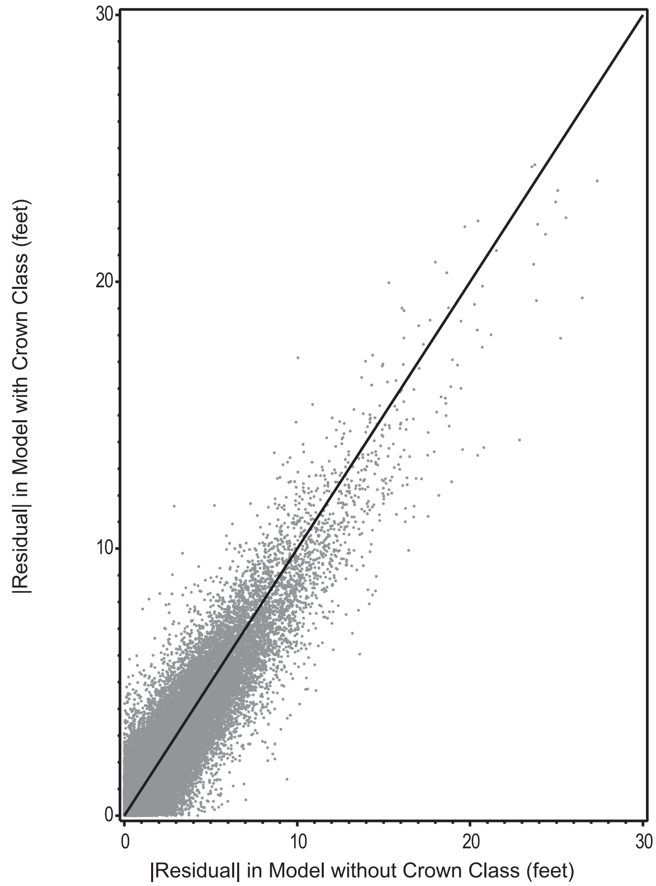

Figure 2. The relationship between the absolute value of the residuals of an equation including crown class and the absolute value of the residuals of an equation that does not include crown class. The diagonal line is where both models are identical. Points to the right of the line are observations where the crown class model is better, and points to the left are where the model omitting crown class is better.

for baldcypress improved the fit of his model. Adding basal area per acre to Equation 43 produced a statistically significant ($\alpha = 0.05$) improvement but in actual terms the RMSE improved by only 0.0133 ft, whereas the bias increased by 0.039 ft. Staudhammer and LeMay (2000) also found a decreased RMSE and increased bias with some of their equations and species. One reason for this apparent lack of a density effect is that the quadratic mean diameter of a given stand is an indirect measure of density, because trees tend to have larger diameters at a given age for a lower density.

Similarly Zhang et al. (1997) found that thinning should make a difference in height–diameter relationships, but this effect does not show up in a thinning response variable added to a height–diameter function. The current study showed a statistically significant effect of no practical importance, so it neither adds to nor subtracts from previous work on this subject. All that can be said is that including an explicit thinning term in the best model does not improve the model in any meaningful way.

One other useful observation that may help others seeking to repeat this work on the same or different species is that several of the equations are identical algebraically even though they differ in appearance (Table 3). Equations 3 and 5, 6 and 7, 8 and 11, 12 and 13, 17 and 18, 32 and 37, and 38 and 39 are identical. In addition, after removing nonsignificant terms, Equation 24 reduced to Equation 23. Furthermore, Equation 20 is different from Equation 18 only in the subtraction of a constant that did not affect the RMSE of the fitted equations. Equation 19 is similar.

In conclusion, a function that predicts height for a given diameter in stands of thinned and unthinned longleaf pine plantations that is quite accurate has been identified. However, as suggested by Curtis (1967), there are many models that are nearly as good as the best. Perhaps a more important result is that the use of relative diameter and relative height provides a great improvement in the fit of the equation regardless of the equation used.

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been used. For example, Jordan and Ducey (2007) measured four crown radii, the first in the direction away from the plot center and the remaining three at 90, 180, and 270° clockwise; Francis (1988) measured eight radii on the directions north, northeast, east, southeast, south, southwest, west, and northwest; and for the Urban Forest Effects model, Nowak et al. (2005) specify crown diameter measurements in two directions, north–south and east–west. Biging and Wensel (1988) observed that basal areas for eccentric trees based on a measurement of the longest axis or on an average that included the longest axis yielded less accurate estimates of the true basal area than estimates of the basal areas based on the length of the minor, or shortest, axis of the bole. Thus, to eliminate any potential bias in crown area estimations or other applications using an estimated crown width, further investigation should be made into the interaction between crown measurement protocols and the method for computing average crown width.

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CORRECTION

LEDUC, D., AND J. GOELZ. 2009. A Height–Diameter Curve for Longleaf Pine Plantations in the Gulf Coastal Plain *South. J. Appl. For.* 33(4):164–170.

It was discovered after publication that the δ and λ parameters were incorrectly transcribed from the analysis to the article even though they were used appropriately in all of the calculations. This results in changes to Table 5 and Equation 44, which are presented in their corrected forms below.

Table 5. Statistics that describe the estimated parameters for Equation 43

Parameter	Estimate	Approximate standard error
α	1.0491	0.00184
β	0.2172	0.00472
δ	4.0543	0.09710
λ	0.2541	0.00935

$$h = 4.5 + (90.3 - 4.5) * 1.0491\left(\frac{((11.0 - 0.5)/11.6)^{4.0543}}{0.2172 + ((11.0 - 0.5)/11.6)^{4.0543}}\right)^{0.2541}. \tag{44a}$$

This simplifies to

$$h = 4.5 + 85.8 * 1.0491\left(\frac{(0.9052)^{4.0543}}{0.2172 + (0.9052)^{4.0543}}\right)^{0.22541} \tag{44b}$$

and further to

$$h = 4.5 + 85.8 * 1.0491\left(\frac{0.6677}{0.2172 + 0.6677}\right)^{0.2541} \tag{44c}$$

then

$$h = 4.5 + 85.8 * 1.0491 * 0.9309, \tag{44d}$$