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The Optimal Forest Rotation:

A Discussion and Annotated Bibliography

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ABSTRACT

The literature contains six different criteria of the optimal forest rotation: (1) maximum single-rotation physical yield, (2) maximum single-rotation annual yield, (3) maximum single-rotation discounted net revenues, (4) maximum discounted net revenues from an infinite series of rotations, (5) maximum annual net revenues, and (6) maximum internal rate of return. First-order conditions for maximization show the rotation effects of the criteria. Various authors have extended basic models and discussed effects of externalities, imperfect markets, changing parameters, and taxes.

Keywords: Forest economics, forest valuation, harvest scheduling.

Introduction

What is the optimum rotation length for a forest stand? This question has been debated for more than a century, and numerous competing theories and practices have been developed in attempts to answer it. These theories have used combinations of physical, biological, and economic criteria for determining the optimal rotation. Confusion over what is the proper criterion continues to this day. At one extreme, as witnessed by the legal requirement of the National Forest Management Act of 1976, the Forest Service, U.S. Department of Agriculture, must manage national forests on a maximum physical sustained-yield rotation.[1] Meanwhile, economists and other critics argue for economic criteria that maximize discounted net revenues and generally imply much shorter rotations (Hirshleifer 1974; Hyde 1980; Samuelson 1976).

In the economics literature, the Faustmann (1849) optimal decision rule for maximizing discounted net revenues is accepted as superior for society under many conditions (e.g., Mitra and Wan 1985, 1986). In order to understand these conditions more clearly, a number of important topics related to the model's results have been examined. These topics include the delineation of the comparative statics of the optimal solution and the loosening of the assumptions that characterize the optimal formulation. By extending the results in these directions, researchers have been able to make much stronger statements about the behavior of the forest sector in the economy.

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[1] U.S. Department of Agriculture, Forest Service. 1983 rev. The principal laws relating to Forest Service activities. Agric. Handb. 453. Washington, DC. (p. 450). 591 pp.

In the first section of this report, I discuss six alternative criteria for determining the optimum rotation length: (1) maximum physical yield of a single rotation, (2) maximum annualized yield of a single rotation, (3) maximum discounted net revenues of a single rotation, (4) maximum discounted net revenues from an infinite series of like rotations, (5) maximum undiscounted net revenues from a regulated forest, and (6) maximum rate of growth of capital from an infinite series of like rotations. The ordering of these criteria assists in their understanding. The first two formulations give purely biological and physical management rules, while the other four combine economic and biological information of differing complexities. Excellent surveys of the criteria for the optimal forest rotation have already been performed (Bentley and Teeguarden 1965; Chang 1984; Gaffney 1957; Goundry 1960; Pearse 1967; Samuelson 1976), and this section only reflects their results. The second section of the report discusses major extensions and developments of these models and suggests directions for future research. The final section is an annotated bibliography.

Comparison of Criteria for Rotation Length Determination

An optimal forest rotation criterion should maximize net economic benefits to either society or the private forest owner. Since competitive markets and an absence of externalities are assumed in this discussion, the interests of individual owners and society will coincide (Anderson 1981). Other goals may be to maximize personal or social utility or maximize physical (either market or nonmarket) benefits. These goals may be useful for various purposes, but net benefit maximization allows for clear comparison of criteria and has been the general (though often unstated) goal used in the literature.

Assumptions are necessary to a comparative forest rotation analysis. There are six basic assumptions for the present analysis:

1. Even-aged management. Starting from bare ground, a single stand of trees is grown and all trees are cut at the same time. The land has an unchanging growth potential, and the technology available for growing trees does not change. Finally, the land is capable of being regenerated (at some fixed cost) instantaneously after harvest. The rotation starting from bare land is not essential. Faustmann (1849) showed that whether a regulated forest or a single stand is examined, the optimal rotation length is unaffected and the per-acre returns are the same (plus the capitalized value of the currently standing trees). The even-aged management assumption is important. Chang (1981) and Hall (1983), among others, have shown that the optimal conditions will change when an uneven-aged stand is managed.

2. Perfect certainty regarding the stand's growth function, future market prices, interest rates, and costs. All are assumed constant.

3. Access to capital markets is unlimited, and money can be borrowed or lent at the same market interest rate (r). This assumption is dropped for rate of return maximization criterion.

4. Net stumpage price (p) is not a function of tree quality but rather is constant per unit of volume produced. This assumption is not completely necessary, but its inclusion avoids the potential problem of multiple optimum rotation ages. Multiple optima can arise when management objectives favor different end products or when discontinuous jumps in prices create nonconcave sections in the total revenue function. Also, only timber is valued; returns from other nonmarket and public goods derived from the forest are ignored. Finally, the timber is sold in a competitive market; any quantity of timber output can be sold without changing the market price.

5. Regeneration costs (wE, where w is the cost per unit of silvicultural effort, E) are the only expenses and occur immediately after harvesting. Intermediate management costs and returns can be included without loss of generality. A substantial body of literature has examined the effects of thinning on optimal management strategies (Reed 1986).

6. Timber production is a function of time alone or of time and silvicultural effort. Early analyses (Gaffney 1957; Goundry 1960; Pearse 1967; Samuelson 1976) describe total merchantable forest production (F) as a concave function of time (T) only. Thus:

$$F = Q(T)$$

where: $Q_T > 0$

$Q_{TT} < 0$

where the subscripts denote the first and second partial derivatives of the timber production function.

Later analyses examine the comparative statics of the optimal rotation choice to assess the management tradeoffs inherent in the decision process. Quantity is not a function of time alone but also of silvicultural effort (Chang 1983; Graham-Tomasi 1983; Hyde 1980; Jackson 1980; Morgan 1974). Thus:

$$V = Q(T,E)$$

where: Q_T and $Q_E > 0$

Q_{TT} and $Q_{EE} < 0$

and $\quad Q_{TT}Q_{EE} > Q_{TE}Q_{ET}$

In economic terms, the inclusion of effort allows for the substitution of labor (E) for capital (T) in the optimization procedure.

Each of these assumptions is quite restrictive. Real world conditions are ignored or altered, calling the validity of the analytical results into question. The assumptions are made, in part, to ease the mathematical manipulations but more importantly because the world of 19th century Germany, where the models were first presented, fit many of these static factor assumptions. The assumptions are continued because the analytical results are primarily for theoretical demonstrations and comparisons, which can then be applied under more general conditions.

Maximization of Gross Yield (V1)

The maximum gross-yield (MGY) criterion has no present theoretical support. It is included for historical development and because it has some plausibility where interest approaches zero. It is an outgrowth of the mercantilist concept of management, in which a country's economic policy was guided by the desire to maintain as large a stock of precious metals as possible. Maximum physical yields were desired to minimize importing of wood and spending of gold. In one form or another, it has been used as a rotation criterion until relatively recently in Germany and other countries (Thomson 1942). It implicitly assumes a zero interest rate and zero costs, so the formulation is:

(1) $\quad V1 = \max(T)\ \{pQ(T)\}$

where max(T) denotes maximization of the expression with respect to T and V1 is the solution for gross-yield maximization. The price term is included only to conform with later formulations. Prices have no effect on the optimal formulation.

The first-order condition is:

(2) $\quad V1_T = pQ_T = 0 = Q_T$

The rotation is optimal when the value of the marginal product (VMP = price times the marginal physical product), or more simply since price is constant, when the marginal physical product alone is equal to 0. This criterion is unacceptable because it ignores all costs involved in growing the stand, interest costs in holding the timber, and treats the land as if it had no value. What is more, it fails to achieve its intended goal of maximum total wood production because the annual production from this model is less than the maximum average production.

Maximization of Annual Yield (V2)

This criterion, known to foresters as the culmination of mean annual increment (CMAI) or the biologically maximum sustained yield (MSY), produces the greatest total physical product from a site permanently managed for forestry. It maximizes the average annual physical yield per acre. It is the criterion by which the Forest Service is mandated to manage its timber. The total production, on a long-term basis, is greater than that of V1 because MSY implicitly assumes that the site will continue to produce trees after harvest and thus contribute to the gross yield. Use of this criterion will allow the achievement of a country's desired timber production goals with the least amount of land devoted to forestry (Goundry 1960). It, like V1, assumes zero costs and interest rates so the desired formulation is:

(3) $$V2 = \max(T) \{pQ(T)/T\}$$

The first-order condition is:

(4) $$V2_T = [pQ_T T - pQ(T)]/T^2 = 0$$

(5) $$pQ_T = pQ(T)/T$$

(6) $$Q_T/Q(T) = 1/T$$

At the optimum, the marginal growth rate is equal to the average growth rate. The problems with this method are that, again, costs and interest charges are ignored. Since price cancels out of the optimal solution, the optimal rotation is unaffected by changes in market conditions. Thus, while this criterion is relatively simple to plan and administer, it is inflexible in its operation.

Maximization of Discounted Net Revenues From a Single Rotation (V3)

Such great economists as Fisher, Jevons, and Wicksell have proposed this model as the optimal criterion for forestry rotation determination. They saw the problem as being related to the classical economic problem of optimizing wine aging. The model is often simply called the present net worth (PNW) model, but Duerr and others (1956) called it the financial maturity model. They claimed it was easier to use and only slightly less precise than the Faustmann formulation, which will be described later. Gaffney (1957) showed that the loss in precision is variable and is influenced by a number of factors, including the length of the optimal rotation, market price, and interest rates. Hirshleifer (1970) states this formulation is not incorrect per se, but that it specifies a different set of productive opportunities for the landowner than does the Faustmann formulation.

This criterion assumes that the owner will maximize the PNW of the forest investment but does not account for future uses of the land when the rotation is completed. Even though harvesting returns are received at discrete intervals, their value is discounted at a continuous rate. Thus the equation includes the exponential function (e) in order to discount future returns, which may then be compared with the initial investment.

The formulation is:

(7) $$V3 = \max(T)\ \{pQ(T)e^{-rT} - wE\}$$

The first-order condition is:

(8) $$V3_T = [pQ_T - rpQ(T)]e^{-rT} = 0$$

(9) $$pQ_T = rpQ(T)$$

(10) $$Q_T/Q(T) = r$$

The rotation length is optimal when the rate of change of growth is equal to the market interest rate or when the VMP is equal to the opportunity cost of delaying the harvest one period. Neither price nor costs affects the optimal solution, but the PNW is affected by these factors. Its major error is that no opportunity cost is associated with the land itself. Thus, all rents accrue to management and none to the fixed factor, land.

Maximization of the Discounted Net Revenues From an Infinite Series of Like Rotations (V4)

This formulation was derived first by Faustmann (1849) and later by Pressler, and finally by Ohlin (Lofgren 1983). It is known to foresters by such terms as "soil rent," "soil expectation value" (SEV), or "land expectation value" (LEV). It is the superior decision criterion of those presented for several reasons. It assumes that the site will stay in forest production in perpetuity so that there is an explicit awareness of the effect that present decisions have on future possibilities. It optimizes the use of all inputs according to their guiding market prices and leaves any excess returns as a rent to the site. Thus, unlike the PNW formulation, this one accounts for changes in stumpage price and unit silvicultural cost in computing the optimal rotation length. The optimal solution includes a separate term for the land-rental charge. This value often is not large for long rotations, but its inclusion in the rotation determination is nonetheless necessary because it assigns a value to the opportunity cost of the land from delaying the harvest one period.

Faustmann first derived the formulation to determine just compensation to forest-land owners when their land was appropriated by the State for conversion to agriculture. Its importance was quickly grasped by other workers in the field and led to a rather bitter debate over its use over other criteria (Thomson 1942). The formulation assumes that after a harvest is completed, the site will be immediately regenerated, and that this process will be continued into the infinite future. The resulting stream of net revenues is discounted back to the present. Since all the variables are fixed, the optimum rotation age will also be constant (Heaps and Neher 1979). The formulation is:

$$(11)\quad V4 = \max(T)\ \{[pQ(T)e^{-rT} - wE](1 + e^{-rT} + e^{-r2T} + ...)\}$$
$$= \max(T)\ \{[pQ(T)e^{-rT} - wE]/(1 - e^{-rT})\}$$

The first-order condition is:

$$(12)\quad V4_T = \{(1 - e^{-rT})[pQ_T - rpQ(T)] - r[pQ(T)e^{-rT} - wE]\}/(1 - e^{-rT})^2$$

$$(13)\quad Q_T/[Q(T) - wE/p] = r/(1 - e^{-rT})$$

$$(14)\quad pQ_T = r(pQ(T) - wE)/(1 - e^{-rT})$$

$$(15)\quad pQ_T = rpQ(T) + r[pQ(T)e^{-rT} - wE]/(1 - e^{-rT})$$
$$= rpQ(T) + r(V4)$$

The rotation age is optimal when the VMP from letting the stand grow one more period is equal to the cost of holding the growing stock (the amount of interest that could be earned by harvesting the stand and investing the returns at the interest rate for one period) plus the cost of holding the land (the interest costs of delaying revenues from future harvests one period). As the planting costs increase or the price decreases, the rotation length increases.

Samuelson (1976) has shown that the Faustmann formulation can be expressed as in equation (6) if an explicit land-rent term is included. This equation must sum to 0 because the land rent (R) will absorb all excess profits in a competitive market. Thus:

$$(16)\quad 0 = \max(T)\ \{pQ(T)e^{-rT} - wE - R\int_0^T e^{-rt}dt\}$$
$$= pQ(T^*)e^{-rT^*} - wE - R^*[1 - e^{-rT^*}]/r$$

where R* and T* are the optimal land rent and rotation length, respectively. Solving for R* gives:

(17) $$R^* = r[pQ(T^*)e^{-rT^*} - wE]/(1 - e^{-rT^*}) = r(V4)$$

Dividing (14) by r yields an infinite stream of annual rental payments that is just equal to the Faustmann formulation or the LEV of the site. As shown in (14), this rental payment is exactly equal to the opportunity cost of the land from the right-hand side of (12).

Binkley (1987) has shown that comparing the maximum sustained-yield rotation with the optimal Faustmann rotation length gives ambiguous results, depending on the relative costs and the interest rate. In particular, he shows that with positive profits, all that is necessary is an interest rate less than $1/T_{MSY}$, as with the PNW criterion, to give a longer Faustmann rotation.

Using a similar analysis, it can be shown that the optimal Faustmann rotation is bounded from above by the PNW rotation. Reexpressing equation (11) so that it can be compared with the optimal condition for the PNW criterion (equation (8)) yields:

(18) $$Q_T/Q(T) = r[1 - wE/pQ(T)]/1 - e^{-rT}) = r$$

Rearranging terms, we find that in order for the PNW and Faustmann rotation to be equal in length, it must be the case that:

$$1 - wE/pQ(T) = 1 - e^{-rT}$$

For the PNW rotation to be shorter, it must be the case that:

$$wE > pQ(T)e^{-rT}$$

This condition implies losses on forestry investments because discounted revenues are necessarily greater than or equal to costs. Thus, the Faustmann rotation will never be larger than the PNW rotation, Q.E.D.

Maximization of Annual Net Revenues (V5)

This criterion, also known as forest rent, was proposed, in part, to justify longer rotations than those prescribed by the soil-rent criterion. It is meant to be used in a fully regulated forest – a forest where equal annual harvests are made and the land is immediately returned to timber production. Thus, it is claimed, there are no holding costs on the initial planting costs; these are covered as an operating expense to be taken from the cash-flow generated by the harvest receipts. This assumption is faulty because it fails to recognize that the decision to invest by planting to start a new rotation is entirely separable from the harvest decision. Saying that the land will continue to be managed for timber production is not the same as saying that it has no opportunity costs. The major effect of the assumption is that it avoids the discounting of future revenues against present costs. As shown by Bentley and Fight (1966) and others, it is a special case of the soil-rent calculation under the condition of zero interest rate. Samuelson (1976) called this criterion the "forester's maximum sustained (net) yield." This is something of a misnomer because MSY is generally derived only in biological terms, ignoring financial constraints.

The formulation of the problem is similar to equation (3) except that planting costs are included:

(19) $$V5 = \max(T) \{(pQ(T) - wE)/T\}$$

The optimal solution is:

(20) $$V5_T = [pQ_T T - (pQ(T) - wE)]/T^2 = 0$$

(21) $$Q_T = [Q(T) - wE/p]/T$$

(22) $$1/T = pQ_T/[pQ(T) - wE]$$

At the optimum, the VMP equals the average annual net revenues. Both prices and costs affect the optimal rotation length. An increase in p increases the right-hand side of (17) and thus decreases the optimal rotation length while an increase in wE lengthens the rotation. Since the numerator of the right-hand side of (17) is less than that of (4) but larger than (2), the optimal rotation for the forest-rent criterion is bounded from below by CMAI and from above by the MGY rotation.

Maximization of the Rate of Growth of Capital (V6)

This criterion is attributed to Boulding (1955) and commonly known by such terms as "internal rate of return" (IRR) or "rate of return" (ROR) maximization. It assumes that capital, not land, is fixed and it returns all rents to the capital investment and maximizes the rate of return on that investment. It assumes that the returns can be reinvested at this maximal rate forever and thus has an initial formulation similar to the Faustmann formulation of (9). The equation is maximized with respect to the interest rate, however. The optimal formulation is:

$$(23) \quad V6 = \max(i)\ \{[pQ(T)e^{-iT} - wE]/(1 - e^{-iT})\} = 0$$

where i = IRR.

Solving for i and then taking the first derivative of that expression, we have:

$$(24) \quad i^* = \max(T)\{(1/T)\ln[pQ(T)/wE]\} = Q_T/Q(T)$$

$$(25) \quad pQ_T = i^* pQ(T)$$

where the star signifies the maximum IRR. The rotation length is optimal when the rate of growth is maximized. This is then the maximum rate of return of the IRR. If $i^* > r$, the IRR criterion will give a shorter rotation than the PNW criterion. If $i^* < r$, then no investment will be made as a negative PNW arises.

There are many conceptual problems with the use of this criterion. The primary problem is that maximizing the IRR maximizes something besides the owner's wealth. Although there may be a relation between the two, it is not necessarily direct. IRR maximization assumes that the amount of land available for forestry is infinite and that access to all capital markets is closed. Thus, the owner will continue to invest all the returns from the original investment and never consume from the earnings.

When the IRR is greater than the interest rate, the potential total earnings from investment by this criterion are infinite. Since infinite returns do not exist, market forces must act to raise the interest rate and drive the present value of the investment to zero. Thus, this criterion leads to either an absurd conclusion, infinite wealth increment, or a trivial one, zero wealth increment (Hirshleifer 1970).

Another problem with the IRR criterion is that decisions relating project scale and timing may be incorrect and that changes in prices or the market interest rate do not affect the optimal rotation or the level of investment. If the planting costs are equal to zero, then the IRR is undefined and if intermediate costs and benefits are received, there will be multiple IRR's that solve equation (19) (Bierman 1968).

Relationships Among the Six Criteria

Model formulations and basic results for the six optimal rotation criteria are summarized below in a condensed form so that they can be more easily referenced and compared. Equation numbers are those found in the text.

Let:

T = time; p = net real stumpage price

r = real discount rate; i = internal rate of return

wE = labor price (wage) times unit regeneration effort

$Q(T)$ = timber growth, a concave function of time with zero regeneration lag and fixed technology, $Q_T > 0$, and $Q_{TT} < 0$

Maximization of gross yield (also: maximum physical yield, MGY)

(1) $V1 = \max(T)\ \{pQ(T)\}$

(2) $pQ_T = 0 = Q_T$

Maximization of annual yield (also: CMAI, MSY)

(3) $V2 = \max(T)\ \{pQ(T)/T\}$

(5) $Q_T = Q(T)/T$

(6) $pQ_T/pQ(T) = 1/T$

Maximization of single rotation discounted net revenues (also: PNW)

(7) $V3 = \max(T)\ \{pQ(T)e^{-rT} - wE\}$

(9) $pQ_T = rpQ(T)$

(10) $Q_T/Q(T) = r$

Maximization of infinite rotation discounted revenues (also: Faustmann's soil rent, LEV, SEV)

(11) $V4 = \max(T) \{[pQ(T)e^{-rT} - wE](1 - e^{-rT})\}$

(13) $Q_T/[Q(T) - wE/p] = r/(1 - e^{-rT})$

(14) $pQ_T = r(pQ(T) - wE/(1 - e^{-rT})$

(15) $pQ_T = rpQ(T) + r(V4)$

Maximization of annual net revenues (also: forest rent)

(19) $V5 = \max(T) \{(pQ(T) - wE)/T\}$

(21) $Q_T = [Q(T) - wE/p]/T$

(22) $1/T = pQ_T/[pQ(T) - wE]$

Maximization of the rate of growth of capital (also: IRR, ROR)

(23) $V6 = \max(i) \{[pQ(T)e^{-iT} - wE]/(1 - e^{-iT})\} = 0$

(24) $i^* = \max(T)\{(1/T)\ln[pQ(T)/wE]\} = Q_T/Q(T)$

(25) $pQ_T = i^* pQ(T)$

Table 1 is derived from Schumacher and Coile's yield equations for loblolly pine.[2] It displays the effects of the choice of criterion on the optimal rotation age. The table assumes a price of $0.30 per cubic foot, planting costs of $100, and an interest rate of 5 percent. The connected values in the table show the approximate time at which the necessary equality condition for the optimal rotation age of each criterion is reached. The rotation length is shortest with the IRR criterion and longest with the MGY criterion. Use of the MSY criterion in place of the soil-rent criterion lengthens the rotation by approximately 20 percent, a significant increase. The soil-rent rotation length lies between the IRR and PNW rotations.

Figures 1, 2, and 3 depict the information in table 1. The optimal rotations are easily computed graphically. From figure 1, the MGY rotation occurs where the yield function is at its maximum point (the slope is equal to 0), or in figures 2 or 3 where the marginal growth function equals 0.

A ray drawn from the origin to the yield curve in figure 1 gives the average yield at any point in time. Therefore, the MSY rotation occurs when this ray is tangent to the yield curve. MSY can also be found in figure 2 of the graph where the marginal and average yield curves cross. The forest-rent rotation length is necessarily longer than MSY as the ray is vertically displaced from the origin by the amount of the regeneration cost.

The IRR rotation occurs at the inflection point of the yield function in figure 1 or where the slope of the marginal growth in figure 2 or figure 3 is equal to 0.

The PNW and Faustmann criteria explicitly contain economic information in their solutions and so cannot be estimated using physical yield curves alone. Figure 3 graphs the left- and right-hand sides of the six optimal solutions so that they can be estimated directly.

The MGY, MSY, and forest-rent criteria share the common shortcoming that interest costs are ignored. Thus, no rents are returned to the land, because it is assumed that the opportunity cost of the land is zero. They therefore imply that capital is costless and there are no other uses for the land, including the growing of a new stand of trees. In addition, they assume that the forest owner and society are indifferent between revenues earned now and later.

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[2] Francis Schumacher and Theodore Coile. 1960. Growth and yields of natural stands of the southern pines. Durham, NC: T.S. Coile Inc. 115 pp.

Table 1.--Mathematical determination of the optimal rotation age for loblolly pine by solution of the six optimal conditions

Age	Q(T)	Q_T	Q(T)/T	$Q_T/Q(T)$	$\frac{Q_T}{Q(T)-wE/p}$	i*	1/T	$\frac{r}{1-e^{-rT}}$
4	31	15	7	0.484	0	--	0.25	0.276
6	105	36	17	.343	-.18	--	.16	.193
8	338	116	42	.343	3.05	0.002	.125	.152
10	715	188	71	.263	.453	.079	.10	.127
12	1,175	230	97	.196	.263	.111	.08	.111
14	1,665	244	118	.147	.178	.122	.07	.099
16	2,147	241	134	T6 [.112] ←	.131	→ [.123]	.06	.091
18	2,600	226	144	.087	.098	.120	.055	.084
20	3,013	206	150	.068	T4 [.076] ←	.116	.05	→ [.079]
22	3,383	184	153	.054	.060	.111	.045	.075
24	3,707	162	154	T3 [.044]	.048	.106	.041	.072
26	3,988	T2 [140] ↔	[153]	.035	T5 [.038] ←	.100	→ [.038]	.069
28	4,229	120	151	.028	.031	.095	.035	.066
30	4,433	101	147	.023	.024	.090	.033	.064
32	4,603	84	143	.018	.020	.086	.031	.063
34	4,743	69	139	.015	.016	.081	.029	.061
36	4,855	56	134	.012	.012	.077	.028	.060
38	4,943	43	130	.009	.009	.074	.026	.059
40	5,009	33	125	.007	.007	.070	.025	.058
42	5,056	23	120	.005	.005	.067	.024	.057
44	5,085	14	115	.003	.003	.064	.023	.056
46	5,100	7	110	.001	.001	.061	.022	.056
48	5,101	T1 [0]	106	0	0	.058	.021	.055
50	5,090	-6	101	--	--	.056	.02	.055

p = \$0.30 per cubic foot, wE = \$100, and r = 5 percent. Connected values indicate approximate equality of the functions.

Source: Volumes are calculated from equations in Francis Schumacher and Theodore Coile's "Growth and Yields of Natural Stands of the Southern Pines." Durham, NC: T.S. Coile Inc. 1960. 115 pp.

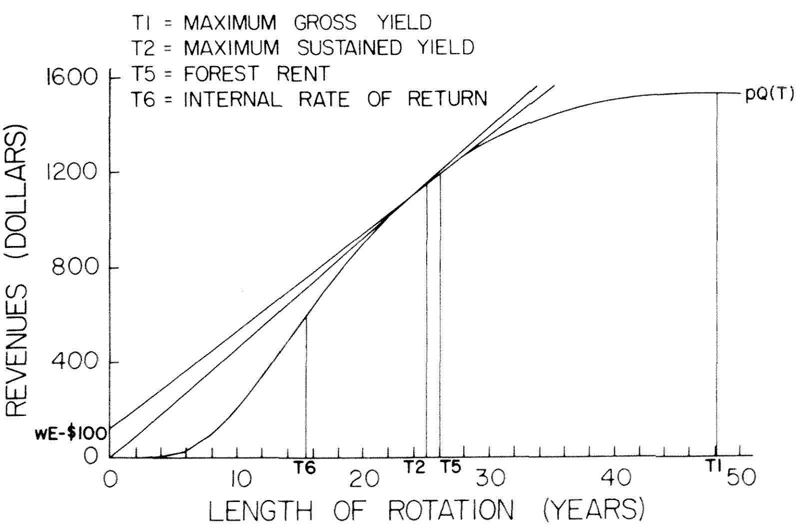

Figure 1.--Total yield determination of the optimal forest rotation.

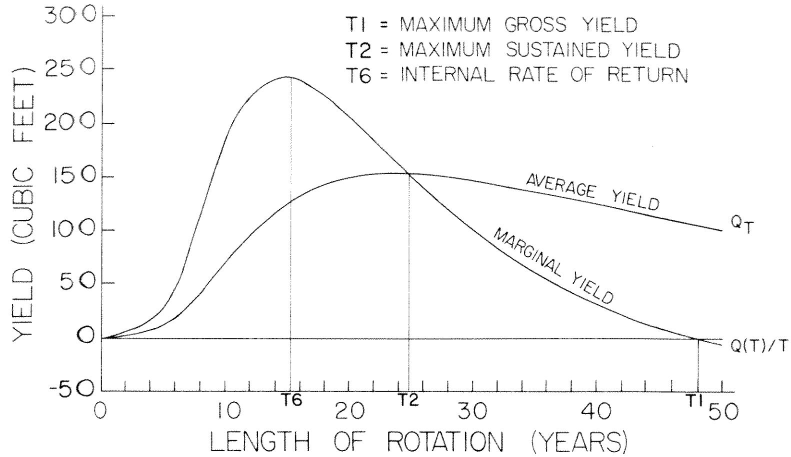

Figure 2.--Marginal and average yield analysis of the optimal forest rotation.

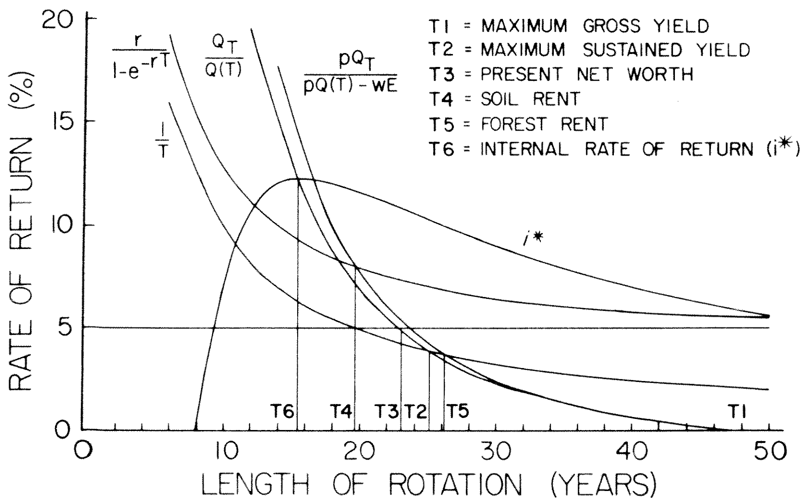

Figure 3.--Marginal determinations of the optimal rotations by the six criteria.

Only the forest-rent and soil-rent rotation lengths are affected by changes in prices or costs. A higher price shifts the curve $\{Q_T/[Q-wE/P]\}$ to the left because a higher price increases the size of the denominator, making the value of the curve smaller for any t. The result is a shortened optimal rotation for both criteria as the curve will cross its equality condition at an earlier period. A higher regeneration cost shifts the curve to the right, causing an increase in the optimal rotation age. Finally, increased interest rates shift those functions inward, causing shorter rotations.

Extensions of the Faustmann Model

The primary use of the Faustmann model is to gain a better understanding of the effects that various market perturbations and assumptions will have on landowner behavior. Although market situations differ from the idealized assumptions used in the discussion presented above, this does not alter the usefulness of the basic results. What is important is that the Faustmann model presents a standard neoclassical approach to management that is internally consistent with general economic analysis.

Some authors have criticized the general applicability of the Faustmann model in an operational sense. Duerr and others (1956) discuss the difficulty of finding easy marking guidelines for fieldworkers to use, whereas Grainger (1968) discusses the fact that modern forestry organizations need more accurate accounting to value their land than the Faustmann formulation offers. Neither argument is particularly persuasive and both can lead to incorrect conclusions when indiscriminately applied.

The rest of this report discusses attempts to broaden the model by deriving the comparative static results of the formulation (Chang 1983; Clark 1976; Graham-Tomasi 1983; Howe 1979; Hyde 1980; Jackson 1980; Morgan 1974) or extending the model to more accurately reflect real world conditions. Extensions of the model have examined the effects on the optimal timber rotation of such factors as: externalities (Berck 1981; Bowes 1983; Calish and others 1978; Hartman 1976; Nguyen 1979; Strang 1983), risk and uncertainty (Kao 1982; Martell 1980; Nostrom 1975; Reed 1984; Routledge 1980), changing price and cost assumptions (Bare and Waggener 1980; Gregersen 1975; Hardie and others 1984; McConnell and others 1983; Newman and others 1985), taxation (Chang 1982; Klemperer 1979; Pearse 1967), and imperfect markets (Murphy and others 1977; Nautiyal and Fowler 1980).

Comparative Statics

The basic results given by authors who have looked at the effect of changes in the exogenous parameters of the model, on the optimum rotation age, and silvicultural effort are shown in table 2. The two most thorough presentations are by Chang (1983) and Graham-Tomasi (1983), who show general agreement in their results, though they used slightly different formulations in setting up the problem. Both authors denote volume as a function of not only time but silvicultural effort as well. Chang uses seedling planting density as a proxy and divides the effort into site preparation and planting variables while Graham-Tomasi uses only a single, general effort variable for his formulation.

The major cause for the uncertain answers in table 2 is that it is often unclear, due to the use of a general growth function, whether time and effort can be considered as substitutes or complements and to what degree. Therefore, it is necessary in those uncertain cases to empirically estimate a stand's growth response for the absolute determination of the sign. Also, in the case of changes in the interest rate, as shown in (16), both factors in the right-hand side are affected, but in different directions. An increase in the interest rate increases the cost of holding the stock (rpQ(T)) while lowering the cost of delaying the harvest (rV5 decreases), since the value of managing the land for timber (V5) decreases with an increase in the interest rate.

One use for these comparative static results is to help determine the ultimate supply responses from changes in the exogenous variables. For instance, Hyde (1980) states that since the optimum rotation age will generally decrease as price increases, each acre in a managed forest will produce less volume on a sustained-yield basis. If acreage is held fixed, the ultimate supply elasticity from a price increase would therefore be negative. Hyde counters this argument in both the short and long run. The short-run response will necessarily be positive as timber owners reduce the rotation length of their stands by increasing, temporarily at least, the amount of timber cut. The long-run supply response will likely also be positive as two other factors will come to bear to increase total production: (1) that increasing land-intensive silvicultural effort becomes profitable, increasing yields on all acres, and (2) more acres that were previously submarginal or in other uses are brought into timber production. Binkley (1987) found that with a fixed land base, the supply curve will be negatively related to price only for rotation lengths less than the MSY rotation.

Another use of the comparative static results is in the analysis of the effects of different tax policies on landowner behavior (Chang 1982; Pearse 1967). The results for the major forestry taxes are:

Uncertain or Changing Value Parameters

A major assumption of the Faustmann formulation is that of perfect certainty with respect to the parameters of the model. This assumption may have been justifiable when dealing with a relatively static economy like that of 19th century Germany. However, in our present economy, where new technologies and events continually affect production and changing market conditions affect demand, it becomes necessary to investigate the changes in the optimal conditions that will occur in this dynamic setting or in a situation of uncertainty.

As a less complex first approach, Gregersen (1975) and Klemperer (1979) examined the case of simple price inflation (i.e., no change in real values over time) and its effect on forestry investments. They found that as long as the inflation is accounted for in the calculations, either by using real or current values for prices and costs, there is no real effect on returns or rotation lengths as all costs and returns are growing at the same rate. Thus, the inflated values in the equation will simply cancel out. In cases with taxation, however, real distortions may occur. For instance, in situations with capital gains taxation, short-term investments are affected to a greater real extent than long-term investments as the tax-deductible purchase costs make up a greater percentage of the final sale value of the timber crop.

Although the optimal behavior designated by the Faustmann conditions will hold under static conditions, supply forces are at work that seem to ensure that real timber prices will rise. Hotelling's (1931) theory of the mine states that as the quantity of a nonrenewable resource decreases, its price will increase over time at a rate equal to the interest rate. This price rise will continue until the time it reaches the price of a substitute resource, at which point a steady-state condition will ensue with the nonrenewable resource essentially used up.

This scenario has been applied and discussed in the forestry literature with regards to the dwindling supply of the basically non-renewable old-growth timber resource and its substitute, second-growth timber (Berck 1979; Lyon 1981). The theory has been substantiated by the findings of rising real stumpage prices over the past 100 years.[3] It has been used in various timber supply projections that predict con-

[3] Manthy, Robert S. 1978. Natural resource commodities: a century of statistics. Baltimore: Johns Hopkins University Press. 240 pp.

tinued price increases for the forseeable future.[4] The price level, as shown by Lyon (1981), can rise at a rate greater than, less than, or equal to the interest rate, depending on assumptions relating to transportation costs, the rate of volume growth, and the amount of regeneration.

This information was used by McConnell and others (1983) to examine the effects of varying real prices and costs on the optimal rotation decision. Their model used a dynamic optimal control framework in which the land could, at some point in the future, switch from forestry to agricultural production when the costs of forestry became too high. They found that if price is increasing at a constant rate (q) and planting costs (c) are increasing at a faster rate ($c > q$), the optimal rotation is no longer constant but increases over time. If $c < q$, then the optimal rotation decreases over time. These results pertain because with costs rising at a greater rate than prices, it is worth it to the owner to have shorter rotations earlier on because the relative costs of later harvests will have increased. With prices rising faster than costs, the owner postpones early harvests in order to receive the price benefits of delay but then decreases the rotation length due to discounting effects. The net price rise must also be less than the discount rate. If it does not do so, harvesting would be delayed indefinitely because the present value of the stand would be continuously increasing.

Hardie and others (1984) extended this model to examine the effect of ignoring changing prices on the net forest returns. When price changes were ignored, they fould that large losses resulted as, incongruously, too short rotation lengths were used with the Faustmann equation. Newman and others (1985) attributed the lengthened optimal rotations to two countervailing effects. Rising prices lead to a shortened rotation, but rising prices in relation to regeneration costs serve to lengthen the rotation because delay reaps the benefits of progressively decreasing relative regeneration costs.

A related form of parameter change occurs from genetic or other biotechnological progress, which causes shifts in the biological production function. Lofgren (1983) and Johansson and Lofgren (1985) examine one-time shifts in the yield function as well as continuing exponential growth in productivity. Their results are similar to those found by Newman and others (1985) because continuing yield increases have the same effect as continuing price increases. Thus, rotation lengths initially decrease and then gradually increase over time.

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[4] U.S.Department of Agriculture, Forest Service,1982. An analysis of the timber situation in the United States: 1952-2030. For. Resour.Rep. 23. Washington, DC. 499 pp. (p. 205)

Nostrom (1975) examined uncertainty with respect to price, and Kao (1982 and 1984) looked at risk and uncertainty in the growth function to see how the optimal rotation was affected. Using the MSY criterion, Kao found that increasing uncertainty in the growth function leads to a shorter rotation. Nostrom, using the PNW criterion, found that ignoring price uncertainty leads to lower net present values and an uncertain change in the optimal rotation.

Work in the general area of value uncertainty has been hampered by the relative intractability of the problems to simple mathematical solutions. Therefore, it is necessary to use large amounts of computing effort in order to solve the dynamic optimization problem for specific cases. Kao found that solving the deterministic case took just 3 seconds of computing time, whereas the uncertain cases took from 235 to 365 seconds. As faster, more powerful computers become available, it can be expected that more research into this topic will be performed.

A final form of uncertainty that has been extensively examined in the silvicultural literature, but rarely with regards to optimal rotation effects, is the effect of fires or other natural catastrophes that can either alter or destroy the forest's growing stock. Martell (1980), Reed (1984), and Routledge (1980) examined the rotation effects by proposing various stochastic modeling schemes to explain the occurrence of fires. Using a simple probability structure, Routledge (1980) found that if the burned stand could be harvested without value loss after the fire, the optimal rotation would be lengthened from the no fire case. This result pertains because the land-rent term in the Faustmann solution (equation 16) is decreased due to occasional premature harvests; thus, a lower growth rate is necessary to maintain the basic equality. Risk of stand loss beyond salvage, however, fairly substantially decreases the rotation length.

Martell (1980) used a Markov decision model with constant fire probabilities to determine the general effects of fire and found that shortened rotations would occur. Using an analysis with fire independent of stand age and the destruction total, Reed (1984) treated the average rate at which fires occurred as a risk factor that could be added to the interest rate. Thus, the shortened rotation length is due to a higher effective interest rate. In tests of more complex fire cases, he found the optimal rule less clear.

Market Failure - Externalities

A major complaint leveled at all the economic criteria used for rotation determination is that insufficient attention is given to the nonmarket benefits derived from growing timber. Critics claim that since timber is often the only output from the forest producing direct dollar returns, its growth requirements receive the highest priority in management. Other goods from which society derives utility (often in common or public form and thus not directly priced) such as water, recreation, wildlife and other amenity goods, whose highest value rotation may not concur with the economic rotation, often are not accounted for in the final management decision. The Forest Service partially justifies its use of the MSY criterion in this manner. Since the Forest Service is obligated to manage national forests for multiple use, it is claimed that longer rotations will best ensure their multiple objectives.

A few authors have looked at the externality problem that arises in forestry. Berck (1981) discussed the possible use of taxation or subsidies to ensure the optimal production of the external goods, whereas Nguyen (1979) used a selection management system and developed optimal cutting conditions for that situation.

In response to criticisms of using the MSY criterion to ensure the production of externalities by Hirshleifer (1974) and Samuelson (1976), Hartman (1976) first adapted the Faustmann equation to include nontimber amenity benefits. The formulation became:

$$(26) \qquad V7 = \max(T)\ \{[pQ(T)e^{-rT} - wE + \int_0^T F(n)e^{-rn}dn]/(1 - e^{-rT})\}$$

where V7 is the solution for the maximized joint production of timber and external benefits, F(n) is the functional form for the growth of the amenity values derived from the growing forest, and all other terms are the same as in the previous equations. The first-order condition gives:

$$(27) \qquad [pQ_T + F(T)]/[pQ(T) - wE + \int_0^T F(n)e^{-rn}dn] = r/(1 - e^{-rT})$$

which is very similar to equation (15) from the Faustmann formulation. It was decomposed by Bowes (1983) to show more clearly the effects of amenity value on rotation age:

$$(28) \qquad a[F(T)/\int_0^T F(n)e^{-rn}dn] + (1 - a)[pQ_T/(pQ(T) - wE)] = r/1 - e^{-rT}$$

where $a = \int_0^T F(n)e^{-rn}dn/[pQ(T) - wE + \int_0^T F(n)e^{-rn}dn]$

If the nontimber share of benefits (a) is low, the optimal rotation approaches the Faustmann rotation. If a is high, however, the optimal rotation will be longer or shorter than the Faustmann rotation, depending on whether the amenity value increases or decreases with stand age. The formulation also shows that it may be worthwhile to delay harvest forever, if the rate of growth of amenity values is high enough.

Calish and others (1978) tested the Hartman (1976) formulation using derived yield equations for seven nontimber goods in the Douglas-fir timber region. The individual effect of each of these goods confirmed Hartman's results in regards to the shape of the amenity value benefit stream and its effect on the optimal rotation length. Using sensitivity analysis to value the nontimber goods, they found the economic optimum rotation given by the Faustmann equation was quite robust. When amenity goods were added, the optimal rotation changed only slightly. Large economic losses, however, generally resulted from the use of the MSY criterion.

A major problem with the Hartman formulation is that it focuses the multiple use benefits on a single stand, ignoring the effects of activities in adjacent stands on the derived amenity values. Incorporating the forestwide rotation effects is a linear programming problem that has been investigated by Bowes (1983).

Market Failure - Imperfect Markets

This general topic has received relatively little research effort, in part because the applicability of these cases seems somewhat limited. In imperfect stumpage markets, local, or even regional, monopolies may exist, but two factors serve to blunt attempts to assert this monopoly power. First, where such monopoly power exists, it is most often held by government, as in western Canada, the Pacific Northwest of the United States, or in socialist countries where all land is publicly held. It is not necessarily in the public's interest to maximize profits from stumpage sales at the cost of a decrease in total social welfare. Second, many close substitutes for the monopoly-held stumpage may exist. One close substitute is similar stumpage supplied from nearby areas. Some monopoly power may still exist, but the demand curve is fairly elastic and the shipping costs necessary to import this external stumpage places an upward bound on any price increase that can be charged.

Nautiyal and Fowler (1980) used a Faustmann model to compare the rotation length and welfare losses that occur where pure monopoly power exists. They model the behavior of a pure monopolist, a social welfare maximizing central planner, and the standard atomistic (competitive) ownerships. In an unregulated forest with a constant area of managed forest land, they find that the monopolist uses the shortest rotation and atomistic owners use the longest. The monopolist shortens rotations to reduce the amount of stumpage in the market. Forcing competitive behavior where monopoly power exists is inefficient because the marginal conditions are violated (value of the marginal product is less than the marginal cost). The societal optimum, therefore, is found to lie between the two extreme cases and closer to the monopolistic rotation. In the case of a regulated forest, the differences in rotation length among the three cases virtually disappear, but the ordering of the lengths stays the same as for the unregulated cases.

Murphy and others (1977) examined the more common situation of imperfect capital markets. In this situation, either the amount of money that can be borrowed is restricted or a multiplicity of borrowing and lending rates apply. Their policy criterion was the maximization of the liquidation value of the firm with a finite 100-year planning horizon. They found that higher borrowing than lending rates lowered the optimal rotation length and greatly reduced the liquidation value of the firm. This result was caused by the need for the landowner to restrict early borrowing and acreage planted in order to build up sufficient capital reserves to finance increased acreage levels.

Summary, Conclusion, Future Research Areas

One would think that a topic with such a long written history had been completely described and understood by now, but the theory of the optimal forest rotation continues to attract new ideas and research. The majority of research centers on the comparison of the various criteria for rotation determination and the derivation of the comparative static conditions of the basic model. Although these problems have been thoroughly covered, relatively scant attention has been placed on extending the model by challenging its assumptions or extending them to better reflect actual market conditions. This situation is changing. Research begun in the last 10 years is the first to examine many of these extensions. The increasing mathematical sophistication of both researchers and those using the research make it likely that this trend will continue.

Likely, new areas of research include changing price scenarios and the effects of uncertainty on the optimal rotation. Some good work on these topics has already been done, but both a clearer generalization of the problem and specific analyses using various production functions are needed to gain a better understanding of the changes that uncertainty can cause. Some topics that could be investigated for optimal behavior could be: uncertainty about the value parameters, either in a pure stochastic sense (where the mean and the variance of the parameters are known) or in a Bayesian sense (where neither the mean nor the variance is known); the assessment of risk on forestry rotations and whether risk premiums are assigned to the forestry situation.

Much of the research that is needed to deal with these topics would necessarily be more applied and computer intensive. Programming methods, which can optimize not only the harvest timing but other silvicultural treatments as well, are beyond the scope of this report. The object here is to investigate the theoretical underpinnings of the general case rather than to become engrossed in specific methodological solutions. It now appears that since the general theory has been well discussed, it will be necessary to incorporate these programming methods into the investigation of specific problem areas.

The effects of technical change on forest production and how forestry fits into a technologically evolving society are important concerns. Unequal access to new technologies may create substantial contrasts in management behavior. Owners will manage land for different objectives as access to new techniques become available. Discerning the rotation effects from the presence of these new conditions will be challenging.

A final area for future research is in market imperfections. Unequal knowledge of market conditions between buyers and sellers is a common condition for nonindustrial owners, and inadequate knowledge can influence the decision to harvest. Oligopolistic or monopsonistic markets seem to be more prevalent than purely monopolistic markets and could be expected to produce substantial disturbances from the theoretical optimum state.

In conclusion, the controversy and debate that Faustmann began more than 150 years ago between economic and biological criteria for forest management continues. As society's uses of its forest resources expand and evolve, new debates and issues will continue to occur.

Annotated Forest Valuation Bibliography

Anderson, F.J. 1976.
Control theory and optimum timber rotation. Forest Science 22(3):242–246.

The net present value maximization problem of Faustmann is compared with the optimal control framework for renewable natural resources. The optimal control model gives the same answers about rotation age as the earlier intuitive formulations. That is, shorter rotations than maximum sustained yield when r > 0. The article points out some problems of the Faustmann formulation in relation to beliefs on changing prices and other non-steady-state conditions.

Anderson, F.J. 1981.
Optimum forest rotation: comment. Land Economics 57(2):293–294.

This paper is a comment on a paper by Nautiyal and Fowler (1980). Anderson felt they implied that the atomistic (competitive) rotation length is different from the social optimum. He shows that unless other issues are introduced (externalities, etc.), this cannot be the case as the marginal conditions of the Faustmann equation ensure optimality.

Bare, Bruce B.; Waggener, Thomas R. 1980.
Forest land values and return on investment. Forest Science 26(1):91–96.

The paper examines increasing price scenarios and their effect on the land expectation value (LEV) of a site. The major point is that increasing future prices leading to higher future values will already be incorporated into the LEV; thus return on investment (ROI) will stay the same. The paper errs in postulating a 60-year rotation for Douglas-fir under all scenarios. The constant rotation example works for their comparisons but is basically incorrect because the LEV could be increased by altering the rotation length in a period of rising prices, thus giving a higher ROI.

Bentley, William R.; Fight, Roger D. 1966.
A zero interest comparison of forest rent and soil rent. Forest Science 12:460.

Using L'Hopital's rule (although not cited), the authors mathematically prove that the forest-rent calculation is a subset of the land expectation value (soil rent) calculation. Thus, when the interest rate approaches zero, the soil rent approaches the forest rent.

Bentley, William R.; Teeguarden, Dennis E. 1965.
Financial maturity: a theoretical review. Forest Science 11:76-87.
A more basic explanation of Gaffney (1957) using graphs. The paper compares the primary models of financial maturity: (a) one rotation maximum revenue model with and without interest, (b) Faustmann equation model, and (c) Boulding's maximization of internal rate of return. The paper has little math and no proofs but presents a clear nontechnical exposition of the models in an economic framework.

Berck, Peter. 1979.
The economics of timber: a renewable resource in the long run. Bell Journal 10(2):447-462.
Analyzes two questions: (1) Have private timber owners over time depleted stocks at a faster-than-optimal rate? and (2) Have public owners depleted stocks at a slower-than-optimal rate? Using rational expectations for future prices, Berck sets up a supply and demand system which essentially assumes that supply equals demand throughout time. He finds that private owners have been using a 5 percent discount rate which is lower than for other industries and lower than the 10 percent rate that he uses as a determinant for past overexploitation. He hypothesizes that nonmarket goods which public owners value could explain differences between public and private cutting rates.

Berck, Peter. 1981.
Optimal management of renewable resources with growing demand and stock externalities. Journal of Environmental Economics and Management 8(2):105-118.
Applies optimal control techniques to situations of a renewable resource. In the general model, an optimal solution occurs when the rate of change of net price plus the marginal production equals the interest rate in the case of constant costs. With growing demand, a steady state will be reached as long as demand growth is less than the interest rate. In the presence of positive externalities, either a subsidy or a tax can be used to bring about the desired increase in stocks to give the socially optimal use over time.

Bierman, Harold, Jr. 1968.
The growth period decision. Management Science 14(6):302-309.
Compares present value maximization and internal rate of return (IRR) solutions. Problems associated with using IRR as an investment guide with respect to scale and intensity of forest management are discussed. The presentation leads circuitously to the Faustmann rule without stating it as such.

Binkley, Clark S. 1985.
The role of capital in long run timber supply. Geneva: International Institute of Applied Systems Analysis (IIASA) Working Paper, 26 pp.
Develops a supply function based on the Faustmann optimal rotation in a manner similar to Hyde (1980). With a fixed technology and land area, the supply curve is elastic only for rotations longer than the maximum sustained yield (MSY) and will otherwise be negative for rotations shorter than MSY. Negatively sloped supply can lead to an unstable equilibrium, which is demonstrated. The capital/output ratio is related to interest rates.

Binkley, Clark S. 1987.
When is the optimal economic rotation longer than the rotation of maximum sustained yield? Journal of Environmental Economics and Management 14(2):152-158.
Examines the situation that would cause the Faustmann rotation to be longer than that prescribed by the maximum sustained yield (MSY). Conditions that will bring this result are an interest rate (r) less than the inverse of the MSY rotation length (1/T*) along with a positive profit. These conditions are most likely to occur in fast- growing species such as southern pine or tropical plantation species. An example verifies the result. The importance of this result is in determining the sign of the elasticity of supply in response to a change in stumpage price.

Boulding, Kenneth E. 1955.
Economic analysis. New York: Harper and Bros.
Chapter 30 discusses maximization of the internal rate of return as the decision criterion for rotation length determination.

Bowes, Michael D. 1983.
Economic foundations of public forestland management. Discussion Paper D-104. Washington, DC: Resources for the Future. 80 pp.
Presents a model based on Hartman (1976) examining optimal management for public ownerships who must account for nonmarket benefits. He shows that if the nonmarket benefits are growing (declining) with age, the rotation is lengthened (shortened). The major stated problem with this type of model is its focus on the single stand, whereas the manager must manage the whole forest. A full working example using the Douglas-fir region illustrates the problem. The paper is an excellent technical overview of the problem of rotation determination with and without nonmarket benefits present.

Calish, Steven; Fight, Roger D.; Teeguarden, Dennis E. 1978.
How do nontimber values affect Douglas-fir rotations? Journal of Forestry 76(4):217-222.

Using derived equations for seven nontimber yields, the authors compare the land expectation value (LEV), economic optimum rotation, biological optimum rotation (maximum sustained yield), and joint economic optimum rotations. Opportunity costs are significant in comparisons of MSY with the joint optima for all options at a wide range of nontimber valuations. Including nontimber yields into the LEV can either lower or increase the economic optimum rotation length, depending on the stream of nontimber returns. At moderate valuations of the nontimber returns, the changes in rotation length from the economic optimum are generally small.

Chang, Sun Joseph. 1981.
Determination of the optimal growing stock and cutting cycle for an uneven-aged stand. Forest Science 27(4):739-744.

Demonstrates that optimal rotation length for uneven-age management can be derived from a formulation very similar to the Faustmann equation. Opportunity cost on the growing stock represents the regeneration costs of the stand. As with even-aged management, increased interest rates lead to shorter cutting cycles and lower stocking rates. Here, however, changing prices do not affect the optimal conditions as the prices are canceled out of the optimal condition. Finally, with increasing interest rates,uneven-aged management is the preferable alternative.

Chang, Sun Joseph. 1982.
An economic analysis of forest taxation's impact on optimal rotation age. Land Economics 58(3):310-323.

A rigorous examination of five types of taxes and their effect on rotation age. They are (1) unmodified property tax, (2) site value tax, (3) timber tax, (4) yield tax, and (5) severance tax. Capitalizing a tax and shifting the incidence of the tax forward to higher valued stumpage will have opposite effects on rotation age. A graphical analysis, based on the technique first demonstrated by Clark (1976), shows the effects on the optimal harvest period. Property taxes reduce the rotation age while yield taxes delay the harvest. Poses questions regarding the concept of tax neutrality and current timber taxation.

Chang, Sun Joseph. 1983.
Rotation age, management intensity, and the economic factors of timber production: do changes in stumpage price, interest rate, regeneration cost, and forest taxation matter? Forest Science 29(2):267-278.

Analyzes the comparative statics of timber under a very general framework. Yield is set as a function of rotation length and planting density, and a Faustmann formulation, including site preparation and planting costs, is used to examine the following conditions: (1) higher site preparation costs, (2) higher planting costs, (3) higher stumpage prices, (4) higher interest rate, (5) ad valorem tax, (6) site value tax, and (7) yield tax. Uncertainties in the analysis come about due to the generality of the growth function and thus uncertainty with respect to the resulting shape of the yield function.

Chang, Sun Joseph. 1984.
Determination of the optimal rotation age: a theoretical review. Forest Ecology and Management 8:137-147.
Basically, a modernization of earlier works by Gaffney (1957) and Bentley and Teeguarden (1965). The four major competing criteria for rotation length (soil rent, present net worth, forest rent, and the traditional biological model) are presented in such a way that makes derivation understandable to foresters with as little as one semester of calculus. The cases are clear and the explanation of the results is very sound. A good synthesis of the material.

Clark, Colin. 1976.
Mathematical bioeconomics: the optimal management of renewable resources. New York: John Wiley & Sons. 352 pp.
Chapter 8 presents the Faustmann model and the first-order conditions for optimality. The left- and right-hand sides of the optimality equation are explained graphically. The book is an excellent introduction into optimal control techniques and their uses for renewable resources (particularly fisheries) management.

Comolli, Peter M. 1981.
Principles and policy in forestry economics. Bell Journal 12(1):300-309.
From the basic Faustmann formulation, casually derives an industry supply function to analyze policy issues relating to forest taxes. A tax will raise prices and lower quantities because it acts like an increase in costs to the producers. An excise tax will raise prices more than a property tax if land's share is less than 50 percent of total income.

Comolli, Peter M. 1984.
A proposition on imperfect stumpage markets. Land Economics 60(3):297-300.
Extends the argument of Nautiyal and Fowler (1980) to that of a monopolist with imperfect knowledge of future demand. In such a case, the monopolistic rotation length may not always be shorter than the

competitive optimum rotation. The rotation for the monopolist will be longer if the rate of price increase is expected to be greater than the discount rate.

Dasgupta, Partha. 1982.
The control of resources. Cambridge, MA: Harvard University Press. 223 pp.
Chapter 9 discusses optimal forest management strategies using a single-tree stand model to derive the Faustmann rule. The author discusses the effects of nonlinearity of the social benefit rule on optimal decisions.

Duerr, William A.; Fedkiw, John; Guttenberg, Sam. 1956.
Financial maturity: a guide to profitable timber growing. Tech. Bull. 1146. Washington, DC: U.S. Department of Agriculture. 74 pp.
Discusses the use of a modified present net worth (PNW) criterion to determine the optimal rotation age. With the modified criterion, marking guides can be decided upon in the field by using a simple rate of growth determination. The appendix compares PNW with the Faustmann formulation. The authors use a synthetic growth function and find a relatively small change in the optimal rotation when the PNW criterion is used. This result was shown by Gaffney (1957) to be a result of the growth function chosen rather than the general case.

Faustmann, Martin. 1849.
On the determination of the value which forest land and immature stands pose for forestry. In: Gane, M., ed. Martin Faustmann and the evolution of discounted cash flow. Paper 42. Oxford, England: Oxford Institute; 1968. 54 pp.
Written as a reply and extension of an earlier article by von Gehren, this paper sets forth the basic economic tenets by which timberland should be valued; that is, the land is worth the present value of an infinite series of like rotations. A path-breaking paper that painstakingly explains the concept that the forest-land value will remain unchanged, regardless of whether there are trees growing on the site or not, as the growing tree's value would necessar-ily be capitalized into any sales price. Derives the formula used today for calculating land expectation value. The foreword by Gane is an excellent historical background and sets the place of Faustmann and his contemporaries within the stream of early economic thought.

Fortson, James C. 1972.
Which criterion? Effect of choice of the criterion on forest management plans. Forest Science 18(4):292-297.

Compares the internal rate of return (IRR) and maximization of net present value (NPV) criteria. The IRR and NPV analyses assume that cash-flows will be invested at the rate of return and the discount rate, respectively. Thus, IRR holds as a criterion only in the situation when excess amounts of identically productive timberland are available.

Gaffney, M. Mason. 1957.
Concepts of financial maturity of timber and other assets. Agric. Econ. Inf. Ser. 62. Raleigh: North Carolina State University. 105 pp.
The first definitive analysis of the various methods for gauging financial maturity of forest stands. The Faustmann derivation is compared with the Jevons/Allen/Fisher single rotation, Boulding's maximization of internal rate of return, and other variants. In mathematical terms, the Faustmann rotation is clearly shown as the superior to the other criteria which make special assumptions or have special intentions. The marginal conditions for optimality are set, and the basic comparative static results regarding price, costs, and tax effects are analyzed.

Goundry, G.K. 1960.
Forest management and the theory of capital. Canadian Journal of Political Economics 26(Aug.):439-451.
Compares maximum sustained yield (MSY), maximum discounted net revenue (present net value), maximum present value of the infinite stream (land expectation value), and maximum rate of growth of capital (internal rate of return (IRR)) criteria for rotation length determination. The expected ordering of optimum rotation lengths is reported with IRR shortest and MSY longest. The author assumes that capital is the fixed factor in production and as such, an owner would want to maximize the return to it rather than the return to the land. He claims that land is not the fixed factor because it is not managed intensively enough to justify that assumption. This logic was shown to be incorrect by Samuelson (1976). The article does include a good literature review.

Graham-Tomasi, Ted. 1983.
The comparative statics of the Faustmann model of forest management. Staff Paper P-83-23. Minneapolis: University of Minnesota Department of Agriculture and Applied Economics. 32 pp.
The author uses a model that differs from Chang (1983) in the manner in which regeneration costs are included (Chang more explicitly) and proves similar results. The model is then used to describe timber supply in cases of restricted (short run) or unrestricted (long run) entry. Changes in volume supplied in response to a demand shift depend on the original rotation age and may increase or decrease, depending on

the relation of the optimum rotation length to the maximum sustained-yield rotation. In addition, if the rate of change of price is positive, then rotation length increases but effort decreases.

Grainger, M.B. 1968.
Problems affecting the use of Faustmann's formula as a valuation tool. New Zealand Journal of Forestry 13:168-183.
The per-acre land values of the Faustmann equation may not be applicable in the situation of a large forest enterprise that is just beginning afforestation operations. Since the Faustmann formula gives a value nearly twice as large for a large tract of land than a financial accounting, the latter method is recommended for valuation with explicit coverage of costs. The major problem is that the Faustmann equation was developed for small ongoing operations and is not applicable to today's large forest enterprises which have more intricate cost and revenue formulations. Other problems stated are delayed planting for large operations, capitalization procedure, administrative costing, etc., but the value of the Faustmann calculation is recognized for comparative purposes.

Gregersen, Hans M. 1975.
Effect of inflation on evaluation of forestry investments. Journal of Forestry 73(9):570-573.
Elementary article explains how inflation does not affect real returns in an investment analysis. Problem for forestry investments is that real timber returns on investments are often compared with nominal alternative investments (i.e., savings deposits) and as a result show poorly.

Gregory, Gordon R. 1972.
Forest resource economics. New York: Ronald Press Co. 548 pp.
Chapter 14 presents a discussion of financial criteria for rotation age. Soil rent, forest rent, and financial maturity formulations are compared through graphical analysis. Management objectives of various landowner types are also compared.

Guttenberg, Sam. 1953.
Financial maturity versus soil-rent. Journal of Forestry 51(10):714.
Rebuts a paper by Worrell criticizing the financial maturity criterion. The primary benefit of the financial maturity criterion is ease of use in the field.

Haley, David. 1966.
The importance of land opportunity cost in the determination of financial rotations. Journal of Forestry 64(5):326-329.

Discusses the effect and characteristics of the land opportunity cost and its importance in the determination of the optimal rotation. If the function for the marginal value growth has a steep negative slope, the difference between the present net worth and the Faustmann rotation will be small. However, the flatter the slope, the greater the effect on the rotation.

Hall, Dale O. 1983.
Financial maturity for even-aged and all-aged stands. Forest Science 29(4):833-836.
Generalizes the Bentley-Teeguarden (1965) present net worth formulation to the case of an all-aged forest stand. In contrast to Chang (1983), the stand is viewed immediately after rather than prior to harvest. This procedure seems to work better than the use of yield tables for stand simulation.

Hardie, Ian W. 1984.
Comparative rents for farmland and timberland in a subregion of the South. Southern Journal of Agricultural Economics 16(4):45-53.
Uses the framework developed in Hardie and others (1984) to compare the potential rents of forest and agricultural lands. With 50-year projections of costs and prices, the expected dynamic equilibrium results with respect to the behavior of the optimal rotation age is achieved. For high-site land, forestry compares quite favorably with agriculture at all reasonable discount rates while on lower sites the discount rate assumes a larger role in determining the relative rents. Results call into question the validity of relying on forestry as only a residual land use.

Hardie, Ian W.; Daberkow, Julie N.; McConnell, Kenneth E. 1984.
A timber harvesting model with variable rotation lengths. Forest Science 30(2):511-523.
In order to include changing prices and costs, the Faustmann formula is modified into a dynamic optimization framework. Optimal rules are developed for rotation determination where an eventual steady-state condition is assumed. In comparisons of rotation lengths and present values when changing prices are forecast, ignoring the prices leads to rotations that are too short and to large revenue losses. Also, changing interest rates of the basic Faustmann formula to cover value changes will not work to get closer to optimal rotation lengths.

Hartman, R. 1976.
The harvesting decision when the standing forest has value. Economic Inquiry 14(1):52-58.

Broadens the general Faustmann model to the case where environmental benefits are included as a return in the valuation stream. In the one-period case, the harvest period is lengthened or even bypassed completely if the environmental benefits are high enough. In the Faustmann case, a similar result as in the one-period case is found. Thus, harvest age is probably increased from the original Faustmann solution and again potentially delayed infinitely. An important paper that has served as a source reference for numerous papers that have followed it.

Heaps, Terry. 1981.
The qualitative theory of optimal rotations. Canadian Journal of Economics 14(4):686-699.

Examines the effect of harvesting and other site quality costs on the optimal rotation. Results are similar to changes in regeneration costs in their effects. Thus, increased hauling costs lead to longer rotations; higher site leads to shorter rotations. Paper has a good comparative statics analysis of the general case.

Heaps, Terry. 1984.
The forestry maximum principle. Journal of Economic Dynamics and Control 7(2):131-151.

When the optimal rotation decision is completely generalized into a dynamic control framework, the Faustmann rotation is the steady-state solution of the more general case. The generalizing is accomplished by loosening of the Faustmann assumptions of constancy by (1) allowing the net stumpage value to vary with the age of the trees and with the harvest rate and (2) allowing an arbitrary initial age distribution of the site.

Heaps, Terry; Neher, Philip A. 1979.
The economics of forestry when the rate of harvest is constrained. Journal of Environmental Economics and Management 6(4):297-319.

Using control theory, broadens the Faustmann derivation to the situation where the standing timber cannot be harvested instantaneously and is cut over a transition period. When looking at a single rotation, the harvest goes as a bang-bang solution where harvesting is done at the maximum rate when the rate of change of value is less than or equal to the interest rate (r) and 0 when $>$r. With ongoing forest, assumes the first harvest period is the transition period, after which the manager attempts to manage as closely as possible to the Faustmann rule. When costs are a function of harvest level, it is not optimal to conduct the first harvest at a constant rate but rather to increase rotation length with time.

Hirshleifer, Jack. 1970.
Investment, interest and capital. Englewood Cliffs, NJ: Prentice-Hall. 620 pp.

In chapter 3, Hirshleifer looks at models of duration and replacement using the forestry paradigm. Maximization of the internal rate of return (IRR) leads to the unsatisfactory conclusion of either infinite or zero profits, depending on the viewpoint. The Fisher/Jevons (one period) model and the Faustmann model present no basic conflict because each presents a different specification of productive opportunities for the land manager. The Faustmann solution's optimal age must fall between the IRR and the one-period solution.

Hirshleifer, Jack. 1974.
Sustained yield versus capital theory. In: Dowdle, Barney, ed. Economics of sustained yield forestry. Seattle: University of Washington. 35 pp.

Discusses the economic consequences of sustained yield with non-declining flow constraints on harvesting. Since timber and timber investments are capital goods, they should be managed at a rate of return equal to the return of other capital investments in the economy; i.e., at the market interest rate. The use of allowable cut will therefore lead to economic fallacies in production decisions.

Hotelling, Harold. 1931.
The economics of exhaustible resources. Journal of Political Economy 39:137-175.

Discusses the optimal exploitation of nonrenewable resources, such as old-growth timber and derives the "golden rule" of resource exploitation.

Howe, Charles W. 1979.
Natural resource economics. New York: John Wiley & Sons. 350 pp.

Chapter 11 of this general resource economics text discusses forest management. The Faustmann model is developed and the effects of six factors on the optimal rotation age are discussed: (1) increased interest rate, (2) increased price, (3) increased planting costs, (4) severance tax, (5) annual property tax, (6) increased distance to the mill.

Hyde, William F. 1980.
Timber supply, land allocation and economic efficiency. Baltimore: John Hopkins University Press for Resources for the Future. 245 pp.

Chapters 3, 4, and appendix B present a strong analysis of the timber production decision and its effect on timber supply. This analysis is one of the first to present both first- and second-order conditions on optimality. It also gives a good historical synthesis of the development of forest economics from the German tradition to present scenarios. It examines rotation and supply effects that may occur as a result of rising prices and technological change.

Jackson, David H. 1980.
The microeconomics of the timber industry. Boulder,CO: Westview Press. 136 pp.
Chapter 2 presents the theory of the private timber firm. The comparative statics are performed using a format similar to Chang (1983), but an artificial logistic growth function is assumed in which planting density is seen as a partial substitute for time. Because of this substitution effect, the long-run effect of an interest rate change is to increase rotation age as producers shift to a lower yield function by planting less. The effect on firm supply of either an increase in price or interest rate is negative. If, however, planting density and rotation length are complements, a change in price leads to an increase in density, rotation length, and supply.

Johansson, Per-Olov. 1984.
Disequilibrium cutting rules. Canadian Journal of Forest Research 14(3):321-325.
Derives cost-benefit rules for examining whether unemployment and other market imbalances provide justification for national forest managers to depart from the Faustmann rotation rule. Because of the intertemporal nature of the forest production process, it is generally not socially optimal to increase production in a single period as a result of unemployment or excess demand.

Johansson, Per-Olov; Lofgren, Karl G. 1983.
Six different results on the properties of the timber supply function. pp. 169-181. In: Andersson, Åke E.; Olsson, Mats-Olov; Zackrisson, Uno, eds. Forest sector development: issues and analysis.Res. Rep. Sweden: University of Umeå: 1983:1. 195 pp.
Presents six basic propositions about the harvesting decision under uncertainty. A simple two-period model is developed to examine the effects of uncertainty with regards to price level, price variability, lump-sum taxes, and profit taxes.

Johansson, Per-Olov; Lofgren, Karl G. 1985.
The economics of forestry & natural resources. New York: Basil Blackwell Inc. 292 pp.
Chapters 4 and 5 discuss the basic optimal rotation period decision and extensions. A history of the formulation of the problem includes

Faustmann (1849) and Pressler and Ohlin (Lofgren 1983). Extensions of the problem include (1) taxes, (2) once and for all changes in biotechnology, and (3) regularly improving biotechnological change.

Kao, Chiang. 1982.
Optimal stocking levels and rotation under risk. Forest Science 28(4):711-719.
A probabilistic dynamic programming model is constructed to evaluate rotation and thinning under a criteria of maximized mean annual increment. Increasing variances of the growth prediction lead to shorter optimal rotations, lower stocking levels, and lower mean annual increments.

Kao, Chiang. 1984.
Optimal stocking levels and rotation under uncertainty. Forest Science 30(4):921-927.
Using a maximum sustained-yield criterion, the paper models the optimal stocking control when the growth function of a species is initially unavailable and the decisionmaker is risk neutral. As with risk (Kao 1982), uncertainty leads to lowered stocking levels and lowered mean annual increments.

Kemp, Murray C.; Long, Ngo V. 1983.
On the economics of forests. International Economic Review 24(1):113-131.
Using land and labor as inputs, discusses optimum rotation length in terms of a welfare maximizing central planner. In a number of cases where labor is redundant, the Faustmann rotation is asymptotically optimum. If land is redundant, the Jevons (one period) rotation is asymptotically optimum. In other cases, it may be optimal to diversify rotation lengths to keep both land and labor occupied.

Kemp, Murray C.; Moore, Elvin J. 1979.
Biological capital theory: a question and a conjecture. Economic Letters 4(2):141-144.
It is shown that for single-tree plots, Faustmann's linear utility function, or even a strictly convex function, the optimum rotation is a periodic constant. For a strictly concave function, however, the cutting times on the plots across the forest spread out, leading to a sustained-yield harvest scheme.

Klemperer, W. David. 1979.
Inflation and present value of timber income after taxes. Journal of Forestry 77(2):94-96.

Discrete analysis shows that inflation will not affect the real returns from timber investments with property or yield taxes. Investments with capital-gain taxes are affected by inflation, leading to an overestimate of real returns if inflation is ignored. Inflation bears more heavily on short-term investments because the purchase cost makes up a greater share of the basis.

Lofgren, Karl G. 1983.
The Faustmann-Ohlin theorem: a historical note. History of Political Economy 15(2):261-264.
Recalls how Bertil Ohlin, while still a student, independently derived the Faustmann formulation for optimal rotation determination in order to be admitted into a graduate seminar led by Eli Hecksher. Needless to say, Ohlin was admitted to the seminar.

Lofgren, Karl G. 1985.
Effect on the socially optimal rotation period in forestry of biotechnological improvements of the growth function. Forest Ecology and Management 10(2):233-249.
Analyzes how genetic progress affects the optimal rotation period and the value of the forest land. A one-time change in genetic quality is similar to a price change and leads to a shorter rotation and a higher land value. A regularly improving technology leads to an initially shorter rotation that increases over time. The final conclusion is that it is not the improved growth as such but the known possibilities of future improvements of the growth function which motivate a shortening of the optimal rotation period.

Lyon, Kenneth S. 1981.
Mining of the forest and the time path of the price of timber. Journal of Environmental Economics and Management 8(4):330-345.
With Hotelling's (1931) theory of mine depletion as a starting point, an optimization model is used to analyze the rate of growth of price in situations of zero and increasing transport cost for the cutting of old-growth timber. With zero costs, the standard result of price increase (p) equal to interest (r) holds only in the case of no growth or regeneration. If the old growth is growing, $p < r$. If regeneration costs are considered, $p > r$. For the model with transport costs, the first case has the same answer as the no-cost case only if it is the growth of net price (price - marginal cost) that is compared to r. However, with a continuously increasing cost function, the increase in $p < r$. Finally, at a stationary point, price will no longer grow and the Faustmann situation is optimal.

Markus, R. 1967.
Ostwald's relative forest rent theory. Munich, Germany: Neue Press.
Critiques the soil-rent theory for (1) considering only the soil, and not including the growing stock, as fixed and thus deserving of rents, and (2) considering only the single stand and not the effects on the whole forest. Develops Ostwald's relative forest-rent theory with a numeric example in a not altogether clear manner. The major point seems to be to include the cost term as a cost of harvesting rather than separating the two decisions. The result is that with revenues thus lowered, the rotation is lengthened.

Martell, D.L. 1980.
The optimal rotation of a flammable forest stand. Canadian Journal of Forest Research 10(1):30-34.
Describes a variation of the Faustmann model that includes a stochastic element for potential catastrophic losses (particularly fires). With dynamic programming, as the risk of fire increases, the optimal rotation age and soil expectation value (SEV) decrease. A method is given for gauging the efficiency of fire management activities that reduce the risk by estimating changes in the SEV.

McConnell, Kenneth E.; Daberkow, Julie N.; Hardie, Ian W. 1983.
Planning timber production with evolving prices and costs. Land Economics 59(3):292-299.
Draws conclusions for optimal rotation length under conditions of dynamic price (p) and cost (c) assumptions. With constant real price: (1) increasing c relative to p increases optimal harvest length, (2) decreasing c relative to p decreases optimal harvest length, and (3) uncertain results with constant c and p changing either way. Assumes constant technology in the presence of changing costs.

McConnell, Kenneth E.; Phipps, Tim. 1984.
Optimum forest rotation in an imperfect stumpage market: comment. Land Economics 60(2):215-217.
Comment on paper by Nautiyal and Fowler (1980). States that while competitive optimum is longer than the social optimum in an imperfect market, it is uncertain whether the monopolistic optimum is always shorter than the social optimum. It does hold for a linear demand curve where elasticity decreases with an increase in volume. It does not hold for constant elasticity demand functions for which the social and monopolistic rotation lengths are the same.

Medema, E. Lee; Lyon, Gary W. 1985.
The determination of financial rotation ages for coppicing tree species. Forest Science 31(2):398-404.
Presents a model for determining the optimal harvest period and number of harvests for coppicing tree species. The decision rule is a modified Faustmann solution in which the individual coppice rotations may decline over time. The solution for the number of cycles to manage the stand is found through an optimal search program.

Mitra, Tapan; Wan, Henry Y., Jr. 1985.
Some theoretical results on the economics of forestry. Review of Economic Studies 52(2):263-282.
The paper poses the following general question: Suppose the owner of a piece of land obtains utility in any time period, and this utility is determined by the timber content of trees harvested in that period. If the owner wishes to maximize the discounted sum of utilities from any initial forest, what pattern of planting and harvesting system should be used? Systematic examination using linear and concave utility functions shows the Faustmann formulation to be generally correct.

Mitra, Tapan; Wan, Henry Y., Jr. 1986.
On the Faustmann solution to the forest management problem. Journal of Economic Theory 40(2):229-249.
Examines optimal solutions to the forest management problem when future utilities are not discounted. The asymptotic properties of such solutions indicate: (1) if the utility function is linear, then the Faustmann solution is optimal, and (2) if the utility function is increasing and strictly concave, an optimal solution converges to the maximum sustained-yield solution.

Morgan, William F. 1974.
The theory of the firm and the behavior of woodland owners: a reappraisal based on empirical evidence of owner's behavior. Durham, NC: Duke University School of Forestry. 172 pp. Ph.D. Dissertation.
Possibly the first presentation giving the complete, and correct, comparative static derivation of the Faustmann model. The derived results provide positive tests of landowner behavior with regard to wealth maximization. The actual rotation ages of the major management types, derived from Forest Survey data, are consistent with the predicted results. Farm owners and industry have an apparent average rate of return of 5 to 5.5 percent. The Forest Service appears to be using no economic rationale in its management.

Murphy, Paul A.; Fortson, James C.; Bethune, James E. 1977.
Timber management decision making under imperfect capital markets. American Journal of Agricultural Economics 59(2):302-310.

Examines the effect of imperfect capital markets, using dynamic programming, on the criteria of maximization of liquidation value of the firm. In a perfect capital market, the globally optimal rotation is constant. Under imperfect capital markets, optimum rotations vary.

Näslund, Bertil. 1969.
Optimal rotation and thinning. Forest Science 15(4):446-451.

Discusses principles for solving the problem of determining optimal rotation length and other managerial inputs (specifically thinning) simultaneously. Pontryagin's maximum principle is used to find the general optimal conditions.

Nautiyal, Jagdish C. 1983.
Towards a method of uneven-age forest management based on the theory of financial maturity. Forest Science 29(1):47-59.

Advances the work of Duerr and others (1956) by including all relevant costs inherent in growing a forest stand. Rules are developed for harvesting mature trees and for thinning using both economic and biological (in the form of potential growth form) criteria. Effects on the ultimate rotation length of quality variations that influence price are clearly discussed.

Nautiyal, Jagdish C.; Fowler, Kenneth S. 1980.
Optimum forest rotation in an imperfect stumpage market. Land Economics 56(2):213-226.

Develops a Faustmann type model where the landowner has a monopolistic position in the market such as the state in western Canada. For the owner, maximization of profit rather than present net value should be the objective. This change leads to a shorter rotation than the atomistic solution. Societal optimum lies between these two points. In a regulated forest, however, the differences between the two ownership types diminish and the question may not have much relevance.

Newman, David H.; Gilbert, Charles E.; Hyde, William F. 1985.
The optimal forest rotation with evolving prices. Land Economics 61(4):357-394.

Using a dynamic optimization model, adapted from McConnell and others (1983), examines the case of increasing stumpage prices and their effects on rotation lengths. With constant costs and exponentially rising prices growing at a rate less than the interest rate, the initial rotation length is longer than if the static Faustmann rotation, using constant prices, had been chosen. Later rotation lengths are found to decrease over time to a steady state as the relative cost effect decreases. A dynamic optimization model is presented that reflects these theoretical results.

Nguyen, D. 1979.
Environmental services and the optimum rotation problem in forest management. Journal of Environmental Management 8(2):127-136.

Builds on the Hartman (1976) model, with a modified selection harvesting system so that the current total benefits from a stand never drop to zero but rather to some minimum level. The optimal rotation lengthens in the presence of environmental services but generally not to infinity as the optimum harvest time must lie on the rising part of the growth curve.

Nostrom, Carl J. 1975.
A stochastic model for the growth period decision in forestry. Swedish Journal of Economics 77(3):329-337.

For a one-period harvest with price fluctuations as the only form of uncertainty, the paper develops a model for examining the rotation decision. The optimal stochastic policy leads to a higher present value than if uncertainty is ignored. It is unclear from reading the article but it appears the increase comes about from shorter rotations in the stochastic case.

Pearse, Peter H. 1967.
The optimum forestry rotation. Forestry Chronicle 43(June):178-195.

Uses a classical derivation of the Faustmann equation based on the Gaffney (1957) study to show that the rotation giving the highest land rent is the optimum. Land rent is defined as the annualized value of the infinite stream of like rotations. Since land is the fixed factor in the forestry enterprise, its return must be maximized. At the optimum, the change in marginal stumpage value must equal the annual opportunity cost plus the interest on the growing stock at the optimum. Other issues examined include: (1) fixed annual costs, (2) precommercial and commercial thinning, and (3) property and severance taxes.

Reed, William J. 1984.
The effects of the risk of fire on the optimal rotation of a forest. Journal of Environmental Economics and Management 11(2):180-190.

Develops a Faustmann type model where fires occur with a Poisson distribution. With a zero salvage value for the remaining stand, the op-

timal rule is the same as the Faustmann rule with the interest rate increased by the average probability of fire occurrence. Thus, rotations are shortened. Also examined are cases with partial salvage and where the probability of fire is not homogeneous through time.

Reed, William J. 1986.
Optimal harvesting models in forest management - a survey. Natural Resource Modeling 1(1):55-79.
An extensive survey of the literature of optimal timber harvesting models for even-aged and uneven-aged stands, single-stand models, and forest-level scheduling models.

Routledge, R.D. 1980.
The effect of potential catastrophic mortality and other unpredictable events on optimal forest rotation policy. Forest Science 28(3):389-399.
Modifies the Faustmann optimal condition to include the probability of a catastrophic fire and the degree of salvage after the fire. If salvage value is low, the optimal rotation will decrease as the probability of fire increases. With a high salvage value, the optimal rotation lengthens. Ignoring risks can lead to large over-estimates of the land expectation value of the site.

Samuelson, Paul. 1976.
Economics of forestry in an evolving society. Economic Inquiry 14(Dec.):466-492.
Probably the most frequently cited article on forest valuation. The internal rate of return, one period, and maximum sustained-yield formulations are dismissed as possible criteria for optimal rotation length determination. Maximizing soil rent leads to the optimum rotation, and this value can be included in the present net value calculation in a one-period model. This wide-ranging article looks at the economic rationale of a number of forestry concerns such as sustained yield and the bogey of compound interest. An excellent annotated bibliography is provided.

Strang, William J. 1983.
On the optimal forest harvesting decision. Economic Inquiry 21(Oct.):576-583.
Extends the Hartman (1976) analysis of optimal rotations where the standing timber itself provides increasing value over time. While the first-order conditions give a rotation length longer than the standard one-period or Faustmann solution, the resulting present value must be compared with the corner solution value of no harvest at all. This step determines whether a global maximum exists. For a mature standing

forest, if the forest is less than the optimal rotation age, it should be cut when it reaches that age. If, however, it is older than the optimal age, it may be optimal to never harvest the stand.

Tait, David E.N. 1987.
The good fairy problem: one more look at the optimum rotation age for a forest stand. Forestry Chronicle 63(4):260-263.
Questions the differences between economic and physical optimal rotations through the metaphor of the good fairy giving an owner the choice of initial forest conditions. With the use of a heuristic proof, the paper shows the difficulty of reconciling a sustained-yield forest with stands managed for optimal economic rotation lengths.

Thomson, Roy B. 1942.
An examination of basic principles of forest valuation. Bull.6. Durham, NC: Duke University School of Forestry. 99 pp.
An interesting historical account of the German, British, and American precedents for forest valuation comparing gross-yield, soil-rent, and forest-rent doctrines. Some of the economic analysis is flawed, but the paper's main value is chapter 2, which deals with the historical development of forest valuation.

Walter, G.R. 1980.
Financial maturity of forests and sustainable yield concept. Economic Inquiry 18(2):327-331.
Applies the Faustmann equation to a situation where rotation lengths are not constant over time and the forest enterprise may eventually stop. The optimal rule in this case is the old Faustmann rule with explicit attention given to the lengthening of the rotation.

Wan, Henry Y., Jr. 1978.
A generalized Wicksellian capital model: an application to forestry. pp.141-153. In: Smith, V.L., ed. Economics of natural and environmental resources. New York: Gordon and Breach.
Using Wicksell's wine-aging model, the author derives the one-period optimal condition and a variation of the Faustmann rule. The effect of a negatively sloped demand curve on rotation is examined with relaxed competitive price assumptions. The discounted marginal revenue over current cost must equal the imputed rent, and the optimal rotation length may change over time.

Worrell, Albert C. 1953.
Financial maturity: a questionable concept in forest management. Journal of Forestry 51:711-714.

Discusses the problems of using the financial maturity methodology of Duerr and others (1956) for optimizing the rotation length. Use of only the alternative rate of return ignores the opportunity cost of the land and gives rotation lengths that are too long. The combination of the optimal rotation and capital will be such that the highest annual economic rent is received.

Zivnuska, John A. 1949.
Some aspects of the economic theory of forestry. Land Economics 25(2):165-172.

Thoughtful paper on the economic structure of forestry. The long production process, the single identity of the product and the production process, and the presence of different cost trees (virgin vs. second growth), lead to market stumpage prices that may have little to do with the costs of production. Results are pronounced short-term variations in supply in response to market price, while the physical quantity of standing timber changes slowly over the long run. Long lags in production lead to problems in the maintenance of equilibrium because extremely long reaction times are required to meet changes in demand.

Newman, David H.

1988. The optimal forest rotation: a discussion and annotated bibliography. Gen. Tech. Rep. SE-48. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 47 pp.

The literature contains six different criteria of the optimal forest rotation: (1) maximum single-rotation physical yield, (2) maximum single-rotation annual yield, (3) maximum single-rotation discounted net revenues, (4) maximum discounted net revenues from an infinite series of rotations, (5) maximum annual net revenues, and (6) maximum internal rate of return. First-order conditions for maximization show the rotation effects of the criteria. Various authors have extended basic models and discussed effects of externalities, imperfect markets, changing parameters, and taxes.

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The Forest Service, U.S. Department of Agriculture, is dedicated to the principle of multiple use management of the Nation's forest resources for sustained yields of wood, water, forage, wildlife, and recreation. Through forestry research, cooperation with the States and private forest owners, and management of the National Forests and National Grasslands, it strives—as directed by Congress—to provide increasingly greater service to a growing Nation.