CAN LONGLEAF PINE PLANTATIONS BE MODELED BY CALIBRATING MIXED-EFFECTS MODELS OF OTHER SPECIES?

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ABSTRACT

A growth-and-yield model system applicable for the relatively greater growth rates of more recently planted longleaf pine (*Pinus palustris* Mill.) is needed. The present analysis uses mixed-effects models of other pines fit using measurements across a more complete range of stand development stages to predict the future development of contemporary longleaf pine plantations. This approach could be useful when there are insufficient measurement ages across a rotation for newer silvicultural treatments. Testing data were obtained of container-planted longleaf pine in central Louisiana. Mixed-effects models both published and newly fit were used for testing purposes. Calibration ages of 6, 7, 8, 9, and 11 were helpful in "molding" trajectories of other species to the characteristics of these longleaf plots at age 20. However, to obtain accurate predictions at ages beyond 20 may require calibration measurements of longleaf pine at older, but still mid-rotation ages. It appears that there are likely "optimum" ages to conduct an inventory such that models of other species can be calibrated to produce accurate predictions at ages 25, 30, 35, etc.

INTRODUCTION

A complete growth-and-yield model system for the relatively greater growth rates of more recently planted longleaf pine (*Pinus palustris* Mill.) silvicultural systems is needed, particularly on cutover sites. These newer silvicultural systems often plant containerized seedlings, conduct herbaceous weed control treatments, and are established using relatively intensive site preparation, etc. This mixed-effects modeling approach asks if absolute age can be modified such that it is a relative measure rather than an absolute measure, and if so, perhaps the biological growth patterns can be applied across a range of species.

Thus, when entering data into traditional growth-and-yield models (for example, a stand table), the simulator can be considered to be "calibrated" to site-specific conditions. However, mixed-effects models could be advantageous because calibration can include more than one temporal observation that may allow for a better calibration and ultimately prediction of the future basal area trajectory.

VanderSchaaf and others (2020) showed a variety of mixed-effects basal area species models following calibration produced reasonable estimates of stand development for other species. The thought process was perhaps mixed-effects models could be calibrated using younger data from a stand of interest to produce reasonable estimates at older, more common rotation ages. Could you in a sense "stretch", "pull", etc., through calibration of a mixed-effects model, a growth trajectory of a model developed using data with measurements across stand development stages of another species to produce reasonable projections for a trajectory of the species of interest? As shown in figure 1, can we take observed data from a longleaf pine stand and use it, through the calibration process, to "mold" a loblolly pine model (or any species or combination of species) such that we obtain a more appropriate trajectory through time for our (e.g., longleaf) stand of interest? Alternatively, we can think of it as can we "borrow" information from a loblolly pine model to better regulate how young data "mold" the stand of interest's trajectory through time.

Basal area is thought to be advantageous as the variable of calibration, as opposed to volume, to ultimately produce volume of the species of interest because basal area measurements are not dependent on other relationships such as individual tree volume equations. The objective of this analysis is to use mixed-effects models of other species to predict the future development of more recently established longleaf pine plantations. It also examines if using more than one measurement to calibrate the models increases predictive ability.

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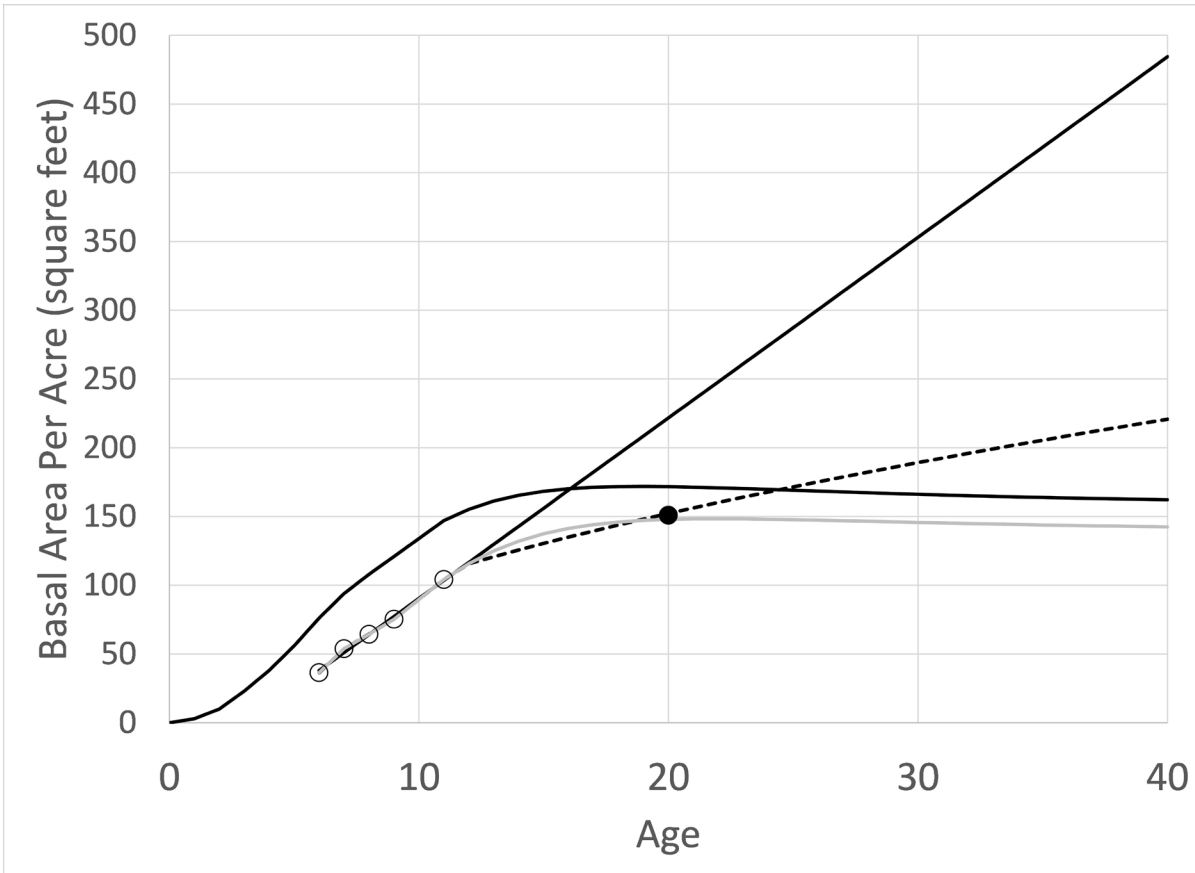

Figure 1—Black curve is a loblolly pine basal area trajectory (in this example from VPI). The white circles are observed basal areas (used in calibration) and the black circle is an observed basal area from block 4 used to test this modeling approach. The black line is a linear regression line fit using observations from ages 6 to 11. The dashed line, following calibration, are future projections by age obtained by always using observed data at age 11 as the previous basal area. The gray curve, following calibration, depicts basal area at age 12 using observed data from age 11, predicted basal area at age 13 using predicted basal area at age 12, and then subsequent annual predictions of basal area using the previous predicted annual basal area.

METHODS

Two linear mixed-effects equations presented in VanderSchaaf and others (2020) were used in this analysis. One fit using loblolly pine (*Pinus taeda* L.) from east Texas:

$$lnBA_j = (2.6179+u_{0i})+(-0.1782)ln(Age_{j-1}/Age_j)+(0.4583+u_{2i})lnBA_{j-1} \quad (1)$$

where

ln is natural logarithm; BA is basal area (square feet per acre); the variances of u_{0i} and u_{2i}, are 0.1505, and 0.006589, respectively, there is also a covariance of -0.02998, and the constant random error variance is 0.02071; *i* indexes a specific stand, and *j* indexes the current and previous measurements. The dataset used to fit these equations is further referred to as ETX.

And the same equation form fit using a combined dataset of loblolly pine from east Texas and central Mississippi, ponderosa pine (*Pinus ponderosa* Lawson and C. Lawson) from the inland Northwest, and red pine (*Pinus resinosa* Aiton) from Ontario:

$$lnBA_j = (2.1755+u_{0i})+(-0.4686)ln(Age_{j-1}/Age_j)+(0.5508+u_{2i})lnBA_{j-1} \quad (2)$$

where

ln is natural logarithm; BA is basal area (square feet per acre); the variances of u_{0i} and u_{2i}, are 0.2563, and 0.01236, respectively, there is also a covariance of -0.05446, and the constant random error variance is 0.01518; *i* indexes a specific stand, and *j* indexes the current and previous measurements. The dataset used to fit these equations is further referred to as ALL.

Data Used in Both Model Testing and Model Development

Study Site Description

The study site is on two soil complexes on the Kisatchie National Forest (KNF) within the humid, temperate, Coastal Plain and flatwoods province of the West Gulf Region of the Southeastern United States. The site is on uplands suitable for restoring longleaf pine forests. Site preparation consisted of a hot burn prior to planting.

Treatment Establishment

Research plots were established in a randomized complete block design of four blocks with three treatments (three pine species) each installed in the fall of 1997. Each of the 12 research plots measured 96 by 96 feet and contained 16 rows of 16 seedlings arranged in 6- by 6-feet spacing. Blocking was by soil complex (two blocks on each complex) and topographic location within each complex. The three southern pines studied, loblolly, longleaf, and slash pine (*Pinus elliottii* Engelm. var. *elliottii*), were randomly assigned to different plots within each block.

Seeds for all three species were supplied by the Stuart Seed Orchard, KNF, Louisiana, and were open-pollinated native Louisiana parent trees. Seedlings were grown in containers by Forest Service, U.S. Department of Agriculture personnel in Pineville, LA. Seedlings, 1-0 container stock, were planted in March 1998.

Hexazinone was banded over the rows of planted pine seedlings in April 1998 on all 12 plots. A triclopyr tank-mix was applied as a directed foliar spray to hardwood trees and shrubs in April 1998 and June 1999.

More information can be found in Haywood and others (2015). Table 1 provides summary statistics by species and age. This dataset will further be referred to as LSLL, excluding age-20 data of longleaf pine. The age-20 data of longleaf pine is used for model testing purposes. The longleaf pine data from ages 6 to 11 exclusively will further be referred to as LL.

Table 1—Summary statistics of data used in both model testing and fitting (longleaf pine) and model fitting (slash and loblolly pine), (*n* = 4 per species)

Slash pine	Quadratic mean diameter				Basal area per acre				Total height			
	inches				*square feet per acre*				*feet*			
Age	Mean	SD	Min.	Max.	Mean	SD	Min.	Max.	Mean	SD	Min.	Max.
6	4.0	0.46	3.6	4.4	84	16.46	70	103	26.7	1.85	24.9	28.5
7	4.4	0.37	4.1	4.8	103	14.00	91	120	28.7	1.99	27.0	30.8
8	4.7	0.41	4.4	5.2	117	15.73	104	136	31.3	2.11	29.4	33.2
9	5.0	0.41	4.6	5.4	129	15.73	116	149	36.6	2.46	34.0	38.8
11	5.4	0.41	5.1	6.0	150	12.87	141	168	42.3	2.90	39.4	45.4
20	6.6	0.56	6.0	7.3	199	11.49	191	216	57.5	4.99	53.2	63.1
Longleaf pine	**Quadratic mean diameter**				**Basal area per acre**				**Total height**			
	inches				*square feet per acre*				*feet*			
Age	Mean	SD	Min.	Max.	Mean	SD	Min.	Max.	Mean	SD	Min.	Max.
6	3.1	0.26	2.8	3.4	42	6.41	36	51	18.5	0.56	17.9	19.1
7	3.7	0.25	3.4	4.0	59	7.65	53	69	21.8	0.43	21.4	22.3
8	4.1	0.29	3.7	4.4	71	8.99	64	83	25.0	0.65	24.2	25.7
9	4.4	0.31	4.0	4.8	84	9.12	75	95	30.1	0.98	29.1	31.2
11	5.1	0.32	4.7	5.5	112	11.55	101	126	36.3	1.22	34.8	37.5
20	6.7	0.42	6.3	7.3	169	16.92	151	191	58.3	2.89	55.3	62.2
Loblolly pine	**Quadratic mean diameter**				**Basal area per acre**				**Total height**			
	inches				*square feet per acre*				*feet*			
Age	Mean	SD	Min.	Max.	Mean	SD	Min.	Max.	Mean	SD	Min.	Max.
6	3.9	0.43	3.4	4.3	76	13.59	60	92	25.7	1.70	23.3	27.2
7	4.3	0.36	3.9	4.7	94	12.43	80	110	27.4	1.80	25.0	29.2
8	4.7	0.43	4.2	5.2	108	14.10	92	126	30.1	2.22	27.2	32.0
9	4.9	0.45	4.5	5.5	121	14.76	104	140	35.5	2.42	32.3	37.6
11	5.5	0.47	5.0	6.1	147	13.71	131	164	40.7	2.92	37.1	43.4
20	6.7	0.74	6.0	7.5	190	22.64	161	211	56.8	7.05	49.2	63.2

SD=standard deviation; Min=minimum; Max=maximum.

Validation Analyses

Prediction errors, following transformation back to the original units, were compared between equations (equations 1 and 2) using the validation process proposed by Arabatzis and Burkhart (1992). The difference between the predicted and observed basal area per acre (e_i = PBA_i - BA_i, respectively, for each individual plot [i]) was calculated for all equations. The mean residual ($\bar{e}$) and the sample variance (v) of residuals were computed and considered to be estimates of bias and precision, respectively. An estimate of mean square error (MSE) was obtained combining the bias and precision measures using the following formula:

$$MSE=\bar{e}^2 + v \qquad (3)$$

To account for logarithmic transformation bias, the procedure recommended by Baskerville (1972) was used. All validation statistics presented in this paper are based on untransformed errors.

Additional Models

After preliminary analyses, it was thought to test mixed-effects models using other, more complete, and available datasets in terms of measurements across a range of stand development stages on the same site. Hence, data from a long-term loblolly pine planting density study (VanderSchaaf and Burkhart 2012) was used to fit the same model form (equation 1) described above using the model fitting approach and criteria utilized by VanderSchaaf and others (2020). Quadratic mean diameter and trees per acre were measured annually between ages 5 and 21 on one of the Coastal Plain sites and to age 22 on the other site. On the Piedmont sites, measurement ages end at 18 at one location and 21 at the other. At the latter Piedmont site, one replication had measurements to 22 years of age. This dataset will be further referred to as VPI. Parameter estimates of equation 1 for this additional model are shown in table 2.

Additionally, datasets of two datasets were created and used to fit equation 1. The VPI dataset was combined with ETX, further referred to as ETXVPI, and the loblolly pine, ponderosa pine, and red pine dataset (ALL), further referred to as ALLVPI. Furthermore, LL was combined with these datasets. It was combined with the VPI dataset (further referred to as LLVPI), the ETX dataset (further referred to as LLETX) and the ALL dataset (further referred to as LLALL). The reason for combining the younger LL data with these

Table 2—Parameter estimates and standard errors (in parentheses) of equation (1) by model-fitting dataset for mixed-effects models

Dataset	b_0	b_1	b_2	Var	b_0Var	b_2Var	Covar (b_0,b_2)	neg 2LL	AIC	n
LLALL	2.14191	-0.44908	0.55833	0.01589	0.57486	0.02670	-0.12230	-688.3	-674.3	202
	(0.11228)	(0.08794)	(0.02241)	(0.00087)	(0.10517)	(0.00494)	(0.02276)			
ALLVPI	1.49807	-0.44110	0.69682	0.00500	0.40034	0.01874	-0.08587	-7650.9	-7636.9	390
	(0.04514)	(0.05018)	(0.00908)	(0.00013)	(0.03839)	(0.00179)	(0.00825)			
LLALLVPI	1.53334	-0.42332	0.68983	0.00519	0.53497	0.02470	-0.11421	-7463.3	-7449.3	394
	(0.04927)	(0.05041)	(0.00998)	(0.00013)	(0.04941)	(0.00230)	(0.01060)			
LLETX	2.77348	-0.05602	0.43115	0.02210	0.36516	0.01559	-0.07399	-191.1	-177.1	97
	(0.12458)	(0.13020)	(0.02267)	(0.00171)	(0.06847)	(0.00341)	(0.01507)			
ETXVPI	1.63685	-0.24032	0.67124	0.00442	0.37698	0.01780	-0.08142	-7189.8	-7175.8	285
	(0.04740)	(0.05335)	(0.00949)	(0.00012)	(0.03502)	(0.00168)	(0.00763)			
LLETXVPI	1.68533	-0.22630	0.66132	0.00467	0.49457	0.02307	-0.10633	-6981.4	-6967.4	289
	(0.05157)	(0.05401)	(0.01038)	(0.00013)	(0.04501)	(0.00216)	(0.00981)			
LLVPI	1.14065	-0.74988	0.77134	0.00258	0.29338	0.00997	-0.05405	-8184.2	-8170.2	194
	(0.05763)	(0.09654)	(0.01016)	(0.00007)	(0.01253)	(0.00030)	(0.00198)			
VPI, b_0	1.20082	-0.42958	0.76229	0.00261	0.00121	–	–	-8611.3	-8601.3	191
	(0.02767)	(0.06862)	(0.00461)	(0.00007)	(0.00015)	–	–			
VPI, b_2	1.15535	-0.53679	0.76970	0.00276	–	0.00004	–	-8496.7	-8486.7	191
	(0.02639)	(0.06697)	(0.00443)	(0.00008)	–	(0.00001)	–			

b_0Var and b_2Var are estimates of the random effects variance components when applicable, Covar (b_0,b_2) is the covariance estimates between b_0 and b_2 when applicable, neg 2LL is -2 negative log-likelihood, and AIC is Akaike Information Criterion. For the VPI dataset models, since convergence criterion was not obtained when including both b0 and b2 as random effects, there is no covariance between the random effects of b0 and b2; – = lack of parameter estimates; LL = longleaf pine age 11 and younger in Central Louisiana; VPI = loblolly pine in the Atlantic Coastal Plain and Piedmont; ETX = loblolly pine in east Texas; ALL = ETX plus loblolly pine in Mississippi, ponderosa pine in the inland Northwest, and red pine in Ontario.

For both model fitting criterion, more negative numbers are superior. n is the number of clusters (or plots) used in model fitting.

Table 3—Parameter estimates and standard errors (in parentheses) of equation (1) by model-fitting dataset for fixed-effects models

Dataset	b_0	b_1	b_2	Var	neg 2LL	AIC	n
LSLL	0.91222	-0.43080	0.82224	0.00185	-193.5	-185.5	56
	(0.07966)	(0.04290)	(0.01854)	(0.00035)			
LL	0.74564	-2.15966	0.79986	0.00084	-67.8	-59.8	16
	(0.10592)	(0.24302)	(0.02670)	(0.00030)			

neg 2LL is -2 negative log-likelihood; AIC is Akaike Information Criterion; LL = longleaf pine age 11 and younger in Central Louisiana; LSLL = all loblolly and slash pine ages, longleaf pine age 11 and younger in Central Louisiana.

For both model fitting criterion, more negative numbers are superior. *n* is the number of individual basal area observations (not plots) used in model fitting.

other datasets is to determine if by combining observed younger data with more biologically-complete datasets using a mixed-effects modeling approach, can we then achieve better predictions beyond the observed younger data of the species of interest (longleaf in this case)? When modeling newer silvicultural treatments of the species of interest, it may be best to use models fit of other species that were also intensively managed. Here, we were not necessarily able to do that per se, although the VPI dataset was relatively intensive at the time of study establishment.

Finally, datasets of three datasets were created and used to fit equation 1. ETXVPI was combined with LL, further referred to as LLETXVPI and ALLVPI was combined with LL, further referred to as LLALLVPI. Datasets were combined to help determine if predictive ability is improved following calibration for the species of interest when there is more variability in the model fitting dataset (e.g., if there is more variability in the random effects, is that beneficial to produce better calibrated projections). Validation analyses were conducted for these models in a similar fashion as described above.

Additionally, for comparative purposes to the calibrated mixed-effects models, a fixed-effects model using only ages 6 to 11 of the longleaf pine data (LL) was fit. Additionally LSLL, which is loblolly and slash pine data ages 6, 7, 8, 9, 11, and 20 and the longleaf data from ages 6, 7, 8, 9, and 11 (excluding age 20), was used to fit a fixed-effects model. Parameter estimates of equation 1 for this additional model are shown in table 3.

RESULTS

Model Fitting

For most datasets, models with two random parameters (both a fixed and mixed component) achieved convergence criteria for equation 1 (table 2). There is some variability among the fixed-effects parameter estimates (b_0, b_1, and b_2) among the datasets. For all datasets the random-effect covariance was negative. For the VPI dataset, model fitting criteria could only be met when using one random parameter. The model with b_0 random had better model fitting statistics but often predictive ability of an independent dataset doesn't necessarily agree with model fitting ability. Hence, both model forms were reported and tested.

Entirely fixed-effects models of equation 1 (table 3) had fairly different parameter estimates than mixed-effects models presented in table 2. Differences are likely due to the young nature of the data (table 1). Additionally, models presented in table 3 are technically not biologically correct because they ignore the behavior of individual stand trajectories when estimating parameters, often one of the advantages of mixed-effects models is their ability to account for individual stand behavior when estimating the population average (PA) trend, or the fixed-effect component of the model.

Model Testing

Most mixed-effects models, whether calibrated or not, underpredicted observed basal areas at age 20 (table 4). The best performing models in terms of both bias (*e*) and MSE was the age 11-only calibration and the PA of the VPI, Random b_2 model and the LLVPI model when calibrating using only ages 7, 8, and 9. However, based on the summation of MSE across all calibration age sets, the LLVPI model behaved the best followed by the VPI, Random b_2 model. Excluding the age 11-only calibrations, additional observations used in calibration substantially improved predictive ability for most models, the exception being the LLVPI model when age 11 was also used in calibration along with ages 7, 8, and 9. However, excluding the age 11-only calibrations, in many cases the PA model or uncalibrated model for a particular dataset performed the best. Models fit using some amount of the VPI dataset generally performed the best while models containing the ETX dataset generally performed the poorest (fig. 2).

In many cases the use of only age 11 in calibration improved predictive ability. The LL Fixed model greatly overpredicted stand development because the mid-rotation inflection point of the basal area trajectory was not yet observed (figs. 1 and 2).

Table 4—Model validation results following calibration of equation (1) by dataset and ages used in calibration, n = 4

	VPI, Random b0			VPI, Random b2			ALL			ETX		
Calibration ages	e	v	MSE	e	v	MSE	e	v	MSE	e	v	MSE
PA	-12.09	60.15	206.2	-3.44	55.88	67.7	-11.10	92.56	215.7	-35.09	133.31	1364.4
7	-53.03	60.13	2872.4	-40.70	49.50	1706.1	-46.58	65.42	2235.1	-72.69	97.67	5380.9
7, 8	-45.54	39.90	2114.0	-33.59	33.72	1162.2	-44.89	44.06	2059.4	-69.53	75.38	4909.9
7, 8, 9	-37.35	46.13	1441.0	-26.38	40.40	736.4	-39.96	49.86	1646.8	-63.82	77.69	4150.7
7, 8, 9, 11	-15.81	60.33	310.3	-7.03	58.33	107.8	-23.09	61.93	595.1	-47.11	78.19	2297.9
11	-9.22	63.12	148.2	-1.37	59.40	61.3	-10.13	68.67	171.3	-33.13	92.97	1190.4

	LLALL			ALLVPI			LLALLVPI			LLVPI		
Calibration ages	e	v	MSE	e	v	MSE	e	v	MSE	e	v	MSE
PA	-12.59	92.08	250.7	-12.67	68.29	228.8	-13.96	70.04	264.8	17.71	51.16	364.8
7	-49.14	64.77	2479.3	-52.69	66.78	2842.6	-54.58	67.96	3047.4	-14.96	73.04	297.0
7, 8	-45.49	43.69	2112.9	-46.71	39.05	2221.1	-48.27	40.04	2369.6	-11.54	40.81	174.0
7, 8, 9	-39.35	51.56	1599.8	-38.95	45.61	1562.8	-40.26	46.76	1667.5	-5.43	37.27	66.8
7, 8, 9, 11	-19.80	63.29	455.4	-14.17	52.39	253.3	-15.02	51.96	277.6	11.48	62.04	193.9
11	-11.45	70.14	201.2	-8.57	64.84	138.4	-10.20	64.71	168.7	18.99	77.83	438.5

	LLETX			ETXVPI			LLETXVPI		
Calibration ages	e	v	MSE	e	v	MSE	e	v	MSE
PA	-40.97	145.20	1824.0	-27.86	82.58	858.6	-28.78	84.98	913.5
7	-82.94	103.96	6983.3	-71.93	81.43	5255.7	-73.18	82.58	5438.5
7, 8	-78.09	83.96	6181.6	-64.18	54.98	4173.5	-65.21	56.00	4308.0
7, 8, 9	-71.01	88.32	5130.6	-55.09	62.28	3097.0	-55.94	63.32	3192.2
7, 8, 9, 11	-51.47	89.83	2738.9	-27.95	58.83	840.0	-28.59	58.87	876.5
11	-39.78	103.72	1686.1	-23.71	67.66	629.7	-25.08	69.26	698.4

	LL Fixed		
	e	v	MSE
Fixed	164.90	198.39	27388.9

	LSLL Fixed		
	e	v	MSE
Fixed	-12.91	55.25	222.0

PA =population average prediction for a model-fitting dataset where no calibration of the mixed-effects model occurred; e = residual, or the difference between the predicted and observed basal area; v = variance of residuals; MSE = mean square error (see equation 3); LL = longleaf pine age 11 and younger in Central Louisiana; LSLL = all loblolly and slash pine ages, longleaf pine age 11 and younger in Central Louisiana; VPI = loblolly pine in the Atlantic Coastal Plain and Piedmont; ETX = loblolly pine in East Texas; ALL = ETX plus loblolly pine in Mississippi, ponderosa pine in the inland Northwest, and red pine in Ontario.

For the mixed-effect model, calibration ages are those ages used in calibration. For age 7, the previous measurement age and basal area was age 6.

For the LLVPI model when calibrated using ages 7, 8, and 9, the VPI, Random b_2 PA model, and the VPI, Random b_2 when calibrated using ages 7, 8, 9, and 11, and when calibrated using just age 11 data, absolute percent errors of 3.50 percent, 1.83 percent, 4.42 percent, and 3.81 percent, respectively were produced.

DISCUSSION

The VPI dataset likely performed best since the data consists of annual measurements by stand (plot) trajectory to common rotation ages which likely captures the entire biological behavior of a stand basal area trajectory. Plus, models fit using this dataset likely behaved well because of relatively greater early management activities and planting genetic stock (Amateis and others 1988) as compared to the ETX (Lenhart and others 1985) and ALL datasets. These observed longleaf pine trajectories are relatively productive at age 20 for this species (table 1) (e.g., Cram and others 2010, Haywood 2015). Perhaps for less productive longleaf pine sites in this region the ETX dataset may perform better. Since loblolly has been shown to be relatively more productive than longleaf when previous generation management activities were used (numerous studies), at least early in the rotations, it was thought that the ETX dataset would be more appropriate for more recently established longleaf pine sites, since more recently established longleaf pine sites were thought to be more productive relative to longleaf plantations established using previous generation management activities. However, as stated earlier, these longleaf plots at age 20 are highly productive for this species and apparently more productive on average than previous generation loblolly from the Western Gulf. The observed basal areas at age 20 were 163, 191, 171, and 151 square feet per acre.

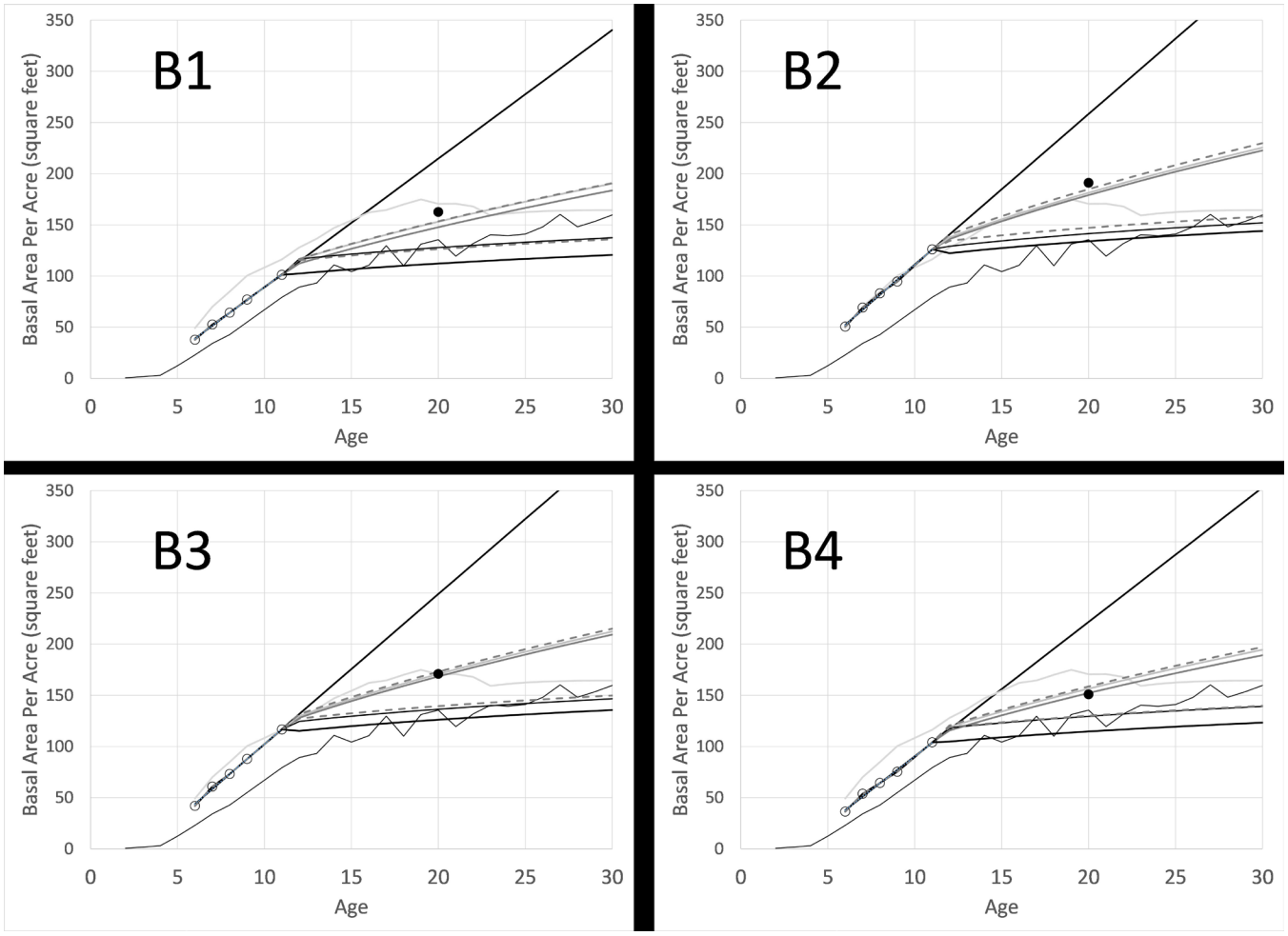

Figure 2—Observed and predicted longleaf pine basal area per acre by block (B). The gray and black unsmooth curves are the average basal areas by age from the VPI and ETX datasets, respectively—these curves ignore individual stand (or plot) behavior and merely are included to provide some indication of average predicted behavior across all stands. The black line is a linear regression using only ages 6 to 11 in model fitting, the upper and lower dashed lines are the VPI, Random b_2 and ETX models calibrated using age 11-only observations, respectively. Bold gray and black curves are the VPI, Random b_2 and ETX models, calibrated using ages 7 to 11, respectively. Light gray and black curves are the population average trends for the VPI, Random b2 and ETX models, respectively. The white circles are observed basal areas and the black circles are observed basal areas at age 20. Curves extend to age 20, and beyond, by using age 11 observed basal area per acre as the regressor value in the equations (e.g., equation 1).

Following calibration, the PA behavior of the mixed-effects models added an inflection point to the predicted longleaf pine trajectories after age 11 basically remedying the substantial overprediction that would have occurred at age 20 if the general trend from ages 6 to 11 (LL) had been maintained (fig. 2). Hence, the calibration data and the PA behavior of the mixed-effect models are working in unison to produce a more biologically correct basal area trajectory for ages beyond those used in calibration. We are in a sense "borrowing" information from other species to produce a more likely basal area trajectory through time of our longleaf pine plantations.

In many studies where longleaf was established with loblolly and slash pine, the basal area trajectories of longleaf eventually caught up to those other species and in some cases exceeded production of those other species. Table 1 shows longleaf basal area per acre is starting to approach those of loblolly and slash, having on average, the greatest basal area periodic increments from ages 9 to 11 and then 11 to 20. Most likely each species is going to have some range of calibration ages that is going to result in the calibration process producing relatively accurate future basal area trajectory estimates. For instance, perhaps the inclusion of age 15 (or by itself) into the calibration ages used for this dataset would produce better predictions for longleaf, but perhaps for ponderosa pine the "optimum" age for calibration may be around 25 or 30 since in general ponderosa pine has relatively longer biological and traditionally managed (e.g., for timber production) economic rotation ages.

How do we as managers optimize our resources in terms of time, costs, etc., to conduct inventories that will result in our ability to best predict future stand development? In order to get accurate results at ages 35 or 40, if desired, does spending money to conduct an inventory at age 10 help, or is

an inventory at age 20 sufficient, yet, you have to wait those additional 10 years to produce sufficient long-term estimates of stand development. This additional waiting time can be problematic if one is trying to project research studies such as silvicultural options and genetic selection to financially and biologically mature ages. Additionally, these "optimum" calibration ages may vary by site quality and management activity by species. Perhaps a longleaf pine dataset of relatively lower productivity at age 20 than that used here would have younger, or older, "optimum" ages.

For the VPI dataset models, generally the observed longleaf data were less than the average basal area per acre behavior across time, while for the ETX dataset the observed data were greater (fig. 2). Most likely, for the VPI datasets, the younger data resulted in underprediction at age 20 because it pulled the VPI PA trend down, while for the ETX datasets (and likely the ALL datasets), the average behavior (and/or model parameter estimates) were not flexible enough to allow the observed longleaf data to "pull it up." In many cases the use of only age 11 in calibration improved predictive ability because of less underprediction (table 4, fig. 2). Perhaps for longleaf pine when calibrating these models using more observations, of younger ages, calibration can be problematic because of the observed data's ability to pull the calibrated trajectory down.

Clearly, calibration is in a sense a "give-and-take", or a "power struggle," between the observed data, which is being used to calibrate, and the PA trend of the mixed-effects models. What is often referred to as "shrinkage" of the cluster-specific or plot/stand-specific behavior to the PA trend. The ability of the observed data to "mold" or "alter" the path of a PA trend likely greatly depends on the variability of the random effects, in addition to the calibration sample size. Even though a PA curve may be grossly incorrect for a plot of a particular species, if the random effects have large variances they will likely be flexible enough to be adjusted to the behavior of the calibration (or species of interest) species through the calibration process, particularly if the calibration sample size is large.

For this model form (e.g., equation 1), whether the oldest calibration age and basal area or, alternatively, predicted ages and basal areas, are used as the previous prediction age and basal area can influence projections (fig. 1). Based on these results, as utilized in figure 2, it appears best to always use the oldest calibration age and basal area (the last observed age and basal area) as the previous age and basal area (in this particular case the observed basal areas at age 11). This is in opposition to using predicted basal areas to predict future basal areas.

Results here are promising but calibration among species may not be a complete panacea and without issues. It appears different trajectory shapes among species, and to some extent stands in general, reduces predictive ability. Further study needs to be conducted using ages of 15 and 20 in calibration and study needs to be conducted of other longleaf plantations. These longleaf pine plots were relatively productive for this species.

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