CAN A MIXED-EFFECTS BASAL AREA EQUATION FIT USING ONE SPECIES PRODUCE ACCURATE ESTIMATES FOLLOWING CALIBRATION USING DATA OF ANOTHER SPECIES?

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Abstract—Stand-level basal area mixed-effects models are commonly being developed for species around the World. We wanted to determine if a stand-level basal area mixed-effects model fit using one species (for example, loblolly pine (*Pinus taeda* L.) plantations where biological and economical rotation ages can range from 20 to 40 years) could produce accurate estimates following calibration using data from another species. This approach would be useful in those situations where there is limited data available for a species (for example, naturally regenerated black spruce (*Picea mariana* (Mill.) Briton, Sterns & Poggenb.) where biological and economical rotation ages can range from 75 to 150 years) to model basal area across the entire range of biological or economical rotation ages. Perhaps a mixed-effects model fit using data of a species where repeated measurements have been made across the entire biological or economical rotation age could be calibrated to produce accurate predictions for those species where data is limited at, say, mid-rotation or final rotation ages. When entering data into traditional growth-and-yield models, in a sense the model is "calibrated." However, mixed-effects could be advantageous because calibration can include more than one temporal observation that may allow for a better prediction of the basal area trajectory. Additionally, this approach could be useful where there are insufficient measurement ages across a rotation for newer silvicultural treatments.

INTRODUCTION

Growth-and-yield models are integral components of forest management. Models help managers identify the productive capability of a particular species and how different cultural treatments will likely affect financial returns. Mixed-effects models are becoming popular modeling tools to provide more site-specific predictions of stand development. Mixed-effects models provide an efficient means to obtain cluster-specific, or for this particular example, stand-specific, parameters through the prediction of cluster-specific random effects.
For example, basal area can be predicted using the following equations:

$$\ln BA_j = \beta_0+\beta_1 \ln(Age_{j-1}/Age_j)+\beta_2 \ln BA_{j-1} + \varepsilon \quad (1)$$

$$\ln BA_j = \beta_0+\beta_1 \ln(Age_{j-1})+\beta_2 \ln(Age_j)+\beta_3 \ln BA_{j-1} + \varepsilon \quad (2)$$

where

ln is natural logarithm; BA is basal area (square feet per acre); β_0, β_1, β_2, and β_3 are parameters to be estimated; and ε is random error where it is assumed $\varepsilon \sim N(0, \sigma^2 I)$.

Equations (1) and (2) provide what is often termed a population average estimate of BA given ages and the previous BA. The parameters β_0, β_1, β_2, and β_3 are assumed to be fixed, or that the parameter estimates apply to every experimental unit (for example, stand) in a population. Whether a stand is located in Florida or Arkansas, the parameter estimates are assumed to be correct. However, stand-specific characteristics such as soil type, nutrient status, competition from herbaceous vegetation, elevation, aspect, genetic stock, etc., may result in the parameters differing across stands. Thus, specific stands may have what are generally termed "random parameters" in mixed-effects model terminology. Equations (1) and (2) can be altered by adding stand-specific random effects to the population average parameters to produce stand-specific parameters:

$$\ln BA_j = (\beta_0+u_{0i})+(\beta_1+u_{1i})\ln(Age_{j-1}/Age_j)+ (\beta_2+u_{2i})\ln BA_{j-1} + \varepsilon \quad (3)$$

$$\ln BA_j = (\beta_0+u_{0i})+(\beta_1+u_{1i})\ln(Age_{j-1})+ (\beta_2+u_{2i})\ln(Age_j)+(\beta_3+u_{3i})\ln BA_{j-1} + \varepsilon \quad (4)$$

where u_{0i}, u_{1i}, u_{2i}, and u_{3i} are stand-specific random effects, assumed to be $N(0, \sigma^2_0)$, $N(0, \sigma^2_1)$, $N(0,\sigma^2_2)$,

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and $N(0, \sigma^2_3)$, respectively; (β_0+u_{0i}) is the stand-specific intercept; (β_1+u_{1i}), (β_2+u_{2i}), and (β_3+u_{3i}) are stand-specific slopes; *i* indexes a specific stand; and all other variables as previously defined.

Additionally, a covariance can be assumed to exist between any of the random effects. Although the parameter estimation efficiency of mixed-effects models is an advantage, often the greatest advantage is the ability to calibrate the model using data independent of those used in model fitting.

Although several basal area mixed-effects models have been developed, no study has examined whether parameters fit using data from one species can be used to obtain cluster-specific random effects of another species. The advantage of using parameters from a species where more complete data in terms of stand ages, planting densities, site qualities, stand development, etc., exist, is that more reasonable extrapolations of stand development for another species may potentially be obtained. For instance, can a calibration using young data of a black spruce (*Picea mariana* (Mill.) Briton, Sterns & Poggenb.) trajectory "borrow" information from a loblolly pine (*Pinus taeda* L.) trajectory fit using data from across the entire biological rotation age to produce adequate estimates of a black spruce trajectory at older ages? Another advantage (given land tenure issues) is that most industrial timberland owners and managers need short-term solutions to long-term operations, as they are only interested in planning horizons of 5 to 10 years (not 20 to 50 years, as in the past). Hence, they do not want to invest large amounts of capital into long-term growth-and-yield studies.

Cluster-specific (or stand or plot) random parameters depend on the amount of estimated variability in the random effects and the population average parameter estimates. Thus, differences in site requirements, structural constraints, growth habits, etc., among species may not allow for mixed-effects models fit using one species to produce reasonable predictions of stand development for another species. This may occur when the population average parameter estimates of the model-fitting species are not correct for the model calibration species, and/or the variability in the cluster-specific random effects of the model-fitting species is not representative of the variability in the model calibration species.

Loblolly pine is one of the most commercially important species in the Southeastern United States and therefore several long-term studies given economical and biological rotation ages have been initiated. However, for many species, fewer and sometimes even no studies have been established that have measurements across the entire range of biological or economical rotation ages. Hence, this concept asks if absolute age can be modified such that it is a relative measure rather than an absolute measure, and if so, perhaps the biological growth patterns can be applied across a range of species. As shown in figure 1, can a basal area growth trajectory of loblolly pine be "stretched" out, out and up, out and down, up, down, etc., such that it will provide reasonable predictions for another species?

VanderSchaaf (2008) showed that using a loblolly pine plantation mixed-effects height-diameter (H-D) model when calibrated using sweetgum (*Liquidambar styraciflua* L.) data produced reasonable results to predict height of sweetgum plantations. A similar concept was proposed by Huang (2016) in which he uses the term "universal" model to describe calibrating models across species. According to Huang (2016), if you can calibrate a single mixed-effects model across species, then all species can be considered to be in the same population. He found that "composite" H-D equations, or equations fit using several species, following calibration, often produced similar predictions to species-specific mixed-effects H-D models. This "universal" approach is similar in nature to Zeide's (1978) two-point principle. He suggested for several stand-level variables that growth curves common across many species could be tailored for an individual stand by obtaining measurements of that variable in that stand at only two ages.

Thus, when entering data into traditional growth-and-yield models (for example, a stand table), the simulator can be considered to be "calibrated" to site-specific conditions. However, mixed-effects models could be advantageous because calibration can include more than one temporal observation that may allow for a better calibration and ultimately prediction of the future basal area trajectory. Additionally, this approach could be useful where there are insufficient measurement ages across a rotation for newer silvicultural treatments.

The objective of this study was to determine if basal area trajectories fit using different species could be calibrated for another species using younger data to produce reasonable predictions at older ages. Additionally, this study attempted to quantify if the use of different ages in calibration impacted predictive ability at older ages.

METHODS

Data Used in Model Fitting and Prediction

Loblolly pine–East Texas Pine Plantation Research Project (ETPPRP—A total of 178 plots were originally established in 1984 throughout East Texas at planting densities ranging from 350 to 1350 seedlings per acre (Lenhart and others 1985). Plantations represented by these plots ranged in age from 2 to 35 years, 30 to 850

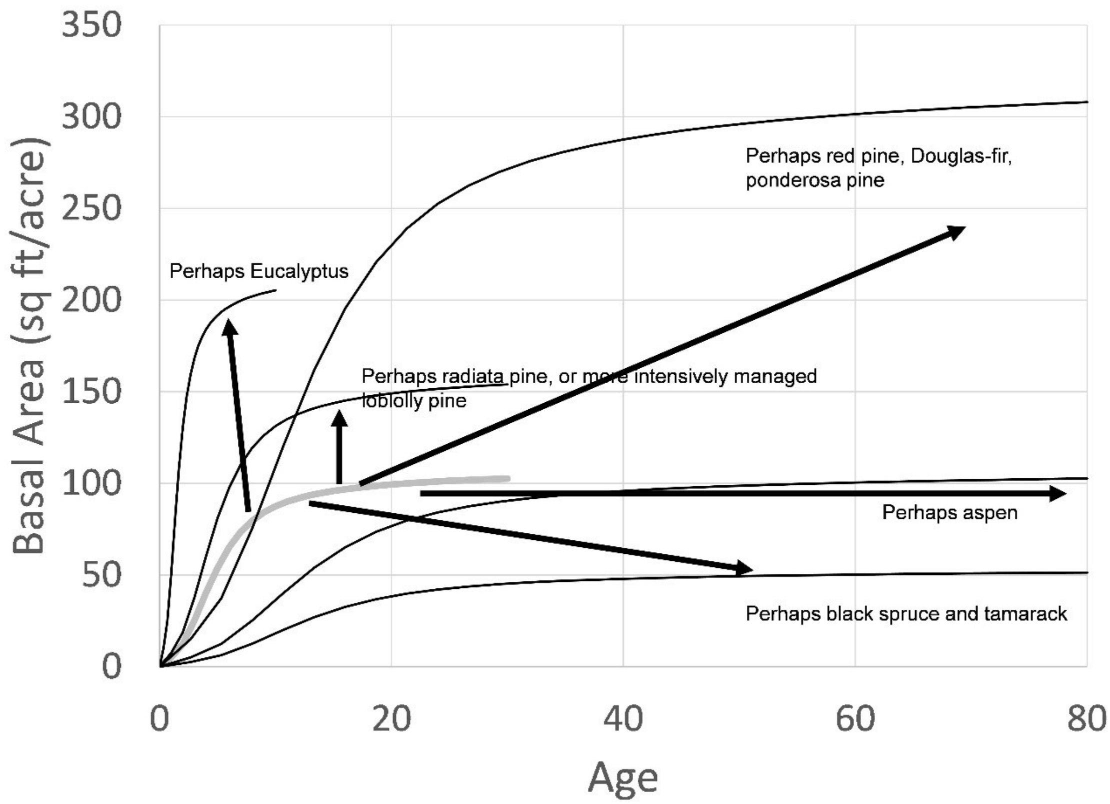

Figure 1—Thick gray curve is a loblolly pine basal area trajectory. Can we, through mixed-effects model calibration using younger ages of another species, "stretch" the loblolly pine trajectory to produce viable estimates of trajectories for other species that have limited data at older ages? Or in some cases, can we "stretch" the trajectory for newer silvicultural practices of loblolly pine plantations? Possible trajectories for aspen (*Populus* spp.), black spruce, Douglas-fir, eucalyptus (*Eucalyptus* spp.), more intensively managed loblolly pine, ponderosa pine, radiata pine (*Pinus radiata* D. Don), red pine, and tamarack (*Larix laricina* (Du Roi) K. Koch) are shown.

trees per acre, and 40 to 90 feet site index (base age 25 years). Measurement ages used during this study ranged from 2 years to 33 years.

Loblolly pine–Mississippi State University (MSU)—Tree- and plot-level measurements were obtained from a 1959 spacing trial located in an old-field on a floodplain in Winston County, MS, using 1-0 local seed source seedlings (VanderSchaaf 2010). Site index was estimated to be near 75 feet (base age 25 years). Three initial densities were established: a 5 feet by 5 feet spacing, a 7 feet by 8 feet spacing, and a 9 feet by 10 feet spacing. For the 7-foot by 8-foot spacing, one of the reps was not used for this study because some trees were removed due to the presence of beetles near the age of 20 years. Thus, a total of 11 experimental units were used (4 replications x 3 planting densities minus the one missing replication mentioned above) for this study. Measurements were conducted at ages 8, 10, 12, 14, 15, 17, 19, 20, 23, 25, 28, and 30 years.

Ponderosa pine–Intermountain Forest Tree Nutrition Cooperative (IFTNC)—In 1985, the IFTNC established 10 nitrogen (N) fertilization trials in stands dominated by ponderosa pine (*Pinus ponderosa* Lawson & C. Lawson) (IFTNC 1998). The trials were located throughout northeastern Oregon and central Washington. In 1987, IFTNC members established six additional ponderosa pine trials in Montana to study the effects of N and potassium fertilization on tree growth and survival. Measurement ages ranged from 32 to 110 years.

Red pine–Ontario—A spacing trial was planted in 1953 on abandoned fields near Chalk River, Ontario (Penner and others 2001). The study site was of high productivity for red pine (*Pinus resinosa* Aiton) with a site index of 80 feet at base age 50 years. Bareroot seedlings were machine-planted in 1953 at 4-foot by 4-foot, 5-foot by 5-foot, 6-foot by 6-foot, 7-foot by 7-foot, 8-foot by 8-foot, 10-foot by 10-foot, and 14-foot by 14-foot spacings. In addition, in 1965 part of the 10-foot by 10-foot spacing area was thinned to a spacing of 20-feet by 20-feet and it was assumed that future growth represented an initial planting configuration of 20-feet by 20-feet. For the plots used in this analysis, no thinnings were conducted. Measurements were conducted at ages 10, 15, 20, 25, 30, 35, 40, and 45 years.

Table 1 provides summary statistics by study.

Model Development and Parameter Estimation

Parameters were estimated using Proc NLMIXED of SAS (SAS Institute Inc., Cary, NC) and the Newton-Raphson algorithm (NEWRAP in NLMIXED). Random errors were assumed to be normally distributed. Full versus reduced model tests were conducted to determine if β_0, β_1, β_2, and β_3 should be considered random and if covariances existed among them.

A linear model form was selected to achieve statistical convergence of a common model form, which will help ensure when comparing predictive ability among different model-fitting datasets that the same model form will be compared. When using nonlinear models sometimes consistent convergence of the same model form/structure is more difficult when using different species in the model-fitting dataset. Hence, using a linear model form should help obtain a more consistent model form/structure across model-fitting datasets that will ensure the ability to determine if the ability, or lack of ability, to calibrate across species is due to what species are used in the model-fitting dataset or due to the number of random-effects in the model (for example, model form/structure). Additionally, it will help us better identify what stand-level variables may be useful in the calibration process. This information may be useful when nonlinear model forms are tested.

Initial Testing

The initial approach to modeling and calibrating was to fit models using the ETPPRP loblolly pine data. Results of calibrating the model for the ponderosa pine and red pine datasets were not as good as desired. Hence, the approach was modified such that all four (loblolly

Table 1—Summary statistics of data used in model fitting and model validation

		Age				TPA				BA			
Dataset	*n*	Mean	Min	Max	SD	Mean	Min	Max	SD	Mean	Min	Max	SD
LP ETPPRP	568	14	2	33	6	465	30	858	144	91	1	215	52
PP IFTNC	392	74	32	110	21	183	55	370	65	133	62	316	46
RP	8	10	-	-	-	1001	193	2300	724	22	4	39	13
		15	-	-	-	877	107	2280	739	74	17	122	39
		20	-	-	-	871	103	2277	739	135	48	192	52
		25	-	-	-	814	100	1858	632	170	78	218	52
		30	-	-	-	794	100	1722	599	192	96	235	52
		35	-	-	-	773	99	1663	570	213	118	253	48
		40	-	-	-	715	99	1334	481	231	131	283	52
		45	-	-	-	684	98	1218	443	244	144	287	48
LP MSU	11	8	-	-	-	911	356	1632	511	85	46	122	26
		10	-	-	-	875	354	1572	471	119	79	158	27
		12	-	-	-	825	346	1482	423	146	112	181	24
		14	-	-	-	783	340	1404	380	167	132	197	23
		15	-	-	-	756	338	1316	351	175	143	203	20
		17	-	-	-	694	332	1164	292	187	157	211	17
		19	-	-	-	647	332	1052	248	195	170	212	13
		20	-	-	-	618	328	984	224	194	174	210	11
		23	-	-	-	520	310	758	147	195	181	204	8
		25	-	-	-	484	302	676	124	199	183	211	9
		28	-	-	-	417	280	542	91	207	185	219	9
		30	-	-	-	369	238	480	73	197	174	220	15

LP ETPPRP = loblolly pine in east Texas, PP IFTNC = ponderosa pine at Intermountain Forest Tree Nutrition Cooperative, RP = red pine, and LP MSU = loblolly pine at Mississippi State University.

LP ETPPRP and PP IFTNC data were obtained from several plots that were remeasured, and thus *n* equals the number of remeasurements across all plots, while RP was obtained from a published figure showing trajectories through time for 8 planting densities, and hence *n* = 8, and LP MSU had a total of 11 plots that were remeasured at the ages reported. Min = minimum; Max = maximum; SD = standard deviation; TPA = trees per acre; and BA = basal area, square feet per acre.

pine–ETPPRP, loblolly pine–MSU, ponderosa pine–IFTNC, and red pine) datasets were combined into one model-fitting dataset and equations (1) and (2) were refit – further referred to as the ALL dataset. Including species into the model-fitting dataset that have longer rotation lengths and maximum carrying capacities will provide more flexibility when trying to calibrate the model for species that have longer rotations and greater maximum carrying capacities relative to loblolly pine.

Initially, it was desired that for both equations (1) and (2) that all parameters would be considered random. However, in most cases convergence was not obtained. When model fitting, the ETPPRP dataset was considered the standard to ensure fair comparison between the mixed-effects model developed using the loblolly pine ETPPRP dataset and the ALL model as well as the mixed-effects models developed using other species. For example, the mixed-effects models fit using the ALL dataset were forced to have the same form/structure as the best model form/structure fit when using only the ETPPRP dataset in model fitting.

Based on likelihood ratio tests and Akaike's Information Criterion (AIC), for equation (1) a model where both β_0 and β_2 were considered to vary across plots and the presence of a covariance term, was found to be best (table 2). For equation (2) a model where only β_0 was considered random was best (table 3).

In many cases random effects account for nearly all autocorrelation among observations when using longitudinal datasets; however, a modeler can also directly model the random error structure. When estimating parameters in a mixed-effects model framework using data from one species and calibrating for another, the measurement intervals may not be the same among the datasets. This can cause problems when trying to estimate covariances of the random errors because a covariance structure that is appropriate for the model-fitting dataset may not be appropriate for the model calibration dataset. For this particular study, since measurement intervals differed among the datasets, the random error covariance-variance matrix was assumed to be $\sigma^2 I$.

Table 2—Parameter estimates and standard errors (in parentheses) of equation (1) by model-fitting dataset for both fixed-effects only and mixed-effects models

Dataset	Model	b_0	b_1	b_2	Var	b_0Var	b_2Var	Covar (b_0,b_2)	neg 2LL	AIC	*n*
LP ETPPRP	Fixed	2.1919	-0.7865	0.5276	0.04722	-	-	-	-100.0	-92.0	465
		(0.0856)	(0.1233)	(0.0144)	(0.00310)	-	-	-			
	Mixed	2.6179	-0.1782	0.4583	0.02071	0.1505	0.006589	-0.02998	-265.1	-251.1	95
		(0.1038)	(0.1218)	(0.0187)	(0.00164)	(0.0352)	(0.001717)	(0.00766)			
PP IFTNC	Fixed	0.1857	-1.0149	0.9672	0.002098	-	-	-	-1009.0	-1001.0	303
		(0.0396)	(0.0894)	(0.0082)	(0.000171)	-	-	-			
	Mixed	-	-	-	-	-	-	-	-	-	-
		-	-	-	-	-	-	-			
RP	Fixed	1.0346	-2.3417	0.7540	0.006669	-	-	-	-119.5	-111.5	55
		(0.1608)	(0.2377)	(0.0242)	(0.001273)	-	-	-			
	Mixed	-	-	-	-	-	-	-	-	-	-
		-	-	-	-	-	-	-			
ALL	Fixed	1.9127	-0.9456	0.6007	0.04152	-	-	-	-326.8	-318.8	951
		(0.0568)	(0.0747)	(0.0104)	(0.00191)	-	-	-			
	Mixed	2.1755	-0.4686	0.5508	0.01518	0.2563	0.01236	-0.05446	-771.0	-757.0	200
		(0.0984)	(0.0858)	(0.0194)	(0.00084)	(0.0615)	(0.00293)	(0.01344)			

b_0Var and b_2Var are estimates of the random effects variance components when applicable, Covar (b_0,b_2) is the covariance estimates between b_0 and b_2 when applicable, neg 2LL is -2 negative log-likelihood, and AIC is Akaike Information Criterion.

LP ETPPRP = loblolly pine in east Texas, PP IFTNC = ponderosa pine at Intermountain Forest Tree Nutrition Cooperative, RP = red pine, and ALL = combined data.

For both model fitting criterion, more negative numbers are superior. For the fixed-effects models *n* is the number of observations (plots and remeasurements of plots) used in model fitting while for the mixed-effects models, *n* is the number of clusters (or plots) used in model fitting.

Prediction errors, following transformation back to the original units, were compared between equations (equations (1) and (2)) and model-fitting datasets (ETPPRP, ALL, red pine, and IFTNC) using the validation process proposed by Arabatzis and Burkhart (1992). The difference between the observed and predicted basal area per acre ($e_{ij} = BA_{ij} - PBA_{ij}$, respectively, for each individual plot (*i*) and age (*j*)) was calculated for all equation and model-fitting dataset combinations. The mean residual ($\bar{e}$) and the sample variance (v) of residuals were computed and considered to be estimates of bias and precision, respectively. An estimate of mean square error (MSE) was obtained combining the bias and precision measures using the following formula:

$$MSE = \bar{e}^2 + v \quad (5)$$

Values of MSE were compared between the equation and model-fitting dataset combinations to determine which one was most appropriate for a particular prediction dataset. To account for logarithmic transformation bias, the procedure recommended by Baskerville (1972) was used. All validation statistics presented in this paper are based on untransformed errors.

RESULTS AND DISCUSSION

In general, equation (2) had slightly better model fit statistics (tables 2 and 3). In almost all cases, parameters were significant at the $p < 0.05$ level. For the mixed-effects model of a particular equation, attempts were made to have all parameters random, but convergence did not occur or the random effects variance components were not significant. This suggests that in most cases for both equations (1) and (2) having all parameters random is not necessary. When convergence for only one or two random parameters was met for a particular equation, the parameter(s) selected to be random was determined based upon the log-likelihood fit statistics for the loblolly pine (ETPPRP) mixed-effect model for that equation. The ETPPRP loblolly pine model was considered the standard because the initial analysis was to see if a loblolly pine model could be calibrated for other species. The same mixed-effects equation fit using a different dataset used the same set of random parameters such that predictive ability is not confounded with the number of random parameters. For a particular equation then (either equation (1) or (2)), when using different datasets, the ability to calibrate mixed-effects models is only due to differences in the model-fitting dataset and not the number of random parameters.

Table 3—Parameter estimates and standard errors (in parentheses) of equation (2) by model-fitting dataset for both fixed-effects only and mixed-effects models

Dataset	Model	b_0	b_1	b_2	b_3	Var	b_0Var	neg 2LL	AIC	*n*
LP ETPPRP	Fixed	2.4690	-0.6540	0.5469	0.5347	0.04665	-	-105.7	-95.7	465
		(0.1440)	(0.1346)	(0.0165)	(0.1618)	(0.00306)	-			
	Mixed	2.5331	-0.2047	0.4052	0.3147	0.02750	0.02655	-188.1	-176.1	95
		(0.1209)	(0.1170)	(0.0193)	(0.1316)	(0.00209)	(0.00536)			
PP IFTNC	Fixed	0.1505	-1.0609	0.9650	1.0711	0.002089	-	-1010.0	-999.9	303
		(0.0506)	(0.0984)	(0.0085)	(0.1026)	(0.000170)	-			
	Mixed	0.1757	-1.0739	0.9584	1.0855	0.001641	0.000447	-1026.0	-1014.0	84
		(0.0603)	(0.0894)	(0.0107)	(0.0925)	(0.000157)	(0.000147)			
RP	Fixed	-2.8154	-5.9350	0.7824	6.8172	0.002838	-	-166.5	-156.5	55
		(0.4608)	(0.4467)	(0.0162)	(0.5442)	(0.000545)	-			
	Mixed	-	-	-	-	-	-	-	-	-
		-	-	-	-	-	-			
ALL	Fixed	1.7025	-1.1775	0.6067	1.2220	0.04106	-	-337.5	-327.5	951
		(0.0853)	(0.1024)	(0.0105)	(0.1121)	(0.00188)	-			
	Mixed	2.4124	-0.4430	0.4556	0.5061	0.02016	0.03038	-604.4	-592.4	200
		(0.0796)	(0.0902)	(0.0117)	(0.0971)	(0.00108)	(0.00397)			

b_0Var = estimates of the random effects variance components when applicable, neg2LL = -2 negative log-likelihood, and AIC = Akaike Information Criterion.

For both model fitting criteria, more negative numbers are superior.

For the fixed-effects models n is the number of observations (plots and remeasurements of plots) used in model fitting while for the mixed-effects models, n is the number of clusters (or plots) used in model fitting.

LP ETPPRP = loblolly pine in east Texas, PP IFTNC = ponderosa pine at Intermountain Forest Tree Nutrition Cooperative, RP = red pine, and ALL = combined data.

Model Prediction/Validation Results

Red pine—For red pine, following calibration, equation (2) produced better predictions relative to equation (1) when only loblolly pine (ETPPRP) was included in the model-fitting dataset, but when including all datasets (ALL), equation (1) generally produced better predictions (tables 4 and 5). Overall, equation (2) performed slightly better. Obviously, by comparing PA (population average) – Not calibrated to Mixed-Calibrated, calibrating the mixed-effects models greatly improved prediction.

After examining initial prediction results, it was desired to see whether a fixed-effects model using initial measurements of the desired model fitting calibration species (red pine) would produce better predictions than calibrating a mixed-effects model fit using other species when using the same initial measurements (table 5) – this is Fixed – RP. Two sets of ages were used in model fitting, either ages 15 to 30 or ages 15 to 35 (table 6). Results showed that using initial measurement ages of red pine to fit a fixed-effects model of the same species produced better results than calibrating a mixed-effects model fit using other species. Future research should concentrate on looking at predictions when using a greater difference in model fitting/calibration ages and predicted ages (e.g. calibrating using ages 15 to 35, but predicting at ages 75 or 80 rather than predicting at ages of 40 or 45). The ability to "borrow" biological relationships (for example, inflection point, carrying capacity or the asymptote, etc.) from other species when calibrating mixed-effects models may prove more useful for longer projections.

Following calibration, unlike the loblolly pine (ETPPRP) and ALL models, the ponderosa pine (IFTNC) model overpredicted future red pine basal area (table 7). The model developed using ponderosa pine (IFTNC) showed poorer results relative to the loblolly pine (ETPPRP) and the ALL datasets (table 7). It was thought that since ponderosa pine and red pine generally have longer rotations than loblolly pine that predictions may be better. However, an issue with the IFTNC dataset is that the range in measurement ages for a particular site is only 12 years at most. Perhaps a model fit using ponderosa pine data that has a greater range in measurement ages at the same plot/site would produce better results following calibration for red pine.

Table 4—Model validation results following calibration of equation (1) using the red pine dataset

Dataset	Fixed–red pine			PA–not calibrated			Mixed–calibrated			
PAge	*e*	*v*	MSE	*e*	*v*	MSE	Calibration ages	*e*	*v*	MSE
ETPPRP										
40	-12.5	174.2	330.0	-67.8	1062.4	5656.2	15, 20	-47.5	495.2	2748.5
							15, 20, 25	-38.9	334.0	1850.0
							15, 20, 25, 30	-32.3	208.3	1249.6
							15, 20, 25, 30, 35	-25.9	154.0	823.0
45	-18.7	136.3	487.5	-73.5	982.8	6386.9	15, 20	-53.0	461.0	3264.8
							15, 20, 25	-43.9	283.5	2209.2
							15, 20, 25, 30	-36.8	158.7	1510.1
							15, 20, 25, 30, 35	-29.9	111.6	1005.0
ALL										
40	-12.5	174.2	330.0	-52.1	734.3	3451.3	15, 20	-31.6	436.7	1432.4
							15, 20, 25	-26.8	291.4	1009.5
							15, 20, 25, 30	-22.8	164.2	685.0
							15, 20, 25, 30, 35	-17.7	122.5	437.0
45	-18.7	136.3	487.5	-56.8	662.3	3888.3	15, 20	-36.4	388.7	1712.4
							15, 20, 25	-31.2	228.0	1202.9
							15, 20, 25, 30	-27.0	108.3	834.9
							15, 20, 25, 30, 35	-21.5	79.0	539.9

The fixed–red pine model used all red pine measurement ages when model fitting (ages 15 to 45) and hence is considered the optimum predictive situation.

PA–not calibrated is the "population average" prediction for a model-fitting dataset where no calibration of the mixed-effects model occurred.

For each predicted age (PAge) and calibration age combination, $n = 8$.

e = residual, or the difference between the observed and predicted basal area and v =variance of residuals

Table 5—Model validation results following calibration of equation (2) using the red pine dataset

Dataset	Fixed–red pine			PA–not calibrated				Mixed–calibrated			Fixed–RP		
PAge	*e*	*v*	MSE	*e*	*v*	MSE	Calibration ages	*e*	*v*	MSE	*e*	*v*	MSE
ETPPRP													
40	-4.6	122.1	143.1	-60.5	1148	4814.1	15, 20	-34.6	255.9	1451.4	-	-	-
							15, 20, 25	-30.4	199.1	1120.8	-	-	-
							15, 20, 25, 30	-26.8	156.2	873.9	-23.0	210.5	737.6
							15, 20, 25, 30, 35	-23.0	133.2	661.4	-4.8	134.9	158.2
45	1.2	55.4	56.9	-64.5	1056.3	5221.9	15, 20	-37.7	280.4	1698.6	-	-	-
							15, 20, 25	-33.2	209.2	1314.7	-	-	-
							15, 20, 25, 30	-29.5	164.6	1035.8	-24.8	152.2	766.3
							15, 20, 25, 30, 35	-25.5	141.3	793.5	1.0	65.1	66.1
ALL													
40	-4.6	122.1	143.1	-59.7	1010.1	4572.2	15, 20	-31.2	205.7	1179.7	-	-	-
							15, 20, 25	-28.6	157.7	978.0	-	-	-
							15, 20, 25, 30	-25.9	123.4	791.8	-23.0	210.5	737.6
							15, 20, 25, 30, 35	-22.3	106.5	603.3	-4.8	134.9	158.2
45	1.2	55.4	56.9	-64.6	928.7	5100.4	15, 20	-35.3	232.4	1476.4	-	-	-
							15, 20, 25	-32.6	163.3	1224.7	-	-	-
							15, 20, 25, 30	-29.7	126.1	1006.7	-24.8	152.2	766.3
							15, 20, 25, 30, 35	-26.0	108.9	783.1	1.0	65.1	66.1

The fixed–red pine model used all red pine measurement ages when model fitting (15 to 45 years) and hence is considered the optimum predictive situation.

PA–not calibrated is the "population average" prediction for a model-fitting dataset where no calibration of the mixed-effects model occurred.

For each predicted age (PAge) and calibration age combination n = 8.

For the mixed-effect model, calibration ages are those ages used in calibration, for the fixed–RP model the ages are those ages used in fitting the fixed-effects model (either ages 15 to 30 or ages 15 to 35 years).

e = residual, or the difference between the observed and predicted basal area and v = variance of residuals

Table 6—Parameter estimates and standard errors (in parentheses) of equation (2) by model-fitting dataset for only fixed-effects models

Dataset	Model	b_1	b_2	b_3	b_4	Var	neg 2LL	AIC	*n*
PP IFTNC	Fixed 1st measure	-0.1618	-3.3813	0.9442	3.4731	0.000666	-376.1	-366.1	84
		(0.3424)	(1.8785)	(0.0092)	(1.9469)	(0.000108)			
RP	Fixed 1530	-1.1198	-4.5903	0.7618	5.0688	0.003291	-89.2	-79.2	31
		(1.2058)	(0.9647)	(0.0190)	(1.2623)	(0.000842)			
	Fixed 1535	-2.6822	-5.7765	0.7670	6.6505	0.003059	-115.1	-105.1	39
		(0.7837)	(0.6771)	(0.0176)	(0.8607)	(0.000698)			

PP IFTNC = ponderosa pine at Intermountain Forest Tree Nutrition Cooperative and RP = red pine.

neg2LL = -2 negative log-likelihood and AIC = Akaike Information Criterion, and n is the number of observations used in model fitting.

For both model fitting criteria, more negative numbers are superior.

For ponderosa pine, Fixed 1st measure only uses data from the first measurement for model fitting. For red pine, fixed 1530 uses data from ages 15 to 30 years, and fixed 1535 uses data from ages 15 to 35 years in model fitting.

Table 7—Model validation results for the red pine dataset following calibration of equation (2) fit using the ponderosa pine (IFTNC) dataset, loblolly pine (ETPPRP) dataset, and the ALL dataset

		Mixed–IFTNC			Mixed–ETPPRP			Mixed–ALL			Fixed–RP		
PAge	Calibration ages	*e*	*v*	MSE	*e*	*v*	MSE	*e*	*v*	MSE	*e*	*v*	MSE
40	15, 20	63.9	295.4	4372.7	-34.6	255.9	1451.4	-31.2	205.7	1179.7	-	-	-
	15, 20, 25	55.9	175.7	3299.9	-30.4	199.1	1120.8	-28.6	157.7	978.0	-	-	-
	15, 20, 25, 30	47.8	128.8	2414.0	-26.8	156.2	873.9	-25.9	123.4	791.8	-23.0	210.5	737.6
	15, 20, 25, 30, 35	41.8	95.8	1846.1	-23.0	133.2	661.4	-22.3	106.5	603.3	-4.8	134.9	158.2
45	15, 20	71.2	442.6	5511.3	-37.7	280.4	1698.6	-35.3	232.4	1476.4	-	-	-
	15, 20, 25	62.8	291.4	4235.8	-33.2	209.2	1314.7	-32.6	163.3	1224.7	-	-	-
	15, 20, 25, 30	54.2	226.5	3163.0	-29.5	164.6	1035.8	-29.7	126.1	1006.7	-24.8	152.2	766.3
	15, 20, 25, 30, 35	47.8	175.6	2462.7	-25.5	141.3	793.5	-26.0	108.9	783.1	1.0	65.1	66.1

Mixed-effects models were calibrated using a variety of ages. For each predicted age (PAge) and calibration age combination $n = 8$.

For the mixed-effect models, Calibration ages are those ages used in calibration; for the fixed-RP model, the ages are those ages used in fitting the fixed-effects model (either ages 15 to 30 or ages 15 to 35 years). Mixed-ETPPRP, Mixed-ALL and Fixed–RP are repeat of table 5.

ETPPRP = loblolly pine in east Texas, IFTNC = ponderosa pine at Intermountain Forest Tree Nutrition Cooperative, RP = red pine, and ALL = combined data.

e = residual, or the difference between the observed and predicted basal area and *v* = variance of residuals

Average bias error in predicted BA ($\bar{e}/\overline{BA}$) following calibration for equation (1) ranged from 7.7 to 23.0 percent while for equation (2) percentages ranged from 9.7 to 16.3 percent. Average error in predicted BA ($\sqrt{MSE}/\overline{BA}$) following calibration for equation (1) ranged from 9.1 to 24.8 percent while for equation (2) percentages ranged from 10.6 to 17.9 percent. Average BA for the prediction ages was 230.8 square feet per acre. As table 7 shows, in general, future stand development was underpredicted when calibrating the loblolly pine (ETPPRP) and ALL models while for the ponderosa pine (IFTNC) model basal area was overpredicted.

An error of 10 percent equates to around an error of 23 square feet per acre. As compared to the Fixed – RP, where errors ranged from 3.5 to 12.0 percent ($\sqrt{MSE}/\overline{BA}$), a 10 percent error is acceptable.

A further analysis of equations (1) and (2) for red pine was conducted to see how well the calibrated mixed-effects models predicted future stand development relative to the optimal prediction system (Fixed-Red Pine – tables 4 and 5). The optimal prediction system is to fit a fixed-effects model for red pine using the entire red pine dataset. Tables 2 and 3 contain parameter estimates. As expected, when models fit using the same data that is predicted, the predictions are very good–for equation (1) calibration using additional ages resulted in the two mixed-effects model results becoming more comparable to the results from Fixed-Red Pine but for equation (2) the Fixed-Red Pine validation statistics were clearly superior (tables 4 and 5) regardless of the number of ages used in calibration.

With that said, it should be recognized that even the ALL models fit using ALL datasets is somewhat optimal because for red pine, for example, the red pine dataset was included when fitting the ALL model. Hence, you are using the data itself to predict red pine data. Thus, for a particular equation, (1) or (2), you would expect the ALL model to be superior to the loblolly pine (ETPPRP) model and the IFTNC model. This is also true for the ponderosa pine analysis.

Ponderosa pine—For ponderosa pine, equation (2) produced better predictions relative to equation (1) when only loblolly pine (ETPPRP) was included in the model-fitting dataset but when including all datasets (ALL) when model fitting equation (1) produced better predictions (table 8). Across all predictions, equation (2) performed slightly better than equation (1). For the mixed-effects models, following calibration, equation (2) always had less bias but for the ALL model-fitting dataset it had more variability relative to equation (1).

After examining initial prediction results, it was desired to see whether a fixed-effects model using the initial two measurements of the desired model fitting calibration species (ponderosa pine) would produce better

Table 8—Model validation results following calibration of equations (1) and (2) using the ponderosa pine dataset (IFTNC)

Dataset	First observation used in model calibration						First two observations used in model calibration					
	Equation (1)			Equation (2)			Equation (1)			Equation (2)		
Model	*e*	*v*	MSE	*e*	*v*	MSE	*e*	*v*	MSE	*e*	*v*	MSE
LP ETPPRP												
Not calibrated	-17.9	942.6	1264.4	1.1	878.7	879.8	-17.9	942.6	1264.4	1.1	878.7	879.8
Calibrated	-15.1	413.0	640.2	-5.2	344.4	371.9	-13.2	243.6	417.2	-6.3	200.4	240.0
ALL												
Not calibrated	-14.2	682.8	884.7	-8.4	830.0	900.1	-14.2	682.8	884.7	-8.4	830.0	900.1
Calibrated	-10.4	200.2	309.0	-9.6	257.2	348.8	-9.0	114.0	194.9	-8.8	145.0	222.9
PP IFTNC												
Fixed 1st measure	-	-	-	13.1	238.3	410.9	-	-	-	-	-	-

LP ETPPRP = loblolly pine in east Texas, PP IFTNC = ponderosa pine at Intermountain Forest Tree Nutrition Cooperative, and ALL = combined data.

Mixed-effects models were calibrated using either the first observation for a plot or the first two observations for a plot. Fixed 1st measure only uses data from the first ponderosa pine measurement age for model fitting.

n, or the number of predicted values, is equal to 140. The number of clusters is 84 (or treatment and replication combination).

e = residual, or the difference between the observed and predicted basal area and *v* = variance of residuals

predictions than calibrating a mixed-effects model fit using other species using the first two observations (table 8). This was only conducted for equation (2). When model fitting (table 6), all parameters were significant when just using the initial measurement age in model fitting but when using both the first and second measurement ages in model fitting some parameters were highly nonsignificant. Hence, only the model fit using the initial measurement age was compared. When comparing a fixed-effects model fit using data from an initial measurement (Fixed 1st Measure) to the calibrated mixed-effects models using the initial measurement age, calibration improved model prediction (table 8).

As expected, the use of two measurement ages in calibration generally improved model prediction. However, bias for equation (2) increased for the loblolly pine ETPPRP model from 5.2457 to 6.2873, but, the variance greatly decreased. Outside of the increase in bias for equation (2) for the loblolly pine ETPPRP dataset, bias was reduced by 7.8 to 13.8 percent. MSEs were consistently reduced by nearly 35 percent. Average bias error in predicted BA ($\bar{e} / \overline{BA}$) following calibration for equation (1) ranged from 6.0 to 10.1 percent, while for equation (2) percentages ranged from 3.5 to 6.4 percent. A prediction bias of only 3.5 percent is fairly good. Average error in predicted BA ($\sqrt{MSE} / \overline{BA}$) following calibration for equation (1) ranged from 9.3 to 16.9 percent while for equation (2) percentages ranged from 10.0 to 12.9 percent. Average BA for the prediction ages was 149.6 square feet per acre. As table 8 shows, following calibration, in general future stand development was underpredicted. An error of 10.0 percent equates to around an error of 15 square feet per acre. As compared to the Fixed 1st Measure, which had a 20.3 percent error, this error is reasonable. However, interestingly the Fixed 1st Measure overpredicted future stand development.

CONCLUSIONS

In conclusion, results from this study demonstrate a mixed-effects model fit using one species but calibrated for another produced reasonable predictions. Future research should concentrate on using different linear and nonlinear model forms. The major issue is to somehow standardize time such that the impacts of growth rates on the timing of various biological growth traits of a basal area (or volume) trajectory such as the asymptote, inflection point, etc., can be eliminated. Greater differences in the calibration and predicted ages may show a greater advantage of using calibrated mixed-effects models relative to fixed-effects models fit using the desired predicted species.

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