OPTIMAL SAMPLE SIZE OR POINT SAMPLING FACTOR BASED ON THE COST-PLUS-LOSS CRITERION

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Extended abstract—The cost-plus-loss principle was used to develop methodology for simultaneously determining optimal sample size and optimal plot size or point sampling factor using the Fairfield-Smith relationship between plot size and the variance among plots. The expected absolute value of the difference between true mean and sample mean volumes per hectare was used to develop a loss function in terms of United States Dollar (USD) value. Sampling costs included USD values of plot measurement and travel time costs. The resulting cost-plus-loss function was minimized by differentiating with respect to plot size and sample size. The optimal plot and sample size values were determined by setting these differentials equal to zero and solving. Example solutions are presented based on realistic stumpage and cost values for the Southern United States.

At least three major approaches have been taken to the determination of optimum sample size for forest inventories: determination of sample size needed to achieve a specified standard error of the mean, determination of sample size that minimizes the standard error of the mean for a fixed total cost, and determination of sample size that minimizes total cost-plus-loss. Determination of the sample size needed to achieve a specified standard error of the mean is probably the most commonly discussed method and is usually presented in forest mensuration texts. The method is often presented as determination of the sample size required to achieve a specified error with a specified confidence level. Another approach to determination of sample size is to minimize the standard error of the mean for a given total sampling budget. This approach has been applied in forestry by Mandallaz (2008). The principle of minimizing standard error subject to a fixed total sampling budget has been used in agriculture. Hamilton (1979) investigated the approach to sample size determination of minimizing the cost-plus-loss for forest sampling plans. Burkhart and others (1978) also used the cost-plus-loss framework to determine the best sampling intensity for inventories to be used for multiple resource planning. Zeide (1980) developed a technique for simultaneously determining optimal plot and sample size using a relationship between plot size and variance due to Freese (1961), which is a special case of the relationship between plot size and sample size presented by Fairfield-Smith (1938).

A cost-plus-loss function was developed by expressing the loss as the USD dollar value of the expected absolute value loss with a normal distribution. The variance of this normal distribution was expressed as a function of plot size using the Fairfield-Smith relationship between plots size and variance among plots (Smith 1938). Plot establishment and travel costs were added to the loss functions. The plot cost was developed by using Zeide's (1980) function for plot measurement time as a function of plot size. This plot cost function was multiplied by the number of plots and the sampling wage rate. An expression for travel time was also added to the loss function and the plot costs. Zeide (1980) indicated that the average travel distance for a sample of *n* plots will be the square root of the product of total tract size and the number of plots. This distance was multiplied by the average rate of travel and the wage rate for sampling personnel. Fixed costs we not included because they do not affect the optimal plot and sample size calculations. This is due to the fact that the mathematical derivative of fixed cost with respect to plot size or sample size is zero.

Reasonable values from the literature were used to parametrize the plot measurement time function and to determine wage rates and stumpage values which would be realistic for the Southern United States. The cost-plus-loss function was minimized by differentiating the cost-plus-loss function with respect to plot size and sample size. The cost-plus-loss function can also be minimized directly using appropriate computing software or within widely available spreadsheet applications. The examples indicate that the cost-plus-loss criterion tends to result in higher sampling intensities than would be typical in the Southern United States. For the example scenarios developed, the cost-plus-loss surface is rather flat near the optimal values of sample and plot size, so

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there are a wide range of combinations of plot and sample size that are associated with cost-plus-loss values close to optimum. Although the cost-plus-loss criterion results in somewhat high sampling intensities, sampling costs under this criterion were only about half of one percent of timber value for the scenarios tested. This level of sampling might be justified in situations where very precise valuations are desired, such as inventories for timber sales.

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