

# COMPARISON OF TWO DIAMETER-BASED MEASURES FOR ESTIMATION OF STAND CARRYING CAPACITY

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**Abstract**—Diameter-based maximum size-density measures are useful to estimate stand basal area carrying capacity. In this study, we used an alternate formulation of relative spacing by replacing average dominant tree height with quadratic mean diameter, denoted diameter-based relative spacing (RSD). We compared the efficacy of RSD and Reineke's (1933) self-thinning rule for estimation of stand basal area carrying capacity. Data from a loblolly pine spacing trial, with planting density ranging from 6727 to 747 trees/ha, were used to estimate the coefficients of both measures. RSD largely eliminated the undesirable property of Reineke's self-thinning rule, which overestimated the maximum stand basal area when stands were young.

## INTRODUCTION

For plant populations, carrying capacity can be regarded as the maximum number or biomass of a species that a certain environment can support. In forestry, stand carrying capacity can be referred to as the maximum possible stocking of a stand, which is similar in concept to stockability or stocking capacity (DeBell and others 1989, Hall 1983). Stand basal area, the sum of cross-sectional areas of all stem diameters at breast height in a unit area, is an informative expression of stand carrying capacity (Burkhart and Tomé 2012, p. 175-177). Diameter-based maximum size-density measures can be used to estimate stand basal area carrying capacity. Yang and Burkhart (2017) indicated that stand basal area carrying capacity implied by three well-known diameter-based measures (Reineke's self-thinning rule, competition-density rule, and Nilson's sparsity index) were close to the actual observations.

Relative spacing (Hart 1926) is defined as the ratio of the average distance between trees to the mean dominant tree height of stands. The expression of relative spacing can be written as:

$$RS = \frac{\sqrt{\frac{10000}{N}}}{\bar{H}_d} = f(A)$$

where

$N$  = number of trees per ha

$\bar{H}_d$  = average dominant tree height (m)

$A$  = stand age (yrs).

Relative spacing is a commonly used measure of stand stocking, which has been widely applied to develop stand density management diagrams (Barrio-Anta and González 2005, López-Sánchez and Rodríguez-Soalleiro 2009). However, unlike diameter-based measures, relative spacing cannot be easily converted to diameter-related stand variables (e.g., stand basal area) by simple transformation or substitution of the equation.

Therefore, we used an alternate formulation of relative spacing by replacing mean dominant tree height with quadratic mean diameter, denoted diameter-based relative spacing (RSD). The purpose of this study was to compare and evaluate the efficacy of RSD and Reineke's (1933) self-thinning rule for estimation of stand basal area carrying capacity.

## MATERIALS AND METHODS

### Spacing Trials

Measurements were obtained from a loblolly pine spacing trial that employs a nonsystematic design introduced by Lin and Morse (1975). In 1983, the experiment was established at four sites, two in the upper Coastal Plain and two in the Piedmont. At each site, three nearly contiguous factorial blocks were established. A spacing factor (F) of 1.2 m (4 feet) was chosen in each block. Four levels of the factor (1F, 1.5F, 2F, and 3F) were randomly assigned to row and column positions. Accordingly, 16 plots made up a compact block: four square plots (1.2×1.2, 1.8×1.8, 2.4×2.4, 3.6×3.6 m), and 12 rectangular plots (1.2×1.8, 1.2×2.4, 1.2×3.6, 1.8×1.2, 1.8×2.4, 1.8×3.6, 2.4×1.2, 2.4×1.8, 2.4×3.6, 3.6×1.2, 3.6×1.8, 3.6×2.4 m). Forty-nine trees

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were planted in each plot. The initial planting densities ranged from 747 to 6727 trees/ha. Other than controlling competing vegetation with chemical herbicides in the first 3 years, no other management treatments were applied. An overview of the study design and field procedures is presented in Amateis and Burkhart (2012).

Groundline diameter (GLD) was measured at ages 1-5 and diameter at breast height (DBH) was recorded annually for ages 5-25. Total tree height was measured annually for ages 1-10 and biennially for ages 12-25. Due to more than half of the plots being damaged by mortality agents other than competition-induced tree mortality after age 18, only the measurements from ages 5-18 were included in the analyses. Quadratic mean diameter, average dominant tree height, number of trees per unit area, and BA per unit area (stand basal area) were computed for annual measurements taken at ages 5-18.

### BA Implied by Diameter-based Relative Spacing

Diameter-based relative spacing (RSD) was defined as the ratio of the mean distance between trees to quadratic mean diameter ( $\bar{D}_q$ ) expressed in cm. By letting RSD be a function of stand age (A), the equation becomes:

$$RSD = \frac{\sqrt{\frac{10000}{N}}}{\bar{D}_q} = f(A)$$

Stand basal area carrying capacity can be obtained by squaring both sides of the preceding equation and then multiplying by the coefficient k ( $\approx 0.0007854$ ). That is,

$$G_{RSD} = \frac{10000k}{[f(A)]^2}$$

where

$G_{RSD}$  is the stand basal area carrying capacity implied by RSD. The details of derivation can be found in Yang and Burkhart (2018).  $G_{RSD}$  was estimated using nonlinear quantile regression (quantile=0.01) in R.

### BA Implied By Reineke's Self-thinning Rule

Reineke's self-thinning rule is a well-known maximum size-density relationship measure to describe potential stand density. Reineke (1933) found that the maximum

stand density (N) in trees per unit area and quadratic mean diameter ( $\bar{D}_q$ ) follows a linear relationship on a log-log scale. Reineke's self-thinning rule is expressed in the form:

$$\text{Ln}N = a_0 + a_1 \text{Ln}\bar{D}_q$$

where

Ln = natural logarithm;  $a_0$ ,  $a_1$  = coefficients. Reineke (1933) indicated that the slope ( $a_1$ ), close to -1.6, was generally consistent among species and regions but that the value of the intercept ( $a_0$ ) varied. However, because subsequent studies questioned the assumption of constant slope (Binkley 1984, Zeide 1985), the slope coefficient was estimated in this study.

Rearranging equation 1 gives:

$$N = e^{a_0} \bar{D}_q^{a_1}$$

Stand basal area carrying capacity implied by Reineke's self-thinning rule can be obtained by inserting N in the equation for stand basal area. That is,

$$G_R = N\bar{D}_q^2 k = e^{a_0} \bar{D}_q^{a_1+2} k$$

where

$G_R$  is the stand basal area carrying capacity implied by Reineke's self-thinning rule.  $G_R$  was estimated using quantile regression (quantile=0.01) in R.

### BA Reference Curve

Stand basal area carrying capacity (G) was estimated directly by fitting the Chapman-Richards equation using nonlinear quantile regression (quantile=0.01) in R:

$$G = b_0(1 - e^{b_1 A})^{b_2}$$

where

$b_0$ ,  $b_1$ ,  $b_2$  = coefficients. G functioned as a reference curve when comparing the stand basal area carrying capacity implied by RSD and Reineke's self-thinning rule.

## RESULTS AND DISCUSSION

As shown in figure 1, in the older stands, stand basal area carrying capacity implied by RSD and Reineke's self-thinning rule are close to the observed values and values estimated by the Chapman-Richards equation. When stands were young, however, the maximum values implied by Reineke's self-thinning rule were much higher than the implied values of RSD. Consequently, Reineke's self-thinning rule overestimated the maximum stand basal area in the young stands. In contrast, RSD mitigated the undesirable property shown by Reineke's self-thinning rule and described the dynamics of basal area stocking more accurately.

## CONCLUSIONS

Stand basal area carrying capacity implied by diameter-based relative spacing (RSD) followed closely the reference curve fitted by the Chapman-Richards equation. RSD is a more appropriate representation of the dynamics of basal area stocking than Reineke's self-thinning rule. Although these initial analyses showed that RSD is a reliable diameter-based measure, further investigations of the properties of RSD are warranted.

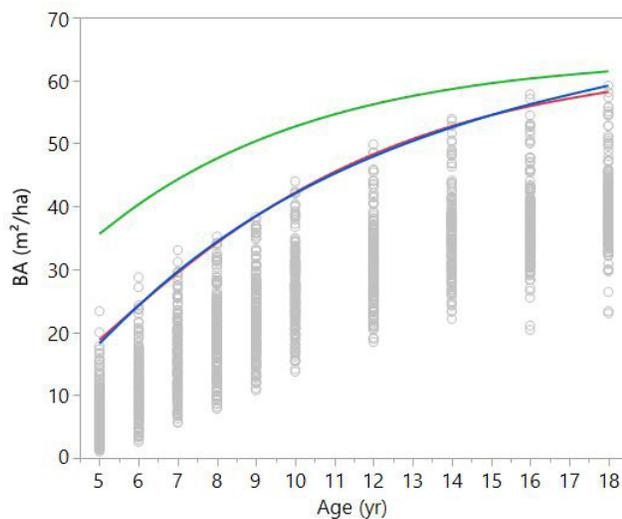


Figure 1 — Comparison of stand basal area carrying capacity estimated by diameter-based relative spacing (RSD) (blue line), Reineke's self-thinning rule (green line), and Chapman-Richards equation (red line).

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