AN *UPDATED* WHOLE STAND GROWTH AND YIELD SYSTEM FOR PLANTED LONGLEAF PINE IN SOUTHWEST GEORGIA

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Abstract—An updated whole stand growth and yield system for planted longleaf pine (*Pinus palustris*) was developed from permanent plot data collected annually over a 13 to 16 year period. The data set consists of 15 intensively managed longleaf pine plantations that are located in Lee, Worth, Mitchell, and Baker counties in southwest Georgia. Stand survival, dominant height, basal area and cubic foot volume yield models were developed for both low and high planting densities. Model prediction error remained low for both planting density classes. Yield models are an improvement over those published in 2006 (Brooks and Jack, 2006), as eight additional growth remeasurements were added which improved projection accuracy for stands older than 10 years. Models are designed for application in unthinned stands (prior to onset of self thinning) in this region between stand age 2 and 25 years.

INTRODUCTION

An updated whole stand growth and yield system was developed for unthinned longleaf (*Pinus palustris*) plantations located in the Flint River Basin of southwest Georgia. The data is from a long term growth and yield study initiated in 1996 based on annual remeasurement data from 15 managed longleaf pine plantations located in Lee, Worth, Mitchell, and Baker counties in southwest Georgia. A system of whole stand projection models were developed for stand survival, dominant height, basal area per acre, and cubic foot volume outside bark (ob) per acre. This system of models updates the previously published models by Brooks and Jack (2006) for the same dataset, however the new models are based on approximately four times the number of growth intervals reported in 2006 and extends plantation prediction age from 18 to 24 years.

METHODS

Study Description

Rectangular fixed area plots were established at different dates and have been remeasured annually; thus the number of measurements available per plot ranges from 7 to 14. Sample plots are approximately 0.1 (mean 0.10585) acre in size with tree age ranging from 2 to 24 years old. Planting densities ranged from 338 to 940 trees per acre. Stand level description of the major variables are shown in table 1. A total of 184 unique non-overlapping growth intervals were available for modeling. This dataset contains two subpopulations, those with planting densities over 750 trees per acre (798 to 940 trees per acre) noted as high density plantings and those with planting densities lower than 450 trees per acre (338 to 445 trees per acre) noted as low density plantings. The groups were separated due to distinct differences in stand survival and dominant height growth patterns. The high density plantings were predominantly cutover sites that were mechanically and chemically site prepared while the low density plantings were old field sites that were mechanically site prepared. All locations were predominantly loamy sands except for two plantings in the high density group which were classified as sands.

At each measurement date, diameter at breast height (d.b.h.) was measured with a diameter tape and recorded for every tree to the nearest 0.01 inch. Total tree height was measured with a height pole or an Impulse laser (depending upon tree size) and recorded to the nearest 0.1 foot. Trees < 15 feet were measured with a height pole, while taller trees were measured with an Impulse 200 laser. Crown class was recorded for every sample tree. The traditional definition of crown class was slightly modified in order to assign crown class to the younger aged stands. The younger plantations generally have wider initial planting spacing, and thus all trees receive full sunlight. The codominant crown class was defined as those trees that make up the average crown canopy, while intermediate and suppressed classes were assigned to those

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Table 1—Range of stand level variables for longleaf plantations in southwest Georgia

	Age (yr)	TPA	BA (ft^2/ac)	HD (ft)	VOL (ft^3/ac)
Low Density					
Min	7	516	36.7	13.7	258.6
Max	24	877	153.5	55.4	4,173.9
High Density					
Min	3	263	4.7	3.9	20.0
Max	20	372	117.1	55.9	3,215.9

TPA = trees per acre, BA = basal area per acre, HD = dominant height and VOL = total cubic foot volume (ob) per acre.

trees that were visually shorter (and usually less vigorous) than the trees that constitute the average crown height. Cubic foot volumes are based on a taper function developed by Brooks and others (2002), utilizing trees sampled from these same plantations.

Survival Projection

Several survival models, commonly employed for loblolly (*Pinus taeda*) and slash pine (*Pinus ellottii*) plantations throughout the South, were tested for their ability to accurately predict stand survival. Models were initially fit independently from other stand level models to test which model forms performed best, based on root mean square error (RMSE) and visual examination of the residuals. Mortality for the high density plantings was greater than the low density plantings and exhibited a typical decrease in survival with age while survival in the low density plantings exhibited negligible mortality by age 20. To provide the best estimates, separate models were fit to the two density classes.

Dominant Height Projection

An algebraic difference equation of a modified Chapman-Richards height/age projection function was tested to model dominant height. This equation is of the form:

$$HD_2 = HD_1 \left[\frac{1 - Exp(\beta_1 * A_2)}{1 - Exp(\beta_1 * A_1)} \right]^{\beta_2} \quad (1)$$

where

A_1 = initial plantation age (yr) at time 1,
A_2 = future plantation age (yr) at time 2,
HD_2 = projected dominant height (ft) at A_2,
HD_1 = current dominant height (ft) at A_1, and
β_1, β_2 = parameters to be estimated from the data.

This equation form has been used successfully in loblolly (Pienaar and Shiver 1980), slash (Pienaar and Shiver 1984) and longleaf (Brooks and Jack 2006) pine plantations. Initial model forms were initially established independent of other stand level models. Evaluation of these independent model forms was based on RMSE and visual evaluation of residuals. Since average dominant height growth differed by planting density group, separate parameter estimates were established for each group. Through age 20, the lower density planting exhibited a higher dominant height development pattern than the higher density planting group.

Basal Area Projection

Several algebraic difference models, commonly employed for loblolly and slash pine plantations throughout the south, were tested for their ability to accurately predict future stand basal area per acre. These model forms were fit as a function of the change in age, trees per acre and dominant height. Again, models were initially fit independently from other stand level models to test which model forms performed best, based on root mean square error (RMSE) and visual examination of the residuals. It became apparent that the same model form would not predict future basal area equally well for both planting density groups.

Volume Projection

Several algebraic difference models, commonly employed for loblolly and slash pine plantations throughout the south, were tested for their ability to accurately predict future stand total cubic foot volume per acre. Again, models were initially fit independently from other stand level models to test which model forms performed best, based on root mean square error (RMSE) and visual examination of the residuals. As found with the individual basal area projection models, the same model form for both planting density groups would not predict future basal area equally well so individual models were developed.

Whole Stand Model Projection Systems

Once the best form of each independent equation was determined, the four models were then fit as a family of projection equations using seemingly unrelated regressions (SUR) using SAS MODEL (SAS 2010). Model forms for each stand parameter were adjusted to provide the best family of projection equations, removing non-significant variables in each individual model form. Final model forms were based on minimum RMSE and visual examination of the residuals for each model. Because of the exhibited differences in growth patterns between the two planting density groups, models were developed separately for the low and high density planting groups.

RESULTS

Parameter estimates and fit statistics are presented by planting density group.

Low Density Planting Group

The final family of projection equations for the low planting density group is of the form:

$$N_2 = Exp\left(Ln\left(N_1\right) + \alpha_1\left(A_2 - A_1\right)\right)$$

$$HD_2 = HD_1 * \left(\frac{\left(1 - Exp\left(\beta_1 A_2\right)\right)}{\left(1 - Exp\left(\beta_1 A_1\right)\right)}\right)^{\beta_2} \tag{2}$$

$$B_2 = Exp\left(Ln\left(B_1\right) + \sigma_1\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \sigma_2\left(Ln\left(HD_2\right) - Ln\left(HD_1\right)\right) + \sigma_3\left(Ln\left(N_2\right) - Ln\left(N_1\right)\right)\right)$$

$$V_2 = Exp\left(Ln\left(V_1\right) + \gamma_1\left(Ln\left(HD_2\right) - Ln\left(HD_1\right)\right) + \gamma_2\left(Ln\left(B_2\right) - Ln\left(B_1\right)\right)\right)$$

where

N_1 = initial stand trees per acre at A_1,
N_2 = future trees per acre at A_2,
B_1 = initial basal area (ft^2/ac) at A_1,
B_2 = future basal area (ft^2/ac) at A_2,
V_1 = initial total cubic feet per acre at A_1,
V_2 = future total cubic feet per acre at A_2,
$\alpha_i, \beta_i, \sigma_i, \lambda_i$ = parameters to be estimated from the data,

All other variables as previously defined.

Model fit statistics are displayed in table 2 and parameter estimates and confidence intervals are displayed in table 3. Residual analysis (fig. 1) indicated a slight positive bias in trees per acre, however most predictions errors were less than 10 trees per acre. Basal area residuals exhibited some negative bias for stands less than 10 years old, although this error was less than 5 square feet per acre. No other residual irregularities were noted. Due to the limited size of the dataset, no independent verification of this prediction system was tested.

High Density Planting Group

The final family of projection equations for the high planting density group is of the form:

$$N_2 = Exp\left(Ln\left(N_1\right) + \alpha_1\left(A_2 - A_1\right)\right)$$

$$HD_2 = HD_1 * \left(\frac{\left(1 - Exp\left(\beta_1 A_2\right)\right)}{\left(1 - Exp\left(\beta_1 A_1\right)\right)}\right)^{\beta_2} \tag{3}$$

$$B_2 = Exp\left(Ln\left(B_1\right) + \sigma_1\left(Ln\left(HD_2\right) - Ln\left(HD_1\right)\right) + \sigma_2\left(Ln\left(N_2\right) - Ln\left(N_1\right)\right)\right)$$

$$V_2 = Exp\left(Ln\left(V_1\right) + \gamma_1\left(Ln\left(HD_2\right) - Ln\left(HD_1\right)\right)\right)$$

Where all variables as previously defined.

Model fit statistics are displayed in table 4 and parameter estimates and confidence intervals are displayed in table 5. Residual analysis (fig. 2) indicated a slight positive bias in trees per acre, however most predictions errors were less than 10 trees per acre. No other residual irregularities were noted. Due to the limited size of the dataset, no independent verification of this prediction system was tested.

DISCUSSION

A new set of whole stand models were developed for longleaf pine plantations in southwest Georgia. Previous models were published (Brooks and Jack 2006), however these earlier models were based on very young stands with limited remeasurement data. The current proposed models are based on unthinned stands from age 2 to 24 with almost 4 times the number of unique growth intervals for model fitting. The dataset consists of both low density plantings, similar to those created under several Conservation Reserve Programs, and high planting densities, similar to densities used in conventional reforestation techniques. To minimize prediction error for both planting densities, separate models or parameter estimates were necessary for each planting density class. These differences were not apparent during the development of the initial models for younger stands in 2006 (Brooks and Jack 2006), but were obvious as stands developed to older ages. Model prediction errors for the fitted dataset were within acceptable ranges and most parameters estimates were significant. The exceptions were with the basal area projection models for both low and high density plantings, where the single parameter estimates representing stand density, trees per acre (TPA), were significant at the 0.18 and 0.07 probability level. These variables were highly scrutinized and finally included in the final models since the prediction models that included these variables were highly significant and that inclusion of the stand density variable (TPA) greatly improved prediction accuracy relative to the stand densities represented in this dataset for basal area projection. Initial comparison with models for west gulf planted longleaf (Lohrey and Bailey 1976) provided less than acceptable results, especially for prediction of dominant height.

Table 2—Nonlinear summary of residual errors for the low density plantation family of projection equations

Equation	SSE	MSE	RMSE	R^2	Adj-R^2
N_2	3,016.2	51.41	7.17	0.9292	0.9300
HD_2	30.4	0.52	0.72	0.9920	0.9920
B_2	661.6	11.71	3.42	0.9619	0.9608
V_2	380,908.0	6,989.10	83.60	0.9852	0.9842

Table 3—Nonlinear seemingly unrelated regressions (SUR) parameter estimates and fit statistics for the low density plantation family of projection equations

Parameter	Estimate	Approx. Std Err	t-value	Approx Pr>|t|
b_1	-0.13122	0.0135	-9.69	<.0001
b_2	2.86595	0.3242	8.84	<.0001
a_1	-0.01180	0.0030	-3.93	0.0002
s_1	-97.90100	30.8709	-3.17	0.0025
s_2	-5.48162	2.0454	-2.68	0.0097
s_3	4.20403	3.1079	1.35	0.1817
l_1	1.20799	0.1686	7.16	<.0001
l_2	0.79831	0.1713	4.66	<.0001

Table 4—Nonlinear summary of residual errors for the high density plantation family of projection equations

Equation	SSE	MSE	RMSE	R^2	Adj-R^2
N_2	9,025.3	139.90	11.83	0.9886	0.9887
HD_2	39.8	0.62	0.79	0.9923	0.9924
B_2	659.5	10.67	3.27	0.9835	0.9829
V_2	394,021.0	6,221.40	78.88	0.9928	0.9927

Table 5—Nonlinear seemingly unrelated regressions (SUR) parameter estimates and fit statistics for the high density plantation family of projection equations

Parameter	Estimate	Approx. Std Err	t-value	Approx Pr>|t|
a_1	-0.01207	0.0022	-5.56	<.0001
b_1	-0.10104	0.0084	-12.09	<.0001
b_2	2.65133	0.2191	12.10	<.0001
s_1	1.16647	0.0706	16.51	<.0001
s_2	0.56564	0.3023	1.87	0.066
l_1	2.08282	0.0733	28.43	<.0001

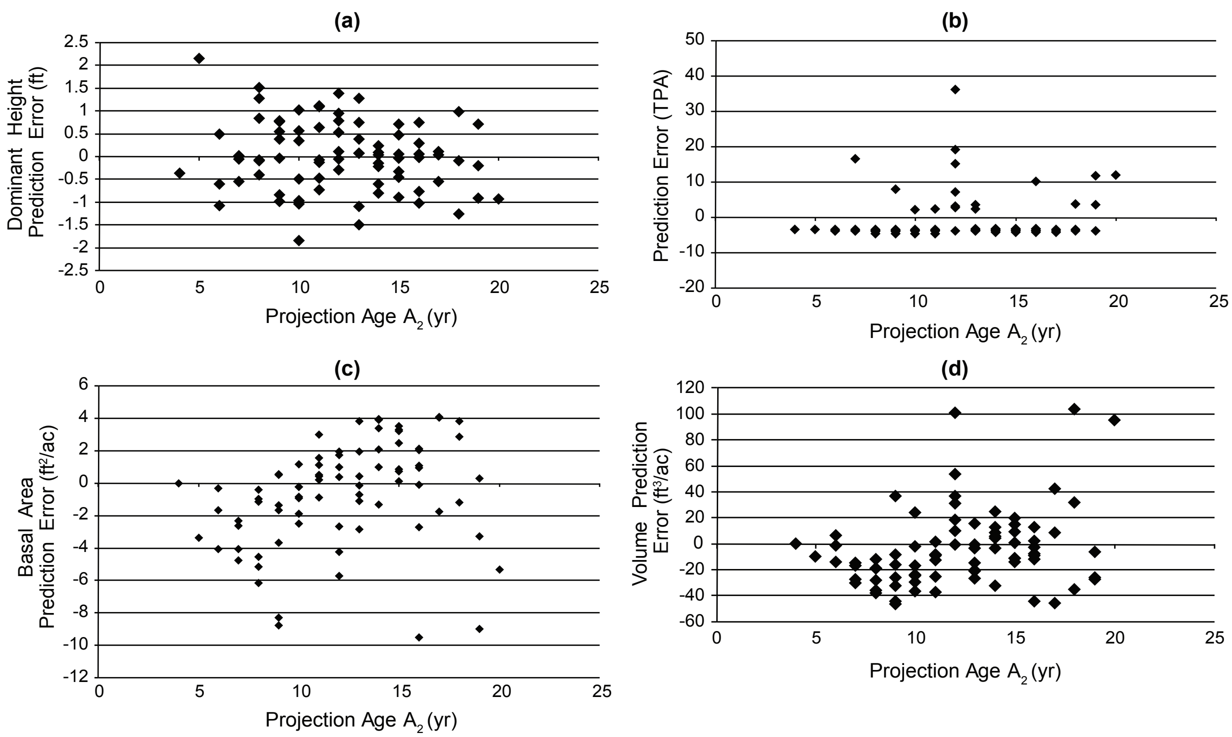

Figure 1—Model residuals for dominant height (a), tree per acre (b), basal area per acre (c), and cubic foot volume per acre (d) for low density plantings.

Comparisons were also made to newly developed whole stand models published by Gonzalez-Benecke and others (2012) for planted longleaf pine plantations in the west gulf region. Using the existing southwest Georgia dataset, their whole stand density (TPA) model compared favorably for both planting density classes while their basal area model tended to over predict this variable. In addition, considerable error and bias was apparent for the whole stand volume models for both planting densities. In an attempt to isolate the source of this difference, the individual tree volume model (Gonzalez-Benecke and others 2014) was compared to the volume models developed for southwest Georgia (Brooks and others 2002). No large differences were found in this comparison. In addition to testing tree volume, the individual tree height model was tested against the measured tree heights from the Brooks and others (2002) study. Differences in average tree height by diameter class was exhibited for all diameter classes, with the west gulf model over predicting tree height from 10 to 15 feet between the 2 and 12-inch class represented in this study.

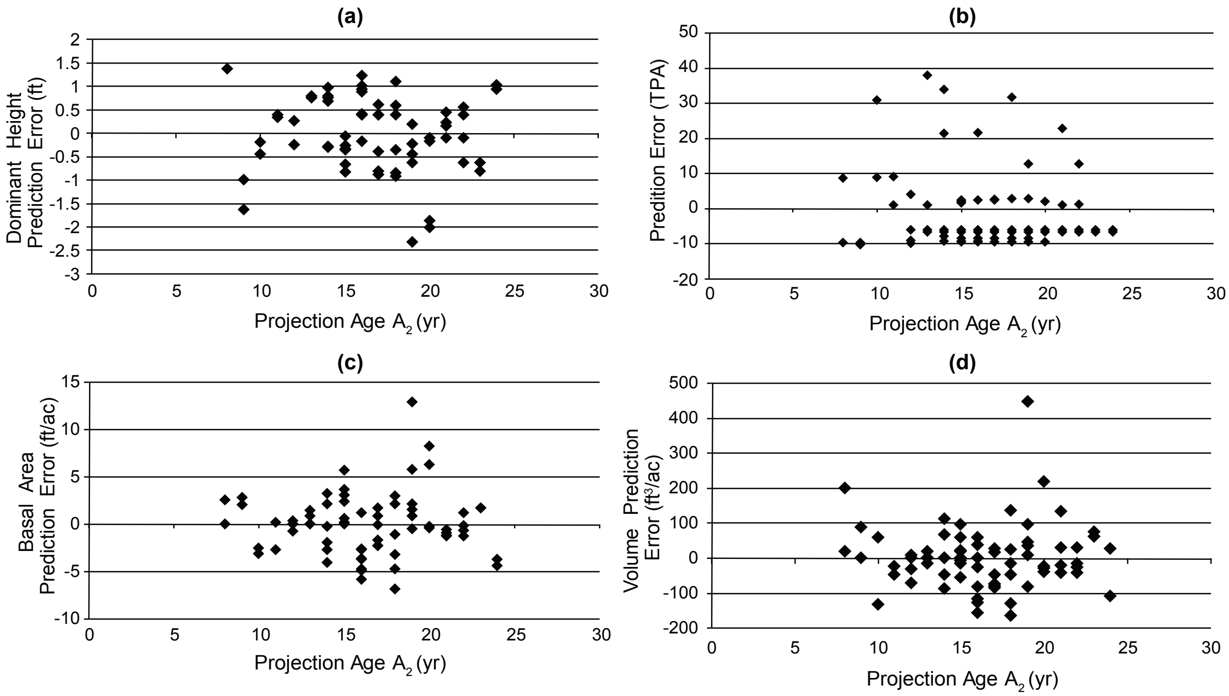

Figure 2—Model residuals for dominant height (a), tree per acre (b), basal area per acre (c), and cubic foot volume per acre (d) for high density plantings.

These models should perform well for planted longleaf in southwest Georgia but caution should be employed if extrapolating to stands outside this region or if heavy self-thinning mortality is evident.

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