AN EVALUATION OF THE PROPERTIES OF THE VARIANCE ESTIMATOR USED BY FIA

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ABSTRACT

The Forest Inventory and Analysis (FIA) program of the U.S. Forest Service currently conducts inventories utilizing the protocols of the national enhanced FIA Program. Due to the permanent locations of the sample plots, the stratification of the population occurs after the selection of sample units, i.e., post-stratification. In situations where the population is of limited areal extent, this may result in small within-stratum sample sizes. The survey literature provides some guidance on post-stratified sample sizes, but does not specifically address the behavior of estimators when sample sizes are smaller than recommended. It is important for FIA to evaluate how estimators perform across a range of sample sizes, such that samples of sufficient size can be constructed to ensure accurate estimates. The variance estimator used by FIA accounts for a secondary source of variation (V_2) due to random within-strata sample sizes that is introduced beyond that obtained from standard proportional allocation of samples to a stratified sample (V_1). Thus, each estimate's variance is composed of two parts. This study utilizes a Monte Carlo simulation to examine the relative contributions of V_1 and V_2 to the total variance (V_{Total}) of the estimate. FIA plots from Pennsylvania were treated as a population from which samples of size *n* are repeatedly drawn and V_1, V_2, and V_{Total} calculated for forest area and cubic volume estimates. The sample size varied from 25 to 200 plots. With increasing sample size *n*, the V_1 variance stabilized at sample sizes greater than 60 plots, whereas the V_2 variance required sample sizes greater than 125. The ratio of the two variance components ($V_{RAT}=V_1/V_2$) was found to increase with increasing *n*, ranging from 6 to 32 plots for the area estimates and from 8 to 45 plots for the volume estimates.

INTRODUCTION

The Forest Inventory and Analysis (FIA) program of the U.S. Forest Service currently inventories forested land across the United States using procedures detailed in Bechtold and Patterson (2005). During Phase 1, remotely sensed information is used to stratify the population to reduce the variance of estimates. This stratification varies by region but generally includes at a minimum forest and nonforest as strata (Bechtold and Patterson 2005). In Phase 2, permanent ground plots are visited and data on numerous attributes are collected at various levels of detail. Plots determined as clearly nonforest from aerial imagery are assessed remotely.

Weights for strata are determined during Phase 1. However, the Phase 2 sample determines strata sample sizes as plots are permanently located without respect to stratum boundaries. This sampling design is considered to be a post-stratified simple random sample (Cochran 1977, Schaeffer and others 2006) and it has an added source of variation due to stratum sample sizes not being fixed in advance. The magnitude of this additional variation within a forest inventory has not been well studied. It is the goal of this study to examine how the use of this post-stratification estimate affects the variances of total area and total cubic foot volume. Specifically, the variance estimates for these values will be split into their components and examined both separately and jointly in order to better understand what role each plays in the total variance under several sample size situations. This is important to FIA to insure that sufficient sample sizes are available for accurate estimates. Sample strata weights will also be tested for agreement with population strata weights using χ^2 tests of agreement.

METHODS

Plot data is from a complete set of panels for Pennsylvania measured from 2003 to 2007. Phase I strata were developed by classifying the percent tree canopy cover from the NLCD 2001 map product (Homer et al. 2004) into five classes. There were 4,628 plots that were treated as the population from which samples of plots were drawn.

VARIANCE ESTIMATOR

The variance of the estimate (Bechtold and Patterson, 2005) is given by

$$v(\hat{Y}_d) = \frac{A_T^2}{n}\left[\underbrace{\sum_h^H W_h n_h v(\overline{Y_{hd}})}_{V_1} + \underbrace{\sum_h^H (1-W_h)\frac{n_h}{n} v(\overline{Y_{hd}})}_{V_2}\right] \quad (1)$$

where

A_T=total area of the population.
$h \ldots H$=strata in the domain of interest.
n=sample size.
W_h=weight for stratum h within the population.
n_h=sample size for stratum h.
$\overline{Y_{hd}}$=mean of attribute of interest (plot proportion forest land or cubic-foot volume) for stratum h
d=domain of interest
$v(\overline{Y_{hd}})$ =variance of the mean for stratum h

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The left side addend within the bracketed sum in (1) will be referred to as V_1. This part of the variance results from the stratification of the population during Phase I. The right side addend within the bracketed sum in (1) will be referred to as V_2. This second part of the variance is a consequence of stratum sizes being random within the strata determined in Phase I. The ratio V_1/V_2 will be defined as V_{RAT} and the total variance as V_{Total}.

MONTE CARLO SIMULATION

The first stage of the Monte Carlo (MC) simulation (Metropolis and Ulam 1949) was to determine how many sets of 50 plots would result in stable values for the V_1, V_2, V_{RAT}, and total variance for both the area and cubic-foot estimates. Each of the four values was calculated for 5,000 draws. Then, the variance of each value was calculated for the first three sets. Subsequently, an additional set was added and the variance recalculated for the specific value. Variances were plotted against the number of sets drawn and it was determined that 5,000 draws were sufficient to stabilize the several measures of interest. As computation length was not extensive, 10,000 draws were performed for each sample size.

Procedures for the second stage of the MC simulation were performed separately for the forest land area and cubic-foot volume estimates. Initially 10,000 sets of plots were selected, for each of a number of selected sample sizes. Plots in a set are drawn without replacement and sample sizes were: 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 125, 150, 175, 200. Every plot in the set had its stratum, proportion of forest, and cubic volume recorded. Each set of n plots had its strata sizes, strata weights, strata means, strata variances, and stratified mean and variance for the total forested area calculated. While each plot in a set is not replaced, at the next iteration (next set of plots) all plots are then replaced, thus all aggregate measures on the plot are considered to be sampled with replacement. The MC variance was then determined as the variance of the 10,000 stratified sample means. These procedures were repeated for cubic-foot volume.

Trends in variance behavior were visually analyzed with the use of boxplots. A boxplot was generated for each set of 10,000 plots at the selected sample sizes for V_1, V_2, V_{RAT} and V_{Total}. Boxplots used the first, second (median), and third quartiles for the lower, middle and upper horizontal lines of the boxes. Minimum and maximum values were represented by the lower and upper whiskers respectively of the boxplots. Means also were calculated and shown as points (triangles). Patterns were examined specifically for the means and medians.

Stratum sample weights were calculated for each set of plots drawn for all sample sizes. A χ^2 test was used to test agreement of stratum sample weights and population stratum weights (H_0: $n_h/n = W_h$) for all sets of plots at all sample sizes. The significance level was set at 0.05. If the test was not significant, the set was recorded to be in agreement with the population stratum weights. Frequencies of agreement for a fixed sample size were calculated for the 10,000 simulations. This agreement testing will be used to assess if deviations from population stratum weights exist and whether they may be influencing the variances of the estimates.

RESULTS

VARIANCES FOR THE AREA ESTIMATES

For the V_1 variance, median and mean values for a given n stabilize around a sample size of 60 plots (Figure 1). Median and mean values are less for a sample size of 50 and below. V_2 variances approximately stabilize for a sample sizes of 125 or greater (Figure 2) and are slightly higher for a sample size lower than 125. V_{RAT} values do not approach a stable point (Figure 3). When considering the median and mean values, V_{Total} approaches an asymptote after 125 samples as well (Figure 4), yet is still decreasing slightly for larger values.

VARIANCES FOR THE VOLUME

Again considering the median and mean values for a given n, V_1, values for the volume estimates also stabilize around a sample size of 60 (Figure 5). As was the case for the area estimates, values for a sample size less than 60 are smaller on average. V_2 values similarly stabilize for sample size 125 and greater (Figure 6). V_{RAT} values are increasing and range from 8 to 45 for sample size 25 and 200 respectively (Figure 7). V_{Total} values approach an asymptote for sample size 125 and greater (Figure 8) when focusing on the mean and median values.

MC VARIANCE ESTIMATES

The MC variance estimates for both area of forest land and volume follow a similar pattern, they decrease at a decreasing rate (Figures 9 and10). As compared to the mean V_{Total} for identical sample sizes, the MC variance is in close agreement.

SAMPLE WEIGHTS

Sample weights were consistently in agreement 95 percent of the time or better for all sample sizes (Table 1). There were no apparent patterns related to sample size, as all agreement levels were either 95 or 96 percent in all cases.

DISCUSSION

Patterns for the V_1, V_2, and V_{Total} values were quite similar between the area and volume estimates. For both area and volume, V_1 increased to a stable value at sample size 60 and above, while V_2 and V_{Total} decreased to a stable value for sample size greater than 125. There were differences in ranges for V_{RAT}, with V_{RAT} for area ranging between 6 and 32 over the given sample size range, while V_{RAT} for volume ranged between 8 and 45 therein.

The increasing values for V_{RAT} stem from minute changes in V_2 relative to V_1 (Figures 1, 2 and Figures 5, 6). While the V_2 values have what appears to be an asymptote, small changes downward are enough to continue inflating the value of V_{RAT}, In regard to V_{Total} however, the overall addition from V_2 is small, and V_{Total} stabilizes when V_2 stabilizes.

Three factors suggest that V_{Total} is biased for smaller samples. First, V_1 increased as it approached 60 samples then approached an asymptote (Figures 1 and 5). Second, V_{RAT} was continuously increasing as well, implying that V_1 dominated V_2 (Figures 3 and 7). Even though V_2 initially decreases, V_{RAT} shows that V_1 is still much greater than V_2, therefore an increasing V_1 offsets the decreasing by V_2. Third, V_{Total} shows a similar pattern as V_1, increasing to an asymptote at 60 (Figures 4 and 8). These factors demonstrate then that for low sample sizes V_{Total} is underestimated. The main factor to this downward bias for appears to be V_1, with minor offsetting by V_2.

Stratum sample weights agreed with population stratum weights for all sample sizes (Table 1). Agreement percentages were 95 percent and above, which is where they should be given that the significance level for the χ^2 test was set at 95 percent as well. It was thought that perhaps the lower sample sizes might fail to generate similar sample stratum weights as compared to the population stratum weight as some of the class sizes were small, but this hypothesis was not supported. Approximately 5 percent of the samples deviated from the population weight and the other 95 percent were similar.

CONCLUSIONS

With increasing sample sizes, the penalty factor for post-stratification, V_2, diminishes greatly compared to the variance component stemming from stratified design (V_1). Cochran (1977) states that the effect of the V_2 variance will be small if the mean number of sampling units per stratum is reasonably large. For these data, asymptotes are approached for V_2 and V_{Total} at sample size of 125. The mean number of sampling units per stratum is therefore 25 here, which may provide some insight of minimum bound for 'reasonably large.' Cochran (1977, p.134) states also that stratum samples greater than twenty are 'reasonably large' and Schaeffer et al. (2006, p. 150) suggest that stratum samples sizes greater than 20 provide "…nearly as accurate sample sizes as stratified sampling with proportional allocation." This may be too conservative a rule of thumb for this data, as the smallest stratum sample weight was about 0.06, resulting in just seven samples on average in that stratum at an overall sample size of 125. Users of FIA data should be aware that stratifications which later have small sample sizes may result in an underestimate of the variance of the intended estimate. Further study may more accurately determine what within-stratum minimums are achievable.

What was not varied in this study was the state from which the plots were located, as this study was conducted using only one population with a specific stratification scheme. Weights for the five strata ranged from 0.06 to 0.38. Results from other populations having differing structures should be examined to determine if the results found in this study are more broadly applicable.

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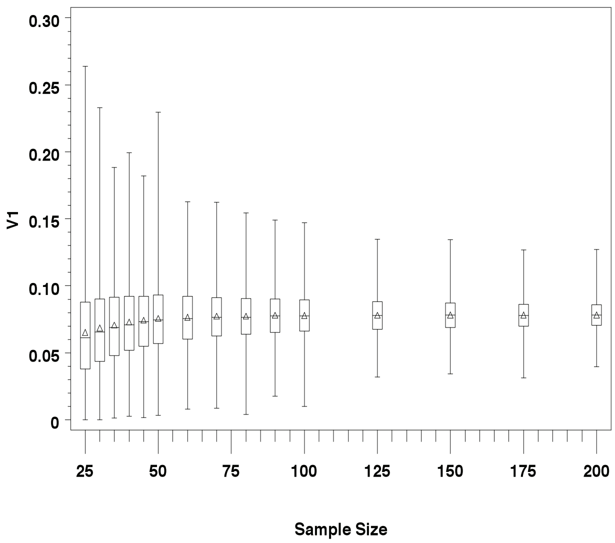

Figure 1—Boxplots for the 10,000 simulations of the area V_1 variance using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edges represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

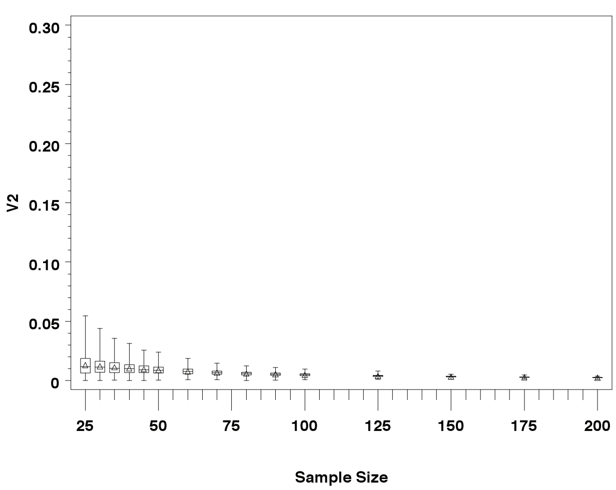

Figure 2—Boxplots for the 10,000 simulations of the area V_2 variance using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edges represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

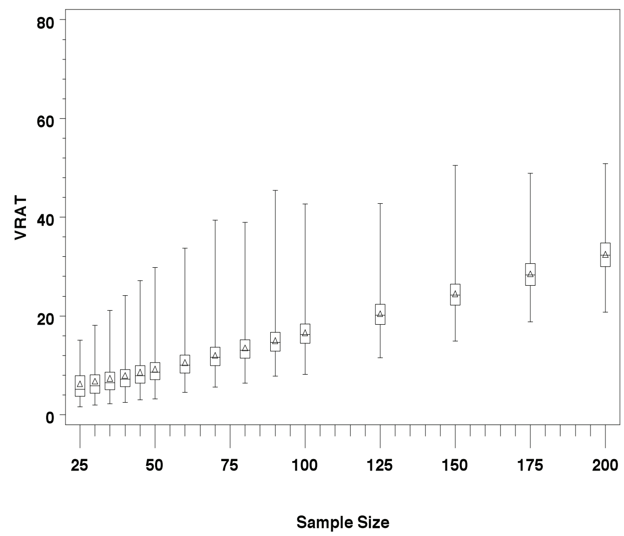

Figure 3—Boxplots for the 10,000 simulations comparing the ratio (V_{RAT}) of the V_2 and V_1 area variances using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edges represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

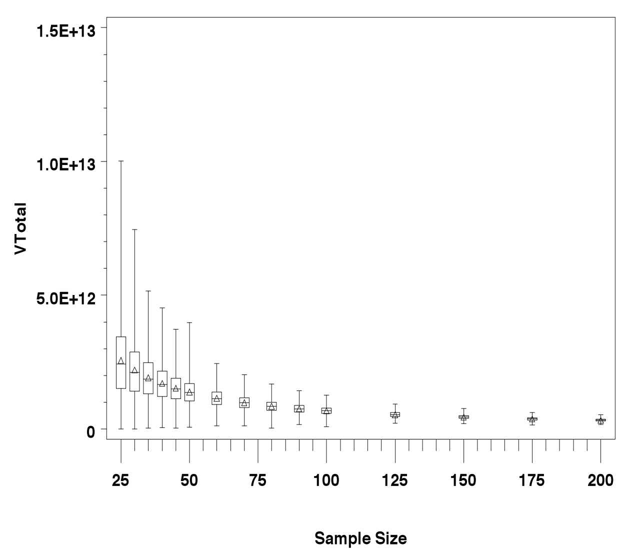

Figure 4—Boxplots for the 10,000 simulations of the total area variance (V_{Total}) using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edges represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

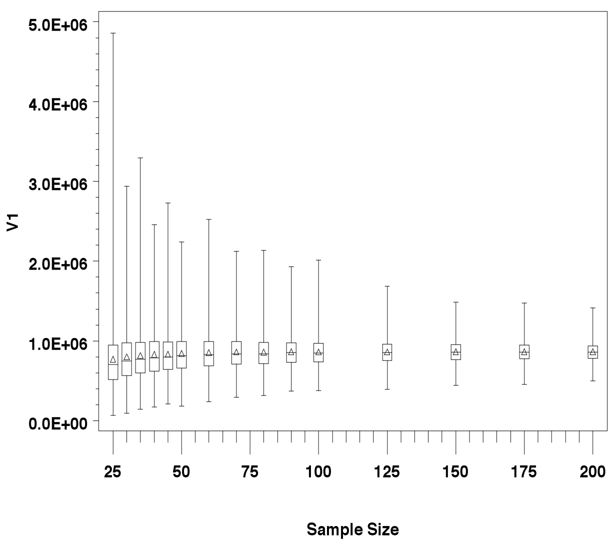

Figure 5—Boxplots for the 10,000 simulations of the cubic volume V_1 variance using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edge represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

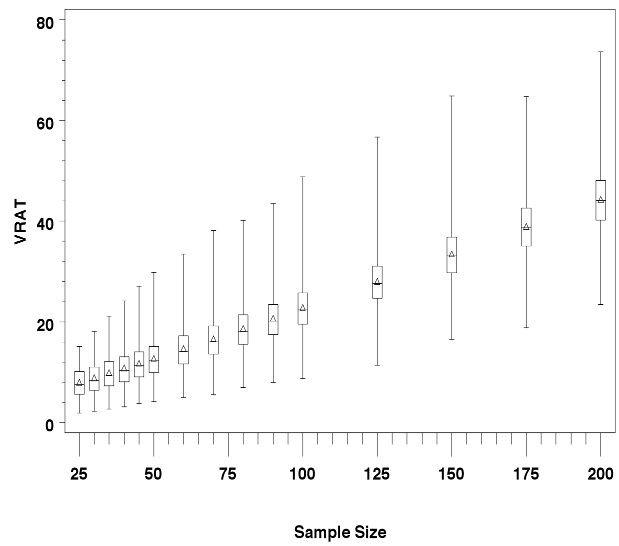

Figure 7—Boxplots for the 10,000 simulations comparing the ration (V_{RAT}) of the V_2 and V_1 cubic volume variances using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edges represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

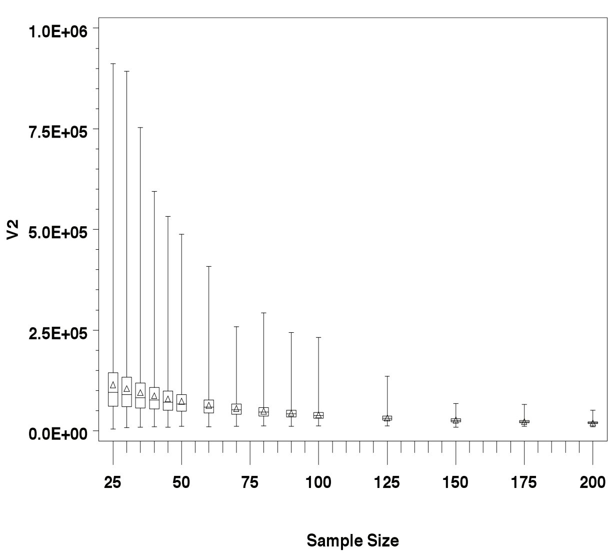

Figure 6—Boxplots for the 10,000 simulations of the cubic volume V_2 variance using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edges represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

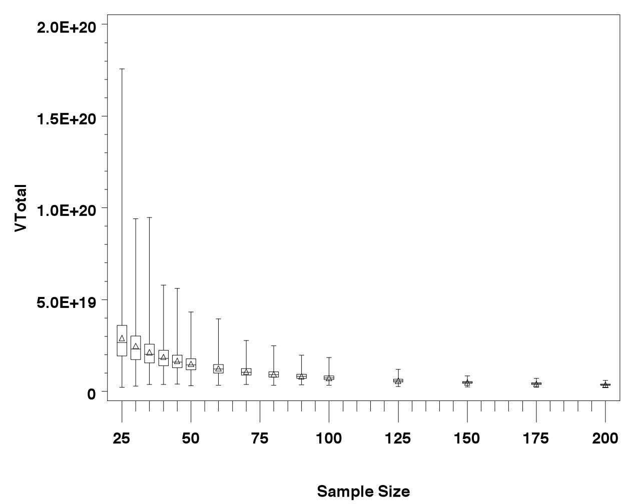

Figure 8—Boxplots for the 10,000 simulations of the total cubic volume variance (V_{Total}) using the given sample sizes. Lower and upper whiskers represent minimum and maximum values. Lower and upper box edges represent 1st and 3rd quartiles, with the median represented by the line inside the box. Means are symbolized with triangles.

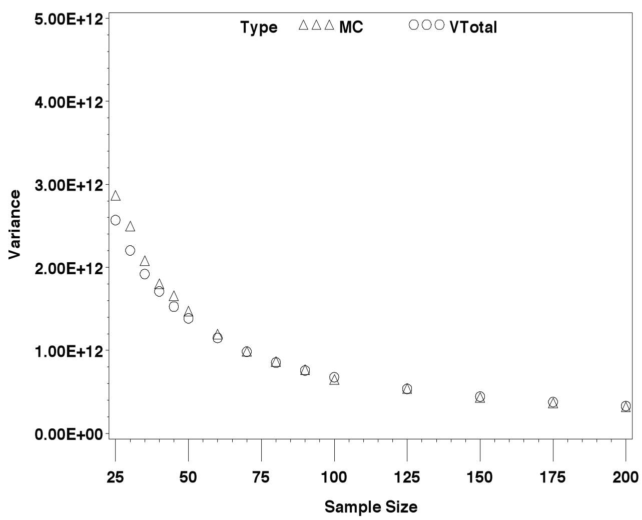

Figure 9—Comparison of the Monte Carlo variance for the 10,000 simulations of the mean total area and mean V_{Total} for a given sample size.

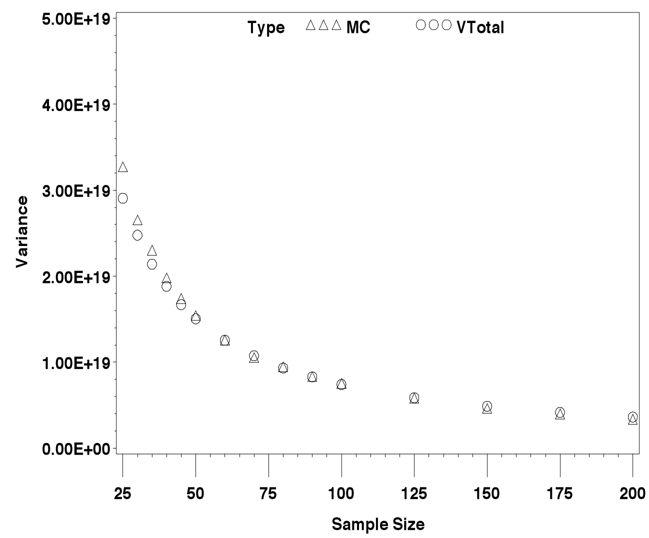

Figure 10—Comparison of the Monte Carlo variance for the 10,000 simulations of the mean total cubic volume and mean V_{Total} for a given sample size.

Table 1—Proportion of simulations where the sample weight agreed with the population weight as tested by a X^2 goodness-of-fit test

	Area			Volume		
Sample size	Lower confidence limit	p	Upper confidence limit	Lower confidence limit	p	Upper confidence limit
25	0.9464	0.9508	0.9550	0.9463	0.9507	0.9549
30	0.9479	0.9523	0.9564	0.9453	0.9498	0.9540
35	0.9525	0.9567	0.9606	0.9482	0.9525	0.9566
40	0.9483	0.9526	0.9567	0.9488	0.9531	0.9572
45	0.9477	0.9521	0.9562	0.9474	0.9518	0.9559
50	0.9482	0.9525	0.9566	0.9460	0.9504	0.9546
60	0.9510	0.9552	0.9592	0.9469	0.9513	0.9554
70	0.9508	0.9550	0.9590	0.9504	0.9547	0.9587
80	0.9523	0.9565	0.9604	0.9462	0.9506	0.9548
90	0.9501	0.9544	0.9584	0.9520	0.9562	0.9601
100	0.9492	0.9535	0.9575	0.9488	0.9531	0.9572
125	0.9512	0.9554	0.9594	0.9533	0.9574	0.9613
150	0.9518	0.9560	0.9599	0.9557	0.9597	0.9635
175	0.9550	0.9591	0.9629	0.9565	0.9605	0.9642
200	0.9515	0.9557	0.9597	0.9545	0.9586	0.9624