SEGMENTED POLYNOMIAL TAPER EQUATION INCORPORATING YEARS SINCE THINNING FOR LOBLOLLY PINE PLANTATIONS

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Abstract—Data from 108 trees felled from 16 loblolly pine stands owned by Temple-Inland Forest Products Corp. were used to determine effects of years since thinning (YST) on stem taper using the Max–Burkhart type segmented polynomial taper model. Sample tree YST ranged from two to nine years prior to destructive sampling. In an effort to equalize sample sizes, tree data were classified into three groups: 2 and 3, 4 and 5, and 6 to 9 years since thinning. A modified Max-Burkhart segmented taper model was fitted to each of the three YST categories in order to detect any trends in the coefficients. Multiple coefficients were then expressed as functions of YST. The resulting three-segment equation included the independent variable of YST. The resulting taper equation predicts expected changes in shape of loblolly pine trees following thinning activities.

INTRODUCTION

Volume equations are useful predictors of total stem volume or merchantable volume within given merchantability constraints. However, taper equations can provide estimates of diameter at any point along a stem. Such equations can be integrated through calculus to derive volume estimates, permitting differing merchantability limits to be specified for a broad and changing array of forest products. Heger (1965) found trees with less taper can yield more volume than like trees having more taper. Several stand related silvicultural processes can alter the rate of taper for individual trees. One of the most important forest management practices is forest thinning. Although thinning has been shown to substantially modify diameter growth, it will generally have negligible effects on height growth for most species (Davis and others 2001).

On the other hand it is possible that thinning may affect tree shape in a way that cannot be completely explained by corresponding changes in the d.b.h.-height ratio. This study seeks to model and quantify the changes in tree taper as the time since thinning operations increase.

TAPER MODELS

Numerous models to explain taper have been developed: Kozak and others (1969), Ormerod (1973), Thomas and Parresol (1991), and Valentine and Gregoire (2001). A segmented polynomial model (SPM) approach was presented by Max and Burkhart (1976). The SPM is able to describe the general shape of a tree by separating the stem into two or more sections. Max and Burkhart's SPM (eq. 1) used three segments to describe the shape of a tree. The bottom bole section, which includes the butt, is modeled as a frustum of a neiloidal solid, the middle of the bole is assumed to take the shape of a frustum of a paraboloidal solid, and the top portion is assumed to be conoid. In this system, three additive models are grafted together by way of incorporating join points between the three segments. A smooth functional form across the join points is obtained by conditioning to ensure continuity and equality of first partial derivatives across the join points. Upper-stem diameters can be estimated anywhere on the bole by using an additive series of mathematical models that enter or exit the model depending on a sequence of indicator variables.

$$\frac{d^2}{D^2} = b_1(x-1) + b_2(x^2-1) + b_3(a_1-x)^2 I_1 + b_4(a_2-x)^2 I_2 \qquad (1)$$

where x is h/H, h is height to upper-stem diameter d, H is total tree height, d is diameter at upper-stem height h, D is d.b.h., and indicator variables are: $I_1 = 1$ if $x < a_1$, otherwise $I_1 = 0$ and $I_2 = 1$ if $x < a_2$, otherwise $I_2 = 0$.

Since its first application to loblolly pine (P*inus taeda*) in 1976 the Max-Burkhart SPM has been used successfully in numerous situations for coniferous species. Valenti and Cao (1986) incorporated the use of crown ratio into the Max-Burkhart SPM taper model to achieve a better fit of stem taper. The a_1 and b_1 parameters in the basic Max-Burkhart SPM were replaced with logarithmic and hyperbolic functions of crown ratio, respectively. Model 2, rewritten in the following form, is:

$$d = D\sqrt{\left(c_1 + \frac{c_2}{CR}\right)z + b_2 z^2 + b_3\left(z - \left(c_3 + c_4 * \ln(CR)\right)\right)^2 I_1 + b_4\left(z - a_2\right)^2 I_2} \qquad (2)$$

where z is (1-h/H), h is height to upper-stem diameter d, H is total tree height, d is diameter at upper-stem height h, D is d.b.h., CR is crown ratio, and $I_1 = 1$ if $z > c_2 + c_3 \ln CR$, otherwise $I_1 = 0$ and $I_2 = 1$ if $z > a_2$, otherwise $I_2 = 0$.

By incorporating crown ratio into the model, upper-stem diameter estimation bias was reduced by five percent and an R^2 of 96.9 percent was obtained.

Valenti and Cao's modification of the Max-Burkhart SPM may have promise for adaptation to variation in stem form due to factors such as thinning. The Max-Burkhart model might be modified by modeling the parameters as functions of years since thinning (YST).

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DATA AND ANALYSIS

Data for this study were from trees felled from Temple-Inland Forest Products Corp. loblolly pine plantations throughout eastern Texas. 108 sample trees were selected across 16 separate stands that spanned ages from 13 to 31 years and had been growing from 2 to 9 years since thinning operations. Each sample tree was prepared after felling as follows: stump height was measured; total height above stump height was measured; at 1, 2.5, and 4.5 feet above the stump, diameter and bark thickness were measured; and finally the stem was marked at five-foot intervals from the stump to the terminal bud. The stem was bucked into 5 foot bolts according to the markings. From each bolt two d.o.b. measurements (0.1 inch) were taken perpendicular to one another with a caliper.

The data were divided into three classes of YST (2 and 3, 4 and 5, and 6 to 9 years) with an approximately equal number of observations. Equation (1) was fitted separately to each class of YST using non-linear regression techniques in SAS NLIN (SAS 2004). The resulting coefficients were plotted against the class means in order to detect any trends. Coefficients that showed trends were replaced in the model using three functions in combinations in a stepwise fashion. Following Valenti and Cao (1986) functions examined in the model were: (a) linear, c_0+c_1CR, (b) hyperbola, c_0+c_1/CR, and (c) logarithm, $c_0+c_1\ln CR$.

Thus far the best combination of replacement parameters was found to be a model with parameters $a_1 = c_3+c_4*\ln(YST)$ and $b_1 = c_1+c_2/YST$. The resulting modified equation has the following form:

$$d = D\sqrt{\left(c_1 - \frac{c_2}{YST}\right)(1-x) + b_2\left(1-x^2\right) + b_3\left((1-x)-\left(c_3+c_4*\ln(YST)\right)\right)^2 I_1 + b_4\left((1-x)-a_2\right)^2 I_2}$$

$$\text{Where}\quad I_1 = \begin{cases} 1, \text{if } (1-x) < c_3 + c_4\ln(YST) \\ 0, \text{otherwise} \end{cases} \quad \text{and} \quad I_2 = \begin{cases} 1, \text{if } (1-x) < a_2 \\ 0, \text{otherwise} \end{cases} \qquad (3)$$

with parameter estimates for equation 3 shown in table 1.

Table 1—Estimates of the parameters from equation 3

Parameters	Parameter Estimates
b_2	2.772079
b_3	-2.968376
b_4	69.259808
a_2	0.909326
c_1	0.295761
c_2	-0.418517
c_3	0.215086
c_4	0.019247

CONCLUSIONS

Modification of parameters in a Max-Burkhart SPM equation adjusted for years since thinning indicated significant differences in stem form due to years since thinning. The equation was modified by replacement of parameters by functions of years since thinning as indicated previously. To date this equation, modified to account for years since thinning, appears to have the best performance of the taper functions that have been fitted to these data. Further testing and validation is warranted using supplementary sample trees and further modifications to the model parameters. In addition, further testing should be evaluated with individual trees and appropriate groupings of trees in the data set.

LITERATURE CITED

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