

# SELF-REFERENCING SITE INDEX EQUATIONS FOR UNMANAGED LOBLOLLY AND SLASH PINE PLANTATIONS IN EAST TEXAS

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**Abstract**—The Schnute growth function was used in this study to model site index for unmanaged or low-intensity managed loblolly pine (*Pinus taeda*, L.) and slash pine (*Pinus elliottii*, Engelm.) plantations in east Texas. The algebraic difference approach was used to derive an anamorphic base-age invariant site function that was fit as a fixed base-age anamorphic site function (base age = 25 years). The dataset was comprised of 1,135 and 502 serially correlated height-age observations of loblolly and slash pine, respectively, which were collected over a 20-year-period as a part of the East Texas Pine Plantation Research Project (ETPPRP). The new site functions represent an improvement over earlier site functions for east Texas, especially for slash pine, primarily because the new function accounted for serial correlation in the data. The new site index equations apply to unmanaged or low intensity managed loblolly and slash pine plantations in east Texas ranging in age from 5 to 40 years.

## INTRODUCTION

Mathematical models or functions have been used extensively to describe site-age relationships. Dynamic site functions are a particular type of mathematical function that are defined by their own value at some reference point in time, which is called the initial condition (Cieszewski 2002). Thus, they are self-referencing (Northway 1985) with the initial conditions defined by data. They also possess the property of base-age invariance (Bailey and Clutter 1974). Base-age invariance means that the selection of a base age (index age) has no effect on the parameter estimates. Bailey and Clutter (1974) introduced a technique to derive dynamic site functions called the Algebraic Difference Approach (ADA). Base-age invariant site functions derived via ADA have the general form:  $H_2 = f(H_1, A_1, A_2)$ , where  $H_2$  = height at  $A_2$ ,  $H_1$  = height at  $A_1$ ,  $A_2$  = Age at time 2, and  $A_1$  = Age at time 1. Fixed base-age site functions have the general form:  $H = f(S, A, A_0)$ , where  $H$  = height at  $A$ ,  $S$  = Site index,  $A$  = age,  $A_0$  = index age for site index. Site index (and index age) must be known prior to model fitting for fixed base-age site functions; however, it need not be known *a priori* for base-age invariant site functions. Cieszewski and others (2000) present a detailed discussion of base-age invariant and fixed base-age site functions.

Sigmoid growth functions (e.g., Chapman-Richards; Chapman 1961, Richards 1959) have been used for decades to predict site index (Pienaar and Turnbull 1973, Newberry and Pienaar 1978, Clutter and others 1983, Lenhart and others 1986). Schnute (1981) generalized these sigmoid growth functions into one model. Coble and Lee (2006) used Schnute's model as the guide curve (Clutter and others 1983) to develop a family of anamorphic site curves for loblolly and slash pine plantations in east Texas. They ignored the serial correlation in the remeasured plot data used to develop the guide curve. The purpose of this study was to derive a base-age invariant version of Schnute's model and use Northway's fitting method to account for serial correlation in the hopes to improve site index estimates over those of Coble and Lee (2006).

## METHODS

### Schnute Growth Function

The integrated form of Schnute's second-order differential equation was used in this study:

$$Y(t) = \left( y_1^b + (y_2^b - y_1^b) \frac{1 - e^{-a(t-\tau_1)}}{1 - e^{-a(\tau_2-\tau_1)}} \right)^{1/b} \quad (1)$$

where,

$Y(t)$  = size of organism at time  $t$ ,  
 $y_1, y_2$  = size of organism at  $\tau_1$  and  $\tau_2$ ,  
 $\tau_1, \tau_2$  = ages at time 1 and 2 (e.g., old and young), and  
 $a, b$  = constants to be estimated via regression  $\neq 0$ .

The Algebraic Difference Approach (ADA) of Bailey and Clutter (1974) was applied to Equation 1 to derive a base-age invariant anamorphic site function. First, solve Equation 1 for the initial conditions,  $H_0$  and  $t_0$ :

$$H_0 = \left( y_1^b + (y_2^b - y_1^b) \frac{1 - e^{-a(t_0-\tau_1)}}{1 - e^{-a(\tau_2-\tau_1)}} \right)^{1/b}$$

Then, solve for the site-specific parameter,  $y_2^b$ :

$$y_2^b = y_1^b + (H_0^b - y_1^b) \frac{1 - e^{-a(\tau_2-\tau_1)}}{1 - e^{-a(t_0-\tau_1)}}$$

Substituting this expression in Equation 1 gives the base-age invariant anamorphic site function:

$$H = \left( y_1^b + (H_0^b - y_1^b) \frac{1 - e^{-a(t-\tau_1)}}{1 - e^{-a(t_0-\tau_1)}} \right)^{1/b} \quad (2)$$

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where all variables are defined as before. Equation 2 represents a base-age invariant anamorphic site function described by the Schnute growth function. The formulation of equation 2 follows that of ADA functions if the following substitutions are made:  $H_2 = H$ ,  $H_1 = H_0$ ,  $A_2 = t$ , and  $A_1 = t_0$ .

### Model Fitting Procedure

Northway (1985) presented a methodology for fitting self-referencing functions to serially correlated data. His procedure requires an estimate of  $H_0$  at  $t_0$  prior to the fitting process, which is a problem since  $H_0$  at  $t_0$  is rarely measured in the field. Northway (1985) referred to this estimate of  $H_0$  and  $t_0$  as site index (S) at the index age ( $t_{IA}$ ). Equation 2 was reformulated as a fixed base-age site function to accommodate this change of variables:

$$H = \left( y_1^b + (S^b - y_1^b) \frac{1 - e^{-a(t - \tau_1)}}{1 - e^{-a(t_{IA} - \tau_1)}} \right)^{1/b} \quad (3)$$

where all variables defined as before. Each remeasured plot provided a growth series from which estimates of S were calculated during the iterative nonlinear fitting process. Each record in the dataset contained a single height-age pair, along with its entire growth series, which is every height-age pair for the specific plot measured over time. As explained below, this growth series was used to estimate S for each height-age pair.

To estimate S for each height-age pair, initial estimates of the regression coefficients (i.e., a and b) were first set in equation 3. These initial estimates corresponded to starting values in the iterative nonlinear fitting process, and they changed with successive iterations. Within each iteration, conditional site index estimates (CSI) were set in equation 3. Heights were predicted for the entire growth series for the values of CSI. The squared differences (observed – predicted) in height were then calculated. The values of CSI for the current iteration that minimized the squared differences were used as final S estimates to estimate new values of the regression coefficients for the next iteration. This process was repeated until the least squares error for the overall regression was minimized (i.e., lowest SSE). Thus, CSI is the estimate of site index that minimizes squared differences of serially correlated observations, given the current coefficient

estimates. Thus, the procedure simultaneously estimates S for the growth series and CSI used in the function. The “throw away” final CSI values are, in fact, excellent estimates of the height at the index age (25 years in this study) for each growth series.

### Data Analysis

This study used the same data as Coble and Lee (2006), where 124 permanent plots were located in loblolly pine plantations, and 56 plots were located in slash pine plantations throughout east TX. The data were compiled differently in this study to work with the Northway (1985) methodology. The ETPPRP study area covers 22 counties across east TX (Lenhart and others 1985). Generally, the counties are located within the rectangle from 30–35 north latitude and 93–96 west longitude. Each plot consists of two subplots: one for model development and one for model evaluation. A subplot is 100 by 100 feet in size, and a 60 foot buffer separates the subplots. All planted pine trees are permanently tagged and numbered. Only the model development plots were used in this study. The average height of the ten tallest site trees and the total age of the plantation were used to represent height and age in the functions. The ten tallest trees per plot (40 trees per acre) were considered site trees if they met the following criteria: 1) free of damage, 2) no forks, and 3) no presence of stem fusiform rust (*Cronartium quercuum* [Berk.] Miyabe ex Shirai f. sp. *Fusiforme*). Plots were remeasured every three years; some plots only provided two observations (six years), while some provided eight observations (24 years). A total of 1,135 remeasured height-age observations for loblolly pine and 502 remeasured height-age observations for slash pine (table 1) were used to fit equation 3. PROC NLIN in SAS version 9.1 was used to run the analyses.

### RESULTS AND DISCUSSION

Equation 3 was fit to the loblolly and slash pine data to produce the coefficients in table 2. All coefficients were significantly different from zero, and the residual plots did not reveal any unusual heteroscedasticity problems (plots not shown). Note that  $y_1 = \tau_1 = 1$ , which corresponds to a one-year-old seedling that is one foot tall; these fixed values were based on measurements of the youngest trees in the datasets. Also, index age =  $\tau_{IA} = 25$  years. The regression coefficients a and b were estimated by SAS. The coefficient values from table 2 were used in equation 3 to produce site

**Table 1—Descriptive statistics for the ETPPRP loblolly and slash pine development plots, where age = total age of plantation and height = average height of the ten tallest site trees on a plot**

Species	Variable	N	Mean	Standard deviation	Minimum	Maximum
Loblolly	Age (years)	1,135	14	7	1	37
	Height (feet)	1,135	44	21	1	94
Slash	Age (years)	502	14	7	1	33
	Height (feet)	502	44	21	2	91

**Table 2—Parameter estimates and fit statistics of loblolly and slash pine site functions (Equation 3)**

Species	Parameter	Parameter estimate	Standard error	Lower 95% confidence interval	Upper 95% confidence interval	Root MSE (feet)
Loblolly	y <sub>1</sub>	1	na	na	na	2.7
	a	0.0690	0.00285	0.0634	0.0746	
	b	0.7291	0.0198	0.6904	0.7679	
	1	1	na	na	na	
	1A	25	na	na	na	
Slash	y <sub>1</sub>	1	na	na	na	2.5
	a	0.0401	0.00423	0.0318	0.0484	
	b	0.8769	0.0314	0.8152	0.9386	
	1	1	na	na	na	
	1A	25	na	na	na	

curves for loblolly pine (fig. 1) and slash pine (fig. 2). These curves range in site index from 40 to 90 feet (index age = 25 years), and they apply to plantations that range from 5 to 40 years of age.

The precision of the parameter estimates (standard errors for a and b) and overall regression (RMSE) were higher for the self-referencing function than the guide curve function of Coble and Lee (2006) for both loblolly and slash pine (table 3). In fact, the guide curve function RMSE was double the value for RMSE of the self-referencing function.

For loblolly pine, the shapes of the self-referencing site curves were similar to those based on the guide curve of Coble and Lee (2006) (fig. 3). Shapes were compared for site indexes of 40, 60, and 80 feet by taking the difference between the site index values of the self-referencing and

Coble and Lee (2006). The largest differences were less than three feet, and these occurred above 30 years of age. For slash pine, the shapes were dramatically different between the self-referencing curves and those of Coble and Lee (2006) (fig. 4). Differences ranged from approximately 3 to 10 feet for ages greater than 30 years. Differences were not as great for younger ages. Thus, the self-referencing site functions seem to better capture the curve shape better for older ages than the functions of Coble and Lee (2006). We attribute this improvement to the self-referencing functions capturing the effect of serial correlation in the data. Both this study and Coble and Lee (2006) used the Schnute (1981) model and the same dataset; however, Coble and Lee (2006) ignored the serial correlation of the data.

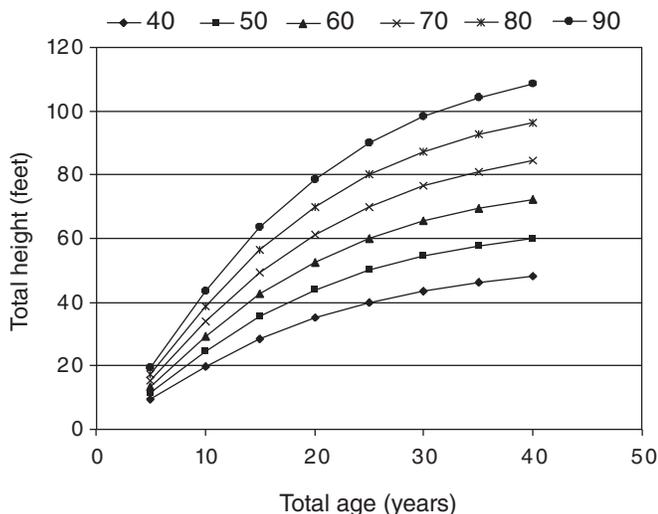


Figure 1—Site index curves (index age = 25 years) for unmanaged loblolly pine plantations in east Texas.

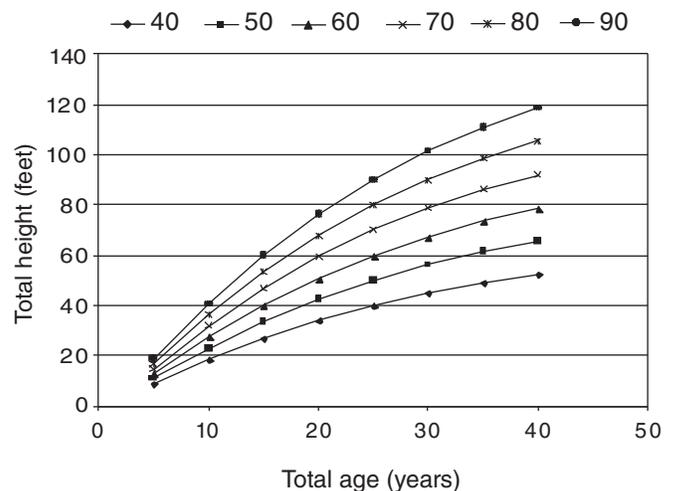


Figure 2—Site index curves (index age = 25 years) for unmanaged slash pine plantations in east Texas.

**Table 3—Comparison of overall precision and the precision of the parameter estimates for the self-referencing site functions of this study and the guide curve functions of Coble and Lee (2006)**

Species	Function	RMSE	SE(a)	SE(b)
Loblolly	Guide Curve	7.3	0.0034	0.0401
	Self-referencing	2.7	0.0029	0.0198
Slash	Guide Curve	6.7	0.0055	0.0648
	Self-referencing	2.5	0.0042	0.0314

Note: RMSE = root mean square error in feet and SE = standard error in feet.

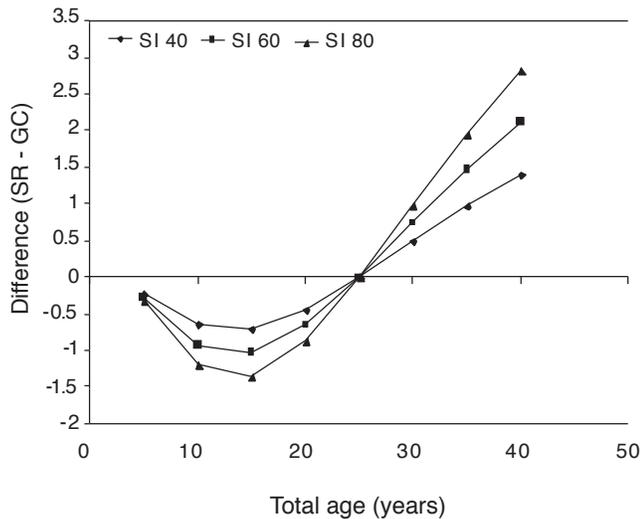


Figure 3—Comparison between the self-referencing curves of this study (SR) and curves of Coble and Lee (2006) (GC) for loblolly pine. Note that SI = site index in feet (base age = 25 years).

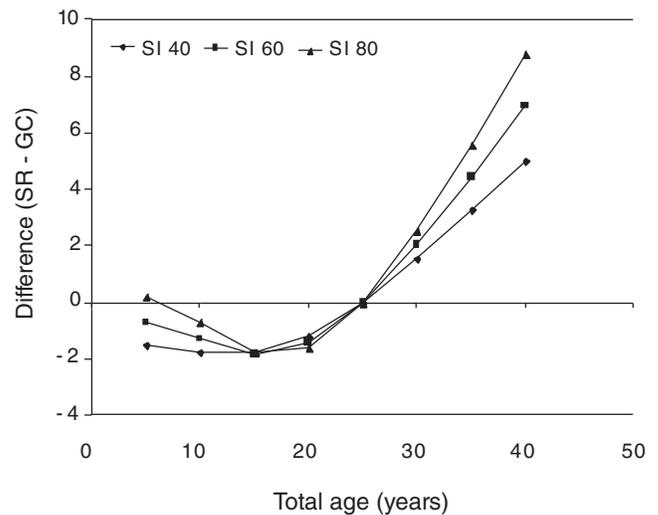


Figure 4—Comparison between the self-referencing curves of this study (SR) and curves of Coble and Lee (2006) (GC) for slash pine. Note that SI = site index in feet (base age = 25 years).

## CONCLUSIONS AND RECOMMENDATIONS

The self-referencing version of the Schnute growth function represents an improvement over Coble and Lee (2006). For both loblolly and slash pine, overall model precision is doubled and standard errors of regression coefficients are reduced for the new function in this study compared to Coble and Lee (2006). Differences in site curve shape between the two functions were most dramatic for slash pine than loblolly pine. The differences were most pronounced for older plantations (age > 30 years). These improvements were attributed to accounting for serial correlation in the data used to build the site function, which Coble and Lee (2006) ignored. The new curves in this study are applicable to unmanaged, or low-intensity managed, loblolly and slash pine plantations in east TX.

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