INDIVIDUAL TREE GROWTH MODELS FOR NATURAL EVEN-AGED SHORTLEAF PINE

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Abstract—Shortleaf pine (*Pinus echinata* Mill.) measurements were available from permanent plots established in even-aged stands of the Ouachita Mountains for studying growth. Annual basal area growth was modeled with a least-squares nonlinear regression method utilizing three measurements. The analysis showed that the parameter estimates were in agreement with respect to both direction and magnitude to those published by Lynch and others (1999) with data for the first two measurements. Since the tree measurements within a plot were correlated, a linear mixed model (with log-transformed basal area growth as a response) with a compound symmetry covariance structure for trees within a plot was also fitted. The mixed model fitted the basal area growth well, although the least-squares nonlinear model and the linear mixed model were not directly comparable.

INTRODUCTION

Shortleaf pine (*Pinus echinata* Mill.) is an important pine species and has the widest range of any pine in the Southeastern United States. Prior to 1985, shortleaf growth studies were limited to temporary plots with full-stocking assumptions. Beginning in 1985, Oklahoma State University's Department of Forestry has cooperated with the USDA Forest Service's Southern Research Station and the Ouachita and Ozark National Forests to study growth and yield of shortleaf pine in even-aged natural stands. Over 200 permanent plots located in shortleaf pine natural stands have been established in southeastern Oklahoma and western Arkansas with repeated measurements (Lynch and others 1999). This paper presents the highlights of preliminary findings based on basal area growth modeling including the third measurement of the plots.

METHODS

Tree measurements were repeated at an interval of 4 or 5 years on fixed-radius circular plots 0.2 acre in size. The details of plot establishment, tree measurements made, and growth and yield modeling from the first two measurements are described in Lynch and others (1999). They estimated parameters in the following nonlinear model:

$$G_i = \frac{b_1 B_i^{b_2} - \left(b_1 B_i / B_M^{1-b_2}\right)}{1 + \exp(b_3 + b_4 B_s + b_5 A + b_6 R_i + b_7 B_i)} \qquad (1)$$

where

G_i = annual basal area growth (square feet) of tree i,

B_i = basal area (square feet) of tree i,

A = stand age,

R_i = the ratio of quadratic mean stand diameter to the d.b.h. of tree i,

B_s = stand basal area (square feet per acre),

B_M = 7.068384 square feet (the maximum expected basal area for a shortleaf pine in managed stands), and

$b_1, b_2, ..., b_7$ = parameters.

This model was updated by including the third series of measurements using nonlinear ordinary least squares with SAS PROC NLIN.

In addition, an attempt was made to fit a linear mixed-effects model using SAS PROC MIXED. A linear mixed-effects model can be written as (Gregoire and others 1995):

$$Y_i = X_i\beta + Z_i\gamma_i + \varepsilon_i \text{ (i=1,2,…,p; p=no. of plots)} \qquad (2)$$

where

Y_i = the vector of natural logarithm of annual basal area growth (square feet per tree per year) for i^{th} plot,

β = the fixed-effects parameter vector, $\beta = [\beta_0 \quad \beta_1 \quad \beta_2 \quad \beta_3]'$,

γ_i = the plot specific vector of parameters for random-effects,

X_i = the t x 4 design matrix of fixed-effects variables stand age, stand basal area and quadratic mean diameter, including intercept;

t = number of trees in a plot,

Z_i = the t_i x 1 design matrix of random-effect variable (random-effect of i^{th} plot), and

ε_i = the plot specific vector of errors.

It is assumed that $\gamma_i \sim MN(0, \sigma^2 B)$, $\varepsilon_i \sim MN(0, \sigma^2 W_i)$, and $cov(\gamma_i, \varepsilon_i) = 0$, where B is a correlation matrix of random effects and W_i is a correlation matrix of within-plot errors.

RESULTS AND DISCUSSION

The findings from the nonlinear ordinary least squares regression by including all the three measurements are presented in tables 1 and 2. All the coefficients were significantly different from 0 at P = 0.05 level of significance. The signs of the coefficients were the same as those found by Lynch and others (1999), and the magnitudes were also comparatively consistent. Standard errors of the estimates are slightly increased compared to previous estimates. One reason for this might be that overall annual basal growth rate for two growth periods (1985 to 1990 vs. 1990 to 1995) was different (0.013 vs. 0.014 square feet per tree per year overall average).

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Citation for proceedings: Connor, Kristina F., ed. 2006. Proceedings of the 13th biennial southern silvicultural research conference. Gen. Tech. Rep. SRS–92. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 641 p.

Table 1—Overall summary statistics for variables used in modeling (n = 14,564)

Variable	Mean	Std. dev.	Minimum	Maximum
Annual basal area growth (ft^2/tree/year)	0.0137	0.01124	0	0.0996
Basal area growth (ft^2/acre/year)	4.1214	2.37575	0	8.7811
Basal area (ft^2/acre)	121.30	35.797	14.9	200.4
Stand site index (ft)	56.9	9.46	38.9	87.5
Stand age (years)	44.9	19.95	18	103
Quadratic mean diameter (QMD, inches)	8.57	3.3979	3.54	20.73
DBH (inches)	8.34	3.9153	1.2	26.6
Ratio of QMD to DBH	1.11	0.3582	0.386	7.03

Table 2—Parameter estimates and their standard errors, along with previous estimates (n = 14,564)

Coefficient	Previous estimate	Current estimate	S.E. (current estimate)
b_1	0.07142	0.1127	0.00589
b_2	0.48038	0.5197	0.01910
b_3	-3.23628	-1.9430	0.15020
b_4	0.01577	0.0092	0.00029
b_5	0.02788	0.0188	0.00083
b_6	1.29452	1.2504	0.04420
b_7	-1.21269	-0.7747	0.04010

Another factor might be that Lynch and others (1999) used additional plots from a shortleaf pine thinning study. The residuals from the fitted model were plotted against 2-inch d.b.h. classes in figure 1. The figure shows that the spread of residuals increases with increasing d.b.h. classes, as expected. Future work will explore possibilities of accommodating the increasing variance with increasing d.b.h. classes. One alternative would be to use weighted least squares instead of ordinary least squares for the regression. Another possibility would be to use a mixed modeling technique.

The parameter estimates and standard errors of the estimates from a preliminary analysis with mixed modeling are (number of plots = 187, total number of observations = 13,792):

$$\left[\hat{\beta}_0 \;\; \hat{\beta}_1 \;\; \hat{\beta}_2 \;\; \hat{\beta}_3\right]' = \left[-1.0155 \;\; 0.004077 \;\; -2.3492 \;\; -0.01024\right]'$$

$$\left[\mathrm{S.E.}(\hat{\beta}_0) \;\; \mathrm{S.E.}(\hat{\beta}_1) \;\; \mathrm{S.E.}(\hat{\beta}_2) \;\; \mathrm{S.E.}(\hat{\beta}_3)\right]' = \left[0.1169 \;\; 0.001405 \;\; 0.04958 \;\; 0.000694\right]'$$

Trees within a plot were assumed to have a compound symmetry covariance structure. This was an improvement on model 1, which assumed independence of trees within a plot. It was also assumed that plots had random effects, and other explanatory variables (stand age, stand basal area and quadratic mean diameter) had fixed effects. These three explanatory variables were found to significantly explain the variation in annual basal area growth, i.e. all the coefficients were significantly different from zero (P<0.004). Parameters were estimated using the restricted maximum likelihood (REML) method. The effect of period or time on annual growth rate, however, was not significant.

Since the study has generated unbalanced longitudinal data, a mixed-effects model has attractive properties. It will be interesting to investigate different covariance structures and

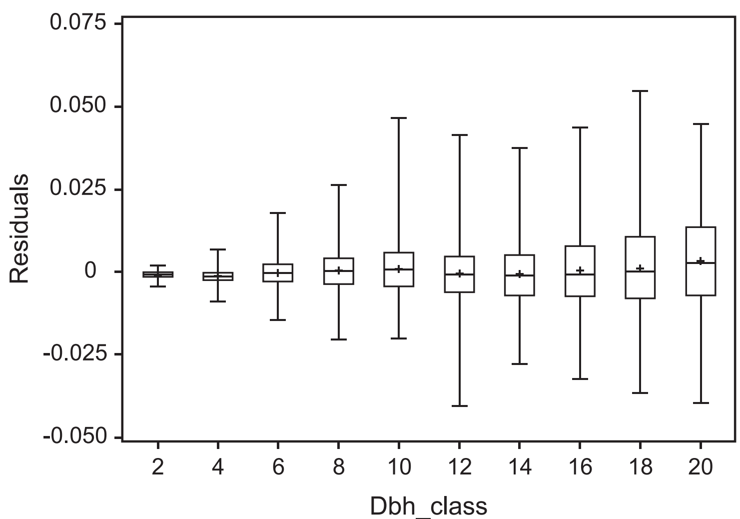

Figure 1—Boxplots of residuals plotted against 2-inch d.b.h. classes.

subsets of explanatory variables to model basal area growth. Because of the nature of the response variable and the relationship of predictors with the response, a mixed-effects model in which parameters enter in a nonlinear fashion might be a better approach to analyze such data despite the model complexity and computing challenges. For example, Hall and Bailey (2001) describe the techniques of nonlinear mixed-effects modeling in forestry problems. In addition to basal area growth, other appropriate response variables can possibly be modeled using the mixed-modeling technique to accommodate errors at different levels.

LITERATURE CITED

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