

# REPRODUCIBILITY AND RELIABILITY: HOW TO DEFINE THE POPULATION OF TREES THAT REPRESENT SITE QUALITY FOR LONGLEAF PINE PLANTATIONS

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**Abstract**—We compared 13 definitions for the subpopulation of “site trees.” Each subpopulation was defined (1) once at base age or at each measurement; (2) by crown class, diameter, or height; and (3) by the number of trees per acre. These subpopulations were applied to base ages of 25 and 50. For base age 25, the subpopulations defined at base age were superior to those defined at each measurement. The subpopulations defined by dominant and codominant trees were slightly superior to the subpopulations defined by the 40 tallest or thickest trees per acre. Generally, subpopulations defined by diameter were superior to subpopulations defined by height, and the subpopulations that included more trees were superior to those that included fewer. For base age 50, there was very little or no benefit from defining the subpopulations at base age. Among the subpopulations defined at each measurement, the one defined as dominant and codominant trees was superior. We selected this subpopulation for our site index modeling work. The results are largely explained by the stability of tree rankings within a plot over time. Ranking with respect to height is much less stable than ranking by diameter. Crown class was unexpectedly stable from measurement to measurement.

## INTRODUCTION

Site index may be broadly defined as the average height of an arbitrarily defined subpopulation of trees in a stand at some arbitrarily determined base age. Site index is the most-used method to distinguish potential productivity of even-aged forests in North America (Carmean 1975, Goelz and Burk 1992). Site index is intrinsically a stand- or plot-level variable (Sharma and others 2002), but it is obtained from measurements of individual trees (Zeide and Zakrzewski 1993). Base age is generally selected so that it is slightly less than the expected rotation age for the species and region (Goelz and Burk 1996); in North America, it is typically between 25 and 50 years, although it may be less for short-rotation species.

One must choose what criterion to use to define the subpopulation of trees that indicate site quality. Various criteria specify the select subpopulation. Generally, regardless of the selection criteria for the subpopulation, damaged or defective trees are excluded, as such trees do not directly indicate site. The selection criteria are typically defined by crown class (Avery and Burkhart 1994), rank in diameter, or rank in height. When crown class is used, either the average height of dominants, or the average height of dominant and codominant trees combined, indicates site quality. When diameter or height is the criterion, either a fixed number of trees per unit of land or fixed proportion of trees on a plot define the subpopulation of trees that represents site quality. “Top height” is another term for the average height of the subpopulation of site trees (Zeide and Zakrzewski 1993). Zeide and Zakrzewski (1993) offered an alternative to using a single definition for the site tree subpopulation. Rather than using either a fixed number of trees per acre or a fixed proportion of trees per plot, they suggested a combined estimator that consisted of the “average of averages” of a fixed number and a fixed

proportion of trees. For their data, this average of the averages cancelled out the biases of the two averages. In this way, it is roughly analogous to an antithetic variate type of estimator.

Site index models may be produced out of a more general growth-and-yield modeling effort, or they may arise out of a site evaluation context. The data for these two contexts tend to differ. In a growth-and-yield context, height-development data arise from remeasured permanent sample plots. Height may be measured on all or some subset of trees on these plots; diameter is typically measured on all trees. In a site evaluation context, height-development data often arise from stem analysis (Duff and Nolan 1953) of selected sample trees. Although the trees may all be within a fixed sample area, other nonharvested trees may not be measured for diameter or height. The term “top height” is more often applied to the growth-and-yield context than the site evaluation context.

It is simple to create a definition for the subpopulation of site trees and to determine the average height of those trees on a given plot. However, the trees that comprise that subpopulation change over time. As individual trees grow at different rates, a tree that was one of the 40 thickest trees at one measurement may not be in that class at some later measurement (Tiarks and others 1998). Given data from permanent sample plots, or even from stem analysis, it is possible to know or infer whether a tree belonged to the defined subpopulation at other ages. As this type of data is used to estimate site index equations, a model can specify membership in the subpopulation at base age, or can specify membership at each measurement. This is a critical decision: whether the trees in the subpopulation of interest should be defined at one time or redefined at each

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measurement. In the first case, we only consider the trees in the subpopulation at base age. In the second case, different trees represent the subpopulation at each age, and, thus, the average height includes different trees from measurement to measurement. Presumably, the first case produces models with smaller variance when fit to data, as one source of variation is removed. However, using only base age might be a false economy; the fit to our data might be improved at the cost of relevance in application. In a site evaluation application, a user cannot be certain which trees were in the defined subpopulation at base age, given measurements at some other age.

So, the choice is between smaller errors in estimation or smaller errors (or objectivity) in application, or between reliability (small error) and reproducibility. Reliability suggests estimates that are more indicative of site quality. Reproducibility suggests greater objectivity, no need to guess which trees will be members of the subpopulation at any other time. In this paper we explore the issue of reliability (smaller error), discuss the processes that contribute to reliability, and incorporate the concept of reproducibility to guide decisions for a definition of site trees for a specific longleaf pine dataset.

## MATERIAL AND METHODS

### Data

The data arise from seven studies in the Gulf Coast States to investigate the effects of spacing and thinning on growth and development of longleaf pine plantations (Goelz and Leduc 2001). The 267 plots have been remeasured multiple times at approximately 5-year intervals; most plots have been remeasured for > 20 years. Some of the stands are now > 60 years old. The protocol varied among the studies. Most recent measurements include height of all trees, and a subset of heights was measured during earlier inventories of some of the studies. The studies have been combined to form a uniformly measured database for growth-and-yield modeling of longleaf plantations (Goelz and Leduc 2002).

### Procedures

We compared 13 definitions for the subpopulation of site trees: (1) dominant or codominant crown class at time of measurement, (2) always dominant or codominant crown class before base age, (3) 40 tallest trees per acre at time of measurement, (4) 40 tallest trees per acre at base age, (5) 20 tallest trees per acre at time of measurement, (6) 20 tallest trees per acre at base age, (7) 40 thickest trees per acre at time of measurement, (8) 40 thickest trees per acre at base age, (9) 20 thickest trees per acre at time of measurement, (10) 20 thickest trees per acre at base age, (11) all trees within 10 percent of maximum height at time of measurement, (12) all trees within 10 percent of maximum height at base age, and (13) dominant or codominant crown class among 40 thickest trees per acre at time of measurement.

Definitions 1 and 2 include trees defined as open grown. More accurately, definition 2 trees were never anything other than dominant, codominant, or open grown; a missing observation of crown class at some age did not exclude trees from definitions 1 or 2. The following model represented our data well:

$$H = \frac{\alpha}{(1 + \gamma A^{-\delta})}, \quad (1)$$

where

$H$  = height

$A$  = age

$\alpha$  and  $\delta$  = parameters common to all trees

$\gamma$  = a parameter unique for each tree.

This model is equivalent to the equation of McDill and Amateis (1992) or, after rearranging terms, the Hossfeld IV equation (Zeide 1993). The estimate for parameter  $\alpha$ , the asymptote, was 113.8217, and the estimate for  $\delta$  was 1.49203. By solving for  $\gamma$ , then setting  $A$  equal to  $A_b$ , the base age (either 25 or 50), setting  $H$  equal to site index ( $S$ ) and substituting back into equation 1, then solving for  $S$ , we get an equation to predict height at base age for individual trees, given an observation of height at a known age:

$$S = \frac{\alpha}{\left(1 + \left(\frac{\alpha}{H} - 1\right) \left(\frac{A_b}{A}\right)^{-\delta}\right)} \quad (2)$$

We used equation 2 to predict height at base ages of 25 and 50, given each observation of height and age. Then, using 13 subpopulation definitions, we calculated predicted  $S$  as the average predicted height of the appropriate trees. For the “true” value of  $S$ , we used the subpopulation definitions to calculate the average height measured at the appropriate base age. For some studies, heights were not measured at age 25 or 50, but at nearby ages. When there was a height measurement between 20 and 30 (for base age 25) or between 40 and 60 (for base age 50), we estimated the “true” value by nonlinear interpolation, employing equation 2.

We calculated error criteria to assess the variability of the estimates of  $S$  for the different subpopulation definitions and the two base ages. We calculated average error (-bias), mean squared error, mean absolute error, and maximum error. For some plots at some measurements (plot-by-measurement combinations comprise an observation), there were few or no trees that qualified for some of the definitions. In some of the studies, few heights were measured during the first few measurement cycles, and there were few surviving trees on some of the older plots. When fewer than two trees fulfilled the definition, the observation was deleted. When different subpopulation definitions produced different numbers of observations, Simpson’s Paradox could provide misleading results. We sought to lessen this effect by deleting all observations for all subpopulations if the observations were not present for particular promising subpopulations. For example, we considered subpopulations 1 and 7 to be promising, so we recalculated the error criteria excluding the observations that were missing for those subpopulations, even though another subpopulation had an observation.

We suspected that the stability of ranks for individual trees would largely determine the variability for the

subpopulations defined by diameter and height. As well, the stability of crown class would largely determine the variability of the subpopulations defined by crown class. If these values fluctuated greatly from measurement to measurement, variability would be high. Thus we explored this source of variability by creating cross-classified tables to assess whether the change in the cumulative rank percentiles differed more for diameter or for height. We used the likelihood-ratio test (Fienberg 1980),  $G^2$ , with an  $\alpha$  of 0.05, to test whether rank percentile changed more for diameter or height; we also tested whether dominants and codominants or intermediate and suppressed trees switched to the alternate, broad crown class more often. These tests help explain stand dynamics in these longleaf pine plantations.

## RESULTS AND DISCUSSION

Error criteria are provided in table 1. For base age 25, bias was very small for all subpopulations defined at base age.

Mean absolute error and mean squared error were also lower for the subpopulations defined at base age than the counterpart subpopulations defined at each measurement. Among the subpopulations defined at base age, the subpopulation defined as dominant and codominant trees was lowest for all error criteria. The subpopulations defined by the 40 tallest and 40 thickest trees per acre were only slightly worse than the dominant and codominant subpopulation. Among the subpopulations defined at each measurement, the subpopulation defined by dominant and codominant trees had the lowest absolute and squared error, with the subpopulation defined by the 40 thickest trees having a slightly greater absolute and squared error, but a lower bias. Regardless of whether the subpopulations were defined at base age or at each measurement, and whether the subpopulations were defined by height or diameter, errors were lower when more trees were included in the subpopulation.

**Table 1—Error criteria for site index predictions using 13 different subpopulations and 2 base ages (25 and 50)**

Subpopulation	Observations <i>no.</i>	-bias	Mean error		Maximum difference
			Absolute	Squared	
<b>Base age 25</b>					
1. DC	958	0.388	2.664	13.173	15.885
2. DC -BA	672	0.068	2.079	9.092	16.199
3. H40	1,046	-0.770	2.939	17.444	27.335
4. H40 -BA	504	0.216	2.162	10.304	17.651
5. H20	1,004	-1.771	3.368	24.582	44.447
6. H20 -BA	488	0.136	2.539	14.111	17.651
7. D40	1,044	-0.075	2.697	13.438	19.364
8. D40 -BA	508	0.216	2.185	10.387	17.699
9. D20	993	-0.302	2.889	16.197	23.831
10. D20 -BA	490	0.499	2.430	13.764	17.699
11. H10 percent	850	1.612	3.765	30.214	37.129
12. H10 percent -BA	150	-0.192	2.705	17.684	16.219
13. DC and D40	940	-0.071	2.873	14.669	19.364
<b>Base age 50</b>					
1. DC	986	-0.529	1.631	6.869	36.487
2. DC -BA	1,029	-0.401	1.686	6.076	14.557
3. H40	1,082	-0.889	1.837	6.796	11.983
4. H40 -BA	630	-0.042	1.703	6.782	15.197
5. H20	1,057	-1.038	2.112	9.062	16.659
6. H20 -BA	613	0.277	1.672	6.569	15.714
7. D40	1,061	-0.830	1.920	7.859	14.284
8. D40 -BA	629	-0.402	1.539	5.054	11.503
9. D20	1,029	-0.984	2.132	9.736	16.659
10. D20 -BA	627	-0.419	1.707	5.932	12.780
11. H10 percent	827	-0.631	2.362	19.425	34.560
12. H10 percent -BA	624	0.088	1.676	6.070	12.697
13. DC and D40	953	-0.857	1.878	7.754	14.284

DC = dominant and codominant trees; H40 = the 40 tallest trees per acre; H20 = the 20 tallest trees per acre; D40 = the 40 thickest trees per acre; D20 = the 20 thickest trees per acre; H10 percent = trees within 10 percent of maximum height on a plot; -BA = subpopulations that were defined at base age rather than anew at each measurement age.

The results for base age 50 were similar to the results for base age 25, except the benefit from defining the population at base age was greatly reduced. In fact, for the subpopulations defined by dominants and codominants, the subpopulation defined at base age had a higher mean absolute error than the subpopulation defined at each measurement. The error criteria differed little between these two subpopulations; a single observation with a large difference (36.487) more than explained the slightly greater mean squared error. Without that single observation, the mean squared error would decrease to 5.524. For base age 50, the overall best subpopulation had the 40 thickest trees at base age.

Because number of observations varied among subpopulations (table 1), some of these results could

potentially be due to a small number of atypical observations in one subpopulation but not another. To lessen this contribution to the results, we recalculated the error criteria, but deleted those observations that had a missing value for either: subpopulation 1 or 7 (table 2), and subpopulation 2 or 8 (table 3).

For base age 25, results were similar among tables 1, 2, and 3. The subpopulations defined at base age were superior to their counterparts defined at each measurement. Among the subpopulations defined by base age, the subpopulation comprised of dominant and codominant trees was clearly the most efficient estimator for base age 25.

**Table 2—Error criteria for site index predictions using 13 different subpopulations and 2 base ages (25 and 50)**

Subpopulation	Observations <i>no.</i>	-bias	Mean error		Maximum difference
			Absolute	Squared	
<b>Base age 25</b>					
1. DC	811	0.579	2.740	13.895	15.885
2. DC -BA	490	0.234	2.295	10.429	16.199
3. H40	798	-0.720	3.298	20.409	27.335
4. H40 -BA	396	0.322	2.512	12.520	17.651
5. H20	764	-1.879	3.781	28.843	44.447
6. H20 -BA	384	0.158	2.981	17.293	17.651
7. D40	811	0.061	2.998	15.518	19.364
8. D40 -BA	396	0.321	2.504	12.414	17.700
9. D20	764	-0.175	3.209	18.829	23.831
10. D20 -BA	378	0.705	2.816	16.844	17.699
11. H10 percent	690	1.847	3.887	30.232	37.129
12. H10 percent -BA	105	0.081	3.160	22.182	16.219
13. DC and D40	811	0.064	2.970	15.521	19.364
<b>Base age 50</b>					
1. DC	811	-0.592	1.615	5.827	13.301
2. DC -BA	777	-0.551	1.676	6.294	14.557
3. H40	808	-1.118	1.820	6.947	11.983
4. H40 -BA	548	-0.211	1.560	5.961	15.197
5. H20	798	-1.321	2.079	9.286	16.659
6. H20 -BA	534	0.088	1.538	5.822	15.714
7. D40	811	-1.021	1.947	8.403	14.284
8. D40 -BA	547	-0.477	1.459	4.666	11.503
9. D20	785	-1.136	2.102	9.764	16.659
10. D20 -BA	545	-0.477	1.617	5.470	12.780
11. H10 percent	645	-0.716	2.164	16.098	34.560
12. H10 percent -BA	545	0.028	1.594	5.732	12.697
13. DC and D40	810	-1.010	1.961	8.499	15.609

DC = dominant and codominant trees; H40 = the 40 tallest trees per acre; H20 = the 20 tallest trees per acre; D40 = the 40 thickest trees per acre; D20 = the 20 thickest trees per acre; H10 percent = trees within 10 percent of maximum height on a plot; -BA = subpopulations that were defined at base age rather than anew at each measurement age. Observations were only included if they were not missing for subpopulation 1 and 7; other subpopulations could have missing observations.

**Table 3—Error criteria for site index predictions using 13 different sub-populations and 2 base ages (25 and 50)**

Subpopulation	Observations <i>no.</i>	-bias	Mean error		Maximum difference
			Absolute	Squared	
<b>Base age 25</b>					
1. DC	234	0.446	2.643	13.927	12.561
2. DC -BA	278	0.293	2.001	9.389	16.199
3. H40	276	-1.021	2.806	17.676	24.681
4. H40 -BA	274	0.472	2.189	10.742	17.651
5. H20	263	-2.063	3.454	30.423	44.447
6. H20 -BA	262	0.226	2.645	15.306	17.651
7. D40	274	-0.036	2.472	12.276	12.561
8. D40 -BA	278	0.520	2.281	11.630	17.700
9. D20	272	-0.311	2.582	14.147	13.095
10. D20 -BA	261	0.422	2.464	13.982	17.700
11. H10 percent	203	2.142	4.260	33.536	18.163
12. H10 percent -BA	44	-0.133	3.192	21.451	12.093
13. DC and D40	230	-0.135	2.804	14.313	12.561
<b>Base age 50</b>					
1. DC	234	-0.452	1.428	4.682	10.073
2. DC -BA	278	-0.353	1.521	4.857	10.733
3. H40	277	-0.795	1.669	6.326	11.983
4. H40 -BA	278	0.374	1.925	8.345	15.197
5. H20	276	-1.024	1.975	8.787	15.714
6. H20 -BA	270	0.773	2.109	9.562	15.714
7. D40	277	-0.549	1.610	5.437	8.816
8. D40 -BA	278	-0.215	1.585	5.270	11.503
9. D20	274	-0.717	1.917	7.927	12.300
10. D20 -BA	277	-0.179	1.837	6.736	12.780
11. H10 percent	186	-0.328	2.022	15.790	26.485
12. H10 percent -BA	273	0.619	1.780	6.858	12.697
13. DC and D40	233	-0.622	1.600	5.587	8.816

DC = dominant and codominant trees; H40 = the 40 tallest trees per acre; H20 = the 20 tallest trees per acre; D40 = the 40 thickest trees per acre; D20 = the 20 thickest trees per acre; H10 percent = trees within 10 percent of maximum height on a plot; -BA = sub-populations that were defined at base age rather than anew at each measurement age.

Observations were only included if they were not missing for subpopulation 2 and 8; other subpopulations could have missing observations.

For base age 50, the results were much less consistent among tables 1, 2, and 3. The subpopulation defined at each measurement by the dominant and codominant trees (subpopulation 1) had the lowest mean absolute error in tables 1 and 3 and the lowest mean squared error in table 3. The subpopulation defined at base age by the 40 thickest trees per acre (subpopulation 8) had the lowest mean squared error in tables 1 and 2 and the lowest mean absolute error in table 2. Results for some subpopulations varied widely among tables 1, 2, and 3. The subpopulation defined at base age of the 20 tallest trees per acre varied from the largest mean absolute error (table 3) to the second lowest mean absolute error (table 2). Bias is sufficiently small for the subpopulations that have lower absolute and squared errors.

A major factor is the stability of the subpopulation over time. Figure 1 represents the change in rank of diameters for one plot over 40 years. Although rank for some trees is relatively unchanging, other trees change considerably over time (Tiarks and others 1998). For some of the trees, the change in rank is directional rather than simply fluctuating. For example, the tree that was initially the largest of the 25 trees fell to 14<sup>th</sup> rank and the tree that was initially 4<sup>th</sup> fell to 21<sup>st</sup> at the most recent measurement. On the other hand, a tree that was initially tied for 20<sup>th</sup> rank climbed to 9<sup>th</sup>, and a tree that was initially tied at the 15.5<sup>th</sup> rank climbed to the 5.5<sup>th</sup> at the most recent measurement. If rank is changing, clearly inclusion of a tree in the subpopulations will also be changing.

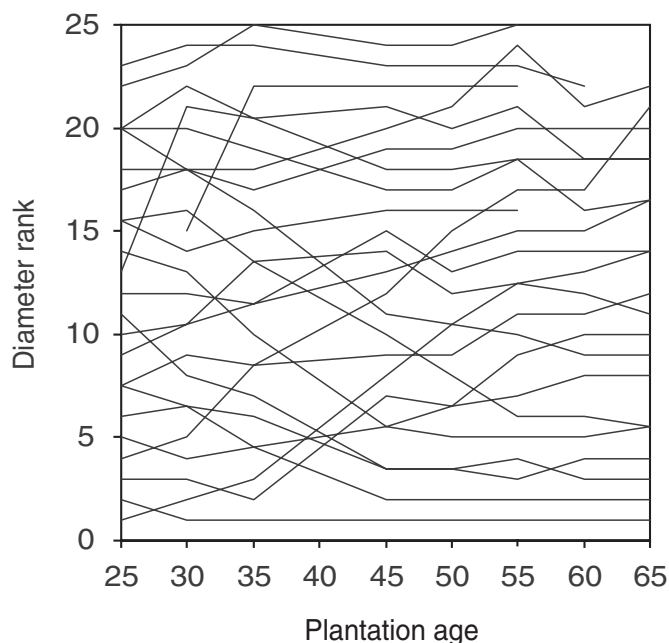


Figure 1—Change of rank in diameter for one arbitrarily selected plot. Each line represents a different tree. Trees present for three or fewer measurements are not included. If a line ends before age 65, the tree died.

The proportion of trees that changed in rank from one measurement to the next was significantly different between height and diameter ( $P$  much < 0.01) (table 4). Both height and diameter have more trees decreasing in percentile rank than increasing, an inevitable consequence of mortality of other trees on the plot. Percentile rank in diameter at breast height was much more stable than percentile rank in height. Ranks in height are less stable because height is measured with more error than diameter, and height can actually decrease if the top is damaged. Therefore subpopulations defined by diameter tend to have lower variation than the subpopulations defined by height.

For the frequency of trees that moved from one broad crown class to another during a measurement interval, there are significant differences between crown classes ( $P$  much < 0.01) (table 5). Unexpectedly, a higher proportion of intermediate and suppressed trees become dominant or codominant than the reverse. As there are more dominant and codominant trees than intermediate and suppressed, there is relatively little net change from measurement to measurement. Change from dominant and codominant trees arises from top damage or from neighbors asserting dominance. Change from intermediate and suppressed trees can occur when neighboring trees die or are harvested. However, much of the change may be explained by many trees being borderline between the crown classes, and the assignment of crown class may change due to different perspectives of different observers at different times. The stability of crown class designation from measurement to measurement, relative to the lower stability of rank of diameter and height, explains the effectiveness of the subpopulation defined at each measurement by the dominant and codominant trees.

Table 4—Percentage of trees changing in rank (expressed as classes of percentile change) for d.b.h. and height, between two successive measurements

Rank change percentiles	D.b.h.	Height
	---- percent ----	
< -17.5	5.4	9.6
-12.5 to -17.5	5.1	6.6
-7.5 to -12.5	10.0	10.0
-2.5 to -7.5	23.8	15.6
2.5 to -2.5	43.2	27.1
2.5 to 7.5	8.8	12.9
7.5 to 12.5	2.4	7.7
12.5 to 17.5	0.7	4.3
> 17.5	0.6	6.3

D.b.h. = diameter at breast height.

Table 5—Frequency in change of crown class between two successive measurements (two broad classes of crown class are used)

Crown class	Stays the same	Changes	Percent changing	Total
Dominant, codominant, or open grown	14,594	773	5.0	15,367
Intermediate or suppressed	6,486	649	9.1	7,135
Total	21,080	1,322		22,502

Sharma and others (2002) compared seven different definitions of top height based on crown class or rank in diameter and Lorey's mean height, which weights height by tree basal area. With regard to the consistency of S estimation over time, although the height of dominants and codominants that had always been such was superior to height of dominants and codominants at the time of measurement, the difference was small. Sharma and others (2002) did not consider the difficulty in applying such a definition in a S prediction context.

## CONCLUSIONS

We chose to specify our site tree subpopulation as the dominant and codominant trees defined at each measurement age. There seems to be little benefit in restricting the subpopulations to those trees that always were dominant or codominant, particularly as that definition would greatly complicate the use of the site curves when predicting S. We chose the base age of 50 partly on these results. Given a base age of 25, our decision would have been to select one of the subpopulations defined at base age; they were superior when S was defined at this base age. It is reasonable to assume that most land managers will have rotations for longleaf pine that approach or exceed

50 years, so there is no real cost in utilizing the older base age. Our decision is made solely for our particular dataset. We suggest similar investigation when a S model is constructed for other species or regions.

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