ANNUAL TREE GROWTH PREDICTIONS FROM PERIODIC MEASUREMENTS

Quang V. Cao[1]

Abstract—Data from annual measurements of a loblolly pine (*Pinus taeda* L.) plantation were available for this study. Regression techniques were employed to model annual changes of individual trees in terms of diameters, heights, and survival probabilities. Subsets of the data that include measurements every 2, 3, 4, 5, and 6 years were used to fit the same tree-growth equations. Two methods of estimating parameters of the annual growth equation from periodic measurements were evaluated. The Constant Rate method assumed a constant tree-survival probability and constant diameter and height-growth rates during the growing interval. In contrast, these annual changes were assumed to be different from year to year in the Variable Rate method. Results indicated that the Variable Rate method out performed the Constant Rate method in predicting annual tree growth from periodic measurements.

INTRODUCTION

A distance-independent individual tree model consists of three main components: tree diameter and height-growth equations and a tree survival function. Equations have been developed to predict tree diameter growth (Amateis and others 1989, Belcher and others 1982, Lessard and others 2001, Mabvurira and Miina 2002, Zhang and others 1997) and height growth (Courbaud and others 1993, Golser and Hasenauer 1997, Lynch and Murphy 1995, Ritchie and Hann 1986). Logistic functions have generally been used in modeling the probability that a tree survives a growing period (Buchman 1979, 1983; Hamilton 1974; Hamilton and Edwards 1976; Monserud 1976; Monserud and Sterba 1999; Zhang and others 1997).

Predicting annual tree growth and survival is not an easy task because trees are not measured every year but often at some interval. McDill and Amateis (1993) developed two interpolation methods for modeling annual growth of one variable (e.g., tree height). One of their methods was later generalized by Cao and others (2002) for predicting many variables (e.g., tree diameter, height, and crown ratio). These interpolation methods were shown to perform better than the Constant Rate method, which assumes a constant growth rate for the entire period. It is particularly difficult to predict annual tree survival from periodic measurements because if a tree was found dead at the end of a period, there was no record of when that actually happened. The survival probability is often assumed to remain constant during the growing period (Hamilton and Edwards 1976, Monserud 1976). Cao (2000) developed an iterative method to account for variable rates of annual survival and diameter growth. The method was later modified to include annual height growth (Cao 2002). The objective of this study is to evaluate the Constant Rate method versus Cao's (2002) Variable Rate method in estimating parameters of an individual tree model that consists of annual tree survival, diameter growth, and height-growth equations.

DATA

Data from two plots in an unthinned loblolly pine (*Pinus taeda* L.) plantation were made available for this study by Dr. Paul Y. Burns, Professor Emeritus of the School of Renewable Natural Resources, Louisiana State University. This plantation was in the School's Lee Memorial Forest, near Bogalusa, LA. There was originally a total of 171 trees per plot planted in a 9-foot by 12-foot spacing, resulting in a plot size of 0.424 acres. Diameter at breast height (d.b.h.), total height, and survival status (dead or alive) of these trees were recorded annually from age 2 to age 21 (fig. 1). Subsets of the above data were created to include measurements every 2, 3, 4, 5, and 6 years.

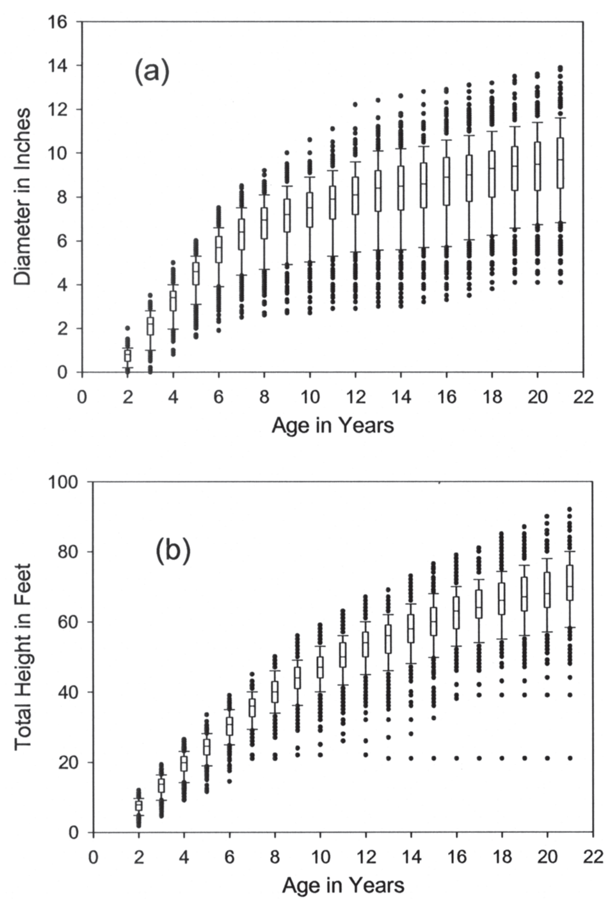

Figure 1—Box plots of tree d.b.h. (a) and total height (b) measured over time.

[1]Professor, School of Renewable Natural Resources, Louisiana State University Ag Center, Baton Rouge, LA 70803.

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METHODS

The following individual tree model comprised of equations predicting annual survival, and diameter and height growth was selected after preliminary analyses:

$$d_{i,t+1} - d_{i,t} = \alpha_1 B_t^{\alpha_2} H_t^{\alpha_3} h_{i,t}^{\alpha_4} \exp(\alpha_5 d_{i,t}/D_{q,t}) + \varepsilon_{i,t} \quad (1.a)$$

$$h_{i,t+1} - h_{i,t} = \beta_1 A_t^{\beta_2} B_t^{\beta_3} H_t^{\beta_4} h_{i,t}^{\beta_5} \exp(\beta_6 d_{i,t}/D_{q,t}) + \varepsilon_{i,t} \quad (1.b)$$

$$p_{i,t+1} = [1 + \exp(\gamma_1 B_t + \gamma_2 H_t + \gamma_3 h_{i,t})]^{-1} \quad (1.c)$$

where

$d_{i,t}$ and $h_{i,t}$ = d.b.h. in inches and total height in feet, respectively, of tree i at age A_t,

$p_{i,t+1}$ = probability that tree i survived the period from age A_t to A_{t+1},

H_t = dominant height (average height of the dominants and codominant heights) in feet at age A_t,

B_t = stand basal area in square feet per acre at age A_t,

$D_{q,t}$ = quadratic mean diameter in inches at age A_t, and

$\varepsilon_{i,t}$ = error term.

Two methods for estimating parameters of the above tree model, the Constant Rate method and the Variable Rate method, will be discussed as follows:

Constant Rate Method

In this method, the growth rates of diameter and height of each tree were assumed to be constant during the growth period from age A_t to A_{t+q}, where q is the length of the period. Similarly, the survival probability was also considered constant during this period. Equations (1.a – 1.c) are rewritten as follows:

$$(d_{i,t+q} - d_{i,t})/q = \alpha_1 B_t^{\alpha_2} H_t^{\alpha_3} h_{i,t}^{\alpha_4} \exp(\alpha_5 d_{i,t}/D_{q,t}) + \varepsilon_{i,t} \quad (2.a)$$

$$(h_{i,t+q} - h_{i,t})/q = \beta_1 A_t^{\beta_2} B_t^{\beta_3} H_t^{\beta_4} h_{i,t}^{\beta_5} \exp(\beta_6 d_{i,t}/D_{q,t}) + \varepsilon_{i,t} \quad (2.b)$$

$$p_i = [1 + \exp(\gamma_1 B_t + \gamma_2 H_t + \gamma_3 h_{i,t})]^{-q} \quad (2.c)$$

where p_i is the probability that tree i survived the period from age A_t to A_{t+q}.

A method suggested by Borders (1989) was used to simultaneously estimate parameters of the diameter and height-growth equations; this fitting procedure involved the use of option SUR (seemingly unrelated regression) of SAS procedure MODEL (SAS Institute Inc. 1993). Maximum likelihood estimation of parameters of the survival equation was obtained using weighted nonlinear regression (Walker and Duncan 1967).

Variable Rate Method

This method allowed the survival and growth rates to vary from year to year as functions of constantly changing stand variables and tree variables. Annual changes in diameter, height, and survival probability were modeled in a recursive manner as follows:

Year (t+1)

$$\hat{d}_{i,t+1} = d_{i,t} + \alpha_1 B_t^{\alpha_2} H_t^{\alpha_3} h_{i,t}^{\alpha_4} \exp(\alpha_5 d_{i,t}/D_{q,t}) \quad (3.a.1)$$

$$\hat{h}_{i,t+1} = h_{i,t} + \beta_1 A_t^{\beta_2} B_t^{\beta_3} H_t^{\beta_4} h_{i,t}^{\beta_5} \exp(\beta_6 d_{i,t}/D_{q,t}) \quad (3.b.1)$$

$$p_{i,t+1} = [1 + \exp(\gamma_1 B_t + \gamma_2 H_t + \gamma_3 h_{i,t})]^{-1} \quad (3.c.1)$$

Year (t+2)

$$\hat{d}_{i,t+2} = \hat{d}_{i,t+1} + \alpha_1 \hat{B}_{t+1}^{\alpha_2} \hat{H}_{t+1}^{\alpha_3} \hat{h}_{i,t+1}^{\alpha_4} \exp(\alpha_5 \hat{d}_{i,t+1}/\hat{D}_{q,t+1}) \quad (3.a.2)$$

$$\hat{h}_{i,t+2} = \hat{h}_{i,t+1} + \beta_1 A_t^{\beta_2} \hat{B}_{t+1}^{\beta_3} \hat{H}_{t+1}^{\beta_4} \hat{h}_{i,t+1}^{\beta_5} \exp(\beta_6 \hat{d}_{i,t+1}/\hat{D}_{q,t+1}) \quad (3.b.2)$$

$$p_{i,t+2} = [1 + \exp(\gamma_1 \hat{B}_{t+1} + \gamma_2 \hat{H}_{t+1} + \gamma_3 \hat{h}_{i,t+1})]^{-1} \quad (3.c.2)$$

Year (t+q)

$$d_{i,t+q} = \hat{d}_{i,t+q-1} + \alpha_1 \hat{B}_{t+q-1}^{\alpha_2} \hat{H}_{t+q-1}^{\alpha_3} \hat{h}_{i,t+q-1}^{\alpha_4} \exp(\alpha_5 \hat{d}_{i,t+q-1}/\hat{D}_{q,t+q-1}) + \varepsilon_i \quad (3.a.q)$$

$$h_{i,t+q} = \hat{h}_{i,t+q-1} + \beta_1 A_t^{\beta_2} \hat{B}_{t+q-1}^{\beta_3} \hat{H}_{t+q-1}^{\beta_4} \hat{h}_{i,t+q-1}^{\beta_5} \exp(\beta_6 \hat{d}_{i,t+q-1}/\hat{D}_{q,t+q-1}) + \varepsilon_i \quad (3.b.q)$$

$$p_{i,t+q} = [1 + \exp(\gamma_1 \hat{B}_{t+q-1} + \gamma_2 \hat{H}_{t+q-1} + \gamma_3 \hat{h}_{i,t+q-1})]^{-1} \quad (3.c.q)$$

where the stand-level variables were predicted from the following equations:

$$\hat{H}_{t+s+1} = \exp\{\lambda_1 + [\ln(\hat{H}_{t+s}) - \lambda_1](A_{t+s}/A_{t+s+1})\} \quad (4.a)$$

$$\ln(\hat{B}_{t+s+1}) = \ln(\hat{B}_{t+s}) + \tau_1 (1/A_{t+s} - 1/A_{t+s+1}) + \tau_2 [\ln(\hat{H}_{t+s+1}) - \ln(\hat{H}_{t+s})] \quad (4.b)$$

$$\ln(\hat{D}_{q,t+s-1}) = \ln(\hat{D}_{q,t+s}) + \delta_1 (1/A_{t+s} - 1/A_{t+s+1}) + \delta_2 [\ln(\hat{H}_{t+s+1}) - \ln(\hat{H}_{t+s})] \quad (4.c)$$

and the probability that tree i survived the period from age A_t to A_{t+q} is given by

$$p_i = \prod_{s=1}^{q} p_{i,t+s} \quad (5)$$

RESULTS AND DISCUSSION

Tree growth (based on data taken at intervals ranging from 1 to 6 years) was projected for a tree measuring 0.7 inches in d.b.h. and 7 feet in total height at age 2. Initial stand variables at age 2 were:

Dominant height = 8 feet

Stand basal area = 1.5 square feet per acre

Quadratic mean diameter = 0.8 inches.

Figure 2a shows that the Constant Rate method underestimated tree-diameter growth. The linear interpolation technique employed by this method always resulted in lower estimates of annual growth. The longer the interval between two measurements, the lower the diameter growth projection curve. Data taken at 6-year intervals yielded a diameter-growth curve that was consistently lower by almost 4 inches between age 11 and age 21.

On the other hand, the Variable Rate method yielded diameter-growth curves that were very close for all interval lengths (fig. 2b). This method produced better predictions for annual diameter growth because it used the model to make interpolations. Curves from 1-year and 6-year intervals were at most 1 inch apart and were less than 0.5 inches different beyond age 11.

Figures 3a and 3b tell a similar story for total height-growth projections. The Constant Rate method produced lower height-growth curves when based on data that were collected at longer intervals (fig. 3a). The curve fitted from height measurements at every 6 years was more than 15 feet lower at age 17 and beyond. Figure 3b shows that height-growth curves constructed using the Variable Rate method were virtually indistinguishable regardless of interval length.

The Variable Rate method also outperformed the Constant Rate method in predicting tree survival probability (figs. 4a and 4b). Modeling the annual probability of tree survival is always a challenge because one never knows exactly when the tree died during the period. Although the tree-survival curves from various interval lengths assumed various shapes, the differences were much less pronounced for the Variable Rate method (maximum difference = 0.10) than for the Constant Rate method (maximum difference = 0.35).

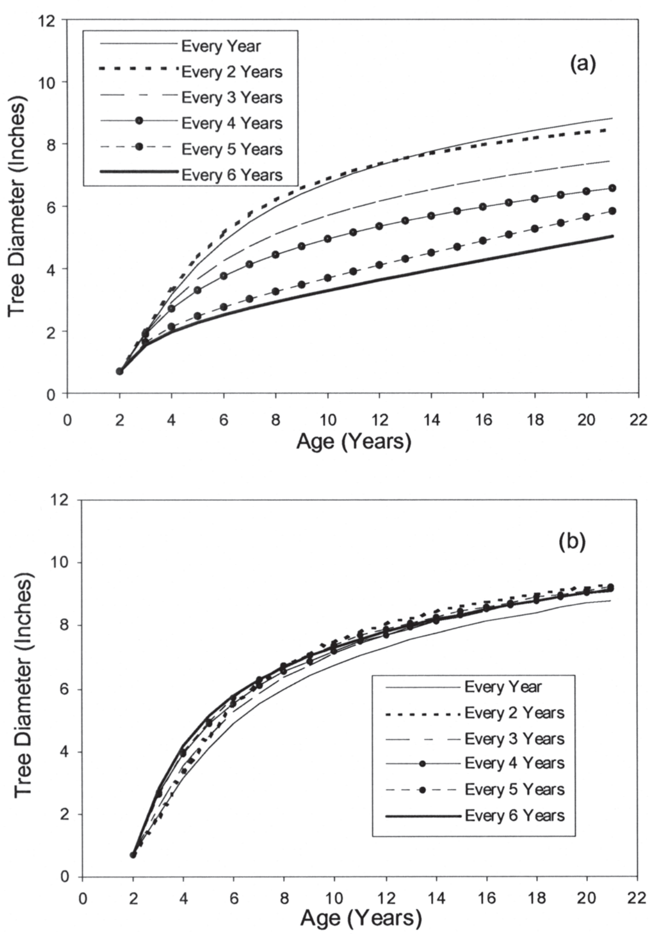


Figure 2—Projections of tree diameter over time based on measurements taken at different intervals. Parameters of the diameter-growth equation were obtained from (a) the Constant Rate method and (b) the Variable Rate method.

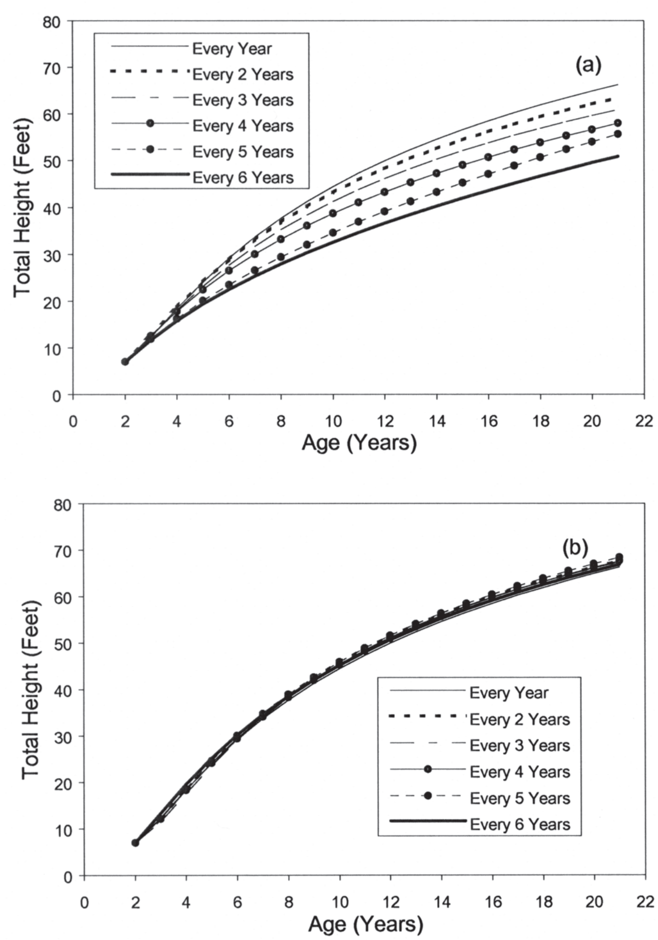
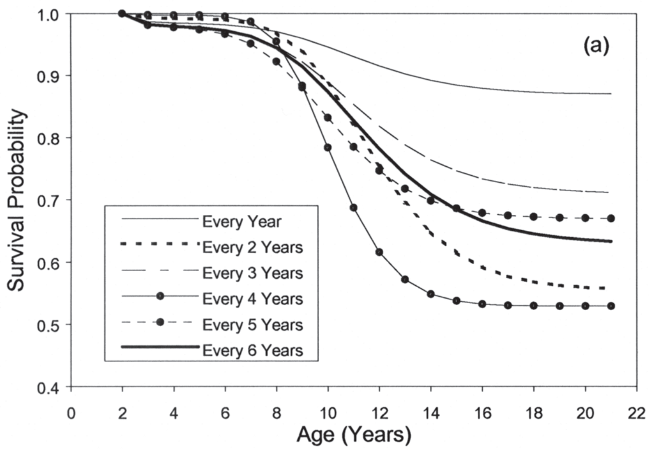


Figure 3—Projections of tree total height over time based on measurements taken at different intervals. Parameters of the height-growth equation were obtained from (a) the Constant Rate method and (b) the Variable rate method.



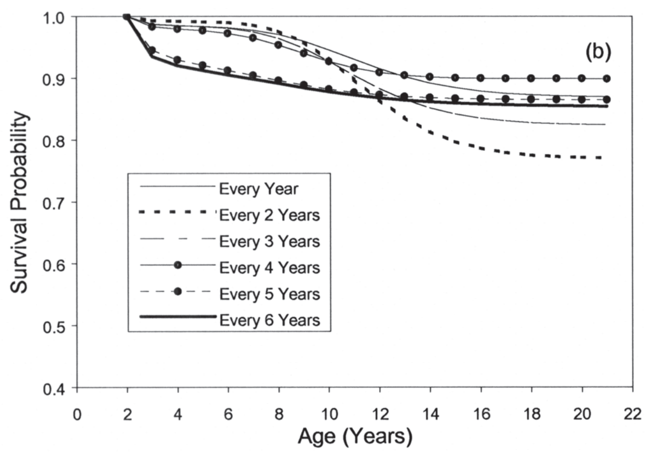


Figure 4—Projections of tree survival probability over time based on measurements taken at different intervals. Parameters of the tree-survival equation were obtained from (a) the Constant Rate method and (b) the Variable rate method.

Because tree and stand variables keep changing every year, a method such as the Variable Rate method that allows diameter growth, height growth, and tree survival probability to vary annually should perform well. This method should be superior to the Constant Rate approach, as clearly demonstrated in this study. Results also indicated that annual tree growth and survival could be successfully modeled using data measured up to 6 years apart. Even though a loblolly pine data set was used in this study, the Variable Rate approach should be applicable to other species as well.

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